

# Quantum Error Correction

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*"To be an Error and to be Cast out is part of God's Design."*

William Blake

- Noise as a longstanding problem in information processing systems
  - e.g., classical computers, modems, CD players, etc.
  - Noise is still a problem in quantum information
- Key idea: to protect a message against noise, *encode* the message by adding redundant information; even if some information is corrupted, redundancy allows us to *decode* and recover the original message

# Project Framework

- Goals:
  - to implement various quantum error-correcting codes
    - we chose the 3-qubit, 9-qubit, 7-qubit codes
  - to analyze and compare their performances
    - *when are they effective?*
    - *when should we use error-correcting codes?*
- Tools:
  - Python's Qiskit package
  - IBM's quantum machines

# 3-Qubit Codes: Classical Inspiration

## Classical Error Correction

- Encoding by *repetition codes*:

$$0 \rightarrow 000$$

$$1 \rightarrow 111.$$

- Decoding by *majority voting*:

$$\text{Ex.: } 001 \rightarrow 0.$$

- Analysis: Let  $p$  be the probability that a bit is flipped. This method fails when 2 or more bits are flipped, which occurs with probability  $3p^2(1-p) + p^3$ , so the probability of error is  $p_e = 3p^2 - 2p^3$ . Then this method is preferred when  $p_e < p$ , or  $p < 1/2$ .

# Noisy Channels: The Bit Flip Channel

- One model for noise is the *bit flip channel* (analogous to classical channel).
- The bit flip channel flips qubits with probability  $p$  and leaves them untouched with probability  $1 - p$ .
- Equivalent to applying  $X$  gate with probability  $p$ .
- We protect qubits from this channel with the *bit flip code*.

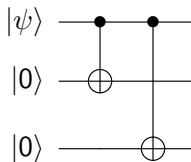
# 3-Qubit Bit Flip Code: Encoding Logical Bits

- The goal is to correct bit flip errors.
- Encoding:

$$|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$$

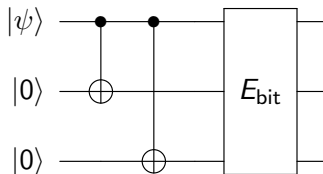
$$|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle.$$

- Encoding circuit for 3-qubit bit flip code:



# 3-Qubit Bit Flip Code: Detecting Errors

- Suppose there is a bit flip error after encoding:

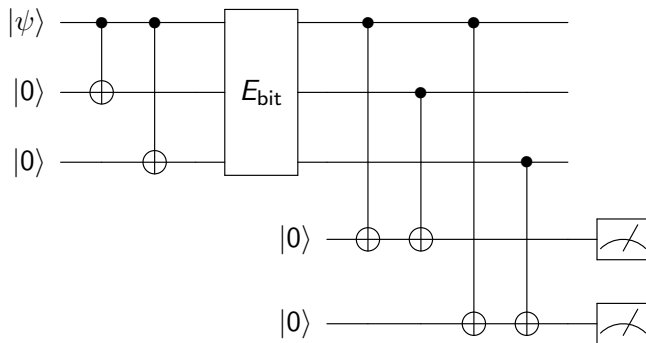


- Error Detection (or *syndrome diagnosis*):
  - we would like to determine which, if any, of the qubits have been corrupted
  - four error syndromes: no error, bit flip on qubit one, bit flip on qubit two, bit flip on qubit three



# 3-Qubit Bit Flip Code: Detecting Errors

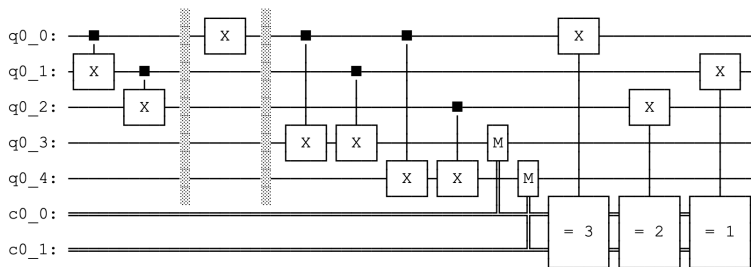
- We can diagnose the syndrome using two ancillary qubits:



- Based on measurement results, we know where the error occurred.

