Modeling Disease Spread - SIR model

Dzineon Gyaltsen, Emily Chen

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Outline

- Motivation
- SIR Model Overview
- Introduction to Dataset and Processing
- Our Approach to Modeling
- Results
- Conclusions / Future Work

Research Questions / Motivation

- How accurately can we model Covid-19 spread with a simple SIR model? What error is accrued with this model over a certain time frame?
- What system-level factors are important to consider? How do we incorporate these into an expanded version of the SIR model?

- ❑ Utilize the techniques learned in class to solve ODEs, such as curve-fitting and optimization
- ❑ Relevant example – plenty of Covid-19 data

SIR MODEL overview

Susceptible S(t), Infectious I(t), and Recovered R(t)

- **Popular model in epidemiology**
- **System of ODEs**
- **Assumptions:**
    - No change in total population, **N**
    - Homogeneous mixing of the infected and susceptible populations
    - Once recovered, can not be susceptible

System of ODEs

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t).$$

S(t) = # of susceptible individuals
I(t) = # of infectious individuals
R(t) = # of recovered/ removed individuals

Parameters:

β = # of contacts by infected individuals per day to spread the disease

γ = fraction of infected individuals recoverings

R0 = reproduction ratio = # of new infection from one infection

N = Total population

S(t) + I(t) + R(t) = N

Dataset

COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University

- Aggregated data from a variety of sources, including World Health Organization (WHO), Center for Disease Control (CDC), and State-level Health Departments (also global)
- Data used to monitor Covid-19 related cases, vaccinations, deaths, and hospitalizations

Data Pre-processing

- Data extracted (Pennsylvania): Confirmed Deaths Recovered
  - Confirmed → $I(t)$
  - Deaths + Recovered → $R(t)$
  - $S(t) = N - I(t) - R(t)$

* stopped recording recovered data after March 2021

- Parameters were estimated using the first 156 days

Our Approach

- Use Scipy's odeint() solver to solve a system of Ordinary Differential Equations

- Added an mse() function to find the mean squared error of SIR model estimation to actual data

```
# SIR model differential equations
def deriv(x, t, beta, gamma):
    s, i, r = x
    dsdt = -beta * s * i
    didt = beta * s * i - gamma * i
    drdt = gamma * i
    return [dsdt, didt, drdt]

# Function to calculate S, I, R using the SIR model equations
def sir_solution(t, beta, gamma, S0, I0, R0):
    x_initial = S0, I0, R0
    soln = odeint(deriv, x_initial, t, args=(beta, gamma))
    s, i, r = soln.T
    return s, i, r

# Given initial guess of beta/gamma params, find mean squared error of
# sir model estimation to actual sir data
def mse(params):
    beta, gamma = params
    S0, I0, R0 = S[0], I[0], R[0]
    S_model, I_model, R_model = sir_solution(t, beta, gamma, S0, I0, R0)
    return np.mean((S_model - S)**2 + (I_model - I)**2 + (R_model - R)**2)
```

Our Approach

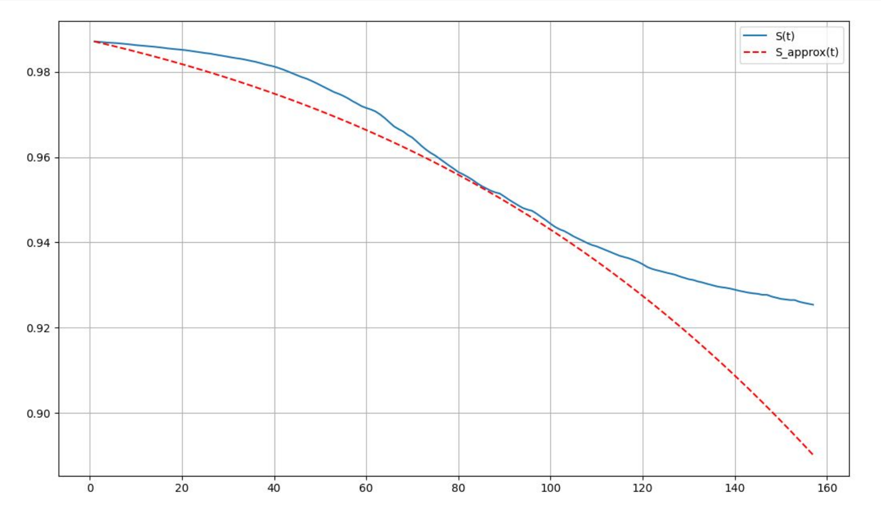
- Using our functions from previous slide, we passed in an initial guess for our parameters

