

# Lecture 3

## Theory of environmental policy

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AEM 6510

# Roadmap

Develop a simple model of

- Pollution damages
- Abatement costs
- Characteristics of efficient pollution allocations

This will guide us in

- Describing the set of policy instruments and their properties
- Information needs for using each kind of policy

# The base model

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# A model of damages and costs

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The firms take output prices as given, and sell electricity in the national market, households buy electricity on the national market



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The model is **non-spatial**:

- All firms' emissions count the same toward aggregate emissions  $E$
- All households experience the same level of pollution  $E$

# The damage function

Assume households have utility:

$$U_i(y_i, E) = y_i - D_i(E)$$

where  $y_i$  is income spent on market goods and  $D_i(E)$  is the household-specific disutility caused by aggregate pollution

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With this utility function, we can interpret  $D_i(E)$  as the *dollar value of lost utility for household  $i$  from aggregate emissions*

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Aggregate damages are then

$$D(E) = \sum_{i=1}^N D_i(E)$$

where  $N$  is the number of households

# Abatement costs

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Define the **abatement cost function** for firm  $j$  by  $C_j(e_j)$

# Abatement costs

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Abatement costs are decreasing in emissions (increasing in abatement)

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As you reduce emissions, the cost of reducing the next unit is higher than the previous<sup>1</sup>

<sup>1</sup> Written another way, if  $\mathcal{C}(A)$  is the cost of abatement, we are assuming  $\mathcal{C}'(A), \mathcal{C}''(A) > 0$

# Abatement costs assumptions

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These assumptions on marginal abatement cost are pretty reasonable:

1. MACs are increasing because firms will choose among different abatement technologies in order of their marginal cost if they are profit-maximizing or cost-minimizing
2. *Weakly* increasing MACs is a reasonable approximation of piecewise constant MAC functions, which is what many MACs look like empirically

# Efficient allocation of emissions

In our setting emissions negatively affects households, and controlling emissions imposes a cost on firms

An efficient outcome optimally balances these two different costs to the economy



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The efficient emission level for each firm  $j$  can be found by minimizing the social costs of emissions:

$$SC(e_1, \dots, e_J) = \sum_{j=1}^J C_j(e_j) + D(E)$$

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$$-C'_j(e_j) = -C'_k(e_k), \quad \forall k, j$$

These are the two fundamental characteristics of the efficient allocation of pollution

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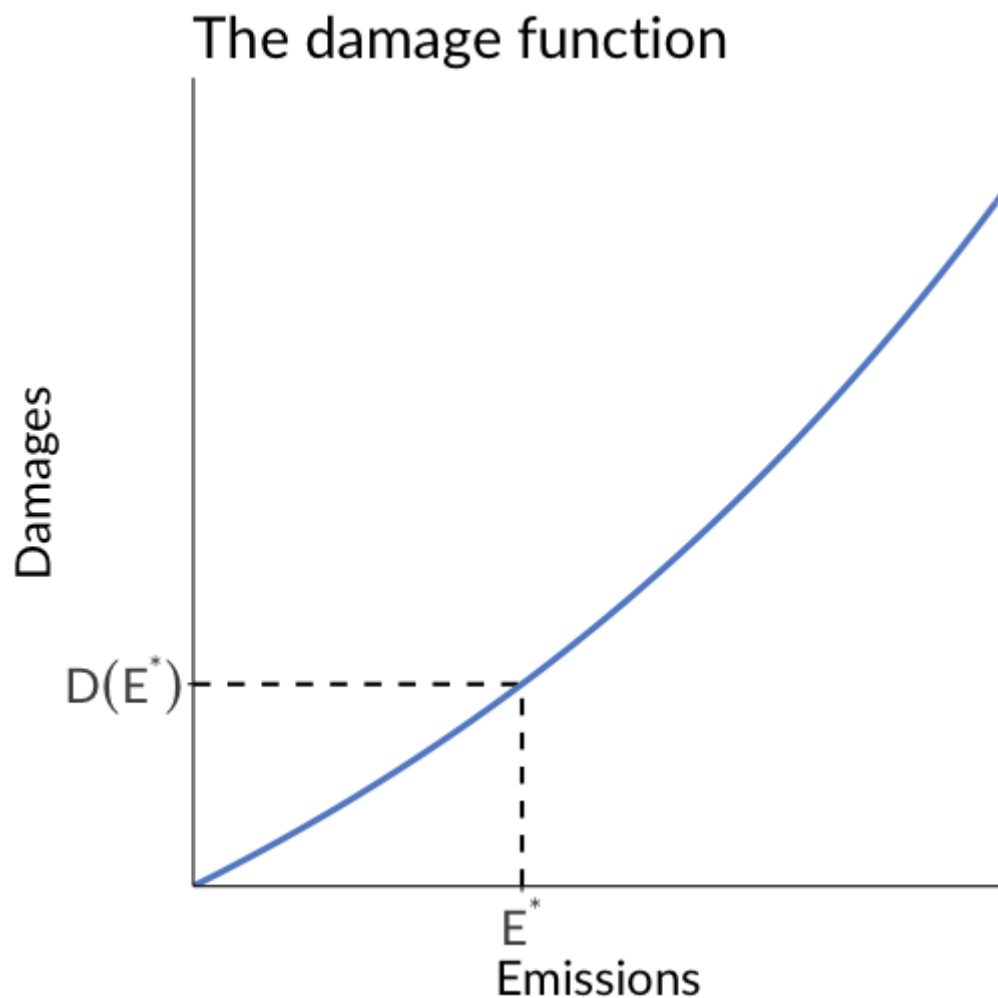
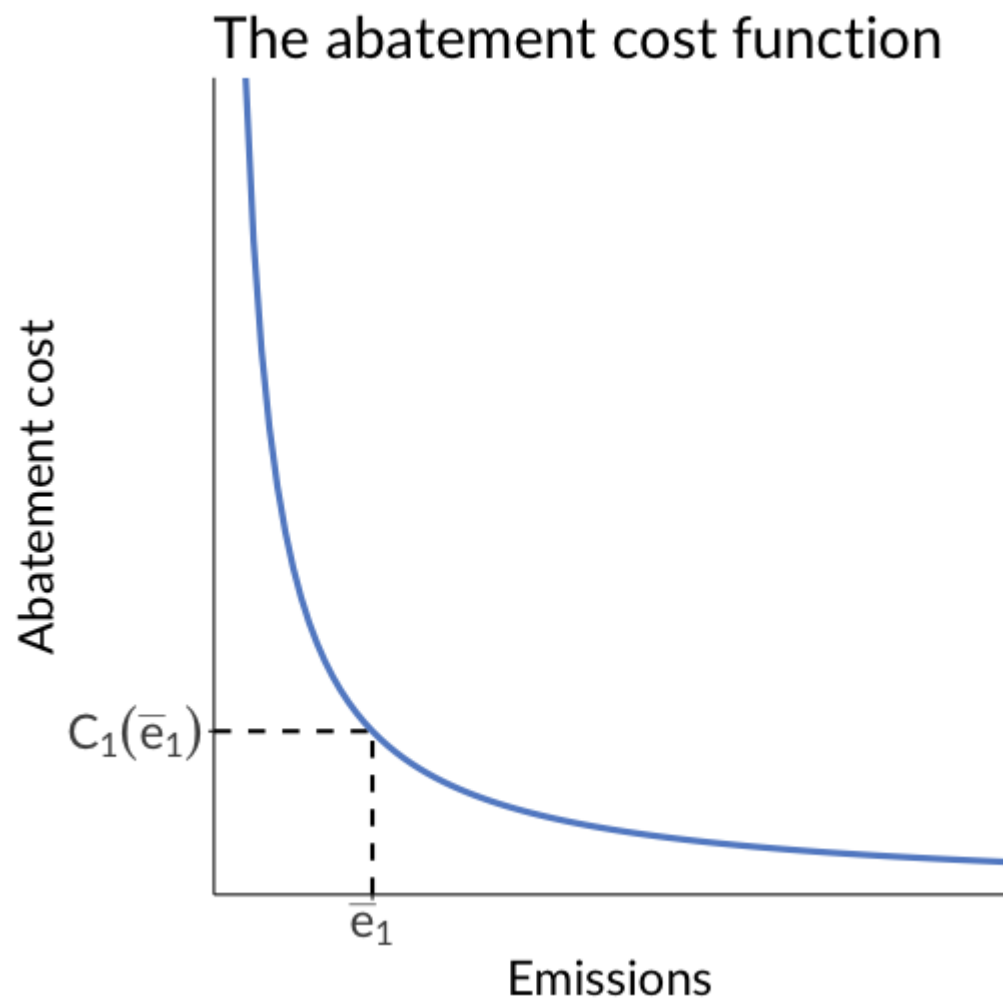
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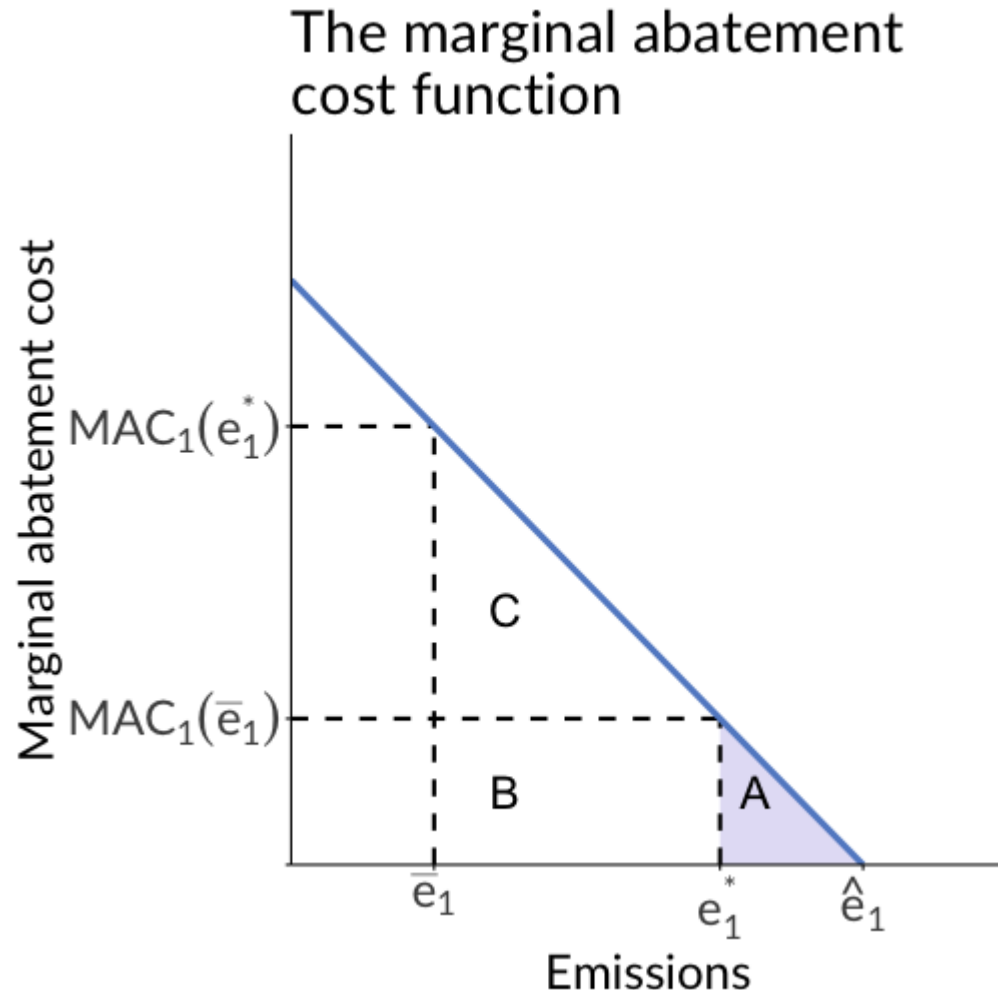
An optimal regulation will satisfy these two condition

