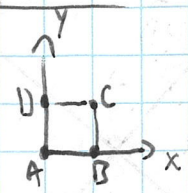
Problem 2

$${}^{A}T_{B} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{B}T_{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}T_{C} = {}^{A}T_{B} \times {}^{B}T_{C}$$

$${}^{A}T_{C} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3 $\quad T = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$



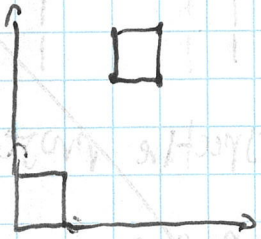
$A_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad B_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad C_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad D_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

Translation:

d, h, l responsible for translation in X, Y, Z.

Translate 2 in x, 3 in y: $H =$

$$H = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad HA = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} \quad HB = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 1 \end{bmatrix} \quad HC = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 1 \end{bmatrix} \quad HD = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

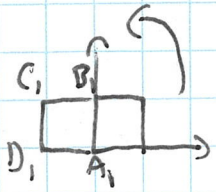


Rotation: a b c, e f g, i j k responsible for rotation

$$rot_{z,\theta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rotate 90° around z: $H = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

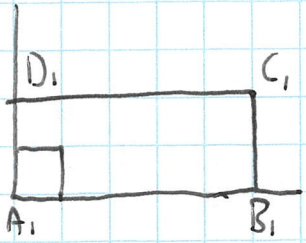
$$HA = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad HB = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad HC = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad HD = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



Scale: diagonal a, f, k responsible for scale in X, Y, Z

Scale 5 in X, 2 in Y

$$M = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad HA = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad HB = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad HC = \begin{bmatrix} 5 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad HD = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$



Perspective: m, n, o responsible for projection from X, Y, Z point

Perspective transform from point (1, 0, 0):

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad HA = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad HB = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} \quad HC = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix} \quad HD = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Divide so last term is 1

$$A_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad B_1 = \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad C_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \\ 1 \end{bmatrix} \quad D_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$