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SUBJECT: Re-Submission of Manuscript #2013WR014667 to *Water Resources Research.*

Ximing Cai, Editor in Chief
Water Resources Research
American Geophysical Union

Dear Dr. Cai

Please find attached an electronic copy of the revised manuscript #2013WR014667 (now titled "Near-optimal alternative generation, visualization, and interaction for water resources decision making") by myself as the sole author with I am resubmitting for publication in *Water Resources Research*.

Below, please also find a listing of the numerous revisions that incorporate and address the suggestions and comments of the 4 reviewers, associate editor, and yourself. Should you need to contact me, please do so via email at david.rosenberg@usu.edu.

Sincerely,

David E. Rosenberg (the corresponding author)
Associate professor

Attachments

- Revised manuscript
- List of revisions and response to reviewer comments

Response to Reviewers' Comments

General Comments

Wow! 13+ single-spaced pages of comments from 4 reviewers, the associate editor, and editor (Dr. Cai) are the most extensive set of comments I have ever received on a submitted manuscript! It has taken me nine months and multiple requests for extensions (and generous grants of them by the editor) to revise in response to all the feedback. Generally the feedback has been extremely helpful and I have made the following major changes:

1. Retitled the paper "Near-optimal alternative generation, visualization, and interaction for water resources decision making".

2. Read and now review in the introduction the prior literature on set-value, interactive, and blended solution methods to multi-objective problems. The review now better leads up to one the manuscript's contribution which is to apply blended alternative generation, visualization, and interaction tools to both: a) identify and communicate the full extent of the near-optimal region to an optimization problem, and b) provide tools for the user to explore the region and streamline the process to elicit un-modelled issues and improve the model formulation.

3. Reordered the presentation of the near-optimal method and tools in sections 3-5 to expand on the blended approach and reflect the actual order of steps taken. Namely:
    a. Stratify Monte-Carlo Markov Chain sample to generate a large number of near-optimal alternatives that comprehensively span the region.
    b. Use a Parallel Coordinate plot to visualize the linked decision and objective components of the generated alternatives, and
    c. Controls on the plot to help users explore the region and features of most interest. The Interactive exploration section (section 5) also now provides specifics additional interactive tools and when to terminate exploration.

4. Highlight one benefit of the interactive tools is to streamline the process to elicit un-modelled issues and improve the model formulation.

5. Removed the mixed-integer case study for Amman, Jordan and added further context for the linear programming example for Echo Reservoir, Utah. Additionally significantly expanded the Echo Reservoir example to:
    a. Compare the near-optimal results to results from the Modelling to Generate Alternatives (MGA) method (this comparison is quite striking!)
    b. Show use of the interactive slider tools to generate a set of alternatives that shift and significantly increase the phosphorus removed by stabilizing stream banks.
    c. Demonstrate the process to elicit an un-modelled issue (increase phosphorus removed), add a new objective to the model, generate the near-optimal region to the updated multi-objective model. Additionally, compare results to both *pareto* solutions and near-optimal alternatives generated by MGA (also quite striking!)

d. Relax the tolerable deviation parameter and show the progressive effects of expanding the parameter value on region composition and ranges of allowable phosphorus removal for each decision variable (a new plotting feature uses a color ramp to do all of this in one plot :)

6. Re-written most text in the manuscript including to de-emphasize use of 1st person narrative and fix other writing issues.

Again, thank you for this opportunity to revise this manuscript. Below, I elaborate on these major changes, list reviewer comments in *blue italic*, and indent my further responses in plain text.

Responses to Reviewer #1 Comments

The paper contributes a novel methodology to support the design of near-optimal solutions allowing decision makers to select among a broader set of promising solutions, considering unmodeled issues. The method is based on a three-step procedure: (1) resolution of the original optimization problem; (2) determine the maximal extents of each decision variable and visualize them in parallel axes plot along with the objective to define tolerable deviations from optimality; (3) fix one decision variable, reducing the problem dimensionality, and solve again the problem. The approach is demonstrated on a linear and a mixed-integer problems (i.e., water quality in the Echo Reservoir, Utah, and supply management in Amman, Jordan). The structure of the paper is appropriate and the topic of the work is interesting. However, I suggest a revision of the paper according to the following points:

> Thanks. These strengths are encouragement to address the numerous subsequent comments.

1- the novel contribution of the paper is not sufficiently framed with respect to the existing decision-making support literature (e.g., Cohon and Marks (1975)). In my opinion, the proposed methodology can be classified as an interactive decision method. The paper might benefit from a comparison of the proposed method with the alternative interactive approaches to highlight strengths and limitations of the proposed approach. The following are just few suggested works to consider…:

> Great and thanks for the suggested works. These suggestions are helpful and as noted in Major Change #2, I have read and now review in the introduction the literature on set-value and interactive solution techniques. In reading, I realized there is a more accurate way to present the new near-optimal methods (see Major Change #3); additionally, the "interactive solution technique" mentioned in the original manuscript is only one of several interactive tools used and solution method is an incorrect description. Thus, I have reordered and now present the overall method as 1) stratify Monte-Carlo Markov Chain sample to generate a large number of near-optimal alternatives that comprehensively span the region; 2) Use a Parallel Coordinate plot to visualize the linked decision and objective components of the generated alternatives; and 3) Interaction controls on the plot help the user explore the region and features of most interest. This

reordering also helps better demonstrate the novelty and contributions of steps #1 and #2 which are:

- Identify and communicate the full extent of the near-optimal region, and
- Show numerous and flexible ways to manage a system to maintain near-optimal performance

These contributions and key points stand even if a user never undertakes the 3rd interactive step.

2- the idea that the solutions of an optimization problem are optimal only with respect to the modeled issues and the decision makers might consider other unmodeled factors in selecting the best alternative is not completely new. The concept of set-valued solutions was firstly introduced by Orlovski et al. (1983) and Orlovski et al. (1984), meaning to solve the problem by identifying a set of solutions (instead of a single one) providing the same performance in the long run. In this way, the decision maker can freely choose the solution that he considers the best with respect to the unmodeled issues, still obtaining the same optimal performance. How the proposed approach is related to the set-value concept? Is it really necessary to attain near-optimal solutions or set-value optimality is sufficient?

I believe set-value optimality is insufficient. I was very interested to learn about the method and how a decision maker can choose from among many alternatives to get the same optimal result. After reading [*Orlovski et al.*, 1983; 1984] and several derivative works [*Aufiero et al.*, 2001; *Nardini and Montoya*, 1995], I found a slight thread of commonality which is to identify a set of solutions from which to choose a preferred one. However, the differences between the set-value and near-optimal methods are many:

- Set-value methods are limited to optimal control problems and as a result the solution techniques make fairly specific assumptions regarding the control rule, time horizon, parameter inputs, and problem structure.
- The solution techniques are also quite specific to the problem structure and assumptions (each set-value paper I reviewed derives a new solution method),
- Set-value methods unanimously focus on finding optimal solutions to multi-objective problems with the sets allowing choice from among multiple possible optimal solutions,
- There is no guarantee that the set will have multiple members and thus offer decision makers a choice (the reviewer also notes this limit in their next comment),
- Un-modelled issues are barely mentioned and when they are it is in reference to choosing from among equally-preferred optimal solutions (i.e., optimal with respect to the modeled objectives).
- Set-value methods do not address an issue core to near-optimal analysis which is when there is concern about one or more modeled objective function.

In contrast, the near-optimal method makes no assumptions about the problem structure, is general and applies to a wide range of optimal control and other optimization problems, and only requires small modifications in the sampling strategy of the alternative generation step to accommodate different problem structures like linear, mixed-integer, etc. Again, near-optimal methods are designed to address concerns including the modeled objective function. The introduction to the paper now briefly cites set-value work but does not discuss any of the above because I wanted to spend the time to review the more relevant near-optimal work blended strategies for *pareto* search.

I did not intend to present as new the idea that decision makers consider un-modeled factors. Brill et al. [1982] long ago highlighted un-modeled issues as a problem in decision making. I think the probable source of confusion was my not citing Brill et al. in the culprit sentence (they were cited in the prior and subsequent sentences). I have re-written most of introduction including this sentence to add multiple citations.