# Abatement costs and damages





# Marginal abatement cost



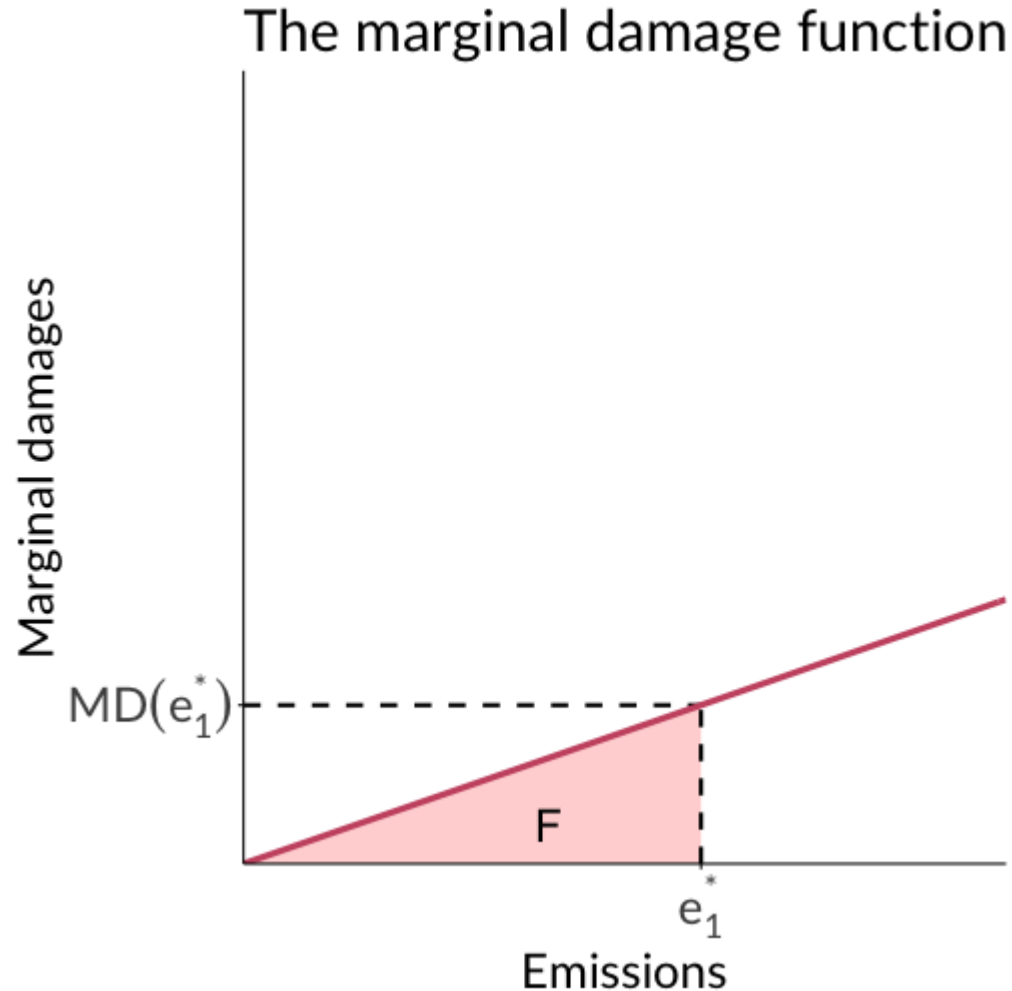
Marginal abatement costs are decreasing in emissions, increasing in abatement

