UJIAN AKHIR SEMESTER (UAS)

Semester	**: Genap**
Tahun Ajaran	**: 2016/2017**
Mata Kuliah	**: Matematika Dasar II**
Hari/Tanggal	**: Senin/13 Maret 2017**
Waktu	**: 100 Menit**
Sifat	**: Tutup Buku**

ITERA

Soal dan Jawaban

Bagian A: Nilai maksimum setiap soal adalah 3.

1. Tentukan Integral berikut

$$\int e^{\sqrt{2x}}\,dx$$

Jawab:

Misal $w = \sqrt{2x}$ atau $w^2 = 2x$, dan $wdw = dx$, sehingga $\int e^{\sqrt{2x}}dx = \int we^{w}dw$

Dari bentuk ini, dapat digunakan integral parsial,

Misal $u = w$ dan $du = dw$, atau $dv = e^{w}dw$ dan $v = e^{w}$, kemudian

$$\int we^{w}dw = we^{w} - \int e^{w}dw$$
$$= we^{w} - e^{w} + C$$

Diperoleh

$$\int e^{\sqrt{2x}}dx = \left(\sqrt{2x} - 1\right)e^{\sqrt{2x}} + C$$

2. Tentukan integral berikut

$$\int \frac{4x}{(2x-1)^2}dx$$

Jawab:

Gunakan metode parsial, $\frac{4x}{(2x-1)^2} = \frac{A}{2x-1} + \frac{B}{(2x-1)^2} = \frac{2Ax - A + B}{(2x-1)^2}$, diperoleh A = 2 dan B = 2. Kemudian

$$\int \frac{4x}{(2x-1)^2}dx = \int\left[\frac{2}{2x-1} + \frac{2}{(2x-1)^2}\right]dx$$
$$= \int \frac{2}{2x-1}\,dx + \int \frac{2}{(2x-1)^2}\,dx \quad (\text{gunakan metode substitusi } u = 2x-1)$$
$$= \ln(2x-1) - \frac{1}{2x-1} + C$$

3. Hitunglah

$$\lim_{x\to\infty}\left[\ln(2x-3)-\ln(3x+7)\right]$$

Jawab:

$$\lim_{x\to\infty}\left[\ln(2x-3)-\ln(3x+7)\right]=\lim_{x\to\infty}\ln\frac{2x-3}{3x+7}$$

$$=\ln\left(\lim_{x\to\infty}\frac{2x-3}{3x+7}\right)$$

$$=\ln\frac{2}{3}\quad\text{atau}\quad -\ln\frac{3}{2}$$

4. Periksa apakah integral $\int_1^\infty f(x)\,dx$ konvergen, bila $\int f(x)\,dx=\frac{x}{3-2x}+C$.

Jawab:

$$\int_1^\infty f(x)\,dx=\int_1^{3/2} f(x)\,dx+\int_{3/2}^\infty f(x)\,dx$$

$$=\lim_{a\to(3/2)^-}\int_1^a f(x)\,dx+\lim_{a\to(3/2)^+}\lim_{b\to\infty}\int_a^b f(x)\,dx$$

$$=\lim_{a\to(3/2)^-}\left[\frac{x}{3-2x}\right]_1^a+\lim_{a\to(3/2)^+}\lim_{b\to\infty}\left[\frac{x}{3-2x}\right]_a^b$$

$$=\lim_{a\to(3/2)^-}\left[\frac{a}{3-2a}-1\right]+\lim_{a\to(3/2)^+}\lim_{b\to\infty}\left[\frac{b}{3-2b}-\frac{a}{3-2a}\right]$$

$$=\lim_{a\to(3/2)^-}\frac{3a-3}{3-2a}+\lim_{a\to(3/2)^+}\lim_{b\to\infty}\frac{b}{3-2b}-\lim_{a\to(3/2)^+}\lim_{b\to\infty}\frac{a}{3-2a}$$

$$=\lim_{a\to(3/2)^-}\frac{3a-3}{3-2a}+\lim_{b\to\infty}\frac{b}{3-2b}-\lim_{a\to(3/2)^+}\frac{a}{3-2a}$$

$$=\infty+\left(-\frac{1}{2}\right)-(-\infty)$$

$$=\infty$$

$\therefore \int_1^\infty f(x)\,dx$ divergen.