> Yet an optimal solution is optimal only with respect to the modelled issues and managers may prefer near-optimal alternatives that also address un-modelled objectives, preferences among objectives, limits, uncertainties, and other shortcomings in the original model formulation [*Brill et al.*, 1982; *Chang et al.*, 1982; *Harrington and Gidley*, 1985; *Rogers and Fiering*, 1986].

3- assuming that set-value optimal solutions do not exist, why the proposed approach should be preferred with respect to enlarging the set of objectives? given the recent advances in terms of computational resources and multi/many-objective optimization techniques, it should be possible to implement the decision makers unmodeled preferences not as constraints on the decision variable but as additional objectives. this latter would also allow tradeoff analysis between the primary objectives and the originally unmodeled preferences. An example of this approach was already proposed in Lotov et al [1992] (http://www.ccas.ru/mmes/mmeda/soft/), Lotov et al [2001], and Lotov et al [2004]

Yes, addressing un-modelled issues requires adding either new constraints or new objectives as needed. The paper now describes and demonstrates both approaches (see Sections 5 and 6.4). At the same time, simply adding a new objective is insufficient because further un-modeled issues can persist. The decision maker cannot discover these further un-modelled issues if their search is limited to the *pareto* solutions of the updated model formulation. The discussion in section 6.4 and Figures 5 and 6 illustrate why it is necessary to further generate, visualize, and explore the near-optimal region to the improved model formulation. Near-optimal identifies additional solutions away from the optimal solution (or pareto-optimal set for multiple-objective problems) and provides a way for the decision maker to systematically explore these solutions while simultaneously clarifying his/her preferences. Tradeoff analysis is naturally embedded in the near-optimal approach because of the way the parallel coordinate visualization tool is constructed. When a decision maker interactively explores the near-optimal region, they simultaneously see how near-optimal alternatives perform according to the modeled objective. And they can use this performance to guide their further exploration.

Note that I base my response having only read the website, *Lotov et al.* [2004], and discussion in *Castelletti et al.* [2010]. I could not find Lotov et al [2001] either in our library or through interlibrary loan.

4- The methodology is presented on linear and a mixed-integer problems, where it computes the solution in O(n) time. However, it requires to solve several optimization problems and the scalability to more complex problems (e.g., non-linear, stochastic, dynamic) can be a significant limitation.

Yes, methods for non-linear, stochastic, and other models remain as further work; this need was stated in the discussion section of the original manuscript and remains in the revised manuscript. The principal scaling issue for non-linear problems is to generate near-optimal alternatives from the non-linear and non-convex region defined by the original and near-optimal tolerance constraints. As discussed in the manuscript, we will either substitute a hit-and-run sampler for the Gibbs sampler because hit-and-run can sample across non-convex regions or revert to an evolutionary algorithm. Initial testing shows both Gibbs and hit-and-run work at about the same complexity level.

MINOR COMMENTS
- line 56: typo in the word quadratically
- box 1: specify the decision variables in the problem formulation, minimize_x f = c(x)
- line 142: missing punctuation mark after "near-optimal region. At the"

Thanks, changes made. Although I already found a period on line 142 after the word *region*.

<u>Responses to Reviewer #2 Comments</u>

In this article the author presents application of the parallel coordinate visualization approach and a stratified sampling approach to explore the near optimal search space in optimization problems. While the motivation of this study is well explained and compelling, the discussion of the author's approach in the manuscript needs improvements. Specifically,

1) The title is vague. It needs to be more focused towards the main context of this paper-perhaps, something along the lines "Exploration of near-optimal alternatives for water resources decision making".

Does the modified title “Near-optimal alternative generation, visualization, and interaction for water resources decision making” work?

2) The article is written in first person language. It should be written in third person or in a manner where the study is central to the discussion instead of the author.

I generally write in first person because first person is more succinct than passive voice and more transparently communicates who did what. At the same time, I agree that the study rather than author should be the focus; thus, I have scaled back the use of first person to only situations where I needed to clarify my role in undertaking the work.

3) The author mentions about various un-modeled preferences, etc. that decision makers have to deal with in selecting final solutions, and then discusses how this tool can be useful in exploring those preferences. While the author demonstrates scenarios of preferences in the example case studies in this paper, he does not make any rigorous assessment on how effective this tool would be when it is actually used by real decision makers. Would the use of this tool help the researchers identify the users' revealed preferences? Would the tool help the users learn about some aspect of their problem? Hence, at this point in time the author has provided no hard experimental/validation data that allows the reviewer to comment on the performance and usefulness of this tool.

This comment is great! A main focus of near-optimal is to present decision makers with alternatives, allow them to explore and choose from among the near-optimal alternatives, and use their choices to help elicit the un-modeled issues that motivate their choice(s). The modeler can then include this new information to improve the model (so the model better reflects and can inform actual decision making). The description of the interactive tools (Section 5) now more explicitly explains the purpose, use, and potential benefits including the statement that "Elicitation will be stronger now because the analyst can work from the manager's revealed preference between the selected alternative, unsatisfactory optimal solution, and prior explored alternatives" (line 339). Also, section 6 now demonstrates two examples that elicit un-modeled issues, update the model formulation, and use the updated formulation to generate new alternatives. These examples include adding new constraints (section 6.3) and an objective function (section 6.4).

The discussion section of the original manuscript noted that further work must test the interactive process with decision makers. This statement persists in the revised manuscript (line 569) and motivated the change in the paper title to focus on contributions related to near-optimal alternative generation, visualization, and exploration (see also my response to your Comment #1). My research group is now working with local stakeholders to develop and apply the near-optimal methods to a model that allocates water for environmental and ecological objectives in the lower Bear River, Utah. However, I also feel the re-organizing of the methods into alternative generation, visualization, and interaction now emphasize contributions that stand independently of testing the interactive tools. These contributions include:

- Show the full extent of the near-optimal region, and
- Identify numerous and flexible ways to manage a system to maintain near-optimal performance

4) The author uses a tolerable deviation factor to narrow down the "near-optimal" space. My major concern with this approach is that how does one decide that a specific value of this factor is no longer an indicator of the "near-optimal" space. Also, "near-optimal" to one decision maker/user might not be "near-optimal" to another decision maker. Also, for objective functions *such as costs, it is easy to perceive the effect of this factor, but how would decision makers make a reasonable choice of this factor for other objective functions that might have different units?*

Another great comment! The interaction section (section 5) now explains how to set the tolerable deviation parameter: start with a default setting (e.g., 110% in the Echo Reservoir example); then if the manager does not find a preferred alternative in the region, relax the parameter value, generate new alternatives, and explore in the expanded region further away from the optimal objective function value. Section 6.5 demonstrates use for increasing the parameter from 110% to 125% and I think you will really, really like the results (Figure 7). Because the stratified sampling and color ramp visualization further show the progressive and continuous effects of increasing the tolerable deviation parameter on each decision variable simultaneously all on one plot! The colors in the ramp demark contours of tolerable deviation through the near-optimal region. Thus, the manager can view the entire region then choose an appropriate parameter value based on the composition and ranges of allowable decisions for each variable that they observe.

5) Step (e) of the sampling approach needs better wording. I can foresee other readers having a hard time understanding how you are using the constraint and gibbs sampling the decision variables for that sub-space.

Reworded by breaking into multiple sentences to separately describe the sub-steps to (i) sample an objective function value and add a constraint, and (ii) Gibbs sample a solution.

6) Do the implementation costs in BMP example also include incentives, opportunity costs, etc.? Please clarify that in your paper, even though it is based on the existing study, because agriculture users using the visualization approach will certainly want to explore the components of the various BMP cost function also.

Yes. Section 6.1 now provides expanded background on the problem including the original source of the cost data from the neighboring Bear River [*Horsburgh et al.*, 2009] as well as the statement, "Costs reflect the full cost for agricultural users to implement a practice and include opportunity costs (such as forgone benefits) for practices like retiring land."

7) In the BMP example, the visualization tool shows the phosphorus removed by each BMP on the green scale. But here are you are assuming that each BMP's performance is independent of other BMPs? What about dependencies between BMPs when their spatial allocation affects their performance as a system of BMPs in a non-linear way?

The model and near-optimal results consider dependencies and mutually exclusive relationships among phosphorus removal practices. These dependencies and relationships were included as constraints in the original linear program (see Alminagorta et al, 2013) and now further described in Section 6.1. The explanation is now included in the revised introduction to this case study. The revision also now further describes the model formulation including the objective function, decision variables, and constraints.