The area under the MAC is total abatement cost

**A:** Total abatement cost of abating  $\hat{e}_1 - e_1^*$  units

**A+B+C:** Total abatement cost of abating  $\hat{e}_1 - \bar{e}_1$  units

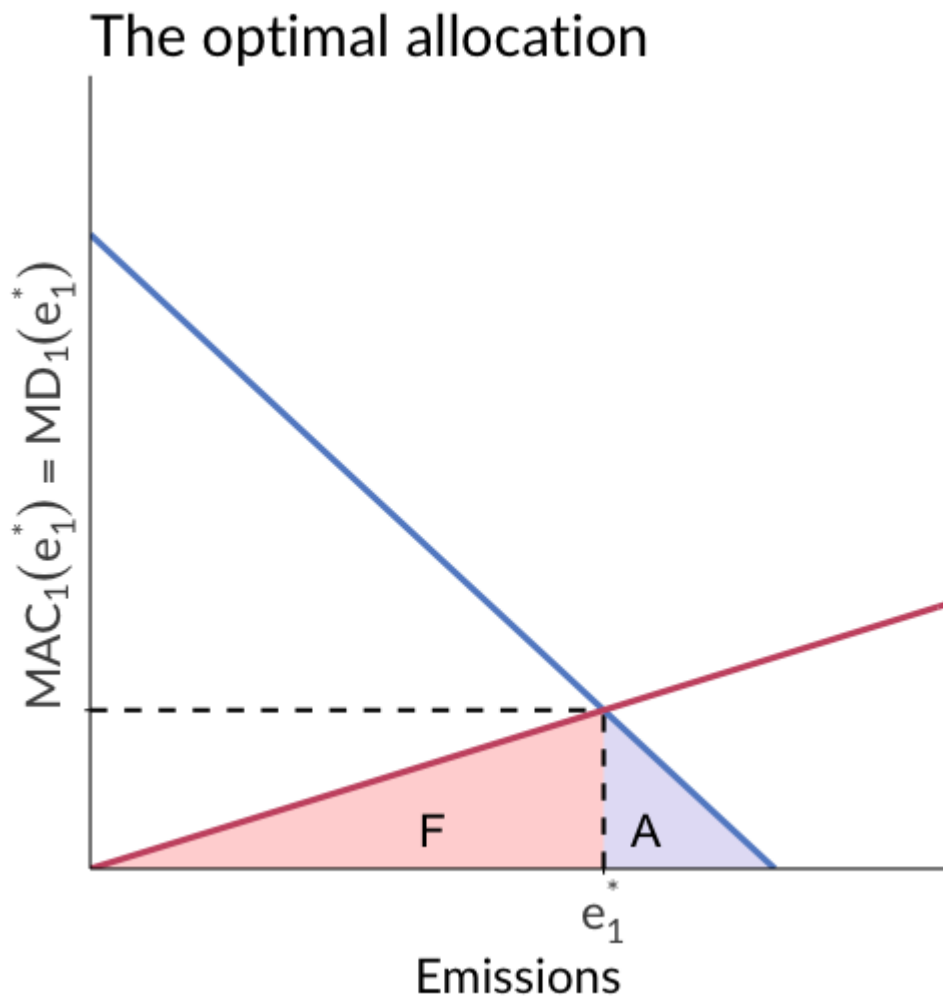
# Marginal damages



Marginal damage curve is increasing in emissions, decreasing in abatement

The area under the MD is total damages

# The optimal allocation



The optimal allocation is where  
MAC and MD intersect

This minimizes the total cost to  
 $A + F$

# Property rights

Do we need government intervention to solve environmental problems?

Is it possible to reach an efficient outcome with negotiation?

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Let's think about a special case of our model: 1 firm and 1 household

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In this setting, simply assigning property rights to the firm or household and allowing for negotiation may lead to the efficient outcome

# Household ownership of pollution rights

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How can we get to the efficient level without government intervention?

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We assume both players are fully informed about each others preferences and technologies

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What contract does the firm offer in equilibrium?

# Household ownership of pollution rights

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This means we can write the firm's total cost as:



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$$-C'(E) = D'(E)$$

Note that this still requires  $tr \leq \underbrace{C(0) - [C(E^*) + D(E^*)]}_{\text{total welfare gain}}$

# Firm ownership of pollution rights

If the firm has the rights to pollution, we just flip the script

The household proposes a contract  $(E, \theta)$  where the firm reduces pollution in exchange for a transfer payment

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The household will then offer the minimum required:  $\theta = C(E)$

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where we again reach the social optimum, as long as:

$$tr \leq \underbrace{D(\hat{E}) - (D(E^*) + C(E^*))}_{\text{total welfare gain}}$$

where  $\hat{E}$  is the firm's initial emission level

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The assignment of property rights **doesn't** matter for efficiency

But it does matter for the distribution of wealth

These observations are known as the **Coase Theorem**

# The Coase theorem

Suppose party A imposes an externality on party B. Provided transactions costs are sufficiently small, irrespective of the initial allocation of property rights: the parties can achieve the socially optimal level of pollution  $E^*$  using a transfer payment  $\theta$  where both parties are at least as well off as they were before

With small enough transactions costs, the party that does not own the property rights can propose a contract that is mutually beneficial

# The Coase theorem: real world

The Coase theorem is not just a useful theoretical exercise:

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Vittel contacted all upstream farmers and negotiated contracts for reducing nitrogen runoff

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In 1972 Switzerland, France, Germany, and the Netherlands contracted to pay MdPA 532 million francs to reduce emissions

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# Policy instruments

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This, i.e. most settings we think about, is where there is a role for public intervention

# Command and control

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We're going to focus on absolute emissions standards

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Is this a realistic option?

Why or why not?

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The regulator needs to know:

- The social damage function
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And the regulator needs to be able to:

- Impose a policy that is different across firms and is unlikely to be politically feasible

# Uniform emission standard

An alternative is to impose a **uniform emission standard** such that  $e_j \leq \bar{e}$  for all firms  $j$

We could imagine setting  $\bar{e} = E^* / J$  where  $E^*$  is the socially efficient level of emissions