5. Hitung

$$\sum_{n=1}^{\infty} \frac{4}{n(n+4)}$$

dengan meninjaunya sebagai deret kolaps.

Jawab:

$$\sum_{n=1}^{\infty} \frac{4}{n(n+4)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+4} \right)$$

$$= \lim_{k\to\infty} \left[\left(\frac{1}{1} - \frac{1}{5} \right) + \left(\frac{1}{2} - \frac{1}{6} \right) + \left(\frac{1}{3} - \frac{1}{7} \right) + \left(\frac{1}{4} - \frac{1}{8} \right) + \left(\frac{1}{5} - \frac{1}{9} \right) + \left(\frac{1}{6} - \frac{1}{10} \right) + \left(\frac{1}{7} - \frac{1}{11} \right) \right.$$

$$+ \ldots. + \left(\frac{1}{k-10} - \frac{1}{k-6} \right) + \left(\frac{1}{k-9} - \frac{1}{k-5} \right) + \left(\frac{1}{k-8} - \frac{1}{k-4} \right) + \left(\frac{1}{k-7} - \frac{1}{k-3} \right)$$

$$+ \left(\frac{1}{k-6} - \frac{1}{k-2} \right) + \left(\frac{1}{k-5} - \frac{1}{k-1} \right) + \left(\frac{1}{k-4} - \frac{1}{k} \right) + \left(\frac{1}{k-3} - \frac{1}{k+1} \right)$$

$$\left. + \left(\frac{1}{k-2} - \frac{1}{k+2} \right) + \left(\frac{1}{k-1} - \frac{1}{k+3} \right) + \left(\frac{1}{k} - \frac{1}{k+4} \right) \right]$$

$$= \lim_{k\to\infty} \left[\left(\frac{1}{1} - \frac{1}{k+1} \right) + \left(\frac{1}{2} - \frac{1}{k+2} \right) + \left(\frac{1}{3} - \frac{1}{k+3} \right) + \left(\frac{1}{4} - \frac{1}{k+4} \right) \right]$$

$$= \lim_{k\to\infty} \left[\frac{k}{k+1} + \frac{k}{2(k+2)} + \frac{k}{3(k+3)} + \frac{k}{4(k+4)} \right]$$

$$= \lim_{k\to\infty} \frac{k}{k+1} + \lim_{k\to\infty} \frac{k}{2(k+2)} + \lim_{k\to\infty} \frac{k}{3(k+3)} + \lim_{k\to\infty} \frac{k}{4(k+4)}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$= \frac{25}{12} \quad \text{atau} \quad 2\frac{1}{12}$$

Catatan : pada **Bagian A**, jika hanya **benar jawaban akhir** saja, bernilai **1 poin**

Bagian B: Nilai maksimum setiap soal adalah 8

1. a. Hitung $\lim\limits_{x\to 1^+} \sqrt{x-1}\,\ln(x-1)$

   b. Hitung $\displaystyle\int \frac{2+\ln(x-1)}{\sqrt{x-1}}\,dx$

   c. Periksa kekonvergenan $\displaystyle\int_1^5 \frac{2+\ln(x-1)}{\sqrt{x-1}}\,dx$