The effects of spatial allocation of phosphorus removal practices across sub-watersheds were found to be linear at the sub-watershed scale for which the Echo Reservoir study and modeling work were done [*Alminagorta et al.*, 2013]. This question regarding linearity was a key question reviewers had for that paper through multiple rounds of review! We ultimately demonstrated the linearity of response by discussing results from prior in-stream water quality modeling work that (i) identified the permissible load for the reservoir and (ii) linear decomposed that load into target loads for the sub-watersheds. These aspects are further described in [*Alminagorta et al.*, 2013]. The current manuscript now adds several sentences and citations in the introduction to the case study to explain how the allowable phosphorus loads were determined and why the optimization model was formulated as a linear program.

Despite the two above answers, yes, I can see how a reader could view the blob of light green in Figure 3 and think it possible that each practice is independent of the others and that any/all combinations are possible within the green region. Certainly all combinations are not allowed (may violate mutual exclusivity and other constraints) and the blob is simply the over-plotting of some 2,500 lines representing the generated near-optimal alternatives. This concern is now addressed in section 6.3 and Figure 4 that show use of the slider controls to further explore the region. The text here explains the slider heights dynamically adjust to show the ranges of allowable phosphorus removal given the model constraints and slider settings.

8) One big missing piece of this work is a comparison study that shows how this interactive solution algorithm is better or worse that existing approaches.

Awesome! So the manuscript now compares results from the new near-optimal alternative generation, visualization, and interaction methods to alternatives generated by *Brill et al.* [1982]'s Modeling to Generate Alternatives (MGA) method. The comparison is quite striking (see Figures 1, 3, and 5) and hopefully summed up well in the last sentence of the abstract: "The flexibility [of the alternatives generated by the new tools] moves beyond the traditional optimal solution and limited alternatives generated by the MGA method." The reason for the striking difference is that MGA seeks alternatives that are maximally different in the decision space and only generates a few alternatives from the region that are not well positioned to elicit un-modelled issues whereas the new tools sample and generate alternatives from throughout the linked decision and objectives spaces. This reasoning and further explanations are now provided in the second paragraph of the discussion (Section 7). The comparison is also now better set up in the re-written introduction that more extensively reviews the *Brill et al.* [1982] and other prior near-optimal work and says they generate a few maximally-alternatives located at select vertices of the near-optimal region that are maximally-different in the decision space.

9) In lines 477-489 the author talks about his planned studies on testing the tool. I would recommend that the author should complete those studies, get their results and then re-submit this paper with some real world validation data. Otherwise this is just a manuscript that merely presents new software.

Hopefully, the new title, my Main Change #3 (reordered presentation of the method to alternative generation, visualization, and interaction), and response to your Comment #3 (above) now address this comment. The paper's contribution focuses on near-optimal alternative generation, visualization, and interaction and the reordered presentation and results show that there are multiple useful results and insights including those listed in the response to your Comment #3.

Yes, the original manuscript clearly stated that testing use with decision makers is an important area of further work and this statement remains in the revision. I could wait a year or more to complete, write-up, and submit the suggested validation studies. However, I feel the key findings of the revised manuscript stand now on their own and are sufficiently validated by the work and results presented. Namely:

- The new alternative generation and visualization tools show the full extent of the near-optimal region

- Interaction tools guide further exploration and streamline the process to elicit un-modeled issues and improve the model formulation

- The tools identify numerous and flexible ways to manage a system to maintain near-optimal performance

10) Lines 518-525 are redundant lines that have already been discussed in the discussion section.

Deleted.

Responses to Reviewer #3

General Comments

The guiding philosophy of the approach is to anchor the decisions on the solution of the model as originally formulated, and then move around in a "near-optimal" region, during which insights are supposedly gained with respect to the original problem. Application in real-life frequently should and do move quite differently, as follows. A model is formulated that bears some resemblance to the actual problem that is being addressed, while the model's structure and mathematics are tailored to the optimization software available (LP and MIP in this case). The solution is then examined and evaluated and the model is modified to bring in or change aspects that have been left out, ignored or simplified in the first round, while still adhering to the original optimization software. At this point the analyst may have the capacity to improve the solution engine so that the model being solved is a closer representation of reality. If this happens then the analyst has provided innovation to the solution methodology. The author may wish to address this matter in the paper.

Thanks for clarifying this important process of model re-formulation which the original manuscript largely side-stepped. Changes in the introduction, presentation of the interactive tools, and discussion of results now show how near-optimal analysis can help streamline the process to elicit un-modelled issues and reformulate the model. Section 6 now provides examples that add new constraints and new objectives. The examples keep the same linear problem structure, but the approach can certainly accommodate a change in structure (e.g., to non-linear) if the un-modelled issue warrants. This change would happen when the analyst steps in to elicit the reason a near-optimal alternative is preferred to the optimal solution and translate the elicited requirement or aspiration into a new constraint or objective. The discussion lists as important work extending the current tools to non-linear problems.

[Incidentally, one of the major pitfalls in systems analysis, is: the model is adjusted to the tool (optimization software) rather than the tool to the real problem that is being addressed].

Agreed.

The paper contains no specific information about the two case studies – Echo Reservoir, Utah and Amman, Jordan. The reader is therefore expected to study these case studies in the original papers (Alminagorta et al., 2013, JWRPM, 2013 and Rosenberg & Lund 2009, WRM, respectively). Understanding the meaning of the results that are presented in the figures of this paper requires familiarity with the case studies, so this paper does not stand on its own, while it discusses in considerable detail the results that are obtained by the methodology and displayed in parallel coordinates. It is clear that this paper cannot contain the same amount of information and data as the original papers of the two case studies, but without some familiarity it is not possible to appreciate the results that are presented.

Section 6.1 beefs up the background on the Echo Reservoir case study so it stands on its own. This background includes a description of the model (decision variables, objective function, and constraints). The revised manuscript no longer includes the Amman, Jordan example.

• Resulting from this observation, it is suggested that one case study should be removed from the paper, since both present the same essentials (even though one is LP and the other MIP). The space saved by removing one example can be used to improve the presentation of the essential information of the case study so the reader can comprehend the meaning of the results.

Excellent suggestion. Done. The manuscript now significantly expands the Echo Reservoir LP example (see also Major Change #5).

The paper rests on a combination of two methodologies:

1. *Generation of alternate near-optimal solutions. It is presented as a way to show DMs solutions that they will "likely find preferable when equity, diversification, or other un-modeled issues exist" (lines 95-96). This is a central point of the whole motivation of the near-optimal solutions generation approach. The paper lacks in providing justification or*

explanation of this "preference", and the two case studies do not elucidate it. See in lines 309-313 the sentences that begin with "should reservoir managers prefer some BMPs over others ….". The explanation that the DMs might prefer alternate solutions because they are concerned about farmers' reluctance/objection is a mere hypothetical. The same thing appears again on lines 324-328. If so, why not include this consideration in the original model, or, if it has been learned only from the solution of the original model then include it ans solve again.

I can see this concern based on the phrasing in the original manuscript. Correct that if an issue is known or articulated a-priori, it should be included in the original model formulation. More generally, include an un-modelled issue whenever it is first articulated. Here, we consider the case that some near-optimal alternatives may perform close to the optimal solution on the stated objective but better address un-modelled issues. The interaction section (Section 5) now more fully describes how the process works for a manager to select a near-optimal alternative as preferred to the optimal solution (for reasons at that particular point that are unknown to the systems analyst), elicit the un-modelled issue that motivates the preference for the near-optimal alternative, and update the model formulation (add new constraints and/or objectives in response). The paper argues that elicitation will be much stronger because the analyst can work from the manager's revealed preference between the selected alternative, unsatisfactory optimal solution, and prior explored near-optimal alternatives. Also, the paper does not claim that this interactive process is new. It's not and has been occurring for as long as people have been making, using, and improving models. Rather the tools contribute to streamline the process – to make selection, updating, and re-generation happen faster, more transparently, and more reproducibly so that managers and analysts can focus on the hard part – eliciting the motivations for a preferred near-optimal alternative and translating those motivations into a model update. (In the case of setting sliders as in Section 6.3, the tools can automatically proceed with the elicitation, updating, and regeneration).

