

Instructions for the Exercise

Today we work with the system of linear equations:

$$\begin{cases} 2x + y = 5, \\ x - y = 1, \\ x + y = 2. \end{cases}$$

Rules of work:

1. Each of you conducts your own conversation with ChatGPT (AI).
2. Save the conversation in a text file.
3. What is graded is primarily the *quality and depth of your questions*, not the final “answer”.

What to pay attention to during the dialogue:

- Does the system have a solution? What does it mean for a system to be *inconsistent*?
- How can we define a “solution” in such a case?
- What does “best approximation” mean, both geometrically and computationally?
- Which tools of linear algebra can help here (e.g., normal equations, matrix decompositions)?

Criteria for evaluation:

1. Precision and inquisitiveness of your questions.
2. Attempts to understand *why*, not only *how to compute*.
3. Ability to connect the problem with a broader context (geometry, approximation methods, stability).

Goal: to demonstrate how you can explore a mathematical problem by asking meaningful questions.

Class Summary

We started from a seemingly simple task: “find the solution of a system of equations.” Adding a third equation made the system inconsistent and left us with no classical solution.

This opens important questions:

- What does it mean to “solve” a problem when exact satisfaction of all equations is impossible?
- How should we understand the idea of the “best approximation” — geometrically and computationally?
- Which tools of linear algebra allow us to formulate and find such solutions?

Today’s conversations with AI were meant to show that the key skill is the *ability to ask questions*. Inquisitive questions lead to new concepts and methods, and thus to deeper understanding.

“Mathematics begins where simple methods stop working.”