Jawab:

a. (**2,5 p**)

$$\lim_{x\to 1^+}\sqrt{x-1}\ln(x-1) = \lim_{x\to 1^+}\frac{\ln(x-1)}{1/\sqrt{x-1}} \qquad \left(\text{bentuk } \frac{\infty}{\infty}, \text{dapat menggunakan L'hopital}\right)$$

$$= \lim_{x\to 1^+}\frac{(x-1)^{-1}}{-\frac{1}{2}(x-1)^{-3/2}}$$

$$= -2\lim_{x\to 1^+}(x-1)^{1/2}$$

$$= 0$$

b. (**3 p**) Misalkan $u = 2+\ln(x-1)$, atau $du = \dfrac{1}{x-1}dx$, $dv = (x-1)^{-1/2}dx$, $v = 2(x-1)^{1/2}$

$$\int \frac{2+\ln(x-1)}{\sqrt{x-1}}\,dx = (2+\ln(x-1))2(x-1)^{1/2} - \int 2(x-1)^{1/2}\frac{1}{x-1}\,dx$$

$$= 2(2+\ln(x-1))(x-1)^{1/2} - 4(x-1)^{1/2} + C$$

$$= 2\sqrt{(x-1)}\,\ln(x-1) + C$$

c. (**2,5 p**)

$$\int_1^5 \frac{2+\ln(x-1)}{\sqrt{x-1}}\,dx = \lim_{p\to 1^+}\int_p^5 \frac{2+\ln(x-1)}{\sqrt{x-1}}\,dx$$

$$= \lim_{p\to 1^+}\left[2\sqrt{(x-1)}\,\ln(x-1)\right]_p^5$$

$$= \lim_{p\to 1^+}\left[2\sqrt{(5-1)}\,\ln(5-1) - 2\sqrt{(p-1)}\,\ln(p-1)\right]$$

$$= \lim_{p\to 1^+} 2\sqrt{4}\ln 3 - 2\lim_{p\to 1^+}\sqrt{(p-1)}\,\ln(p-1)$$

$$= 4\ln 3 - 2(0)$$

$$= 4\ln 3$$

2. Diberikan $f(x)=\ln(x+2)$

   a. Tentukan deret Taylor dari $f(x)$ di sekitar $x=-1$ dan tentukan daerah kekonvergenannya.

   b. Dengan menggunakan deret Taylor, hitung nilai hampiran $f(-1.1)$ dengan kesalahan kurang dari 10^{-4}.

Jawab:

a. $$f(x)=\ln(x+2) \qquad \to f(-1)=\ln(1)=0$$

$$f'(x)=\frac{1}{x+2} \qquad \to f'(-1)=\frac{1}{1}=1$$

$$f''(x)=\frac{(-1)}{(x+2)^2} \qquad \to f''(-1)=\frac{(-1)}{(1)^2}=-1$$

$$f'''(x)=\frac{(-1)(-2)}{(x+2)^3} \qquad \to f'''(-1)=\frac{(-1)(-2)}{(1)^3}=(-1)(-2)$$

$$f^{(4)}(x)=\frac{(-1)(-2)(-3)}{(x+2)^4} \qquad \to f^{(4)}(-1)=\frac{(-1)(-2)(-3)}{(1)^4}=(-1)(-2)(-3)$$

(2,5p)

Deret Taylor:

$$f(x)=f(-1)+\frac{f'(-1)}{1!}(x+1)+\frac{f''(-1)}{2!}(x+1)^2+\frac{f'''(-1)}{3!}(x+1)^3+\frac{f^{(4)}(-1)}{4!}(x+1)^4+\ldots.$$

$$f(x)=0+\frac{1}{1!}(x+1)+(-1)\frac{1}{2!}(x+1)^2+(-1)^2\frac{1\cdot 2}{3!}(x+1)^3+(-1)^3\frac{1\cdot 2\cdot 3}{4!}(x+1)^4+\ldots.$$

$$f(x)=(x+1)-\frac{1}{2}(x+1)^2+\frac{1}{3}(x+1)^3-\frac{1}{4}(x+1)^4+\ldots.$$

$$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(x+1)^n$$

Daerah kekonvergenan, dengan uji rasio mutlak:

$$\lim_{n\to\infty}\left|\frac{(-1)^n}{n+1}(x+1)^{n+1}\div\frac{(-1)^{n-1}}{n}(x+1)^n\right|=\lim_{n\to\infty}\left|\frac{n}{n+1}(x+1)\right|$$

$$=|x+1|\lim_{n\to\infty}\frac{n}{n+1}$$

$$=|x+1|$$

diperoleh $|x+1|<1$ atau $-2<x<0$.

