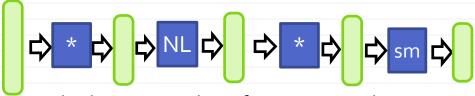
## **Lecture 3: Convolutional Networks**

#### Neural network: parameter overdoze

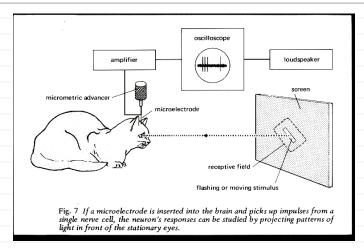


Multiplicative number of parameters: the main problem. Solving the problem:

- Stop optimization early (always keep checking progress on validation set)
- Impose smoothness (weight decay)
- Bag multiple models

**Main avenue**: picking a less generic architecture with less parameters

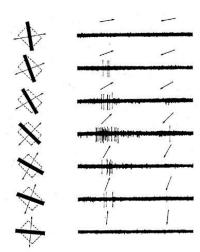
#### **Huber-Wiesel**



The reaction of a neuron is localized to a part of the visual field

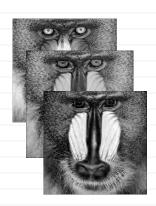
## **Huber and Wiesel 1968**

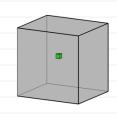
V1 physiology: direction selectivity



#### Idea 1: neuron maps

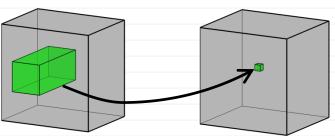






- 1. Organize neurons into map stacks
- 2. E.g. an image is a WxHx3 map stack

## Idea 1: locally-connected layer

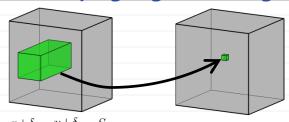


- Organize neurons into maps
- 2. Limit the *receptive field* in the multiplicative layer:

$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K^{x,y,t} (i-x+\delta, j-y+\delta, s) \cdot U(i,j,s)$$

 Huge parameter reduction by a factor of O( (W/2δ)²) compared to fully-connected layers

#### Idea 2: tying together weights

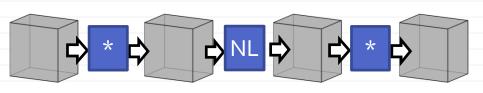


$$V(x, y, t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{s} K^{t}(i - x + \delta, j - y + \delta, s) \cdot U(i, j, s)$$

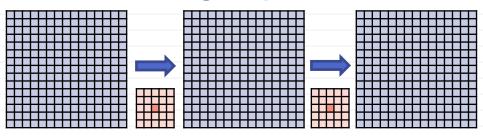
Further dramatic reduction in the number of parameters by a factor O(W<sup>2</sup>) compared to locally-connected layer:

$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K(i-x+\delta,j-y+\delta,s,t) \cdot U(i,j,s)$$

## **Stacking layers**

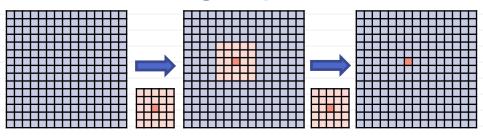


 The layers are stacked and interleaved with non-linearities (e.g. ReLU)



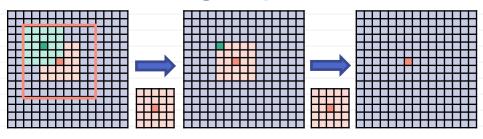
## E.g. with 5x5 filters:

Receptive field is 5x5 after 1 conv layers



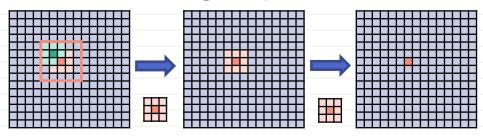
### E.g. with 5x5 filters:

Receptive field is 5x5 after 1 conv layers



## E.g. with 5x5 filters:

- Receptive field is 5x5 after 1 conv layers
- Receptive field is 9x9 after 2 conv layers

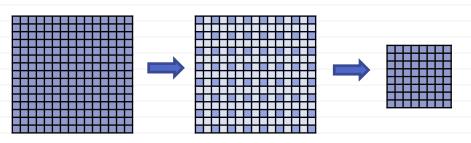


## E.g. with 3x3 filters:

- Receptive field is 3x3 after 1 conv layers
- Receptive field is 5x5 after 2 conv layers

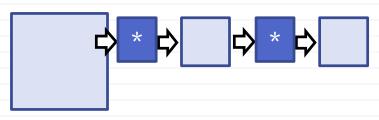
#### Convolution with strides

Having strides s >1 is equivalent to computing every (s-1)th pixel only. For s=2:



$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K(i-x+\delta, j-y+\delta, s, t) \cdot U(i,j,s)$$

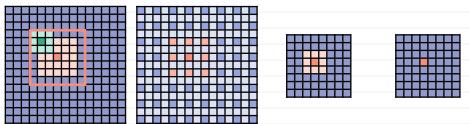
## Accelerating growth: stride



- In the early layers strides are often > 1
- Strides > 1 downsample maps
- Strides > 1 increase the receptive fields

What is the receptive field after [5x5 stride=2] followed by [5x5 stride = 1] convolution?

## Strides lead to growing receptive field

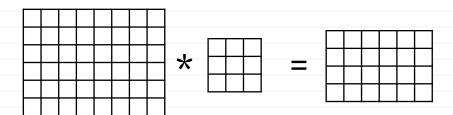


## E.g. with 3x3 filters:

- Receptive field is 3x3 after 1 conv layers
- Receptive field is 5x5 after 2 conv layers

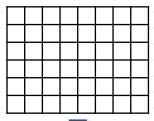
## **Boundary issues**

"Valid" mode:



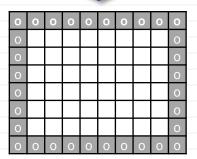
- Complicates implementation and reasoning
- Unequal contribution of elements

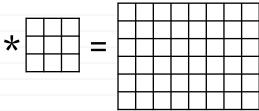
## Boundary issues: padding with zeros



"Same" mode:

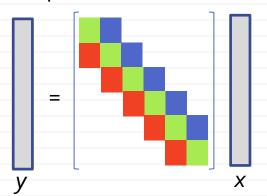
- Solves the problem
- Introduces "false" edges



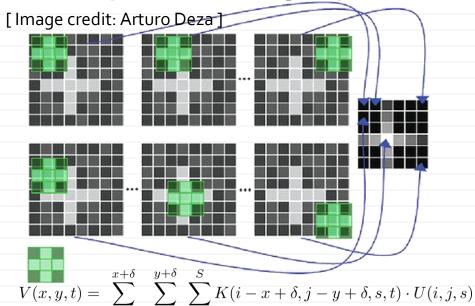


## Conv. layer is still multiplicative

E.g. 1D correlation with is a multiplication over a banded matrix:



## Interpretation: looking for patterns

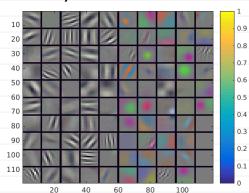


 $i=x-\delta j=y-\delta s=1$ 

## Interpretation: looking for patterns



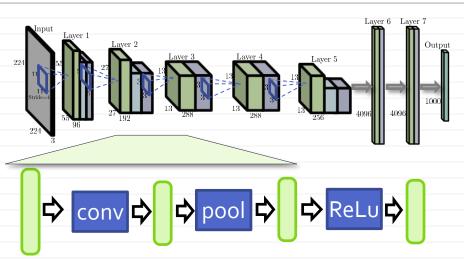
# AlexNet filters of the first layer:



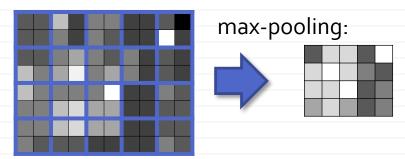
Responses in the first layer



#### What are modern ConvNets made of



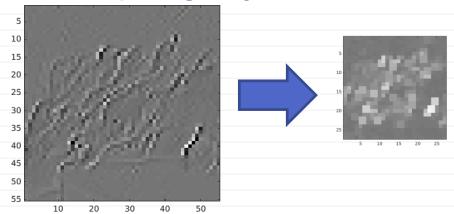
## Third-component: pooling (+subsampling)



Pooling is almost always with subsampling

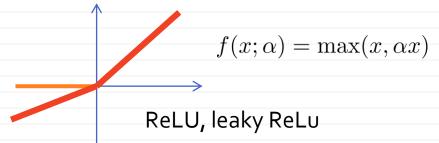
Alternatives: sum-pooling, average pooling, Rapid decrease of map size Parameter-free

## Max-pooling and jitter-invariance



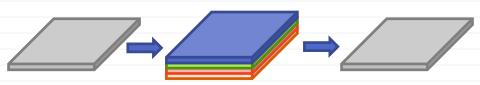
- Usual motivation: adding invariance to small shifts
- Several max-poolings can accumulate invariance to stronger shifts

## **Details:** nonlinearity



# Another formerly popular non-linearity: maxout

$$f(x) = \max(\alpha_1 x + \beta_1, \alpha_2 x + \beta_2, \dots, \alpha_m x + \beta_m)$$



## **CNN** applications

# Pattern finding through convolution/correlation is ubiquitous:

- 2D images (and the like, e.g. speech)
- 1D signals (e.g. time series, speech)
- 3D images
- Videos
- Graphs (more generalized sense)

[Bruna et al. Spectral Networks and Locally Connected Networks on Graphs. ICLR 2014]

## Reminder: layer abstraction

#### Each layer is defined by:

- forward performance: y = f(x; w)
- backward performance:

$$z(x) = z(f(x; w))$$

$$\frac{dz}{dx} = \frac{dy}{dx}^{T} \cdot \frac{dz}{dy} \qquad \frac{dz}{dw} = \frac{dy}{dw}^{T} \cdot \frac{dz}{dy}$$

# Backprop equations: multipicative layer

$$\frac{z(x) = z(f(x; w))}{\frac{dz}{dx} = \frac{dy}{dx}^{T} \cdot \frac{dz}{dy} \qquad \frac{dz}{dw} = \frac{dy}{dw}^{T} \cdot \frac{dz}{dy}$$

$$y = Wx$$
$$\frac{dy}{dx} = W$$

$$\frac{dz}{dx} = W^T \frac{dz}{dy}$$

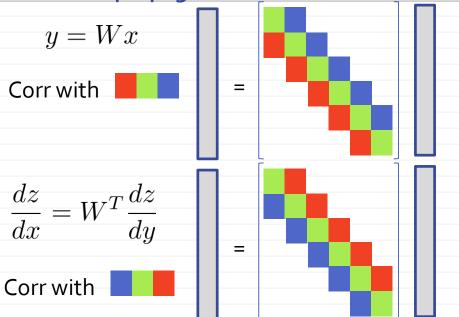
## Backprop equations: convolutional layer

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy}$$

 $y=Wx\,$  - still holds for conv layer

$$\frac{dz}{dx} = W^T \frac{dz}{dy} \quad \text{- also corresponds to some} \\ \text{correlation?}$$