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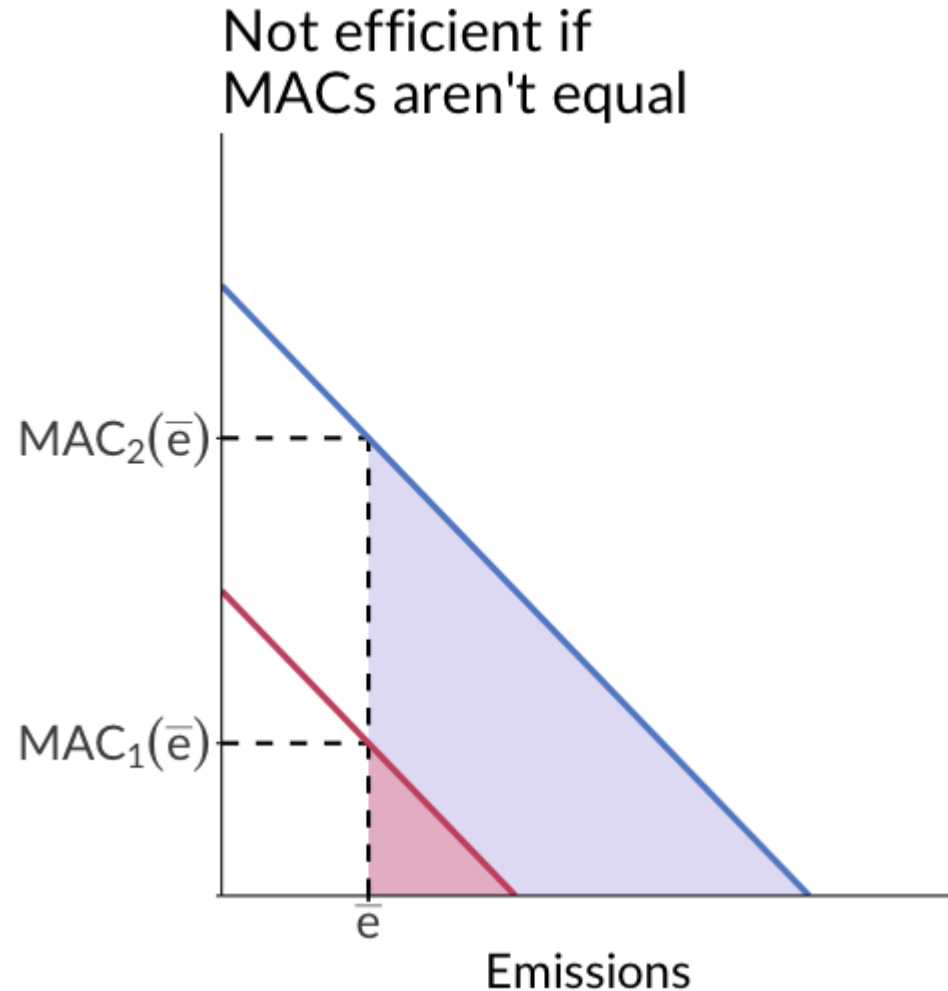
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If firms are identical this achieves the efficient outcome

If they're not identical it won't

# Uniform standard costs



Even though  $\bar{e} \times J = E^*$ , the MACs may not be equal

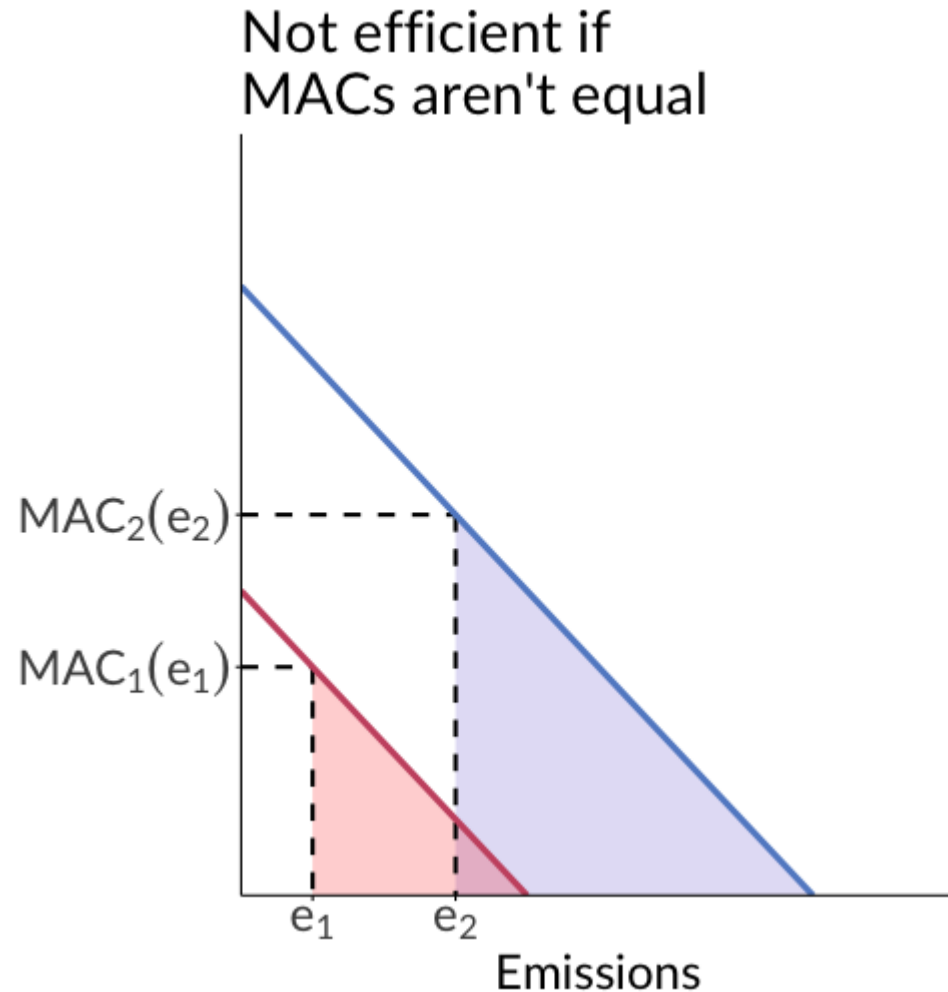
If MACs aren't equal we can maintain  $E^*$  and reduce costs

How?

Firm 1 abates 1 unit more, firm 2 abates 1 unit less



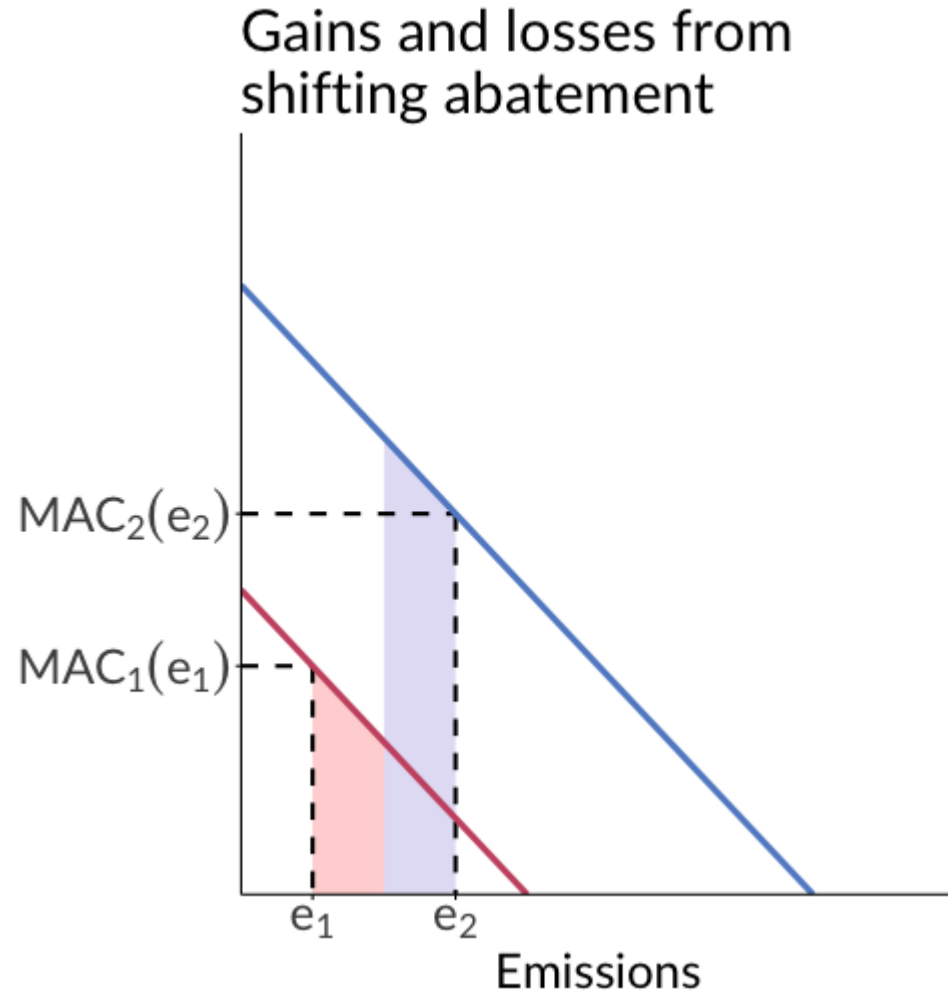
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This changed the total abatement cost