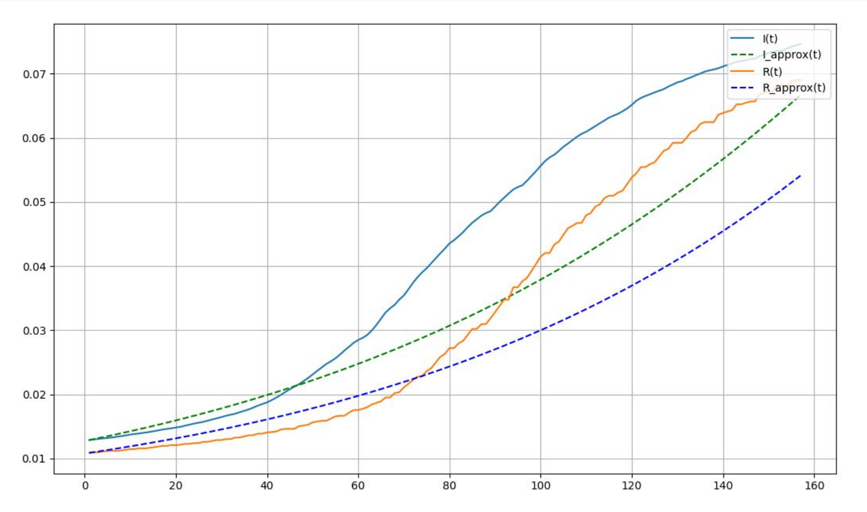
- Found the optimized parameter values that minimized the mean square error

- Utilized the Powell Method from Lecture

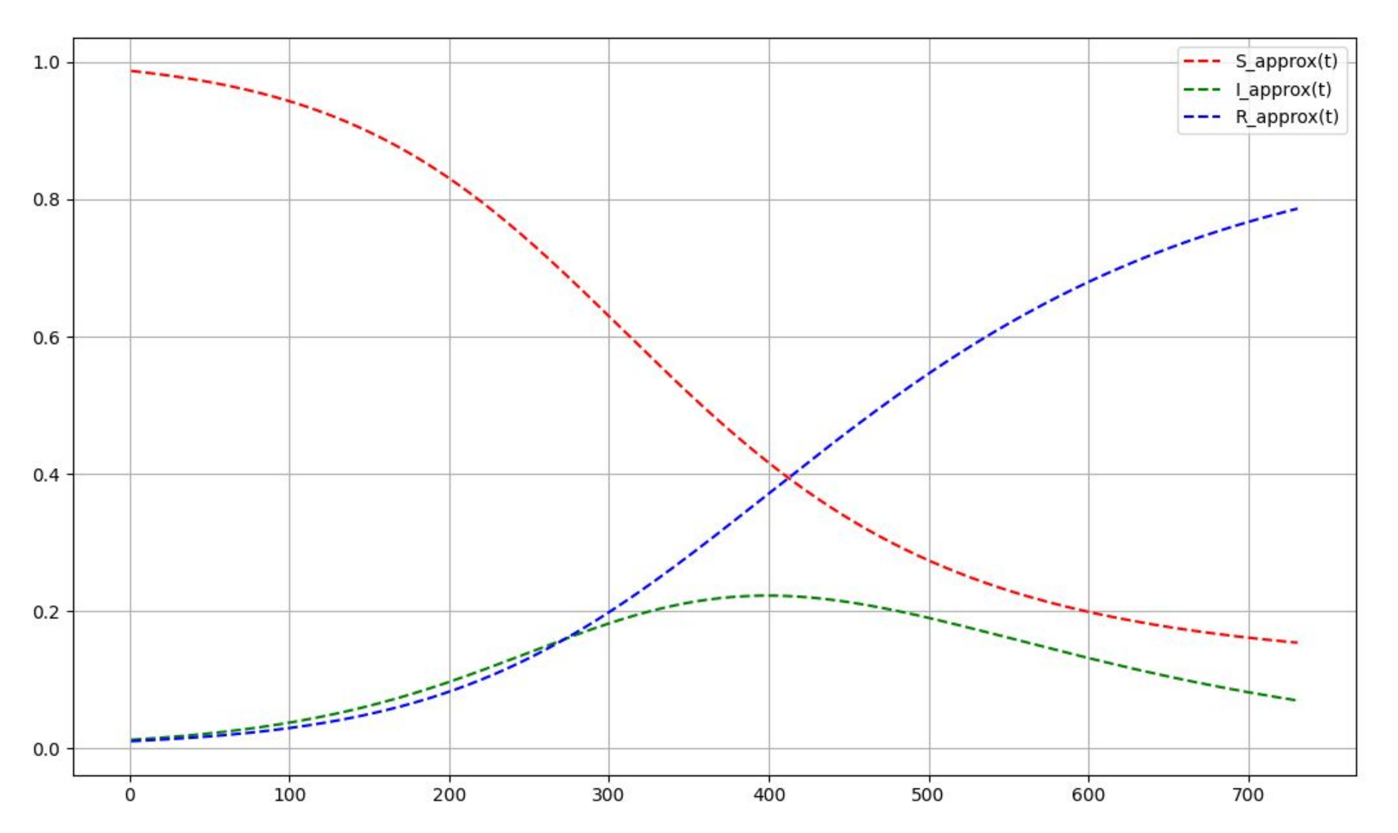
```python
# Initial guess for parameters
beta_guess = 0.2
gamma_guess = 0.1
sigma_guess = 0

param_guess = np.array([beta_guess, gamma_guess, sigma_guess])

result = minimize(mse, param_guess, method='Powell')

beta_optimized = result.x[0]
# print(beta_optimized)
gamma_optimized = result.x[1]
# print(gamma_optimized)
sigma_optimized = result.x[2]
# print(sigma_optimized)

x_initial = S[0], E[0], I[0], R[0]
soln = odeint(deriv, x_initial, t, args=(
    beta_optimized, gamma_optimized, sigma_optimized))
s, e, i, r = soln.T
```