2. *Use of parallel coordinate display as a tool to examine alternate near-optimal solutions. Parallel coordinate display (Inselberg, 2009, Wegman 1990) is an elegant way of overcoming the difficulty of viewing in N-space and for visualizing the variability of the decisions and objective function over a range of their near-optimal variability. The paper does not contribute novelty to this method, but still uses it well. It thus contributes to the dissemination of the value of the method among water resources management researchers and practitioners.*

Thanks. Yes, I agree the Parallel Coordinate display is an elegant way to view N-Space and all credit goes to Inselberg and Wegman. The contribution of this paper in this area is to apply of technique to high dimension optimization problems – to visualize the high-dimensional linked objective and decision spaces on a single plot. This linkage is the first time I have seen Parallel Coordinate plots use in this way. The visualization also serves and the foundation for all the subsequent interaction to explore the near-optimal region and streamline the process to select preferred alternatives, elicit un-modelled issues, etc. A tractable visualization is needed to do all of this. The paper now identifies this contribution in application both in the introduction and in Section 4 that describes the

Parallel Coordinate visualization tool. The text in Section 4 also now explains how to apply Parallel Coordinate plotting to an optimization problem.

It seems that the only novel aspect of the paper is the manner in which near-optimal solutions are generated. And here some questions arise. In particular, the order of the variables and excursion over their feasible is arbitrary, and it is not proven, or at least shown, that (a) the results do not depend on the order in which the variables are treated, and (b) the method used for covering the range of each variable with value points is more efficient than, say, a method of generating value-points simultaneously for all variable in their ranges (i.e., GA-like).

A few points of clarification and response. By "generation" I interpret you to mean the "interactive solution algorithm" referred to in the original manuscript. This algorithm is de-emphasized in the revision and is now simply referred to as an interactive tool to generate a single near-optimal alternative or family of alternatives. It's also one of several tools to help the manager explore the near-optimal region. You are correct that this particular interactive technique is very susceptible to the order variables are selected. This limitation is now prominently mentioned in the discussion (Section 7). At the same time, this limitation is also a benefit – allow managers to jump directly to the decisions or components that most interest them!

The revised manuscript now explains the use of stratified Monte-Carlo Markov Chain sampling as a first step and tool to generate a large number of alternatives that span the near-optimal region. This approach is not susceptible to an ordering bias. This approach was also used in the original manuscript to generate the green mass of near-optimal alternatives on Figure 3 but not emphasized. Hopefully, the reordering of methods (alternative generation, visualization, interaction) and new emphasis better show how several of the paper's main contributions result from the stratified Monte-Carlo Markov Chain alternative generation and visualization and are not dependent on the interaction.

We could use an evolutionary algorithm or similar approach to generate alternatives; however I expect the complexity (i.e., generation clock time) to be similar or longer than the stratified sampling approach for several reasons:

- Still need to generate an initial population
- Must evaluate population fitness at each generation (with fitness quantifying the desired spread of solutions over the near-optimal region), and
- Must apply genetic operators and re-evaluate fitness over successive generations until reaching some defined end-point, and
- The number of final alternatives generated must be the same as for the stratified random sampling approach. This number is driven by the visualization – need to show a smooth, nearly-filled near-optimal region on the parallel coordinate plots (e.g., light green lines in Figure 5).

Given the above, stratified sampling appears as a simpler and less-complex generation method for the well-behaved linear and mix-integer problems addressed in this manuscript. Evolutionary algorithms may prove very useful for generating near-optimal solutions to more complex non-linear and other problems and I look forward to investigate their use when we move on to those problems. Section 7 also now includes discussion of the above points related to the number of required alternatives and use of evolutionary algorithms for non-linear problems.

The captions of the figures include explanatory text that should be incorporated in the text, and/or do not provide satisfactory explanation. Take, for example, Figure 1:

- *The objective function contains only x1, while the constraint has 4 variables. The feasible region is shown on the right for (x1;x2) only. Due to the symmetry of the constraint the feasible region is of the same shape for x3 and x4, but the absence of these in the drawing on the feasible region is confusing and not explained*
- *The value of d for which the shaded region is shaded in the small figure is not stated.*
- *The drawing of the feasible region, on the right hand side, seems to correspond to r=2 but this is not stated.*
- *The statement below the caption "Pin a value on the x1 axis to narrow ranges on the right…." Is obscure. So is the rest of this text.*
- *The caption of Figure is also problematic. What does "… can render geometric objects" mean?*

Thanks for these observations. I have removed this figure from the manuscript, revised subsequent figure titles to be shorter, and moved additional content to the text where the figure is referenced.

<u>Detailed comments, by line number:</u>

The author uses "I … (do)" in many locations in the paper. See Abstract, line 1 and 4, also and later in the paper: Is "I present" "I demonstrate" acceptable style?

This comment was also raised by two other reviewers. I have revised to reduce the use of first person narrative (but not in this response letter :).

• Is line 4: why "existing"?

I meant existing, prior-published work. Actually, this word is not needed and I have deleted it in the revision.

• Key Points: why is parallel coordinate visualization not mentioned among the key points? It appears in the 1st sentence at the top of "research significance". I agree that the paper does not contribute to the method of parallel coordinate visualization (see later comment), yet it deserves to be listed among the key points.

It was not mentioned for space reasons. I have revised the key points; point #1 now mentions “visualization”.

• Key point 3: Is there a way to rank the near-optimal solutions, according to criteria that are not contained in the original objective function? Is there a way to incorporate these criteria in the original optimization problem so that the solution

takes them into consideration?

These are great and interesting questions. Yes, “promising” was ill-defined in the original manuscript and this point has been deleted and replaced with “Tools identify flexible ways to manage a system to maintain near-optimal performance.”

But how to go about ranking… it would be great if one could do this. The problem I see though is that the ranking/criteria are additional un-modelled issues and need to be elicited as new aspirations and modeled as objectives before they can be used! Obviously, once added as a new objective, the ranks can be used to order and more efficiently structure search in a higher-dimensioned representation of the objective space of the problem. So yes, the method can accommodate and use ranks (because it can generally elicit aspirations and add new objectives), but the method cannot automatically determine the ranking criteria (or values) – this requires human intervention. The ranks (essentially preferences) also will be very problem specific. So I would love to hear if you can see a way to automate this process (or reduce the required human intervention)

• Line 9: Except multi-objective that yield a "best compromise" solution.

Yes. I have added “pareto-optimal” to the sentence.

• 11: Add "the" in front of "modeled". • 11: Replace "issues" by "objectives and constraints".

Done.

• 22: "choose" means "fix", and state that the choice of the value is arbitrary, i.e., not guided by any criterion. If there is a criterion for fixing the value then state it.

Thanks. The criteria is what the decision maker desires. This sentence is no longer in the abstract.

• 24: Does the outcome depend on the order in which the variables are fixed? It probably does. If yes or no - there should be an explicit statement about it in the paper.

Yes, certainly. The choice/order of decision variables corresponds guides how the user explores the near-optimal region. Please also see the longer response to your earlier comment.

• 29: "promising" in what sense? This is a critical point in the approach. How does the DM decide/select among alternate solutions? Does she/he have an additional criterion (besides the stated objective function) according to which she/he screens among the near-optimal solutions? If there is, then the problem might be converted to a multi-objective framework. If not, the manner in which "promising" solutions are determined must be stated.

Promising meant one or more near-optimal alternatives address a previously un-modeled issue and the decision maker wants to use (or further explore) the promising alternative(s). However, this use was confusing and the revised manuscript no longer uses the word promising. Instead, the manuscript now uses the word "preferred" as part of the description of the interactive tools (Section 5): a manager selects a near-optimal alternative as preferred to the optimal solution. The analyst elicits the un-modelled requirement or aspirations that motivates the preference, translates them into a new constraint or objective, updates the model formulation, and solves for a new optimal solution (or *pareto* set) and the near-optimal region. So yes, if the un-modelled issue is a criterion, this criterion can and should be added as a new objective and transform the original single-objective problem to a multi-objective formulation. This elicitation and updating process is also now demonstrated in section 6.4 (note when additional un-modelled issues lurk the manager will still prefer near-optimal alternatives to the pareto set).

• 43: Comment on the statement at the end of the line: This is true - not only for single-objective models but also for multi-objective ones. However: MO models can capture more robustly the "other" objectives that are not present in the present model (Box 1) which has a single objective.

Yes. To better encompass both single- and multi-objective problems, I have revised the ending part of the sentence to read: "un-modelled objectives, preferences among objectives, limits … persist." And yes, multi-objective models can capture, but you need to elicit and articulate the issue to model it. The last sentence of the intro now points out this challenge and the response to your prior comment (#29) also takes on this issue.