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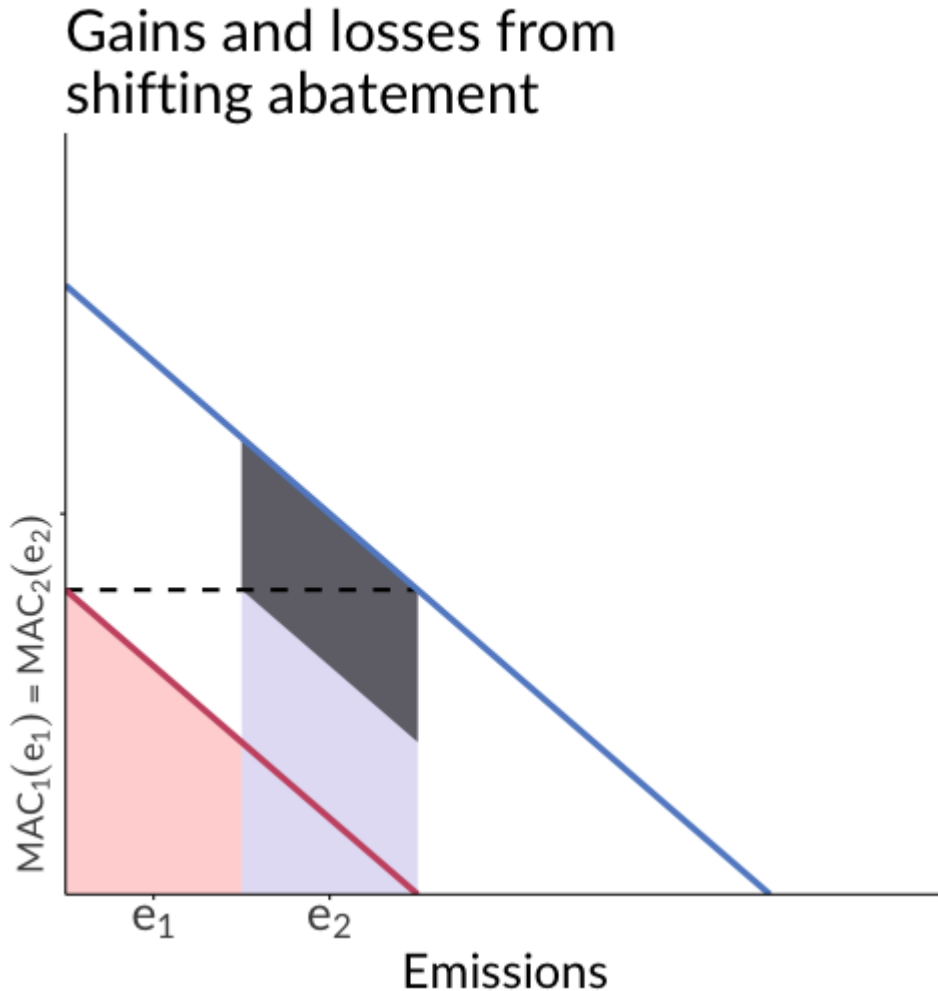
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Firm 1 has costs **increase**

Firm 2 has costs **decrease**

Net effect is a **decrease** in costs

# Uniform standard costs



We can keep obtaining cost reductions until MACs are equal across firms

With net reductions in deadweight loss equal to the dark gray area (light blue minus light red)

We want low MAC firms to abate more than high MAC firms

# Emission taxes

An emission tax is a fee that the pollution firm must pay per unit of emissions

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Suppose the government imposes a tax of size  $\tau$  per unit of pollution

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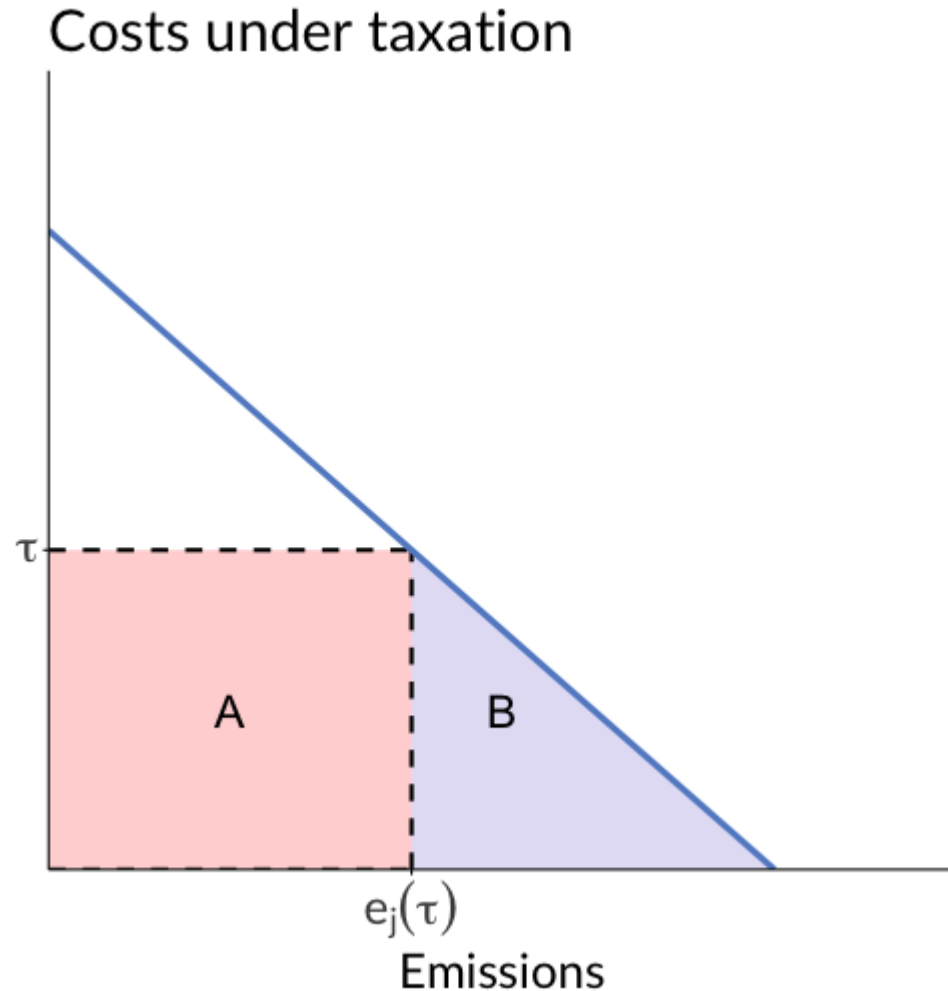
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The firm's optimal choice is to set marginal abatement cost equal to the tax rate

The firm reduces emissions as long as the cost of emissions reductions is less than the alternative: paying the tax

# Emission taxes



Under a tax  $\tau$ , the emission choice is a function of the tax:  $e_j(\tau)$

The firm pays total tax  $A$  and incurs abatement cost  $B$

Now the government has revenue  $\tau \times e_j(\tau)$  that it can use for different purposes, we will look at this more closely in a few classes

# Emission taxes

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This then implies that:

$$MAC_j(e_j) = MAC_j(e_k) \quad \forall j, k$$

Marginal abatement costs across firms are equal and we have obtained the given emissions reduction at least-cost

# Emission taxes

If we change the tax rate what do we expect to happen to emissions?

