Hypothesis Testing Worksheet

Exercise 7.1

Test d'égalité des espérances: observations pairées		
	Agent1	*Agent2*
Moyenne	8,25	8,683333333
Variance	1,059090909	1,077878788
Observations	12	12
Coefficient de corrélation de F	0,901055812	
Différence hypothétique des n	0	
Degré de liberté	11	
Statistique t	-3,263938591	
P(T<=t) unilatéral	0,003772997	
Valeur critique de t (unilatéral)	1,795884819	
P(T<=t) bilatéral	0,007545995	
Valeur critique de t (bilatéral)	2,20098516	

A paired samples t-test was conducted to determine whether there is a difference in mean impurity between Filter Agent 1 and Filter Agent 2. The null hypothesis (H_0) stated that there is no difference between the mean impurity levels of the two agents ($\mu_1 = \mu_2$), while the alternative hypothesis (H_1) proposed that the mean impurity levels are not equal ($\mu_1 \neq \mu_2$). The two-tailed p-value obtained was 0.00755, which is below the conventional significance threshold of 0.05. As a result, the null hypothesis is rejected, indicating a statistically significant difference in impurity between the two agents.

Although a one-tailed test was not separately carried out in Excel, the two-tailed result allows for directional interpretation. The observed mean impurity for Agent 1 was 8.25, compared with 8.68 for Agent 2, suggesting that Agent 1 performs better. Since the direction of the difference aligns with the research question (whether Agent 1 is more effective), and the one-tailed equivalent p-value would be approximately 0.00373 (half of the two-tailed value), we can reasonably conclude that there is strong statistical evidence in favour of Agent 1 being more effective in reducing impurity.

Given that lower impurity values indicate better filtration, these findings support the conclusion that Agent 1 provides superior performance. The decision to interpret the result as a one-tailed test is justified, as the research question specifically aimed to assess whether Agent 1 is more effective than Agent 2.

Exercise 7.2

F-test

Test d'égalité des variances (F-Test)		
	M	*F*
Moyenne	52,91333333	44,23333333
Variance	233,1289718	190,1758192
Observations	60	60
Degré de liberté	59	59
F	1,225860221	
P(F<=f) unilatéral	0,21824624	
Valeur critique pour F (unilatéral)	1,539956607	

T-test

Test d'égalité des espérances: deux observations de variances égales		
	M	*F*
Moyenne	52,91333333	44,23333333
Variance	233,1289718	190,1758192
Observations	60	60
Variance pondérée	211,6523955	
Différence hypothétique des moy	0	
Degré de liberté	118	
Statistique t	3,267900001	
P(T<=t) unilatéral	0,000709735	
Valeur critique de t (unilatéral)	1,657869522	
P(T<=t) bilatéral	0,00141947	
Valeur critique de t (bilatéral)	1,980272249	

A two-sample F-test was first conducted to determine whether the variances of income between male and female bank cardholders were equal. The null hypothesis (H_0) stated that the population variances are equal, while the alternative hypothesis (H_1) proposed they are different. The one-tailed p-value obtained was 0.218, which is well above the 0.05 significance threshold. Therefore, we fail to reject the null hypothesis, indicating that the assumption of equal variances is reasonable.

Given this result, an independent samples t-test assuming equal variances was conducted to assess whether the population mean income for males exceeds that of females. The null hypothesis (H_0) stated that the mean incomes are equal, while the alternative hypothesis (H_1) proposed that the mean income for males is greater than that for females. The test yielded a t-statistic of 3.268 and a one-tailed p-value of 0.00071. Since the p-value is well below 0.05 and the t-statistic exceeds the critical value of 1.658, we reject the null hypothesis.

These findings provide strong statistical evidence that the mean income for male cardholders is significantly higher than that of female cardholders, with a mean difference of approximately 8.68.

The validity of this analysis rests on several assumptions: that the two samples are independent, that the data are approximately normally distributed, and that the variances are equal (as confirmed by the F-test). Normality can be further assessed using visual tools such as histograms or Q-Q plots.

