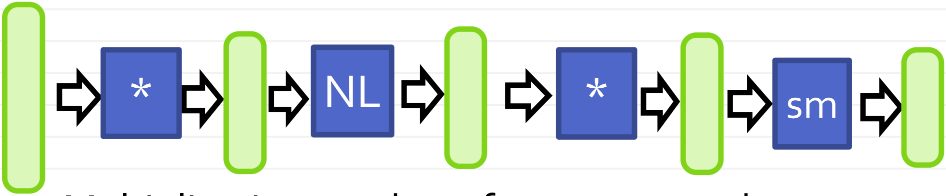
# Lecture 3: Convolutional Networks

# Neural network: parameter overdoze




Multiplicative number of parameters: the main problem. Solving the problem:

- Stop optimization early (always keep checking progress on validation set)
- Impose smoothness (weight decay)
- Bag multiple models

**Main avenue**: picking a less generic architecture with less parameters

# Huber-Wiesel

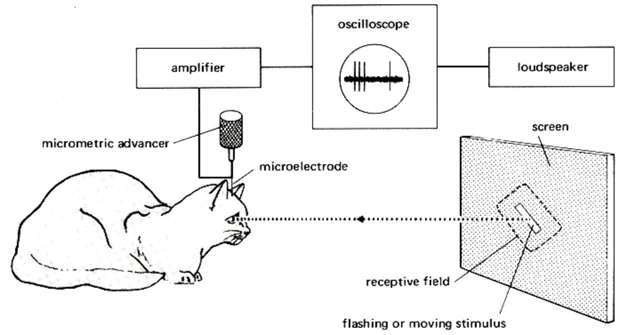
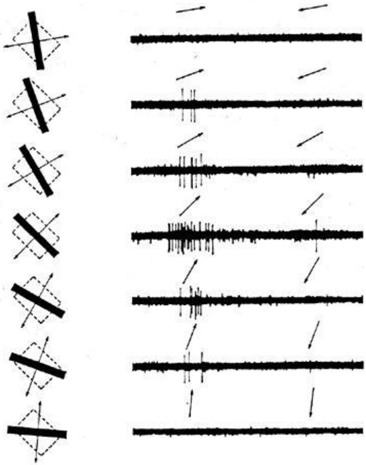



Fig. 7 *If a microelectrode is inserted into the brain and picks up impulses from a single nerve cell, the neuron's responses can be studied by projecting patterns of light in front of the stationary eyes.*

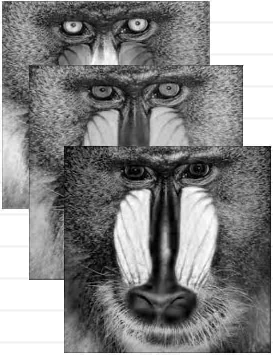
The reaction of a neuron is localized to a part of the visual field

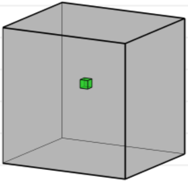
# Huber and Wiesel 1968

V1 physiology: direction selectivity



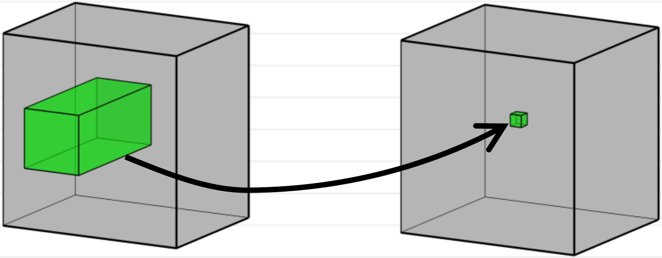
# Idea 1: neuron maps





1. Organize neurons into *map stacks*
2. E.g. an image is a WxHx3 map stack

# Idea 1: limited receptive field

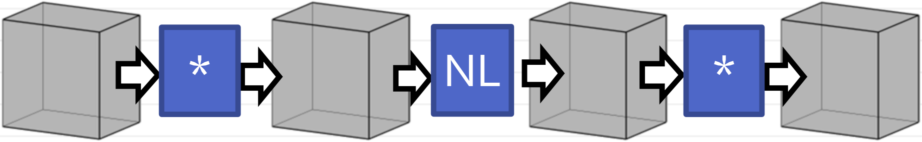


1. Organize neurons into *maps*
2. Limit the *receptive field* in the multiplicative layer:

$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K^{x,y,t}(i-x+\delta, j-y+\delta, s) \cdot U(i,j,s)$$