Backpropagation via convolution



## Backprop equations: convolutional layer

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy}$$

 $y=Wx\,$  - still holds for conv layer

$$\frac{dz}{dx} = W^T \frac{dz}{dy} \quad \text{- also corresponds to conv}$$

During backpass do the same correlations but with flipped kernels

$$K^{\text{back}}(i, j, t, s) = K(2\delta + 1 - i, 2\delta + 1 - j, s, t)$$

## Backprop equations: multipicative layer

$$z(x) = z(f(x; w))$$

$$\frac{dz}{dx} = \frac{dy}{dx}^{T} \cdot \frac{dz}{dy} \qquad \qquad \frac{dz}{dw} = \frac{dy}{dw}^{T} \cdot \frac{dz}{dy}$$

$$y = Wx$$

$$\frac{dz}{dx} = W^T \frac{dz}{dy}$$

# $\frac{\partial z}{\partial w_{ij}} = \left(\frac{dy}{dw_{ij}}\right)^T \frac{dz}{dy} = x_j \frac{\partial z}{\partial y_i}$

$$=$$
  $\left( \right.$ 

$$=\left(rac{dy}{dw_{ij}}
ight)$$

$$\frac{dz}{dW} = \frac{dz}{dy} \cdot x^T$$

## Backprop equations: convolutional layer

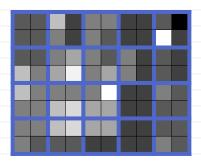
$$\frac{dz}{dW} = \frac{dz}{dy} \cdot x^T \qquad \frac{\partial z}{\partial W_{ij}} = \frac{\partial z}{\partial y_i} \cdot x_j$$

In conv. layer we tie together multiplicative weights corresponding to the same relative position of  $y_i$  and  $x_j$ . So the formula becomes:

$$\frac{\partial z}{\partial K_{m,n,s,t}} = \sum_{k,l} \frac{\partial z}{\partial y_{k,l,t}} \cdot x_{k-m,l-n,s}$$

NB: this is also a convolution between dz/dy and x

## **Backpropagation:** max-pooling





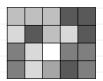


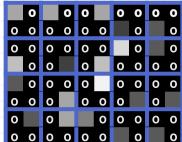


#### backward pass:

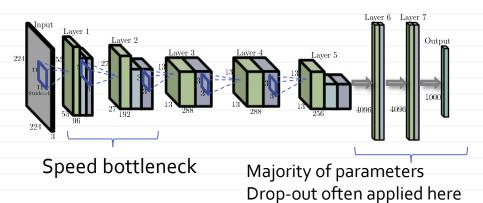
$$\frac{dz}{dx} = \frac{dy}{dx}^{T} \cdot \frac{dz}{dy}$$



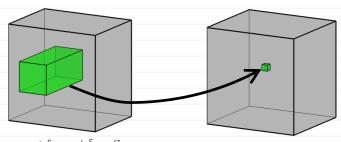




#### **Bottlenecks**



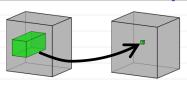
## Efficient implementations: direct



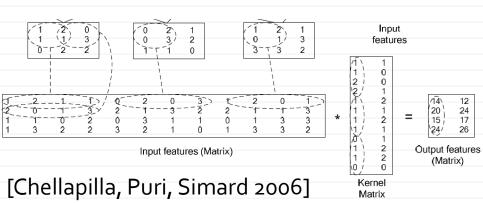
$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K(i-x+\delta, j-y+\delta, s, t) \cdot U(i,j,s)$$

- Loop ordering very important
- Data alignment very important
- NVIDIA cuDNN, Nervana kernels efficient GPU implementations

# Efficient implementations: im2col



**Idea:** reduce all *ST* convolutions to a single matrix multiplication



# **Efficient implementations: Fourier**

$$A * K = \mathcal{F}^{-1}(\mathcal{F}(A) \odot \mathcal{F}(K))$$

- Each map participates in many convolutions (hence FFT is reused)
- Maps are much smaller than images that most FFT codes are optimized for
- Careful implementation needed to get reasonable speed-up
- Memory hungry (why?) (8)
- Does not help for very small filters