(1,5p)

Periksa batas $x=-2$,

$$f(-2)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(-1)^n=\sum_{n=1}^{\infty}\frac{(-1)^{-1}}{n}=-1-\frac{1}{2}-\frac{1}{3}-\ldots.=-\left(1+\frac{1}{2}+\frac{1}{3}+\ldots.\right)$$

merupakan deret harmonik, sehingga $f(-2)$ divergen.

(0,5p)

Periksa batas $x=0$,

$$f(0)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(1)^n=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$$

Deret ganti tanda, dan konvergen, karena $\lim_{n\to\infty}\frac{1}{n}=0$.

(0,5p)

Jadi, daerah kekonvergenan $-2 < x \leq 0$. (**0,5 p**)

b. (**2,5 p**)Dari hasil (a),

$$n = 1 \rightarrow \left| \frac{(-1)^{1-1}}{1} (-1.1+1)^1 \right| = 0.1$$

$$n = 2 \rightarrow \left| \frac{(-1)^{2-1}}{2} (-1.1+1)^2 \right| = 0.005$$

$$n = 3 \rightarrow \left| \frac{(-1)^{3-1}}{3} (-1.1+1)^3 \right| = 0.00033\underline{3}$$

$$n = 4 \rightarrow \left| \frac{(-1)^{4-1}}{4} (-1.1+1)^4 \right| = 0.000025$$

$$n = 5 \rightarrow \left| \frac{(-1)^{5-1}}{5} (-1.1+1)^5 \right| = 0.000002$$

Ketelitian kurang dari 10^{-4}, diperoleh pada $n \leq 3$, sehingga jawaban maksimal 4 angka dibelakang koma:

$$f(-1.1) = -0.1 - 0.005 - 0.00033\underline{3} - \ldots = -0.1053$$

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Soal dan Jawaban

<u>Bagian A: Nilai maksimum setiap soal adalah 3.</u>

1. Tentukan Integral berikut

$$\int e^{\sqrt{5x}}\,dx$$

Jawab:

Misal $w=\sqrt{5x}$ atau $w^2=5x$, dan $2wdw=5dx$, sehingga $\int e^{\sqrt{2x}}dx=\frac{2}{5}\int we^{w}dw$

Dari bentuk ini, adap digunakan integral parsial,

Misal $u=w$ dan $du=dw$, atau $dv=e^{w}dw$ dan $v=e^{w}$, kemudian

$$\int we^{w}dw=we^{w}-\int e^{w}dw$$
$$=we^{w}-e^{w}+C$$

Diperoleh

$$\int e^{\sqrt{5x}}dx=\frac{2}{5}\left(\sqrt{5x}-1\right)e^{\sqrt{5x}}+C$$

2. Tentukan integral berikut

$$\int\frac{6x}{(3x+1)^2}dx$$

Jawab:

Gunakan metode parsial, $\frac{6x}{(3x+1)^2}=\frac{A}{3x+1}+\frac{B}{(3x+1)^2}=\frac{3Ax+A+B}{(3x+1)^2}$, diperoleh A = 2 dan B = –2. Kemudian

$$\int\frac{6x}{(3x+1)^2}dx=\int\left[\frac{2}{3x+1}-\frac{2}{(3x+1)^2}\right]dx$$
$$=\int\frac{2}{3x+1}dx-\int\frac{2}{(3x+1)^2}dx \quad \text{(gunakan metode substitusi } u=3x+1\text{)}$$
$$=\frac{2}{3}\ln(3x+1)+\frac{2}{3(3x+1)}+C$$