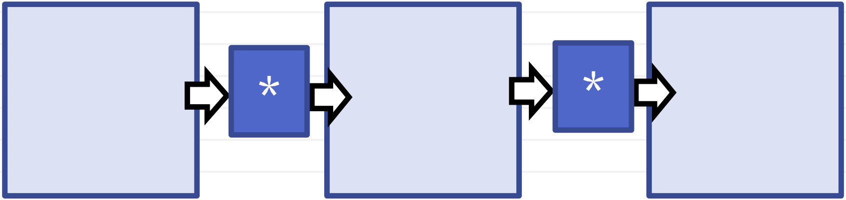
3. Huge parameter reduction O( (W/2δ)²)

# Stacking layers




- The layers are stacked and interleaved with non-linearities (e.g. ReLU)

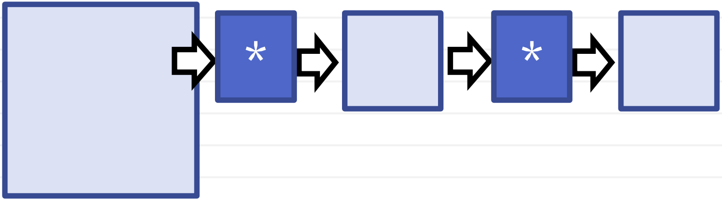
# Growing receptive field




E.g. with 7x7 filters:

- Receptive field is 7x7 after 1 conv layers
- Receptive field is 13x13 after 2 conv layers

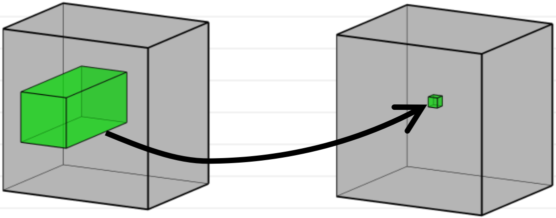
# Accelerating growth: stride




- In the early layers strides are often > 1
- Strides > 1 downsample maps
- Strides > 1 increase the receptive fields

What is the receptive field after [7x7 stride=2] followed by [7x7 stride = 1] convolution?

# Idea 2: tying together weights



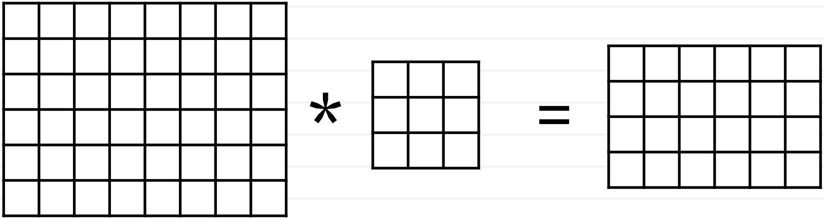
$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K^t(i-x+\delta, j-y+\delta, s) \cdot U(i,j,s)$$

Further dramatic reduction in the number of parameters: O( W²)

$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K(i-x+\delta, j-y+\delta, s, t) \cdot U(i,j,s)$$

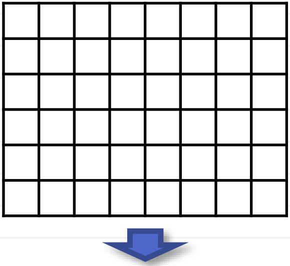
# Boundary issues

"Valid" mode:



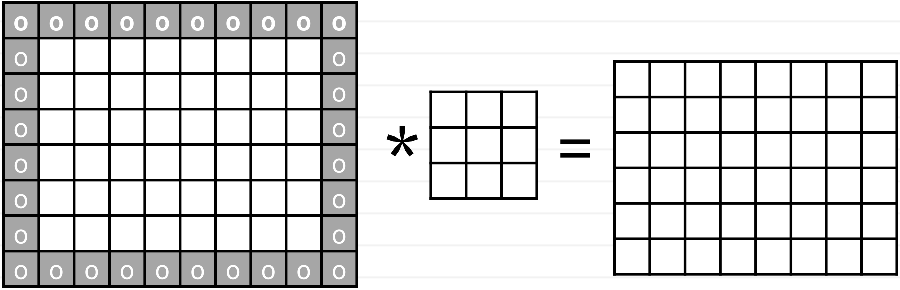
- Complicates implementation and reasoning
- Unequal contribution of elements

# Boundary issues: padding with zeros



"Same" mode:

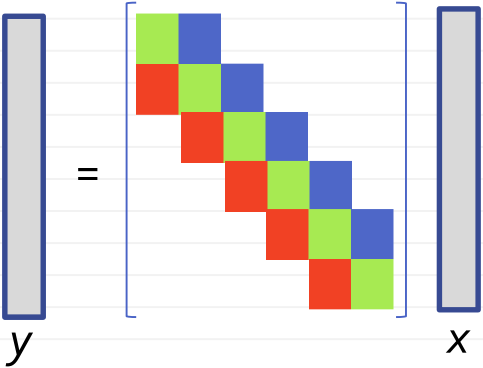
- Solves the problem
- Introduces "false" edges



# Conv. layer is still multiplicative

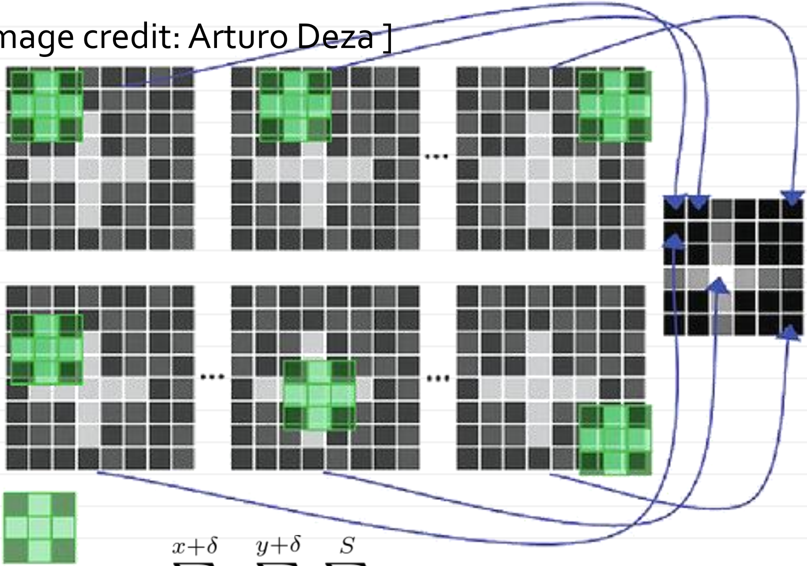
E.g. 1D correlation with 

is a multiplication over a banded matrix:



# Interpretation: looking for patterns

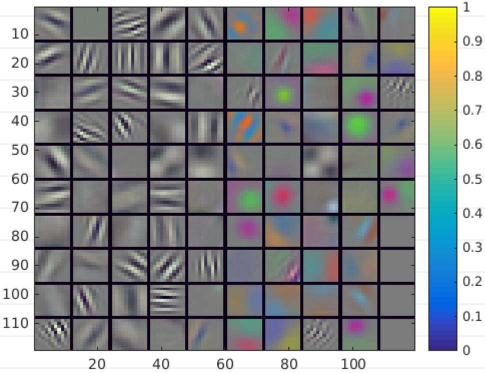
[ Image credit: Arturo Deza ]



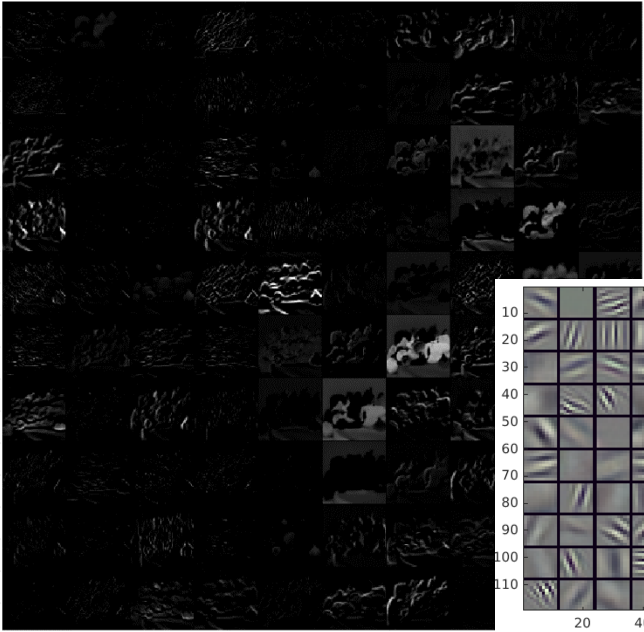
$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K(i-x+\delta, j-y+\delta, s, t) \cdot U(i,j,s)$$

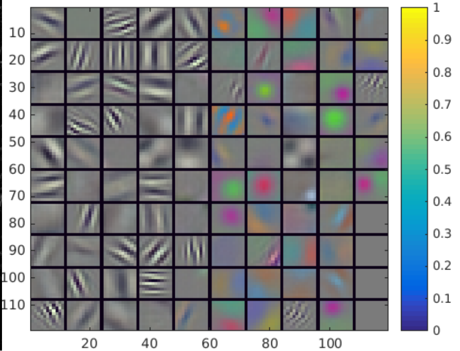
# Interpretation: looking for patterns

AlexNet filters of the first layer:

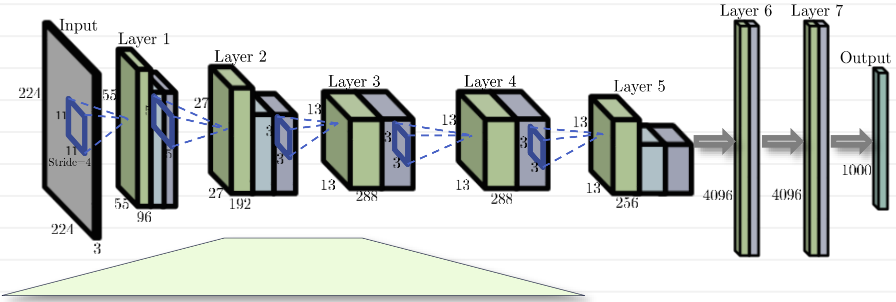


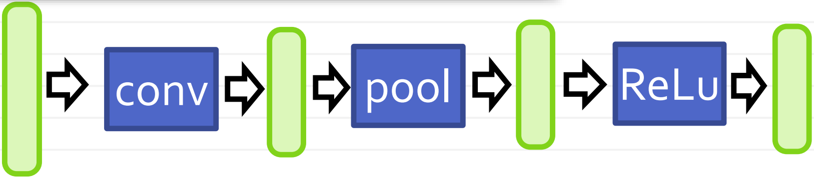
# Responses in the first layer



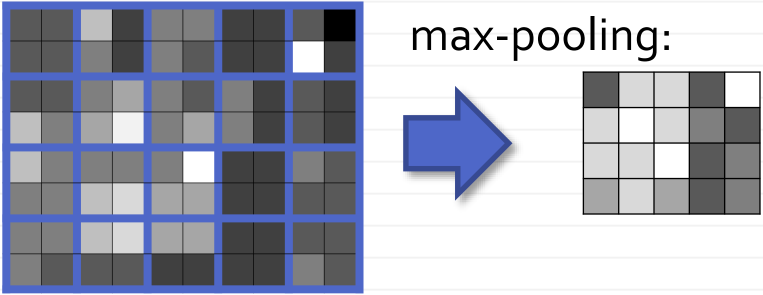


# What are modern ConvNets made of



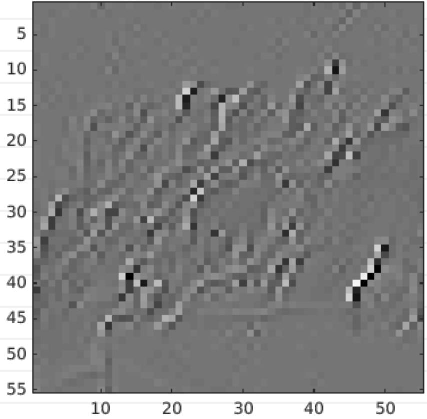
# Third-component: pooling (+subsampling)

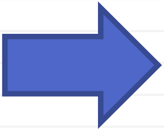


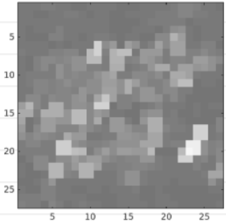

Pooling is almost always with subsampling

Alternatives: sum-pooling, average pooling,
Rapid decrease of map size
Parameter-free

# Max-pooling and jitter-invariance

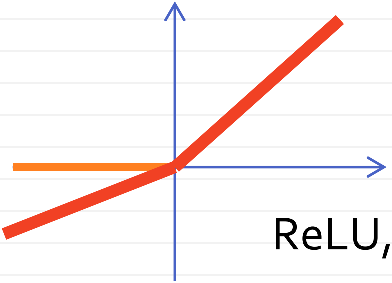





- Usual motivation: adding invariance to small shifts
- Several max-poolings can accumulate invariance to stronger shifts

# Details: nonlinearity

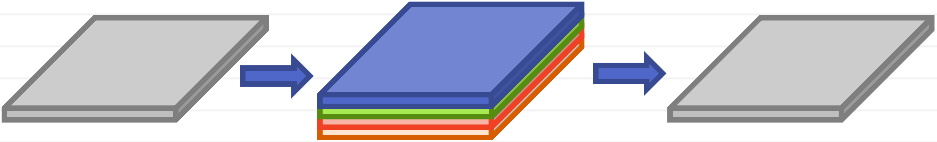


$$f(x;\alpha) = \max(x, \alpha x)$$

ReLU, leaky ReLu

Another formerly popular non-linearity: *maxout*

$$f(x) = \max(\alpha_1 x + \beta_1, \alpha_2 x + \beta_2, \ldots, \alpha_m x + \beta_m)$$