• 51: change the structure of the sentence: " for small linear problems of managing land, farm and forest cases…"

Done.

• 56: close the space inside the word "quadratically".

I removed the entire word.

• 60: Is this true? Can you substantiate this statement? Even though it is more difficult to prove what has NOT been done, at least you can point to the most recent publications that probe the feasible space close to the optimal vertex (for continuous) and close to the discrete optimal solution for an integer problem.

The paragraph before now reviews evolutionary algorithm work in this area and shows that such work has also identified and considered only a few near-optimal alternatives of the many. I have revised the sentence to read: "The above near-optimal methods have generated a few alternatives that address some existent un-modelled issues and shortcomings of the optimal solution. To address more numerous un-modelled issues, a tool must:"

• 68: So 39 and 18 variables in continuous and integer problems, respectively, is considered by the author to be "large" (lines 15, 37, 54, 60). Modest size or at least medium size is probably more appropriate.

> I have removed the word "large" on these lines and in other locations when referring to the case study problems.

• Box 1: why is the optimization problem not presented in-line, as part of the text? Box 1 shows a general non-linear optimization problem, while the models tackled in this paper are linear. I find this misleading. If the only problems that have been solved (except the "toy" model shown in Fig. 1) then Box 1 should present a linear continuous model, and then add that integer variables can be introduced if the problem is discrete.

> I thought the box made it easier for the reader to separate and differentiate the model formulation from the rest of the text. The revised manuscript includes the model formulation in line.
>
> The intention by showing a general model formulation was and remains to show that the near-optimal method is general and can apply to a wide-variety of model types including non-linear programs. It is true that only a linear program is presented later in the paper and that the non-linear case remains for future work. However, the linear/mixed-integer cases require only a small change in the method to generate alternatives that span the near-optimal region. The presentation of the alternative generation (Section 3) now includes more specifics for how to go from the general case shown in Equations 1a and 1b to the linear, multi-objective, and other cases presented later.

• 77: Could reference something more recent than Dantzig, with all due respect.

> If you prefer. Or the reference is not needed. I have removed the citation.

• 90: So you first generate points on the polyhedron that are "close" to the optimal and then test whether they satisfy the original constraints and a specified Gamma in Eq. 1. Is this efficient?

> Maybe the writing was unclear. That was how the prior cited work [*Burton et al.*, 1987; *Chang et al.*, 1982; *Makowski et al.*, 2000] attempted to solve the problem. Yes it's inefficient which is why they were unsuccessful as was later stated on lines 96-99. I have reorganized the material and move some of it to the review of prior near-optimal methods in the introduction to better explain and differentiate this work from the current work.

• 93: The basic optimization problem is presented only in Figure 1, without the extension that generates the points in the figure.

I have removed the extreme points from the figure. They are a distraction and not needed. But note the addition of the single near-optimal alternative generated by MGA! To me, the MGA result was quite a surprise and clearly shows the limit of the prior work to address a broader set of un-modelled issues (e.g., equity) besides those that align with maximally-different. Hopefully the figure and the rewritten text in section 2 better ground the motivation and contribution of the work.

• 95: Why "likely" and not "possibly". There must be a formalism to specify "preferable" not only in terms of the tolerance on the objective function. If the DM has preference for solutions that have both x1 and x2 in them, why does this not show up in the original formulation of the objective function or >dx constraints?

Yes, another reason to delete the entire use of "preferable" throughout the manuscript. This section was largely re-written. The key idea now is: "In contrast, generating and communicating the *entire* near-optimal region (Figure 1, green shaded area) identifies numerous strategies to share water between users and still keep allocation cost near the optimal cost."

• 99: If there are only 2 decision variables, how many optimization problems have been solved to generate the circles and triangles in figure 1?

None. The points on the figure are extreme points which were generated by enumerating basic and non-basic combinations for the primary decision and slack variables (associated with the optimization program constraints). Then testing each combination to see if it is feasible. These steps are part of prior enumeration methods and not central to the near-optimal method and tools proposed in this work. Thus, as noted in the response to line 93, I have removed these points from the figure.

• 100-148: Has the author added something new to the parallel coordinate visualization method, or possibly to the interactive solution? If so: state this explicitly. If not: there is no justification for the explanation. This part is less important than the exposition of the case studies, which has been skipped (see above).

Yes, there are several new contributions: a) Use of parallel coordinates to visualize solutions to a high-dimensional optimization problem, b) simultaneously link objective and decision spaces on a single plot, and c) and place interaction controls on the plot to allow managers to explore the near-optimal region. I have revised and reordered the writing in this section to emphasize these contributions and how they relate to the broader near-optimal method.

• 104: The model depicted in Figure 2 is not presented, so the figure has no meaning to the reader. The caption of Figure 2 is not clear: … can render complex geometric objects". What does this mean? Also, the explanation below the

caption does not elucidate the figure and the relation between its two parts.

The underlying example data are for a line (linear relation). I have revised the figure caption and adjusted the colors to make the emphasis comparing Cartesian and Parallel coordinate plots.

• 113-115: Is it merely an application of a known methodology (parallel coordinates) or is there something new in this paper. Admittedly, even a novel application is a contribution, but this should be made clear.

Sorry for the confusion. This sentence is misplaced and the method (setting sliders to generate a single alternative) is now presented in the Interaction section (Section 5). The description there and citation now indicate this particular alternative generation as a contribution in application. Note again, the manuscript no longer uses "slider adjustment algorithm" as there are many, much larger contributions related to the stratified Monte Carlo Markov Chain sampling to generate alternatives, visualization, and interactive exploration of the near-optimal region.

• 119: Why "extensions"? These are merely applications of the method presented above, right?

I didn't find "extensions" here. Do you mean section 4? Agreed, it is the method for particular problem types. This section has been removed and the contents included in a new, earlier step of the near-optimal method (section 3) now titled Alternative Generation.

• 149: Why is this "extension"? It is the application to LP and MIP.

See prior comment and fix.

• 163: Random sampling is an alternative that should have been discussed earlier, as an option (see bullet point in General Comments).

Ok. I haven't seen any other near-optimal work that uses random sampling. But it is now discussed in introduction in the review of prior methods.

• 188 et seq.: The examples that have been solved are small, medium at beast, while the paragraph begins by (implicitly) justifying the proposed sampling method as applicable to large problems

Clarified per the earlier comment.

.

• 190-196 and Figure 4: The text that relates to Figure 4 is not clear. Why show the (suggested) disadvantage of uniform vs. stratified sampling?

The intent is to justify the stratified sampling approach. I have removed this figure and the discussion in the revised manuscript. The review in the introduction simply describes this limitation.

• 195-196: I do not agree that DMs are necessarily interested in all feasible nearoptimal solutions.

Agreed, decision makers want the relevant solutions and don't want to be overwhelmed. The introduction has been reorganized around goal of solving two long-running and conflicting challenges to provide managers useful near-optimal results: (i) generate alternatives that span the near-optimal region, and (ii) tractably communicate them. All of the prior work is reviewed under this lens. Paragraphs in the introduction, discussion (Section 7) and conclusions (Section 8) also now explain how the new tools address these challenges. Namely, generate a large number of alternatives and visualize them in mass to communicate the full extent of the near-optimal region. Then, provide interactive tools so managers can jump directly to and explore the variable(s) and level(s) of implementation that are of most interest.

• 222: Focusing the search on sub-regions requires justification for selecting the sub-regions. The justification can be stated as: after the DM examines the last iteration she/he can instruct the search to focus on certain sub-regions of the decision (and objective) space.

Great, thanks. This example no longer remains, but I have carried through the idea in the new example that shifts and expands the phosphorus removed by stabilizing stream banks in sections 6.3 and 6.4. This is exactly the idea for *exploration* rather than *search*.

• 263 et seq.: We do not have information about the Utah (and Amman) case study, and hence cannot appreciate the details of the optimization, the near-optimal solutions and the process of moving in the feasible region by changing decision variables. So now have the reader has to go to Figure 5 and try to extract from it the problem being addressed.

I have added more exposition and introduction to the Echo Reservoir case study. See section 6.1. The revision no longer includes the example for Amman, Jordan.