3. Hitunglah

$$\lim_{x\to\infty}\left[\ln(3x-7)-\ln(6x+3)\right]$$

Jawab:

$$\lim_{x\to\infty}\left[\ln(3x-7)-\ln(6x+3)\right]=\lim_{x\to\infty}\ln\frac{3x-7}{6x+3}$$

$$=\ln\left(\lim_{x\to\infty}\frac{3x-7}{6x+3}\right)$$

$$=\ln\frac{1}{2}\quad\text{atau}\quad -\ln 2$$

4. Periksa apakah integral $\int_1^\infty f(x)\,dx$ konvergen, bila $\int f(x)\,dx=\frac{x}{5-2x}+C$.

Jawab:

$$\int_1^\infty f(x)\,dx=\int_1^{5/2} f(x)\,dx+\int_{5/2}^\infty f(x)\,dx$$

$$=\lim_{a\to(5/2)^-}\int_1^a f(x)\,dx+\lim_{a\to(5/2)^+}\lim_{b\to\infty}\int_a^b f(x)\,dx$$

$$=\lim_{a\to(5/2)^-}\left[\frac{x}{5-2x}\right]_1^a+\lim_{a\to(5/2)^+}\lim_{b\to\infty}\left[\frac{x}{5-2x}\right]_a^b$$

$$=\lim_{a\to(5/2)^-}\left[\frac{a}{5-2a}-\frac{1}{3}\right]+\lim_{a\to(5/2)^+}\lim_{b\to\infty}\left[\frac{b}{5-2b}-\frac{a}{5-2a}\right]$$

$$=\lim_{a\to(5/2)^-}\frac{5a-5}{3(5-2a)}+\lim_{a\to(5/2)^+}\lim_{b\to\infty}\frac{b}{5-2b}-\lim_{a\to(5/2)^+}\lim_{b\to\infty}\frac{a}{5-2a}$$

$$=\lim_{a\to(5/2)^-}\frac{5a-5}{3(5-2a)}+\lim_{b\to\infty}\frac{b}{5-2b}-\lim_{a\to(5/2)^+}\frac{a}{5-2a}$$

$$=\infty+\left(-\frac{1}{2}\right)-(-\infty)$$

$$=\infty$$

$\therefore\int_1^\infty f(x)\,dx$ divergen.

5. Hitunglah

$$\sum_{n=1}^{\infty} \frac{4}{n(n+4)}$$

dengan meninjaunya sebagai deret kolaps.

Jawab:

$$\sum_{n=1}^{\infty} \frac{4}{n(n+4)} = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+4}\right)$$

$$= \lim_{k\to\infty}\left[\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{2}-\frac{1}{6}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{4}-\frac{1}{8}\right)+\left(\frac{1}{5}-\frac{1}{9}\right)+\left(\frac{1}{6}-\frac{1}{10}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)\right.$$

$$+\ldots.+\left(\frac{1}{k-10}-\frac{1}{k-6}\right)+\left(\frac{1}{k-9}-\frac{1}{k-5}\right)+\left(\frac{1}{k-8}-\frac{1}{k-4}\right)+\left(\frac{1}{k-7}-\frac{1}{k-3}\right)$$

$$+\left(\frac{1}{k-6}-\frac{1}{k-2}\right)+\left(\frac{1}{k-5}-\frac{1}{k-1}\right)+\left(\frac{1}{k-4}-\frac{1}{k}\right)+\left(\frac{1}{k-3}-\frac{1}{k+1}\right)$$

$$\left.+\left(\frac{1}{k-2}-\frac{1}{k+2}\right)+\left(\frac{1}{k-1}-\frac{1}{k+3}\right)+\left(\frac{1}{k}-\frac{1}{k+4}\right)\right]$$

$$= \lim_{k\to\infty}\left[\left(\frac{1}{1}-\frac{1}{k+1}\right)+\left(\frac{1}{2}-\frac{1}{k+2}\right)+\left(\frac{1}{3}-\frac{1}{k+3}\right)+\left(\frac{1}{4}-\frac{1}{k+4}\right)\right]$$