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Return to the firm FOC:

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and differentiate it with respect to  $\tau$  and recognize that  $e_j$  is a function of  $\tau$ :

$$-C''_j(e_j) \frac{de_j(\tau)}{d\tau} = 1$$

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This gives us that:  $\frac{de_j(\tau)}{d\tau} = \frac{1}{-C''_j(e_j)} < 0$ : higher taxes lower emissions if MACs are decreasing in emissions



# Auctioned pollution permits

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Let there be  $L$  permits for sale, and let  $\sigma$  be the auction price that emerges

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If firms are price-takers then their total cost is given by:

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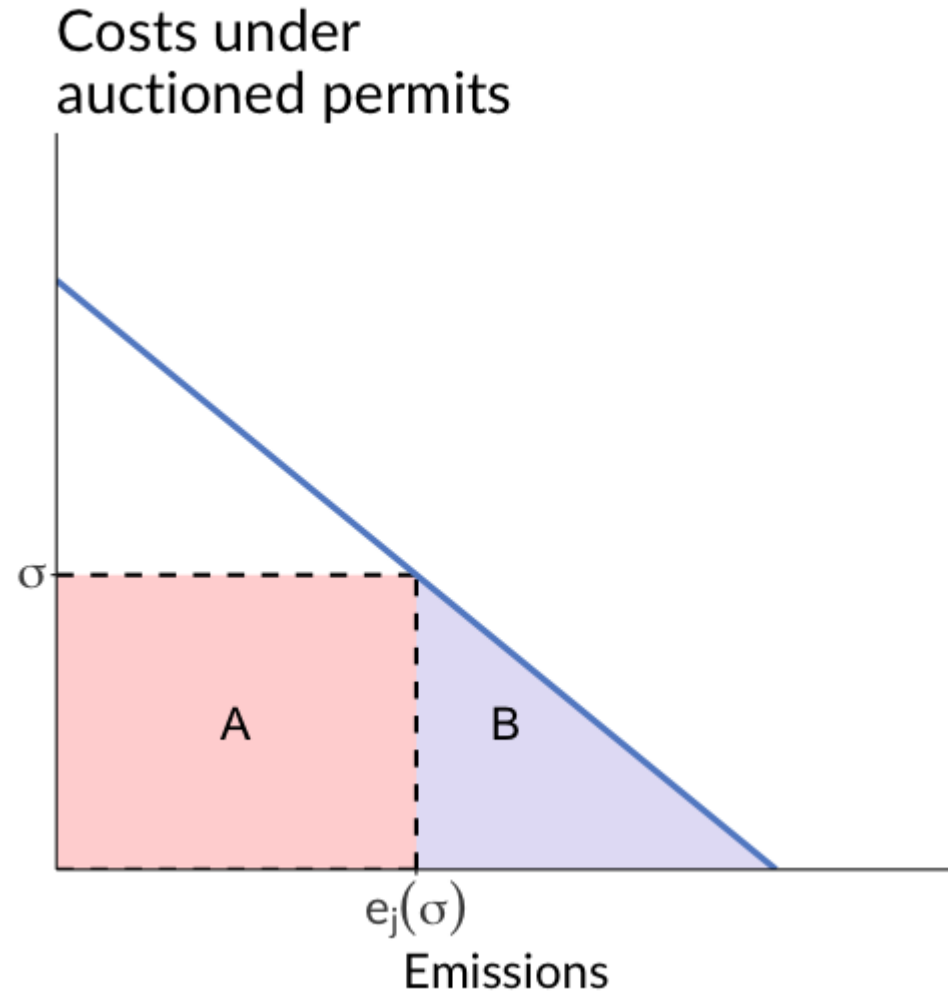
The second term is the permit purchase cost, the first term is the abatement cost

Cost-minimization gives us:

$$-C'_j(e_j) = \sigma$$

which indicates that firms set their MACs equal to the permit price (and implicitly each other's MACs)

# Auctioned permits



Under a permit price  $\sigma$ , the emission choice is a function of the price:

$$e_j(\sigma)$$

The firm pays permit costs  $A$  and incurs abatement cost  $B$

This is **identical** to an emission tax if

$$\sigma = \tau$$

# Auctioned pollution permits

We can then invert the MAC to get the firm's emission-response to permit prices

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What is this expression?

$e_j(\sigma)$  is just firm  $j$ 's **permit demand** as a function of permit price  $\sigma$

Aggregate demand for permits is then the sum of the individual demands:

$$E(\sigma) = \sum_{j=1}^J e_j(\sigma)$$

# Auctioned pollution permits

The price  $\sigma$  that clears the market equates supply of permits  $L$  and demand for permits:

$$L = \sum_{j=1}^J e_j(\sigma)$$

This equation (supply = demand) defines the market equilibrium like the market for any product

# Taxes, permits, and efficiency

Both taxes and permits achieve  $MAC_j = MAC_k \forall j, k$ , so both achieve any given emission reduction at least-cost

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Both taxes and permits achieve  $MAC_j = MAC_k \forall j, k$ , so both achieve any given emission reduction at least-cost

With knowledge of the damage function  $D(E)$ , both can also be used by a regulator to achieve the socially optimal emission level  $E^*$

# Freely distributed transferable permits

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How does this system work?

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1. Regulator sets total amount of pollution
2. Regulator disburses permits
3. Firms can trade permits

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FOCs are:

$$-C'_j(e_j) = \sigma$$

identical to auctioned permits!

