Let r_{sj} be the residual for the jth observation on the sth subject. Make a list of all within-subject normalized cross-products $r_{sj}r_{sj'}/\sigma^2$ whether or not $j = j'$, and a corresponding list of absolute time lags $|t_{sj} - t_{sj'}|$. Let y_i be the ith cross-product on the list, and let x_i be the ith time lag. Our generalized AR-1 model is

$$E(y_i) = \rho^{x_i} + \epsilon_i$$

where ϵ_i is some error with mean zero and finite variance and $0 < \rho < 1$. We rewrite this as

$$E(y_i) = \exp(-\theta x_i)$$

where $\theta = -\log \rho$ and $0 < \theta < \infty$.

So we will try to minimize the loss function

$$Q(\theta) = \frac{1}{2} \sum_{i=1}^{n} (y_i - f(y_i))^2$$

where $f(y_i) = \exp(-\theta x_i)$. Then

$$\frac{dQ}{d\theta} = -\sum_{i=1}^{n} (y_i - f(y_i)) \frac{df(y_i)}{d\theta}$$

We will use Fisher scoring: first argue that

$$E\left(\frac{d^2Q}{d\theta^2}\right) = \sum_{i=1}^{n} \left(\frac{df(y_i)}{d\theta}\right)^2$$

Then to find the place where $\frac{dQ}{d\theta} = 0$, iterate

$$\hat{\theta}_{new} = \hat{\theta}_{old} - \left(E\left(\frac{d^2Q}{d\theta^2}\right)\right)^{-1} \frac{dQ}{d\theta}$$

Here

$$\frac{df(y_i)}{d\theta} = -x_i \exp(-\theta x_i)$$

so

$$\frac{dQ}{d\theta} = \sum_{i=1}^{n} (y_i - \exp(-\theta x_i))(\exp(-\theta x_i))\, x_i$$