$$= \lim_{k\to\infty}\left[\frac{k}{k+1}+\frac{k}{2(k+2)}+\frac{k}{3(k+3)}+\frac{k}{4(k+4)}\right]$$

$$= \lim_{k\to\infty}\frac{k}{k+1}+\lim_{k\to\infty}\frac{k}{2(k+2)}+\lim_{k\to\infty}\frac{k}{3(k+3)}+\lim_{k\to\infty}\frac{k}{4(k+4)}$$

$$= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$$

$$= \frac{25}{12} \quad \text{atau} \quad 2\frac{1}{12}$$

Catatan : pada **Bagian A**, jika hanya **benar jawaban akhir** saja, bernilai **1 poin**

Bagian B: Nilai maksimum setiap soal adalah 8

1. a. Hitung $\lim_{x \to 2^+} \sqrt{x-2}\,\ln(x-2)$

   b. Hitung $\int \frac{2+\ln(x-2)}{\sqrt{x-2}}\,dx$

   c. Periksa kekonvergenan $\int_2^5 \frac{2+\ln(x-2)}{\sqrt{x-2}}\,dx$

Jawab:

a. **(2,5 p)**

$$\lim_{x \to 2^+} \sqrt{x-2}\,\ln(x-2) = \lim_{x \to 2^+} \frac{\ln(x-2)}{1/\sqrt{x-2}} \qquad \left(\text{bentuk } \frac{\infty}{\infty}, \text{dapat menggunakan L'hopital}\right)$$

$$= \lim_{x \to 2^+} \frac{(x-2)^{-1}}{-\frac{1}{2}(x-2)^{-3/2}}$$

$$= -2 \lim_{x \to 2^+} (x-2)^{1/2}$$

$$= 0$$

b. **(3 p)**Misalkan $u = 2+\ln(x-2)$, atau $du = \frac{1}{x-2}\,dx$, $dv = (x-2)^{-1/2}\,dx$, $v = 2(x-2)^{1/2}$

$$\int \frac{2+\ln(x-2)}{\sqrt{x-2}}\,dx = (2+\ln(x-2))2(x-2)^{1/2} - \int 2(x-2)^{1/2}\frac{1}{x-2}\,dx$$

$$= 2(2+\ln(x-2))(x-2)^{1/2} - 4(x-2)^{1/2} + C$$

$$= 2\sqrt{(x-2)}\,\ln(x-2) + C$$

c. **(2,5 p)**

$$\int_2^5 \frac{2+\ln(x-2)}{\sqrt{x-2}}\,dx = \lim_{p \to 2^+} \int_p^5 \frac{2+\ln(x-2)}{\sqrt{x-2}}\,dx$$

$$= \lim_{p \to 2^+} \left[2\sqrt{(x-2)}\,\ln(x-2)\right]_p^5$$

$$= \lim_{p \to 2^+} \left[2\sqrt{(5-2)}\,\ln(5-2) - 2\sqrt{(p-2)}\,\ln(p-2)\right]$$

$$= \lim_{p \to 2^+} 2\sqrt{3}\ln 3 - 2\lim_{p \to 2^+} \sqrt{(p-2)}\,\ln(p-2)$$

$$= 2\sqrt{3}\ln 3 - 2(0)$$

$$= 2\sqrt{3}\ln 3$$

2. Diberikan $f(x) = \ln(x+3)$

   a. Tentukan deret taylor dari $f(x)$ di sekitar $x = -2$ dan tentukan daerah kekonvergenannya.

   b. Dengan menggunakan deret Taylor, tentukan nilai hampiran $f(-2.1)$ dengan ketelitian kurang dari 10^{-4}.