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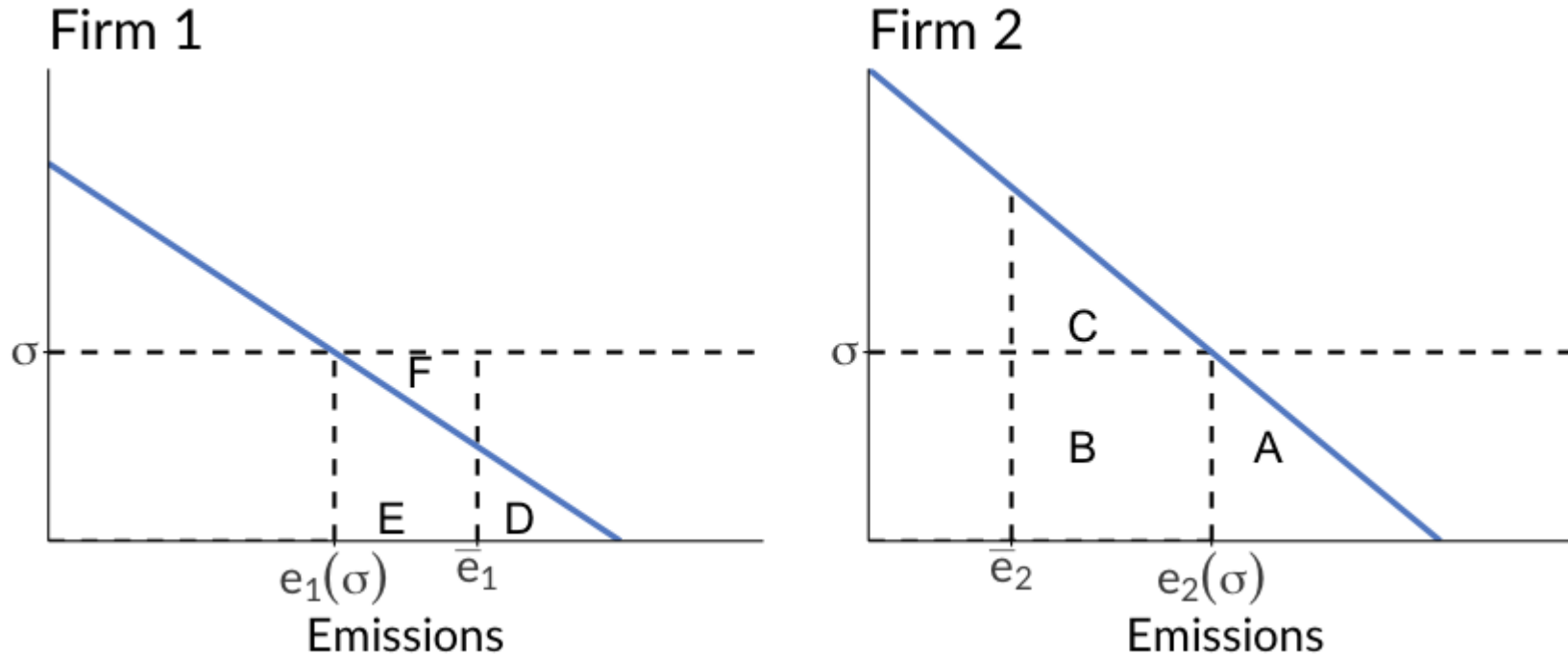
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As before, market equilibrium is given by:

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In short: efficiency is the same, but distributional outcomes will be different

# Freely distributed transferable permits



Firm 1: Abatement cost ( $D \rightarrow D+E$ ); Permit revenues ( $0 \rightarrow E+F$ )

Firm 2: Abatement cost ( $A+B+C \rightarrow A$ ); Permit costs ( $0 \rightarrow B$ )

Total cost reductions:  **$C+F$**  ( $A+B+C+D - (A+D+E) = B+C-E = (E+F)+C-E = C+F$ )

# Subsidies

So far we've put the responsibility of expenditures on firms

But, for political economy reasons, regulators may not want to put this extra burden on firms

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But, for political economy reasons, regulators may not want to put this extra burden on firms

Often regulators subsidize abatement

How does this differ from taxation and permits?

# Subsidies

Suppose we subsidize a firm  $\xi$  for each unit their emissions are below some baseline level  $\hat{e}_j$ , its total costs are now

$$TC_j(e_j) = C_j(e_j) + \xi(e_j - \hat{e}_j)$$

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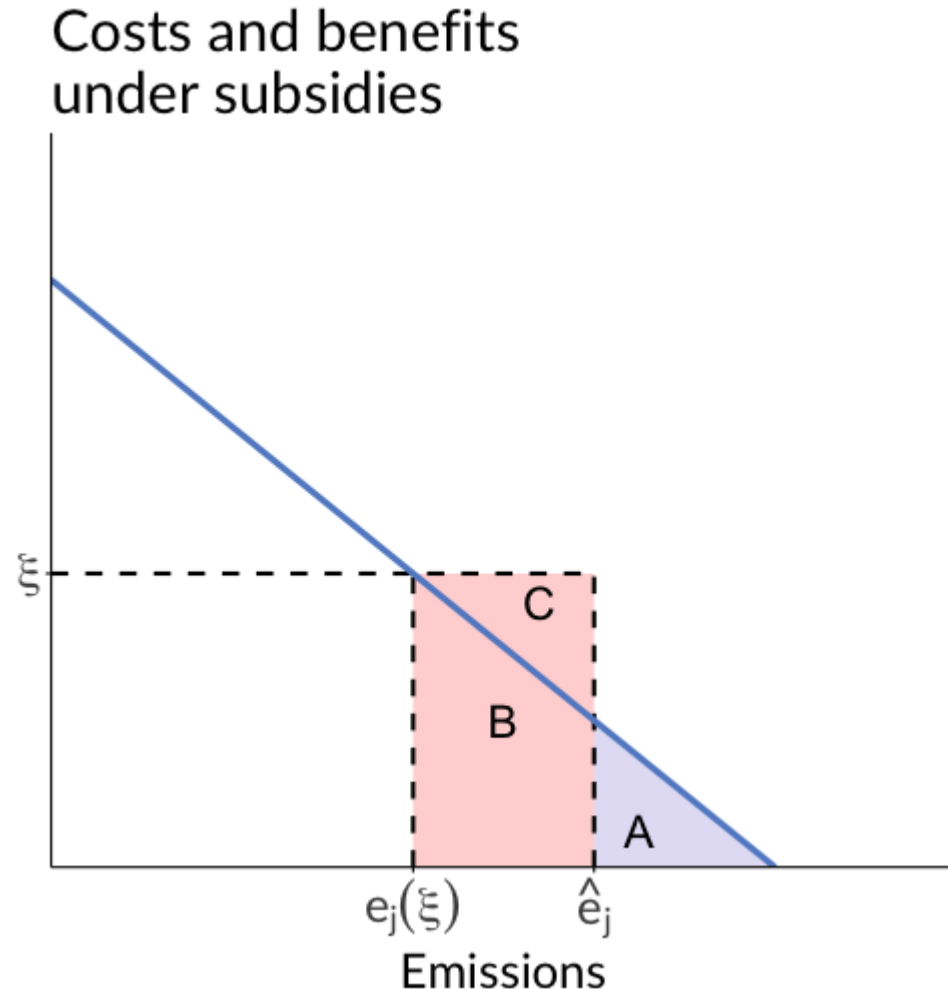
The firm's FOC is then:

$$-C'_j(e_j) = \xi$$

The per-unit abatement subsidy  $\xi$  has the same behavioral effect as a per-unit emission tax  $\tau$ : firms set MAC equal to the subsidy<sup>1</sup>

This is conditional on the total subsidy payment being large enough to induce abatement.