# 3-Qubit Bit Flip Code: Correcting Errors

- Complete circuit for error correction (or *recovery*):



# Analyzing the Bit Flip Code: Simulation

- Let's look at the performance of the 3-qubit bit flip code against bit flip channels of varying error probabilities  $p$ .

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  - 3 run error correcting code
  - 4 measure final state

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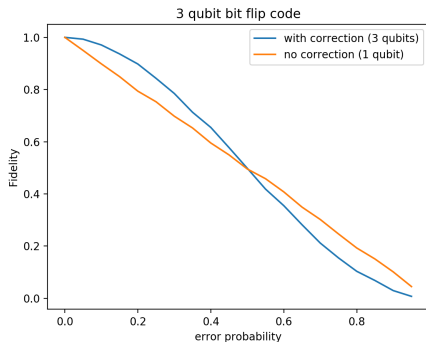
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- We can calculate the accuracy of the error correcting code for a given  $p$  by repeating many times and taking the number of times we measure a correct final state  $|000\rangle$  and dividing by the total number of trials.

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- We can calculate the accuracy of the error correcting code for a given  $p$  by repeating many times and taking the number of times we measure a correct final state  $|000\rangle$  and dividing by the total number of trials.
- We can compare this to the accuracy of a single qubit (without encoding or error correction) that goes through a bit flip channel with the same  $p$  to see when error correction is effective.

# Analyzing the Bit Flip Code: Simulation

- Ran tests on Qiskit's simulator
- Probability  $p$  ranging from 0 to 1; 10000 trials for each  $p$



- Observe crossover point at  $p = 0.5$ .
- For  $p < 0.5$ , error correcting code performs better than a single qubit with no correction.

# Noisy Channels: Phase Flip Channel

- Another quantum channel is the *phase flip* error model.
- With probability  $p$  the relative phase of states  $|0\rangle$  and  $|1\rangle$  is flipped, with probability  $1 - p$  it is left alone.
- Equivalent to applying  $Z$  operator with probability  $p$ .
- We fight this channel with the *phase flip code*.



# 3-Qubit Phase Flip Code

- No classical analog, but it is easy to turn the phase flip channel into a bit flip channel.
- Use  $x$ -basis for encoding:

$$\begin{aligned}|0\rangle &\rightarrow |0_L\rangle \equiv |+++ \rangle \\ |1\rangle &\rightarrow |1_L\rangle \equiv |-- - \rangle .\end{aligned}$$

- Phase flip  $Z$  acts as bit flip for this encoding!

# The Shor Code

# 7-Qubit Code

Encodes 1 logical qubit using 7 physical qubits:

$$|\bar{0}\rangle = \frac{|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle}{\sqrt{8}}$$

$$|\bar{1}\rangle = \frac{|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle}{\sqrt{8}}$$

$$H^{\otimes 7} |\bar{0}\rangle = \frac{|\bar{0}\rangle + |\bar{1}\rangle}{\sqrt{2}}$$

$$H^{\otimes 7} |\bar{1}\rangle = \frac{|\bar{0}\rangle - |\bar{1}\rangle}{\sqrt{2}}$$

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- Of the 16 bit strings above, any two differ by  $\geq 3$  bits
- Intuition: therefore a single bit flip can be recovered
  - $X$  error flips bit in  $|\bar{0}\rangle, |\bar{1}\rangle$
  - $Z$  error flips bit in  $H^{\otimes 7} |\bar{0}\rangle, H^{\otimes 7} |\bar{1}\rangle$

# Example recovery for $X$ error in qubit 3

TODO