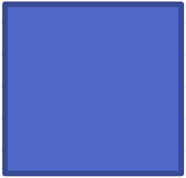
# CNN applications

Pattern finding through convolution/correlation is ubiquitous:

- 2D images (and the like, e.g. speech)
- 1D signals (e.g. time series, speech)
- 3D images
- Videos
- Graphs (more generalized sense)

[Bruna et al. Spectral Networks and Locally Connected Networks on Graphs. ICLR 2014]

# Reminder: layer abstraction



Each layer is defined by:

- forward performance: $y = f(x; w)$
- backward performance:

$$z(x) = z(f(x;\, w))$$

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy} \qquad\qquad \frac{dz}{dw} = \frac{dy}{dw}^T \cdot \frac{dz}{dy}$$

# Backprop equations: multipicative layer

$$z(x) = z(f(x;\, w))$$

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy} \qquad \frac{dz}{dw} = \frac{dy}{dw}^T \cdot \frac{dz}{dy}$$

$$y = Wx$$

$$\frac{dy}{dx} = W$$

$$\frac{dz}{dx} = W^T \frac{dz}{dy}$$

# Backprop equations: convolutional layer

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy}$$

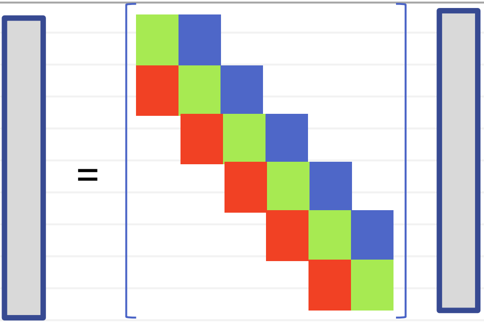
$$y = Wx$$ - still holds for conv layer

$$\frac{dz}{dx} = W^T \frac{dz}{dy}$$ - also corresponds to some correlation?

# Backpropagation via convolution

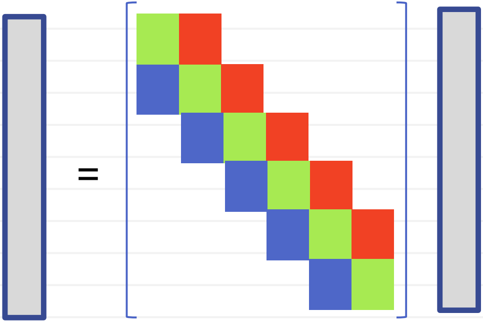
$$y = Wx$$

Corr with 



$$\frac{dz}{dx} = W^T \frac{dz}{dy}$$

Corr with 



# Backprop equations: convolutional layer

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy}$$

$y = Wx$ - still holds for conv layer

$\dfrac{dz}{dx} = W^T \dfrac{dz}{dy}$ - also corresponds to conv

During backpass do the same correlations but with flipped kernels

# Backprop equations: multipicative layer

$$z(x) = z(f(x;\, w))$$

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy}$$

$$y = Wx$$

$$\frac{dy}{dx} = W$$

$$\frac{dz}{dx} = W^T \frac{dz}{dy}$$

$$\frac{dz}{dw} = \frac{dy}{dw}^T \cdot \frac{dz}{dy}$$

$$\frac{\partial z}{\partial w_{ij}} = \left(\frac{dy}{dw_{ij}}\right)^T \frac{dz}{dy} = x_j \frac{\partial z}{\partial y_i}$$

$$\frac{dz}{dW} = \frac{dz}{dy} \cdot x^T$$

# Backprop equations: convolutional layer

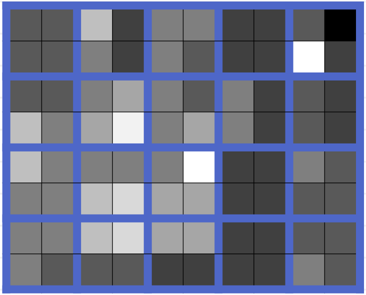
$$\frac{dz}{dW} = \frac{dz}{dy} \cdot x^T \qquad \frac{\partial z}{\partial W_{ij}} = \frac{\partial z}{\partial y_i} \cdot x_j$$