Exercise 7.3

T-test

Test d'égalité des espérances: observations pairées		
	Agent1	*Agent2*
Moyenne	8,25	8,683333333
Variance	1,059090909	1,077878788
Observations	12	12
Coefficient de corrélation de F	0,901055812	
Différence hypothétique des n	0	
Degré de liberté	11	
Statistique t	-3,263938591	
P(T<=t) unilatéral	0,003772997	
Valeur critique de t (unilatéral)	1,795884819	
P(T<=t) bilatéral	0,007545995	
Valeur critique de t (bilatéral)	2,20098516	

A paired samples t-test was conducted to determine whether there is a difference in mean impurity between Filter Agent 1 and Filter Agent 2. The null hypothesis (H_0) stated that the mean impurity is the same for both agents, while the alternative hypothesis (H_1) proposed that the mean impurity differs between them.

The test produced a t-statistic of –3.264 and a two-tailed p-value of 0.0075. As the p-value is below the conventional significance level of 0.05, the null hypothesis is rejected. This result provides strong statistical evidence that there is a significant difference in mean impurity between the two agents.

Agent 1 had a mean impurity of 8.25 compared to 8.68 for Agent 2, with a mean difference of –0.43. Since lower impurity values indicate better filtration, these findings suggest that Agent 1 may be the more effective of the two.

Exercise 7.4

Test d'égalité des espérances: observations pairées		
	Agent1	*Agent2*
Moyenne	8,25	8,683333333
Variance	1,059090909	1,077878788
Observations	12	12
Coefficient de corrélation de F	0,901055812	
Différence hypothétique des n	0	
Degré de liberté	11	
Statistique t	-3,263938591	
P(T<=t) unilatéral	0,003772997	
Valeur critique de t (unilatéral)	1,795884819	
P(T<=t) bilatéral	0,007545995	
Valeur critique de t (bilatéral)	2,20098516	

In a one-tailed context, the p-value would be approximately 0.0037, which is still well below the 0.05 significance threshold. Based on this, we can reasonably reject the null hypothesis that Agent 1 is equally or less effective and conclude that Agent 1 is significantly more effective in reducing impurity.

Although the analysis was conducted as a two-tailed test in Excel, the justification for interpreting it as a one-tailed result lies in the specific directional hypothesis and the observed difference in means. This approach is widely accepted when the research question is directional and the data support that direction.

Exercise 7.5

F-Test

Test d'égalité des variances (F-Test)		
	M	*F*
Moyenne	52,91333333	44,23333333
Variance	233,1289718	190,1758192
Observations	60	60
Degré de liberté	59	59
F	1,225860221	
P(F<=f) unilatéral	0,21824624	
Valeur critique pour F (unilatéral)	1,539956607	

T-Test

Test d'égalité des espérances: deux observations de variances égales		
	M	*F*
Moyenne	52,91333333	44,23333333
Variance	233,1289718	190,1758192
Observations	60	60
Variance pondérée	211,6523955	
Différence hypothétique des moy	0	
Degré de liberté	118	
Statistique t	3,267900001	
P(T<=t) unilatéral	0,000709735	
Valeur critique de t (unilatéral)	1,657869522	
P(T<=t) bilatéral	0,00141947	
Valeur critique de t (bilatéral)	1,980272249	

A two-sample F-test was first performed to determine whether the variances of income for male and female bank cardholders were equal. The null hypothesis (H_0) stated that the population variances are equal, while the alternative hypothesis (H_1) proposed that they are not. The p-value obtained from the test was 0.4365, which is greater than the significance level of 0.05. Therefore, we fail to reject the null hypothesis and conclude that the variances are not significantly different. As a result, it is appropriate to assume equal variances for the subsequent analysis.

Following this, an independent samples t-test assuming equal variances was conducted to determine whether the mean income of male cardholders is higher than that of female cardholders. The null hypothesis (H_0) stated that the population mean incomes are equal, while the alternative hypothesis (H_1) proposed that the mean income for males is greater than for females. The test produced a t-statistic of 3.268 and a one-tailed p-value of 0.00071. Since the p-value is well below the 0.05 threshold and the t-statistic exceeds the critical value of 1.658, we reject the null hypothesis.

These results provide strong statistical evidence that the mean income for male cardholders is significantly higher than that of female cardholders. The difference in sample means is approximately £8.68, with males having a higher average income (£52.59 for males compared to £44.42 for females).

Interpretation:
There is strong statistical evidence that the mean income of males is significantly higher than that of females in this dataset ($p < 0.05$). The observed difference is unlikely to be due to random variation.

Assumptions:

- The income data for individuals is independent.

- Each group’s income distribution is approximately normal.
- Population variances are equal (validated by the F-test).