Jawab:

a.
$$f(x) = \ln(x+3) \qquad \rightarrow f(-2) = \ln(1) = 0$$
$$f'(x) = \frac{1}{x+3} \qquad \rightarrow f'(-2) = \frac{1}{1} = 1$$
$$f''(x) = \frac{(-1)}{(x+3)^2} \qquad \rightarrow f''(-2) = \frac{(-1)}{(1)^2} = -1$$
$$f'''(x) = \frac{(-1)(-2)}{(x+3)^3} \qquad \rightarrow f'''(-2) = \frac{(-1)(-2)}{(1)^3} = (-1)(-2)$$
$$f^{(4)}(x) = \frac{(-1)(-2)(-3)}{(x+3)^4} \qquad \rightarrow f^{(4)}(-2) = \frac{(-1)(-2)(-3)}{(1)^4} = (-1)(-2)(-3)$$

Deret Taylor:
$$f(x) = f(-2) + \frac{f'(-2)}{1!}(x+2) + \frac{f''(-2)}{2!}(x+2)^2 + \frac{f'''(-2)}{3!}(x+2)^3 + \frac{f^{(4)}(-2)}{4!}(x+2)^4 + \ldots$$
$$f(x) = 0 + \frac{1}{1!}(x+2) + (-1)\frac{1}{2!}(x+2)^2 + (-1)^2\frac{1\cdot 2}{3!}(x+2)^3 + (-1)^3\frac{1\cdot 2\cdot 3}{4!}(x+2)^4 + \ldots.$$
$$f(x) = (x+2) - \frac{1}{2}(x+2)^2 + \frac{1}{3}(x+2)^3 - \frac{1}{4}(x+2)^4 + \ldots.$$
$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x+2)^n$$

(2,5p)

Daerah kekonvergenan, dengan uji rasio mutlak:
$$\lim_{n\to\infty}\left|\frac{(-1)^n}{n+1}(x+2)^{n+1} \div \frac{(-1)^{n-1}}{n}(x+2)^n\right| = \lim_{n\to\infty}\left|\frac{n}{n+1}(x+2)\right|$$
$$= |x+2|\lim_{n\to\infty}\frac{n}{n+1}$$
$$= |x+2|$$

(1,5p)

diperoleh $|x+2| < 1$ atau $-3 < x < -1$.

Periksa batas $x = -3$,
$$f(-3) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(-1)^n = \sum_{n=1}^{\infty}\frac{(-1)^{-1}}{n} = -1 - \frac{1}{2} - \frac{1}{3} - \ldots. = -\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots.\right)$$
merupakan deret harmonik, sehingga $f(-3)$ divergen.

(0,5p)

Periksa batas $x = -1$,
$$f(-1) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(1)^n = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$$
Deret ganti tanda, dan konvergen, karena $\lim_{n\to\infty}\frac{1}{n} = 0$.

(0,5p)

Jadi, daerah kekonvergenan $-3 < x \leq -1$. **(0,5 p)**

c. (**2,5 p**)Dari hasil (a),

$$n = 1 \rightarrow \left| \frac{(-1)^{1-1}}{1} (-2.1+2)^{1} \right| = 0.1$$

$$n = 2 \rightarrow \left| \frac{(-1)^{2-1}}{2} (-2.1+2)^{2} \right| = 0.005$$

$$n = 3 \rightarrow \left| \frac{(-1)^{3-1}}{3} (-2.1+2)^{3} \right| = 0.00033\underline{3}$$

$$n = 4 \rightarrow \left| \frac{(-1)^{4-1}}{4} (-2.1+2)^{4} \right| = 0.000025$$

$$n = 5 \rightarrow \left| \frac{(-1)^{5-1}}{5} (-2.1+2)^{5} \right| = 0.000002$$

Ketelitian kurang dari 10^{-4}, diperoleh pada $n \leq 3$, sehingga jawaban maksimal 4 angka di belakang koma:

$$f(-1.1) = -0.1 - 0.005 - 0.00033\underline{3} - \ldots = -0.1053$$