In conv. layer we tie together multiplicative weights corresponding to the same relative position of $y_i$ and $x_j$.
So the formula becomes:

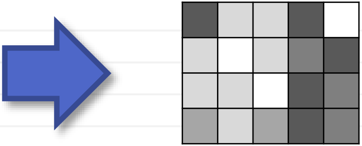
$$\frac{\partial z}{\partial K_{i,j,s,t}} = \sum_{x,y} \frac{\partial z}{\partial y_{x,y,t}} \cdot x_{x-i,y-j,s}$$

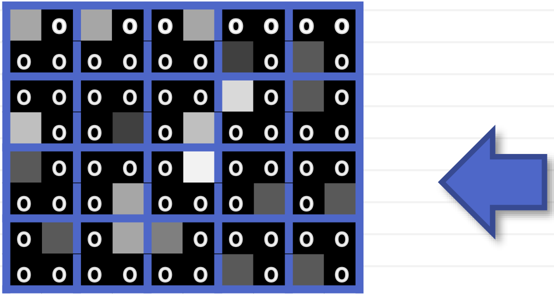
**NB**: this is also a convolution between dz/dy and x

# Backpropagation: max-pooling



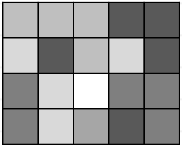
forward pass:



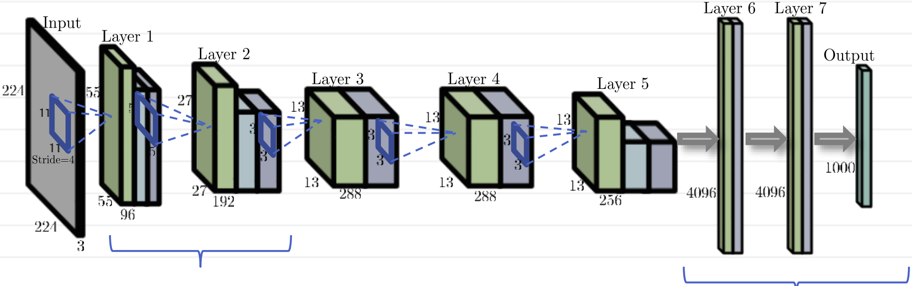


backward pass:

$$\frac{dz}{dx} = \frac{dy}{dx}^T \cdot \frac{dz}{dy}$$



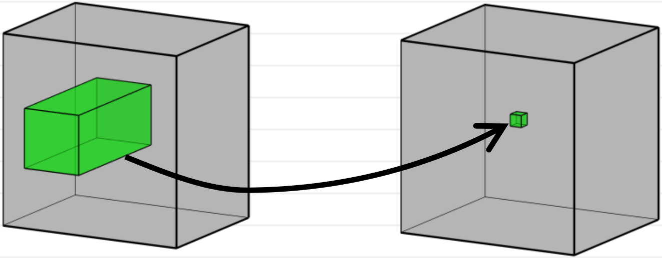
# Bottlenecks




Speed bottleneck

Majority of parameters
Drop-out often applied here

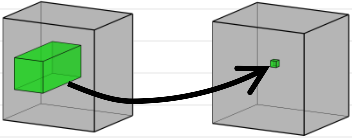
# Efficient implementations: direct



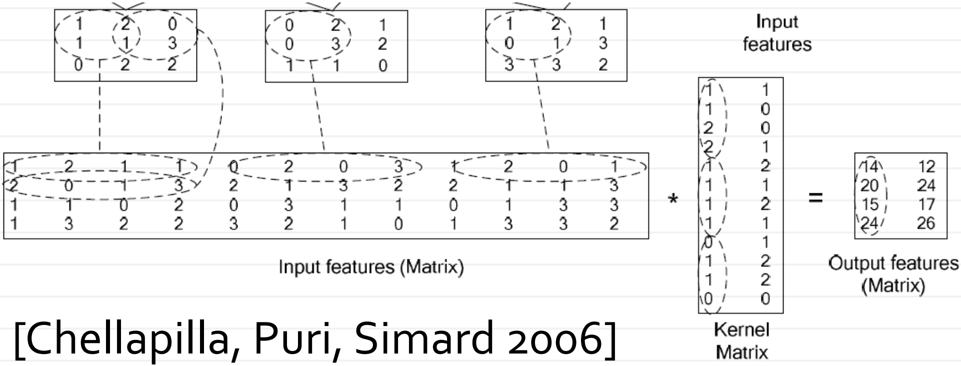
$$V(x,y,t) = \sum_{i=x-\delta}^{x+\delta} \sum_{j=y-\delta}^{y+\delta} \sum_{s=1}^{S} K(i-x+\delta, j-y+\delta, s, t) \cdot U(i,j,s)$$