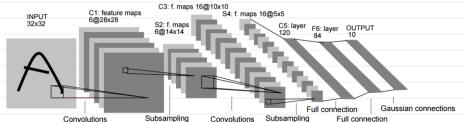
### [Fast Training of Convolutional Networks through FFTs

Michael Mathieu; Mikael Henaff; Yann LeCun, ICLR 2014]

### History: LeNet







[LeCun 89, 98]

### 2012: Image-net



14,197,122 images, 21841 synsets indexed

### Statistics of high level categories

outlotted of high fever outegories										
High level category	# synset (subcategories)	Avg # images per synset	Total # images							
amphibian	94	591	56K							
animal	3822	732	2799K							
appliance	51	1164	59K							
bird	856	949	812K							
covering	946	819	774K							
device	2385	675	1610K							
fabric	262	690	181K							
fish	566	494	280K							
flower	462	735	339K							
food	1495	670	1001K							
fruit	309	607	188K							

(all competitions on much smaller subset 1000x1000)

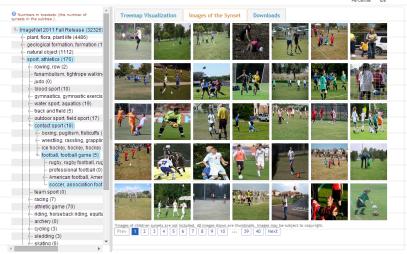
### **Image-net**

### Soccer, association football

A football game in which two teams of 11 players try to kick or head a ball into the opponents' goal







### **Image-net**

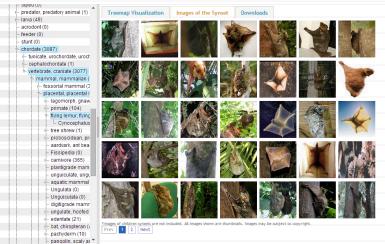
#### Flying lemur, flying cat, colugo

Arboreal nocturnal mammal of southeast Asia and the Philippines resembling a lemur and having a fold of skin on each side from neck to tail that is used for long gliding leaps

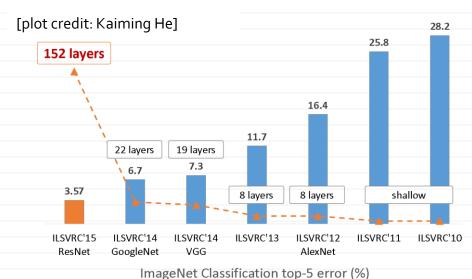




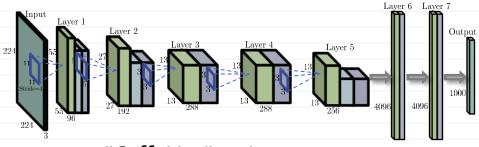




# Building the best network



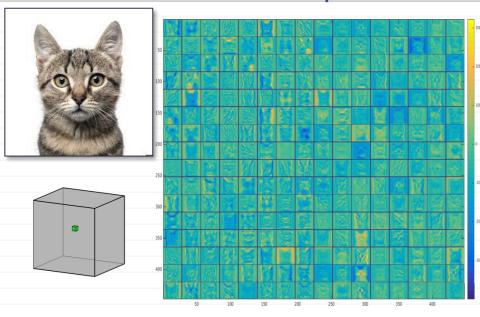
### AlexNet (2012)



- .."CaffeNet" variant
- 5 conv layers (11x11,5x5,3x3,3x3,3x3)
- 6oM parameters
- Learns in 3-5 days on a GPU
- Faster than subsequent architectures

[Krizhevsky et al. 2012]

