

1. Show that if the p-values of the m_0 null hypotheses are independent then $\frac{V(t)}{t} = \frac{\sum_{i=1}^{m_0} 1(p_i \leq t)}{t}$ for $0 \leq t \leq 1$ is a martingale with time running backward with respect to the filtration $\mathcal{F}_t = \sigma(1\{p_i \leq s\}, t \leq s \leq 1, i = 1, \dots, m)$, in other words for $s \leq t$ we have $E[V(s)/s | \mathcal{F}_t] = V(t)/t$.
2. Show that the random variable $T_\alpha(\widehat{FDR}(t))$ is a stopping time with respect to $\mathcal{F}_t = \sigma(1\{p_i \leq s\}, t \leq s \leq 1, i = 1, \dots, m)$.
3. Show that the martingale $\frac{V(t)}{t}$ stopped at $T_\alpha(\widehat{FDR}(t))$ is bounded by $\frac{m}{\alpha}$.