- Loop ordering very important
- Data alignment very important
- NVIDIA cuDNN, Nervana kernels - efficient GPU implementations

# Efficient implementations: im2col



**Idea:** reduce all *ST* convolutions to a single matrix multiplication




[Chellapilla, Puri, Simard 2006]

# Efficient implementations: Fourier

$$A * K = \mathcal{F}^{-1}(\mathcal{F}(A) \odot \mathcal{F}(K))$$

- Each map participates in many convolutions (hence FFT is reused)
- Maps are much smaller than images that most FFT codes are optimized for
- Careful implementation needed to get reasonable speed-up
- Memory hungry (why?) ☹

[Fast Training of Convolutional Networks through FFTs
*Michael Mathieu; Mikael Henaff; Yann LeCun, ICLR 2014]*

# Benchmarking: forward times

VGG-Net (mostly 3x3 convolutions)

| Library | Class | Time (ms) | forward (ms) | backward (ms) |
|---|---|---|---|---|
| Nervana-neon-fp16 | ConvLayer | 254 | 82 | 171 |
| Nervana-neon-fp32 | ConvLayer | 320 | 103 | 217 |
| CuDNN[R4]-fp16 (Torch) | cudnn.SpatialConvolution | 471 | 140 | 331 |
| CuDNN[R4]-fp32 (Torch) | cudnn.SpatialConvolution | 529 | 162 | 366 |
| TensorFlow | conv2d | 540 | 158 | 382 |
| Chainer | Convolution2D | 885 | 251 | 632 |
| fbfft (Torch) | SpatialConvolutionCuFFT | 1092 | 355 | 737 |
| cudaconvnet2* | ConvLayer | 1229 | 408 | 821 |
| CuDNN[R2] * | cudnn.SpatialConvolution | 1099 | 342 | 757 |
| Caffe | ConvolutionLayer | 1068 | 323 | 745 |

https://github.com/soumith/convnet-benchmarks [Soumith Chintala]

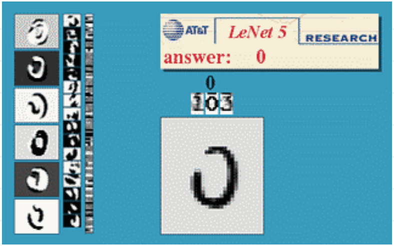
# Benchmarking: forward times

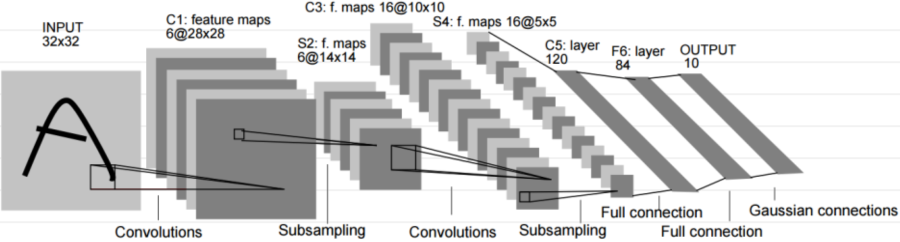
AlexNet (11x11,5x5,3x3 convolutions)

| Library | Class | Time (ms) | forward (ms) | backward (ms) |
|---|---|---|---|---|
| CuDNN[R4]-fp16 (Torch) | cudnn.SpatialConvolution | 71 | 25 | 46 |
| Nervana-neon-fp16 | ConvLayer | 78 | 25 | 52 |
| CuDNN[R4]-fp32 (Torch) | cudnn.SpatialConvolution | 81 | 27 | 53 |
| TensorFlow | conv2d | 81 | 26 | 55 |
| Nervana-neon-fp32 | ConvLayer | 87 | 28 | 58 |
| fbfft (Torch) | fbnn.SpatialConvolution | 104 | 31 | 72 |
| Chainer | Convolution2D | 177 | 40 | 136 |
| cudaconvnet2* | ConvLayer | 177 | 42 | 135 |
| CuDNN[R2] * | cudnn.SpatialConvolution | 231 | 70 | 161 |
| Caffe (native) | ConvolutionLayer | 324 | 121 | 203 |

https://github.com/soumith/convnet-benchmarks [Soumith Chintala]

# History: LeNet




[LeCun 89, 98]