Figure 5:

- *The range of objective values seems to be 985,000 – 1,083,000. It is not made clear what intermediate value of objective value is generated by what mix of decision variables. There are three-four decisions that have a wide range (green peaks), two of them appear in the basic solution (black hats) while two of them do not appear there at all. It is not clear why the green areas (collection of lines) for these variables is generated by the search. Could the search be carried out more effectively without spanning the full range of these variables*

Correct on the range as well as that it is not clear how intermediate values of the objective function correspond to particular decision variable values in that figure (now #3). These relations are now shown in Section 6.5 as part of the example and results that relax the near-optimal tolerance level to 125% (Figure 7). Figure 7 is a cool visualization for a number of reasons including the color ramp from light green (removal cost at 125% of optimal) to dark green (removal cost at optimal cost) allows the viewer to see progressively how the allowable ranges for each decision variable change when increasing the tolerance parameter. Additionally, the color ramp also shows contours through the near-optimal region, allows viewers to compare removal cost to each other axes, and partially addresses a deficiency of parallel coordinate plots that limit comparisons to adjacent axes. Further, the fanning out of color bands from dark to light along an axis indicate that the decision variable is constrained by the near-optimal tolerance level. When the color bands stack on top of each other (dark on top of light), the decision variable level is limited by one of the other model constraints.

Yes and no to the final question of can the search be carried out more efficiently without spanning the full range of the variables. The reorganizing of the methods into alternative generation, visualization, and interaction means that we're not searching in the generation and visualization steps—the goals are simply to identify and communicate the composition and extent of the near-optimal region (e.g., results in Figure 7). At the same time, the identified extents guide the manager and allow him/her to jump directly to explore the features that most interest them. New paragraphs in the introduction, discussion (Section 7) and conclusions (Section 8) also now explain how the new tools partition these responsibilities. Please also see my response above to 195-196.

- *There is no correspondence between specific combinations of variables in the near-optimal range and the value of the objective function. This is obviously seen by the DM as the "slides" are shifted, but the reader of the paper cannot discern this from the figure – so much of the meaning is lost.*

  Yes, correct. The color ramping of results shown in Figure 7 now show the relationships between the objective function and decision variables. Please also see the response to the prior comment.

- *It might have been interesting to indicate the combination of variables that generates the max (110%) value of the objective function. The one that generates the min (100%) is the black line. Comparing these two on the graph may reveal something.*

  Great comment, and yes it does! Color ramping the results (Figure 7) now allow the reader to simultaneously compare the optimal and 110% levels, as well as many more progressive levels between 100% and 125%., all on a single plot! Please also see my responses to your two prior comments.

- *273: It took 2,500 stratified, random-sampled near-optimal solutions to generate Figure 5. Is there no more efficient way to generate a smaller sample?*

The 2,500 stratified, random-sampled alternatives are for the visualization aspect – to show the composition and extent of the near-optimal region as a nearly filled in area in Figure 3. You can use a smaller number of alternatives if patches are acceptable and there is not the need to “fill” the region. At the extreme, you need only solve *2n* (n=number of decision variables) optimization problems as discussed in Section 3 step b to identify the separate and independent minimum and maximum extents of each decision variable and provide bounds for the user to begin interactive exploration. These bounds place lower and upper limits on each decision variable axis (which in one of the interactive tools is also indicated by slider heights). The number of required samples depends on the problem size, structure, and the need for the user to initially view the full extent of near-optimal region before proceeding with interactive analysis. The discussion section now has a paragraph that describes these issues.

313: This is the first time that a rationale for limiting decision variables is presented. Could it be introduced as a constraint into the problem and then solved, rather than "discovered" in the large body of solutions that allow a 10% deviation from the optimal value of the objective function?

Yes, a limit on a decision variable should be included and specified earlier *if it’s previously known to exist*. However, here we are dealing with the case that the limit was not previously known and was only discovered, articulated, and elicited after the manager expressed a preference for a near-optimal alternative. This comment is similar to Reviewer #1’s comment #3 about adding an objective to represent a previously un-modeled issue. The manuscript now includes more description in the abstract, introduction, interaction section, discussion and conclusions to explain that controls also streamline the process to elicit un-modelled issues and improve the model formulation.

317: The explanation of Figures 7 and 8 could be improved to facilitate tracking the black line and the ranges of the green and pink regions.

These figures have now changed to emphasize other aspects of the near-optimal region and interaction.

• 325-330: Same comment: Why consider this post-factum and not introduce this consideration into the definition of the optimization problem?

Similar as before.

• Section 5.2: It is not possible to comprehend that meaning of the solutions obtained, since the basic optimization problem is not available without going back to Rosenberg and Lund [2009] and studying it. Figure 9 does not elucidate the reasons for the diversity between variables. For example: Zara-Maeen emerges with a very large possible value (green triangle) while it does not show in the basic solution (black line). Distant brackish water has a range of values while most other variables are practically fixed. Why? The range of values of the objective function is relatively small. It would be instructive to know which combination of variable given the max?

The discussion of the case study now presents more of the relevant background (see section 6.1).

Also, the manuscript no longer includes the Amman example, but to answer your questions: some of the diversity is due to the fact that most of the decisions are binary (implement; don't implement) whereas new local groundwater, distant brackish water, mobile units, and new local source water are integer (corresponding to the number of wells or plants built). The Zara-Ma'een project, if built, can supply a large volume of water as indicated by the light-green near-optimal solutions. If it's not built, the project will offer no supply (zero value as indicated by the solid black line optimal solution). I can similarly and quickly plot the results with a color ramp (as in Figure 7) if you'd like to see which combination of variables and variable levels give objective function values at various tolerable deviation levels – let me know.

• 415: Delete "all".

• 431: This is not quite accurate, or, at least, a point for discussion. A better alternative would be to discover/uncover aspects that are found missing in the original formulation, then re-formulate (i.e., improve) the model and resolve. Searching in the near-optimal domain is not necessarily the best way, since it remains "anchored" in/near the original optimal solution, whereas expanding the model to include other constraints and/or elements and/or different parameter values in the objective function can improve the definition of the problem and model.

Great, thanks for the clarification. This clarification is now incorporated into the introduction, description of the interaction method, results presented in Section 6.4, and the discussion of results.

• 439: Here this approach is allowed, namely, that if the initial solution elucidates deficiencies in the original definition of the decision issues then the model should be re-formulated and solved rather than "sliding" to it by allowing decision variables to move within the region of "near-optimal". But this aspect is hidden in a late section of the paper, and should have been placed where the approach is proposed.

Yes, noted, and addressed per prior comments.

• 463: Here, for the first time, MO is mentioned. This too, should have been placed in the overall perspective of the proposed methodology.

Yes. Please see also the response to Reviewer #1, Comment #3. The example in section 6.4 now shows the process to elicit an un-modelled issue and transform the single-objective formulation into a multi-objective one.

• 490: The "Conclusions" contain too much repetition of the factual results, which, as stated above, are difficult to follow because the decision problems and models are not present in this paper and the reader must go to the original papers to appreciate the value of the results

obtained here. This section should concentrate on the "Conclusions" relevant to the methodology and its application, and much less (if at all) to the results of the two applications.

Ok. I have revised the conclusions to focus on the contributions associated with the methodology and its application.

• 632-637: The model in Figure 3 is not addressed properly in the text of the paper (lines 124-135) and it therefore difficult to understand the figure and the verbal discussion in these lines. For example, why is the range of x1 different from that of the other variables? All decision variables appear in the same role in the constraint (a 4D ball with radius r, Figure 3). The model seems completely artificial: the objective function depends only on x1 with arbitrary (unspecified) parameters.

The revision no longer has this figure or the associated text. This deletion allows the paper to focus on the more concrete case studies and their results.

<u>Responses to Reviewer #4</u>

Comment 1. It is suggested that the writing style should be revised throughout the paper, as it reads very informally.

1a. The author used the first person pronoun "I" too often. It is a matter of style, but it is suggested to use first person only when necessary, rather than writing the paper as a journal or report of actions taken by the author.

Yes, Reviewer #2 (comment #2) noted this first-person use of “I” as well. I have scaled back the use of first person to only situations where it is imperative to identify my role in undertaking the work.

1b. The writing slipped between declarative tense and imperative tense within paragraphs, so that much of the paper reads like a user's manual. It would be easier to read if algorithmic steps are explained using an imperative tense in a set of numbered steps, rather than mixed within a descriptive paragraph.