# Idea 1: neuron maps



### **VGGNet (2014)**

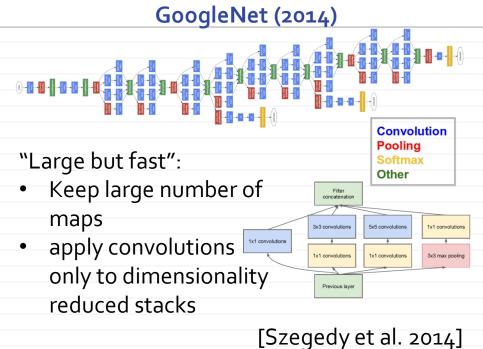
•	upto 16 conv.	layers

- All filters are 3x3
- balances load between layers
- ~140M params
- Stagewise training (before batch norms)
- The highest performance among chain-like models

В	C	D	Е		
weight	16 weight	16 weight	19 weight		
ayers	layers	layers	layers		
$224 \times 2$	24 RGB image	e)	-		
nv3-64	conv3-64	conv3-64	conv3-64		
nv3-64	conv3-64	conv3-64	conv3-64		
max	pool				
rv3-128	conv3-128	conv3-128	conv3-128		
ıv3-128	conv3-128	conv3-128	conv3-128		
max	pool				
w3-256	conv3-256	conv3-256	conv3-256		
w3-256	conv3-256	conv3-256	conv3-256		
	conv1-256	conv3-256	conv3-256		
			conv3-256		
max	pool				
w3-512	conv3-512	conv3-512	conv3-512		
w3-512	conv3-512	conv3-512	conv3-512		
	conv1-512	conv3-512	conv3-512		
			conv3-512		
	pool				
w3-512	conv3-512	conv3-512	conv3-512		
w3-512	conv3-512	conv3-512	conv3-512		
	conv1-512	conv3-512	conv3-512		
			conv3-512		
	pool				
	4096				
FC-	4096				

[Simonyan & Zisserman, 2014]

soft-max



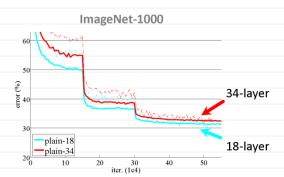
"Deep Learning", Spring 2019: Lecture 3, "Convolutional networks"

## **ResNet (2015)**



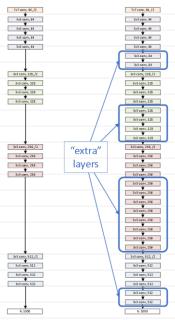
# Simply deepening does not work:

ResNet, 152 layers (ILSVRC 2015)



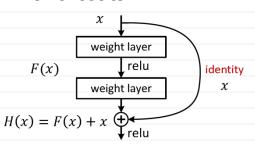
[He et al. 2015]

## **ResNet** (2015)



Q: How to ensure that the training error does not go up?

A: shortcuts



Learn F(x) instead of H(x)
[He et al. 2015]

### 1x1 Convolution

A variant of convolutional layer with 1x1 spatial filter size that is very important in modern architectures:

$$V(x, y, t) = \sum_{s=1}^{S} K(1, 1, s, t) \cdot U(x, y, s)$$

- The 4-dim kernel turns into a 2D-matrix
- No spatial propagation (has to be done by other layers)
- We can think that we apply the same small fullyconnected layer at each location independently (originally 1x1 conv was called "network-in-network")

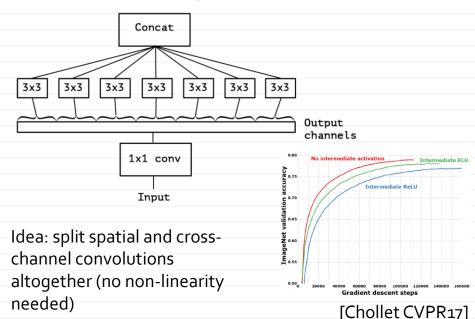
# **Depthwise-separate Convolution**

A variant of convolutional layer with 1x1 spatial filter size that is very important in modern architectures:

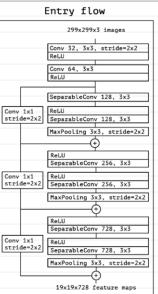
$$V(x, y, t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} K(i - x + \delta, j - y + \delta, t) \cdot U(i, j, t)$$

