**1.** (34.5-1) in CLRS book.

**Hint: use the Clique problem.**

*The* ***subgraph-isomorphism problem*** *takes two undirected graphs G1 and G2, and it asks whether G1 is isomorphic to a subgraph of G2. Show that the subgraph- isomorphism problem is NP-complete.*

*Answer::*

*To do this ,use a certificate is just the injection fromG1 into G2 so the G1 is isomorphic to its image*

*To see it is NPc, do a reduction to clique., to detect if a graph has a clique of size k, just let G1 be a complete graph on k vertices and let G2 be the original graph, if we could solve the subgraph isomorphism problem quickly we could solve the clique problem quickly.*

**2.** (34.5-8) in CLRS book.

In the ***half 3-CNF satisfiability*** problem, we are given a 3-CNF formula with n variables and m clauses, where m is even. We wish to determine whether there exists a truth assignment to the variables of such that exactly half the clauses evaluate to 0 and exactly half the clauses evaluate to 1. Prove that the half 3-CNF satisfiability problem is NP-complete.

Answer: A certificate would be an assignment to input variables which causes exactly half the clauses to evaluate to 1, and the other half to evaluate to 0. Since we can check this in polynomial time, half 3-CNF is in NP.

For NP hard, we’ll show that 3-C

**5.** (34-2) in CLRS book (Bonnie and Clyde).

Bonnie and Clyde have just robbed a bank. They have a bag of money and want to divide it up. For each of the following scenarios, either give a polynomial-time algorithm, or prove that the problem is NP-complete. The input in each case is a list of the n items in the bag, along with the value of each.

1. ***The bag contains n coins, but only 2 different denominations: some coins are worth x dollars, and some are worth y dollars. Bonnie and Clyde wish to divide the money exactly evenly.***
2. ***The bag contains n coins, with an arbitrary number of different denominations, but each denomination is a nonnegative integer power of 2, i.e., the possible denominations are 1 dollar, 2 dollars, 4 dollars, etc. Bonnie and Clyde wish to divide the money exactly evenly.***
3. ***The bag contains n checks, which are, in an amazing coincidence, made out to “Bonnie or Clyde.” They wish to divide the checks so that they each get the exact same amount of money.***
4. ***The bag contains n checks as in part (c), but this time Bonnie and Clyde are willing to accept a split in which the difference is no larger than 100 dollars.***

**6.** (34-1) in CLRS book.

1. ***34-1 Independent set***An ***independent set*** of a graph G D .V; E/ is a subset V 0 V of vertices such that each edge in E is incident on at most one vertex in V 0. The ***independent-set problem*** is to find a maximum-size independent set in G.
2. ***Formulate a related decision problem for the independent-set problem, and prove that it is NP-complete. (Hint: Reduce from the clique problem.)***
3. ***Suppose that you are given a “black-box” subroutine to solve the decision prob- lem you defined in part (a). Give an algorithm to find an independent set of max- imum size. The running time of your algorithm should be polynomial in jV j and jEj, counting queries to the black box as a single step.***
4. Although the independent-set decision problem is NP-complete, certain special cases are polynomial-time solvable.
5. ***c.*** Give an efficient algorithm to solve the independent-set problem when each ver- tex in G has degree 2. Analyze the running time, and prove that your algorithm works correctly.
6. Give an efficient algorithm to solve the independent-set problem when G is bipartite. Analyze the running time, and prove that your algorithm works cor- rectly. (*Hint:* Use the results of Section 26.3.)

**7.** Read the Graph Coloring problem on Wikipedia (<https://en.wikipedia.org/wiki/Graph_coloring)>. You don’t need to read all the content, just understand when it will be a NPC problem and why (Hint: the difficulty of the problem depends on the number of colors *k*).

**3.** (35.2-3) in CLRS book.

***35.2-3***Consider the following ***closest-point heuristic*** for building an approximate trav- eling-salesman tour whose cost function satisfies the triangle inequality. Begin with a trivial cycle consisting of a single arbitrarily chosen vertex. At each step, identify the vertex u that is not on the cycle but whose distance to any vertex on the cycle is minimum. Suppose that the vertex on the cycle that is nearest u is vertex . Extend the cycle to include u by inserting u just after . Repeat until all vertices are on the cycle. Prove that this heuristic returns a tour whose total cost is not more than twice the cost of an optimal tour.

**4.** (35.3-1) in CLRS book.

***35.3-1***

Consider each of the following words as a set of letters: farid;dash;drain; heard; lost; nose; shun; slate; snare; threadg. Show which set cover GREEDY-SET-COVER produces when we break ties in favor of the word that ap- pears first in the dictionary.