Thanks. Most of the algorithmic steps were already written in the imperative (commanding) tense. I switched several sentences in Sections 3 and 4 to the imperative sense.

1c. The text varies between present tense and past tense in lines 182-186. Keeping all in the present tense is easier to read.

OK, change made.

1d. informal usages can be revised: several sentences begin with "Or"; phrases such as "Put another way" line 192, "like" instead of "similar to" line 351; beginning sentences *with a number line 351; sentence fragment presented as a sentence (line 291); use of subjective words "cleverly" and "important".*

Revised. Please note the sentence on line 351 already had the requested change.

1e. Paragraph at lines 477-489 is unnecessary.

Deleted

Comment 2. The author compares this work in the introduction primarily with the MGA approach of Brill. However, it is not convincing that the MGA approach is the approach that should be compared with the one described here. The statement of the contribution of the work should be refined.

Please now see the more full review in the introduction of near-optimal as well as generation, set-value, interactive, and blended search methods for multi-objective problems (see also Major Change #2). One of this paper's contributions is to apply blended techniques of alternative generation, visualization, and interaction previously demonstrated for multi-objective problems to exploration within the larger near-optimal region. I believe the MGA methods are the relevant comparison points because they are the only methods I have seen that generate near-optimal alternatives. The examples now explicitly compare results from the new tools to those obtained by the MGA method (Figures 1, 3, 5, 6). This comparison is extremely important as the MGA method generates very limited alternatives that do no span the near-optimal region or address un-modelled issues. The discussion now includes a paragraph that describes reasons for the difference which centers on the fact that MGA seeks alternatives that are maximally different in decision space and located at select vertices of the near-optimal region whereas the new stratified sampling tools generate alternatives spread through the decision and objective spaces of the region.

2a.The sentence at line 59-60 is not really true if one explores the work done by Ranjithan and others that follow the MGA using evolutionary computation methods. I suggest removing the sentence or updating it to reflect work that has been done for large, mixed-integer, nonlinear, and simulation-based problems.

Sorry and thanks. "Near-optimal regions remain uncharacterized for larger problems…" was an overstatement and not exactly what I meant to say. I have revised the statement to read: "The above near-optimal methods have generated a few alternatives that address some existent un-modelled issues and shortcomings of the optimal solution. To address more numerous un-modelled issues, a tool must: …" This statement follows a review of evolutionary methods to generate alternatives including those by Ranjithan that tackle larger problems. But those methods still use MGA to generate only a few near-optimal alternatives that are maximally different from the optimal solution in the decision space. Again, the key point here is that much more work is needed to identify, visualize, and explore near-optimal regions.

2b. Line 195. Author suggests that decision makers are interested in all (emphasis on the word all) feasible near-optimal solutions. Because this number may be infinitely large, a decision maker is in fact not interested in all feasible near-optimal solutions. This is the advantage offered by the MGA approach, that users would not be overwhelmed by a small number of very different alternatives. Using the approach described here, a secondary algorithm is needed to parse the set of all near-optimal solutions (or a large set of samples of all near-optimal solutions), which seems to be the interactive part of the algorithm.

> Correct, decision makers are not interested in all near-optimal solutions. This comment was particularly helpful as it suggested the way forward to re-organize the methods into separate steps of alternative generation, visualization, and interaction (Major Change #3). The first paragraph of the introduction now better sets up the underlying issue as a key motivating challenges for the paper: "Providing managers with useful near-optimal alternatives requires solving two conflicting challenges: (i) generate alternatives that address the un-modelled issues, and (ii) tractably communicate them." And as you note, yes, the alternative generation requires producing a large number of alternatives to identify and communicate the composition and extent of the near-optimal region. But the visualization and interaction components allow the manager to jump to and explore the features of most interest. The introduction, discussion, and conclusions now also highlights the blended methods of generation, visualization, and interaction.
>
> And further correct that MGA provides a much smaller number of near-optimal alternatives (typically 2 to 4 in the work reviewed) and decision makers may find this small number more manageable. However, there is no guarantee that the MGA alternatives, which are maximally different in the decision space, span the near-optimal region or address the un-modelled issues. The MGA alternatives do not in the examples now considered in the paper (Figures 1, 3, 5, and 6) and this finding is highlighted in the abstract, results, discussion, and conclusions. Another way to see the shortcoming of MGA is that its maximally-different objective is the "criterion" the method uses to search through the near-optimal region. However, this criterion need not (and in many cases does not) align with the criterion one would obtain by eliciting, articulating, and translating an un-modelled goal or aspiration into a new objective. Thus, MGA heads in different direction through near-optimal region which the manager may find useful only for select un-modelled issues. The new tools provide a more flexible way to elicit a wider variety of un-modelled issues.

2c. Comparing this approach with the MGA approach is not compelling. For example, the hypothetical different solutions described in lines 307-338 could be identified by the MGA approach. The important difference in the algorithm that is presented here is that the interaction of the user drives the type of solution that is identified. Identifying near-optimal solutions is important, as it is stressed in the manuscript, because those are the only ones that decision-makers would be interested in. However, if the algorithm just identifies all the potential near-optimal solutions and presents them to the user, then the user is overwhelmed, even if it is on a parallel axis. If the contribution of the approach is that a user can see a large number of *solutions and comprehend those better than if the solutions are presented in an alternative approach, then the paper should experiment with how users are responding. The real contribution of this approach may be that the interaction with the user drives the solution identification, and this is not demonstrated. The approach has merit in its problem statement, but the author can communicate its contribution more clearly and compare the approach to relevant algorithms.*

Thanks for framing the underlying issues; my response follows on to the prior #2b. Again, the revised ordering of the method and blending of the tools (generation, visualization, and then interaction, Major Change #3) clarifies the multiple contributions as:

- New stratified sampling and parallel coordinate plotting tools generate and communicate the composition and extent of the near-optimal region to an optimization problem,
- Interaction tools support further exploration and streamline the process to elicit un-modeled issues and improve the model formulation, and
- The tools identify numerous and flexible ways to manage a system to maintain near-optimal performance

Section 5 now describes the interaction component, goals, and endpoints while the examples in Section 6.3 and 6.4 (Figures 4, 5, and 6) illustrate use of the interactive tools to explore, elicit un-modelled issues, and improve the model formulation.

Comment 3. How important are the different steps and parts of the algorithm? For example, simulate how four different decision-makers interact with the algorithm and arrive at different solutions. Show how the order of decision variables can change the decision-making process of a user.

Yes, agreed. How a user interacts with the visualization (the steps s/he follows, including the ordering of decision variables—axes—on the parallel coordinate plot) will determine the outcomes or alternative identified. This result is to be expected of interactive techniques. Additionally, the tool is programmed specifically to give users the flexibility to quickly order axes on the plot as they want. The manuscript now presents these actions as exploration instead of search. Sections 4, 5, and 7 now discuss this issue.

Comment 4. It is not clear from the description of the methodology - what part of the algorithm presented here is the "new fast interaction solution algorithm" and what has already been developed and presented by Inselberg? In section 3, at line 115-116 and in the paragraph at line 118, is this referring to the new algorithm or the Inselberg citation?

Sorry for the confusion. Inselberg describes the slider adjustment algorithm as fast. The new part is applying the algorithm to explore the high-dimensional near-optimal region of an optimization problem. This “interactive solution algorithm” and search are de-emphasized in the revised manuscript in favor of a larger set of interactive tools to explore the region.

Comment 5. Text at line 94 is confusing, may be more clear: "the optimal point includes only X1 greater than zero, whereas near-optimal solutions include both X1 and X2 greater than zero."

> Actually, not correct. Near-optimal solutions can include X2=0 (see the MGA result!). I reworded this sentence to: "In the example, the optimal solution allocates water only to User 1 (blue circle, bottom right)." (Near-optimal alternatives that better address equity issues have X2 > 0).

Comment 6. What is Step 3 at line 131? Numbering steps would make the methodology easier to understand.

> Step 3 refers to the "Third" on line 126 – choose a value for one decision variable within its maximal extents, reduce the problem dimensionality by one degree, …. I have reorganized the description of this section to put in more algorithmic/numeric form.