SIR Model Results





Actual Data vs SIR Model (156 days)
Left: Susceptible
Right: Infected and Recovered



SIR Model across **730 days (2 years)** using estimated parameters

SEIR MODEL overview

Susceptible S(t), Exposed E(t), Infectious I(t), and Recovered R(t)

- **Exposed = infected - infectious**
    - **** in this project, exposed = infected - death**
- **Assumptions:**
    - No change in total population, **N**
    - Homogeneous mixing of the infected and susceptible populations
    - Once recovered, can not be susceptible

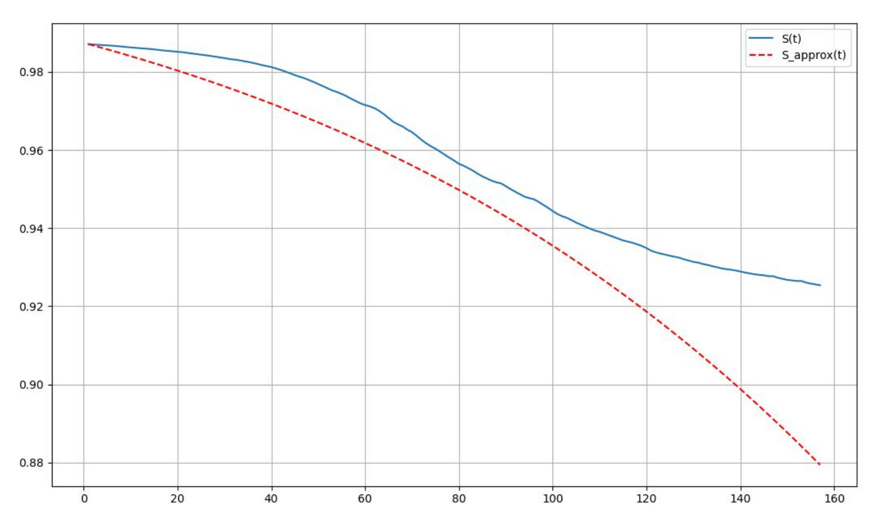
Our Approach

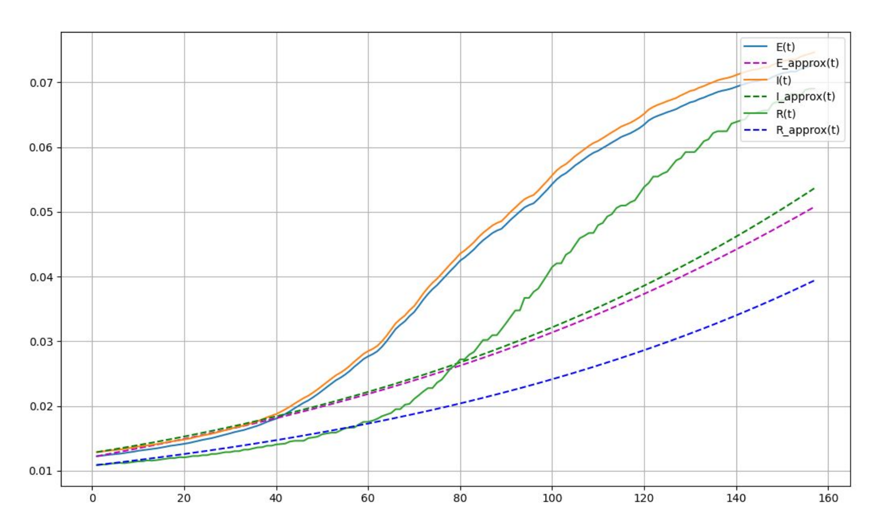
$$
\begin{aligned}
\dot{S} &= \Lambda - \mu S - \beta S \frac{I}{N}, \\
\dot{E} &= \beta S \frac{I}{N} - (\mu + \epsilon) E, \\
\dot{I} &= \epsilon E - (\gamma + \mu + \alpha) I, \\
\dot{R} &= \gamma I - \mu R,
\end{aligned}
\tag{1}
$$

```python