- No propagation across channels (has to be done by other layers)
- S times less operations and parameters than standard convolution

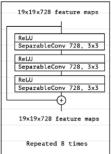
## Xception (2016)



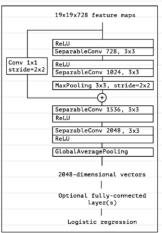
### Xception (2016)



### Middle flow



### Exit flow



	Top-1 accuracy	Top-5 accuracy
VGG-16	0.715	0.901
ResNet-152	0.770	0.933
Inception V3	0.782	0.941
Xception	0.790	0.945

[Chollet CVPR<sub>17</sub>]

# MobileNet (2017)

Table 1. MobileNet Body Architecture

Type / Stride	Filter Shape	Input Size
Conv / s2	$3 \times 3 \times 3 \times 32$	$224 \times 224 \times 3$
Conv dw / s1	$3 \times 3 \times 32 \text{ dw}$	$112 \times 112 \times 32$
Conv / s1	$1 \times 1 \times 32 \times 64$	$112 \times 112 \times 32$
Conv dw / s2	$3 \times 3 \times 64 \text{ dw}$	$112 \times 112 \times 64$
Conv / s1	$1 \times 1 \times 64 \times 128$	$56 \times 56 \times 64$
Conv dw / s1	$3 \times 3 \times 128 \text{ dw}$	$56 \times 56 \times 128$
Conv / s1	$1 \times 1 \times 128 \times 128$	$56 \times 56 \times 128$
Conv dw / s2	$3 \times 3 \times 128 \text{ dw}$	$56 \times 56 \times 128$
Conv / s1	$1 \times 1 \times 128 \times 256$	$28 \times 28 \times 128$
Conv dw / s1	$3 \times 3 \times 256 \text{ dw}$	$28 \times 28 \times 256$
Conv / s1	$1 \times 1 \times 256 \times 256$	$28 \times 28 \times 256$
Conv dw / s2	$3 \times 3 \times 256 \text{ dw}$	$28 \times 28 \times 256$
Conv / s1	$1 \times 1 \times 256 \times 512$	$14 \times 14 \times 256$
5× Conv dw / s1	$3 \times 3 \times 512 \text{ dw}$	$14 \times 14 \times 512$
Conv/s1	$1 \times 1 \times 512 \times 512$	$14 \times 14 \times 512$
Conv dw / s2	$3 \times 3 \times 512 \text{ dw}$	$14 \times 14 \times 512$
Conv / s1	$1 \times 1 \times 512 \times 1024$	$7 \times 7 \times 512$
Conv dw / s2	$3 \times 3 \times 1024 \text{ dw}$	$7 \times 7 \times 1024$
Conv / s1	$1 \times 1 \times 1024 \times 1024$	$7 \times 7 \times 1024$
Avg Pool / s1	Pool 7 × 7	$7 \times 7 \times 1024$
FC/s1	$1024 \times 1000$	$1 \times 1 \times 1024$
Softmax / s1	Classifier	$1 \times 1 \times 1000$

3x3 Depthwise Conv
BN
ReLU
ReLU
1x1 Conv
BN
ReLU

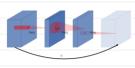
Table 2. Resource Per Layer Type

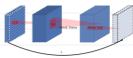
Mult-Adds	Parameters		
94.86%	74.59%		
3.06%	1.06%		
1.19%	0.02%		
0.18%	24.33%		
	94.86% 3.06% 1.19%		

[Howard et al. Arxiv17]

### **MobileNet v2 (2018)**

- Expand the number of layers (to give more work to spatial convolutions)
- Do not use non-linearities in the expansion:
  - (a) Residual block
- (b) Inverted residual block