Comment 7. Lines 299-302 The text describes the conclusions that are drawn from viewing Figure 6. Are the numbers and conclusions supposed to be intuitive, that a decision-maker would look at the graphs and make these same conclusions immediately? Is there some interface where the user would find the value of 735 kg of phosphorus? It is difficult to see from the figure, and because the algorithm is driven by interaction, the explanation of how the user would interact with these graphs is important for this methodology.

> With regard to lines 301-302, there are two interfaces a user can use to find the value of 735 kg for phosphorus. First, the dark green line crosses the blue Manage Ag. nutrients axis at 735 kg of phosphorus removed (read on the right). Second, a user can hover the mouse over the point where the dark green line crosses that axis and the screen will show the value. This latter feature was never discussed in the manuscript but is now mentioned in section 5 as one of the interactive tools.
>
> The reviewer is correct that the original manuscript provided little explanation for how the user would actually interact with the graphs shown in Figures 6-9. The plot content has changed in the but the text in sections 6.2-6.5 now provide more detailed descriptions, including of how the user actually interacts with the plot and controls.
>
> I didn't find numbered results/conclusions in lines 299-302 for Figure 6; do you rather mean Figure 9 and lines 374-381 for the Amman, Jordan example? If yes, the numbers were simply to identify, highlight, and distinguish three findings that derived from the results which characterized the near-optimal region for that problem. I didn't expect that they were intuitive to most readers – which is why the text called attention to them. But please note the revised manuscript no longer includes the Amman example.

<u>Responses to the Associate Editor</u>

The manuscript entitled "Near-optimal management to improve water resources decision making" seeks to facilitate interactive decision making in the neighborhood of optimal solutions.

The paper has received 4 very comprehensive reviews that are strongly consistent in their concerns on the work's methodological weaknesses and limited contributions. Three of four reviews suggest Major Revision while one review recommends Reject. The reviews have provided significant guidance to the authors.

Yes agreed. The guidance has been significant and prompted extensive revisions.

The primary weaknesses in the present manuscript are as follow: (1) the framework is not significantly different from prior published work and represents a test case employing existing approaches;

The introduction now includes a more extensive review of prior near-optimal methods as well as generation, set-value, interactive, and blended methods to identify *pareto* solutions to multi-objective problems. The review now better leads up to one the manuscript's contribution which is to apply blended alternative generation, visualization, and interaction tools previously used for multi-objective problems to explore larger near-optimal regions. Another important difference is that current work generates alternatives from throughout the near-optimal region whereas prior methods generate a limited number of alternatives that are maximally-different in the decision space and located on select vertices of the region. The Case Study (section 6) now compares results from the new blended tools to findings by the prior near-optimal Modelling to Generate Alternatives method. The results are striking and demonstrate numerous important differences and improvements advanced by the new blended tools.

(2) the novelty claims for the approach are disconnected from the historical literature;

I have completely revised contributions. They are:

- New alternative generation and visualization tools show the full extent of the near-optimal region
- Interaction tools guide further exploration and help streamline the process to elicit un-modelled issues and improve the model formulation
- The tools identify numerous and flexible ways to manage a system to maintain near-optimal performance

Each contribution derives directly from the reorganized method of alternative generation, visualization, and interaction. Regarding disconnect from the historical literature, please see my responses to the prior comment.

(3) it is not clear that near optimality as defined in this work is a necessary or sufficient condition to address that weaknesses or concerns in the decision making process.

Near-optimal isn't necessary or sufficient to find a preferred alternative to the optimal solution. The goal of the work is not to guarantee finding a preferred alternative and I don't feel necessary and sufficient are the appropriate or relevant criteria to use to evaluate the current work or any prior near-optimal alternative generation methods. The last paragraph of the discussion section now addresses this concern head on:

> Finally, the new tools help users explore the near-optimal region and streamline the process to elicit un-modelled issues but do not guarantee a user will find a preferred alternative to the optimal solution. First, a preferred alternative may not lie in the near-optimal region and second a user may stop exploring before they find an alternative. Even if a user stops exploration, the blended tools still provide new insights on the composition of the region and problem structure and can elicit more varied un-modelled issues than either the optimal solution or prior generation methods such as MGA.

I expect you were looking for necessary and sufficient conditions because the original manuscript incorrectly framed the problem as a search problem and labeled the new method as an "interactive solution algorithm." These descriptions created the presumption and expectation that the search for a preferred near-optimal alternative must resolve definitively like a search for an optimal solution. A search for a preferred near-optimal alternative will not necessarily resolve definitively (so little time, so many alternatives, little guidance on how to navigate towards preferred. If the analyst knew how to navigate towards preferred the issue would be *modelled* rather than *un-modelled* :). Instead, I feel the more useful evaluation metric is new insight gained from using the new tools. And in this regard, the key points listed above in response to your comment #2 each offer strong insight into the near-optimal region composition and problem structure.

In addition to adding the new paragraph to the discussion, I have also revised and restructured the entire manuscript to de-emphasize the search aspect and more strongly emphasize the blended components and insights to the problem gained by using them. Numerous changes towards this goal include:

- Now describe the method as blended tools to generate near-optimal alternatives, visualize them, and allow users to interactively explore the near-optimal region (Major Change #3).
- State two of the three key points as stemming from the generation and visualization components of the method,
- No longer use the term "interactive search algorithm" and instead say "generate an individual alternative". Also introduce this tool as one of a set of interactive tools to help the user explore the near-optimal region.
- Always use the word "alternative" to refer to the near-optimal; only use the word "solution" to refer to the optimal solution.
- In section 5, clarify the stopping points for interaction as: "Terminate interaction when the manager is satisfied with an optimal solution, near-optimal alternative, or insight gleaned from exploring the near-optimal region."
- Examples in Section 6 emphasize the insights learned about the problem and the structure of the near-optimal region structure. The examples also compare results of the new blended tools to alternatives generated by the MGA method. The

comparisons show the new tools can generate alternatives that address a much wider range of un-modelled issues than the very few MGA alternatives.
- Removed "improve" from the title.

Also, one final note: the weaknesses or concerns are not in the decision making process but rather in the modeling methods (optimization and prior near-optimal approaches) that support decision making.

I believe that the paper in its present form is borderline on Reject but that the authors should be given one round of major revision to significantly enhance its contributions and provide a more rigorous literature-based case for its claimed novelty.

Thank you for this opportunity to revise and resubmit.

Responses to the Editor

Thank you for submitting "Near-optimal management to improve water resources decision making" [Paper #2013WR014667] to Water Resources Research. I have now received the Associate Editor's comments and 4 excellent reviews of your manuscript, which are attached for your reference. As all agree, your work is interesting and it is potentially useful for find approximately solutions of water resources system optimization problems.

Thank you.

However, based on the review comments, I also find there are several major issues that have been raised through the review process, which must be addressed before your paper is accepted for publication. It seems at least another round of review is required. As the AE summarizes, the novelty and contribution need to be addressed by relating your work to previous publications; the "near optimality" should be clarified and justified for decision making problems. I would suggest you to pay special attention to a comment provided by reviewer # 3 about to "prove" or "show" the way to generate near optimal solutions satisfies the criteria of "near optimality." In other words, how will the approach not miss the true near-optimal solutions?

The new blended alternative generation, visualization, and interaction tools can miss true near-optimal solutions if by "true solution" you mean a near-optimal alternative that the manager prefers to the optimal solution. The tools do not necessarily converge to a preferred alternative nor was that ever the intention. Rather, the contribution is to provide better insight about the problem structure, show the varied ways to manage the system that still keep performance close to optimal, and more effectively direct managers towards alternatives that address un-modelled issues than prior near-optimal methods. Please also see my lengthy response to the AE's comment #3 which explain the extensive changes I have made to address this fundamental issue and concern.

Following this context, is the approach subject to any assumptions (i.e., it might be appropriate for certain problems with a special structure?).

Yes, the principal assumption is regarding the original model structure (linear, multi-objective, non-linear, etc.). Currently, the method can only update model formulations for elicited issues that can be translated and modelled as linear or in mixed-integer problems. Extending to non-linear programs is an important area of further work and discussed in Section 7.

By the way, you may find the Tchebycheff algorithm relevant to your approach, which is described in Steuer, R. E. 1986. Multiple criteria optimization: Theory, software and testing examples in decision support systems, Wiley, New York.

Great, thanks. I now review Steuer in the introduction as one of several interactive solution methods to multi-objective problems.

In highlighting these points, it is not my intention to discount other elements of the attached reviews.

Yes, understood.

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