Quantum Error Correction

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Introduction

"To be an Error and to be Cast out is part of God's Design."

William Blake

- Noise as a longstanding problem in information processing systems
  - *e.g.*, classical computers, modems, CD players, etc.
  - Noise is still a problem in quantum information
- Key idea: to protect a message against noise, *encode* the message by adding redundant information; even if some information is corrupted, redundancy allows us to *decode* and recover the original message

Project Framework

- Goals:
  - to implement various quantum error-correcting codes
    - we chose the 3-qubit, 9-qubit, 7-qubit codes
  - to analyze and compare their performances
    - *when are they effective?*
    - *when should we use error-correcting codes?*
- Tools:
  - Python's Qiskit package
  - IBM's quantum machines

3-Qubit Codes: A Review

Classical Inspiration

- Encoding by *repetition codes*:

$$0 \to 000$$
$$1 \to 111.$$

- Decoding by *majority voting*:

$$\textit{Ex.:}\ \ 001 \to 0.$$

- Analysis: Let p be the probability that a bit is flipped. This method fails when 2 or more bits are flipped, which occurs with probability $3p^2(1-p)+p^3$, so the probability of error is $p_e = 3p^2 - 2p^3$. Then this method is preferred when $p_e < p$, or $p < 1/2$.

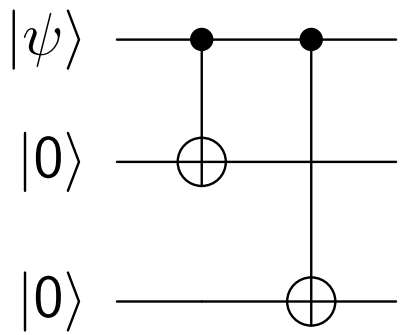
3-Qubit Codes: A Review

The Quantum Version: 3-Qubit Bit Flip Code

- The goal is to correct bit flip errors.
- Encoding:

$$|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$$
$$|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle\,.$$

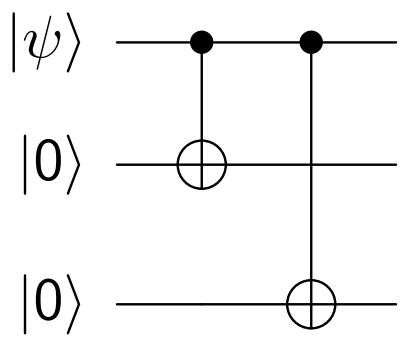
- Encoding circuit for 3-qubit bit flip code:



3-Qubit Codes: A Review

The Quantum Version: 3-Qubit Bit Flip Code

- Suppose there is a bit flip error after encoding:
- Error Detection (or *syndrome diagnosis*):
  - we would like to determine which, if any, of the qubits have been corrupted
  - we will need 2 ancillary qubits:
- Encoding circuit for 3-qubit bit flip code:



The Shor Code

7-Qubit Code