# Abatement subsidies

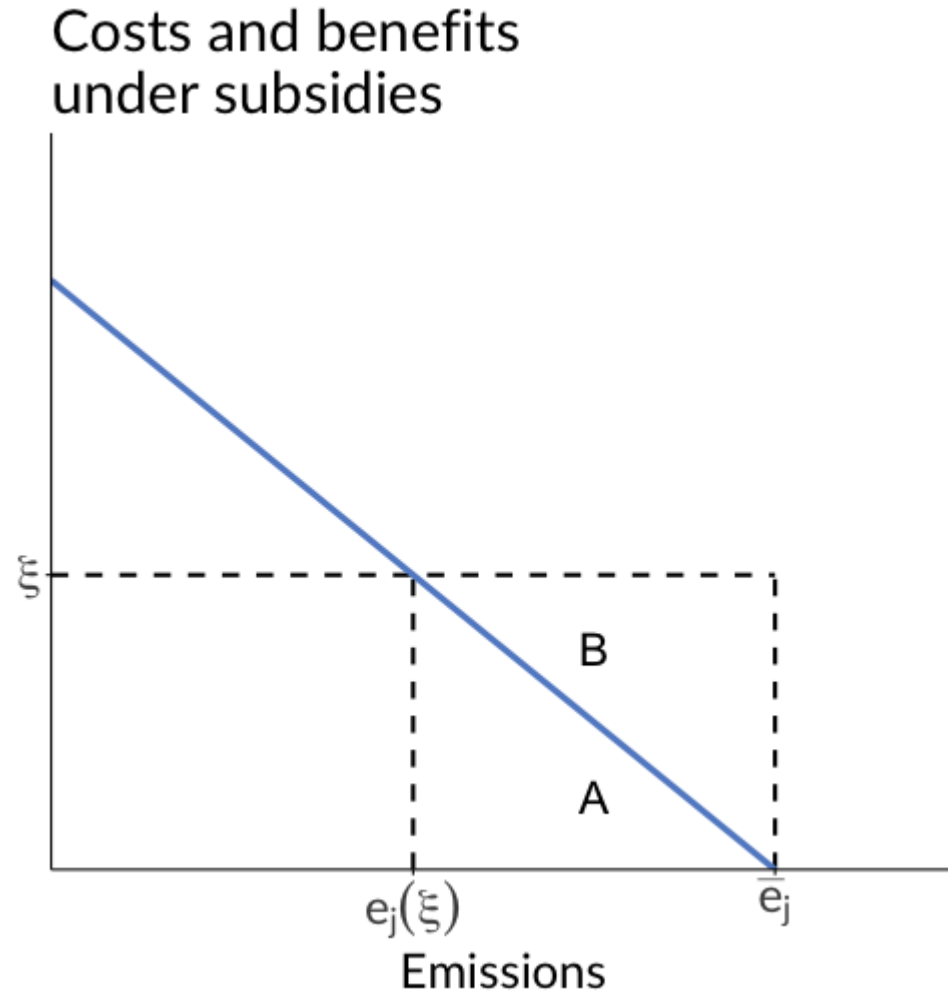


Under a subsidy  $\xi$ , the emission choice is a function of the subsidy:  $e_j(\xi)$

The firm incurs abatement cost  $A + B$  and receives total subsidy  $B + C$  with a baseline level of emissions of  $\hat{e}_j$

Total benefits to the firm are  $C - A$  which needs to be positive for the firm to abate

# Abatement subsidies



If we change the emission baseline to  $\bar{e}_j$  the incentives are identical!

Total costs change:

Abatement cost is now: A

Total subsidy is now: A + B



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Under subsidies, the regulator pays the firms

**The efficiency properties are the same**

# Aggregate marginal abatement cost

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It reflects the industry's marginal abatement cost when an efficient (i.e. least-cost) policy is implemented

Lets develop this formally

# Aggregate marginal abatement cost

Suppose firms pay a per-unit tax  $\tau$ , we know the firm's optimal emission decision is given by:

$$-C'_j(e_j) = \tau$$

with a resulting emission response function  $e_j(\tau) = C_j'^{-1}(-\tau)$  which we can interpret as the firm's **demand for emissions**



# Aggregate marginal abatement cost

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and the **aggregate MAC** is derived by inverting the aggregate demand:

$$AMAC = E^{-1}(\cdot)$$

This allows us to characterize socially optimal emissions in a more direct way

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And re-invert:  $MAC = 3 - \frac{1}{2}E$

What's the last step?



# Aggregate marginal abatement cost

Recall our two MACs are:  $MAC_1 = 4 - e$ ,  $MAC_2 = 2 - e$

Recognize that firm 2 can't abatement any more than 2 units, so any emission reductions for prices greater than 2 **must** come from firm 1

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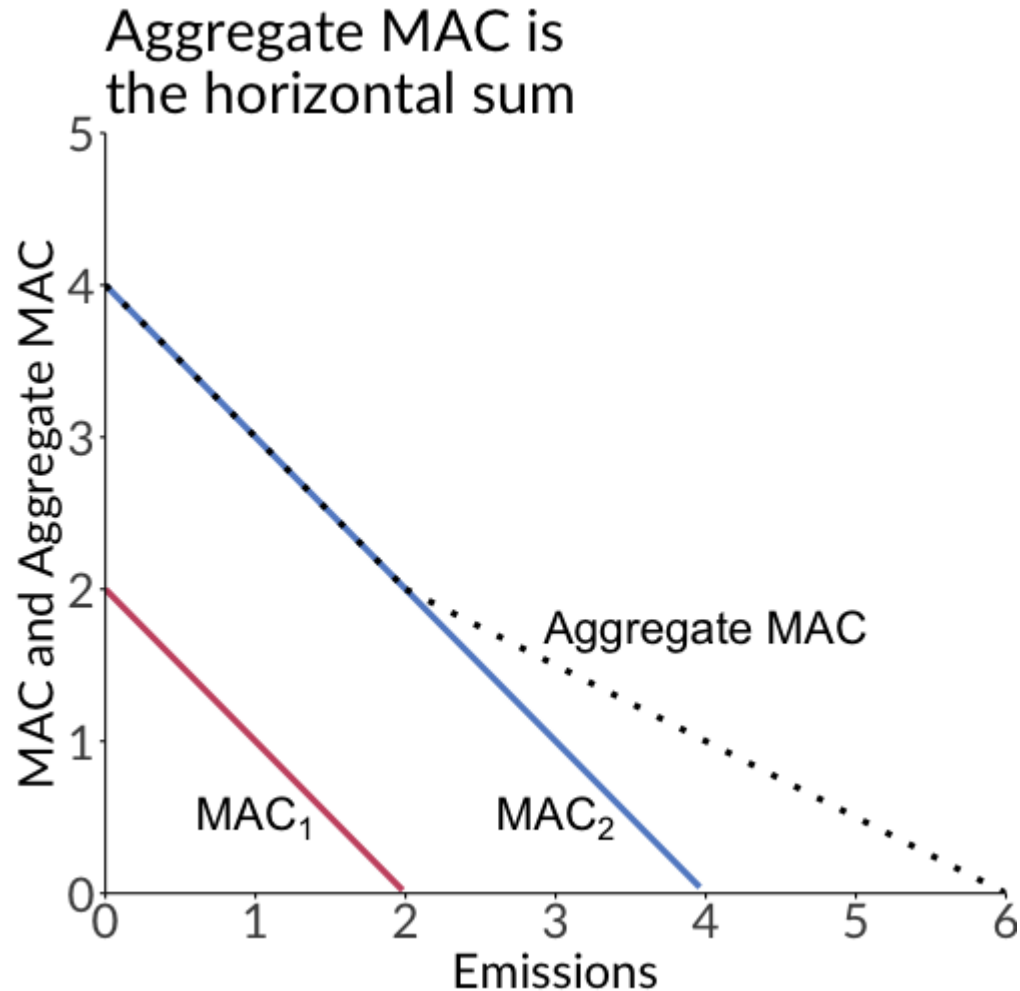
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So  $MAC = 3 - \frac{1}{2}E$  is only defined for  $P \leq 2$ ,  $E \geq 2$

This gives us that:

$$AMAC(E) = \begin{cases} 4 - E, & \text{for } 0 \leq E < 2 \\ 3 - \frac{1}{2}E & \text{for } E \geq 2 \end{cases}$$

# Aggregate MAC



The social objective is to minimize the sum of total abatement costs, so we care about where **aggregate MAC** crosses marginal damage

AMAC tells us: at a given price, what is the total quantity we can abatement?