# SIR model differential equations
def deriv(x, t, beta, gamma, sigma):
    s, e, i, r = x
    dsdt = -beta * s * i
    dedt = beta * s * i - sigma * e
    didt = sigma * e - gamma * i
    drdt = gamma * i
    return [dsdt, dedt, didt, drdt]
```

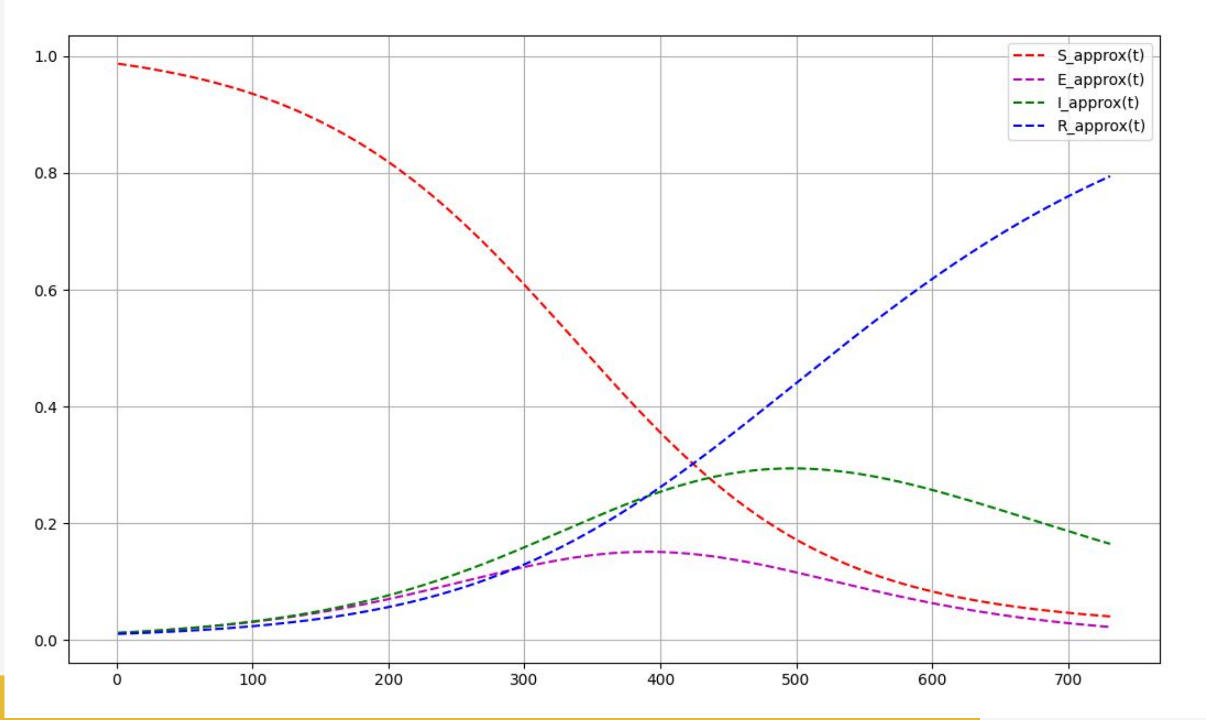
Λ: Per-capita birth rate.
μ: Per-capita natural death rate.
α: Virus-induced average fatality rate.
β: Probability of disease transmission per contact (dimensionless) times the number of contacts per unit time.
ϵ: Rate of progression from exposed to infectious (the reciprocal is the incubation period).
γ: Recovery rate of infectious individuals (the reciprocal is the infectious period).

SEIR MODEL Results





SEIR MODEL Results



Further steps

- Explore other optimization methods
- Model effects of Quarantine and Vaccination
- Using more data to fit parameters
- Change population to take account of deaths and births

Thanks for listening!