Input	Operator	t	c	n	s
$224^2 \times 3$	conv2d	-	32	1	2
$112^{2} \times 32$	bottleneck	1	16	1	1
$112^{2} \times 16$	bottleneck	6	24	2	2
$56^2 \times 24$	bottleneck	6	32	3	2
$28^{2} \times 32$	bottleneck	6	64	4	2
$14^{2} \times 64$	bottleneck	6	96	3	1
$14^{2} \times 96$	bottleneck	6	160	3	2
$7^{2} \times 160$	bottleneck	6	320	1	1
$7^{2} \times 320$	conv2d 1x1	-	1280	1	1
$7^2 \times 1280$	avgpool 7x7	-	-	1	-
$1\times1\times1280$	conv2d 1x1	-	k	-	

Network	Top 1	Params	MAdds	CPU
MobileNetV1	70.6	4.2M	575M	113ms
ShuffleNet (1.5)	71.5	3.4M	292M	-
ShuffleNet (x2)	73.7	5.4M	524M	-
NasNet-A	74.0	5.3M	564M	183ms
MobileNetV2	72.0	3.4M	300M	75ms
MobileNetV2 (1.4)	74.7	6.9M	585M	143ms

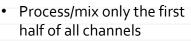
expansion

repetitions

ons [

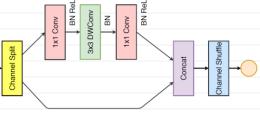
[Sandler et al. CVPR18]

## ShuffleNet v2 (2018)



Second half is kept intact

Shuffle after each block



Output size	I/Cino	Ctuido	Donost	Output channels												
Output size	Ksize	Stride	Repeat	$0.5 \times$	1×	1.5×	$2\times$	500			Xception					
224×224				3	3	3	3	300		<b>60</b> 0	ShuffleNet v1					
112×112	$3\times3$	2	1	94	24	94	94			• 👭	MobileNet v2					
56×56	$3\times3$	2	1	24	24	24	2-4	400		***	ShuffleNet v2 (Ours)					
28×28		2	1	48	116	176	244	Ų.			*					
		1	3	-10	40 110	170 2	110	110	1110	170	170	244	io l			^
1		2	1	96	232	352	488	es			_					
		1	7		50 202 002 400		100	5			<b>*</b>					
		2	1	192 464	464	464	464	464	704	704	704	976	200			· • • • * ·
		1	3							ш			<b>→</b>			
		1	1	1024	1024	1024	2048	100			<b>●</b> △					
1×1	7×7							100								
								٥٤	55	60 65	70 75					
				1.4M	2.3M	3.5M	7.4M			00 00						
	224×224 112×112 56×56 28×28 28×28 14×14 14×14 7×7 7×7 7×7 1×1	224×224 112×112 56×56 3×3 28×28 28×28 14×14 14×14 7×7 7×7 7×7 1×1 1×1 7×7	224×224 112×112 56×56 3×3 2 28×28 28×28 1 14×14 1 7×7 7×7 1 7×7 1×1 1×1 1×1 1×	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Output size KSize Stride Repeat         0.5×           224×224         3         3           112×112         3×3         2         1         24           56×56         3×3         2         1         24           28×28         2         1         48           14×14         2         1         96           7×7         2         1         192           7×7         1         3         192           7×7         1×1         1         1024           1×1         7×7         1000         41M	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Output size RSize Stride Repeat         0.5x         1x         1.5x         2x           224×224         3         3         3         3         3           112×112         3×3         2         1         24         24         24         24           28×28         2         1         3         48         116         176         244         7           28×28         1         3         48         116         176         244         7         24	Output size KSize Stride Repeat   0.5×   1×   1.5×   2×   22×224   3   3   3   3   3   3   3   3   3	Output size KSize Stride Repeat 0.5× 1× 1.5× 2× 2× 224×224					

4 variants

[Ma et al. ECCV18]

### Recap

- Convolutional networks are the most popular and influential model in modern deep learning
- Convolutional layer is a special type of a multiplicative one with greatly reduced number of parameters
- Different ways to compute convolutions exist
- Good ConvNet architectures have been discovered

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