# 2012: Image-net

IMAGENET

*www.image-net.org*

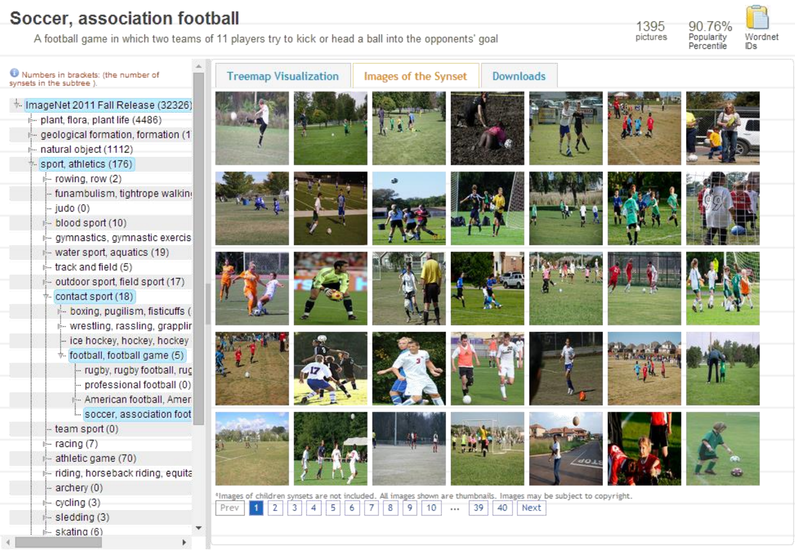
14,197,122 images, 21841 synsets indexed

(all competitions on much smaller subset 1000x1000)

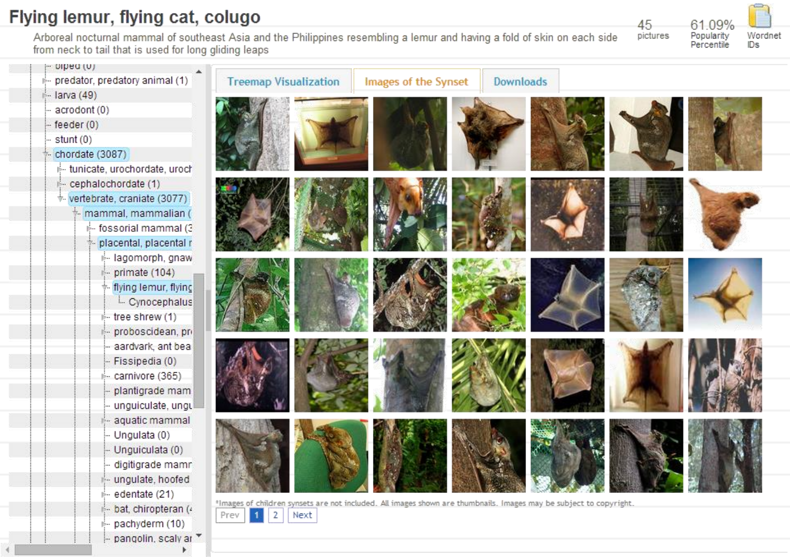
Statistics of high level categories

| High level category | # synset (subcategories) | Avg # images per synset | Total # images |
|---|---|---|---|
| amphibian | 94 | 591 | 56K |
| animal | 3822 | 732 | 2799K |
| appliance | 51 | 1164 | 59K |
| bird | 856 | 949 | 812K |
| covering | 946 | 819 | 774K |
| device | 2385 | 675 | 1610K |
| fabric | 262 | 690 | 181K |
| fish | 566 | 494 | 280K |
| flower | 462 | 735 | 339K |
| food | 1495 | 670 | 1001K |
| fruit | 309 | 607 | 188K |

# Image-net


Soccer, association football
A football game in which two teams of 11 players try to kick or head a ball into the opponents' goal
1395 pictures
90.76% Popularity Percentile
Wordnet IDs
Numbers in brackets: (the number of synsets in the subtree ).
ImageNet 2011 Fall Release (32326)
plant, flora, plant life (4486)
geological formation, formation (1
natural object (1112)
sport, athletics (176)
rowing, row (2)
funambulism, tightrope walkin
judo (0)
blood sport (10)
gymnastics, gymnastic exercis
water sport, aquatics (19)
track and field (5)
outdoor sport, field sport (17)
contact sport (18)
boxing, pugilism, fisticuffs (
wrestling, rassling, grapplir
ice hockey, hockey, hockey
football, football game (5)
rugby, rugby football, rug
professional football (0)
American football, Amer
soccer, association foot
team sport (0)
racing (7)
athletic game (70)
riding, horseback riding, equita
archery (0)
cycling (3)
sledding (3)
skating (6)
Treemap Visualization
Images of the Synset
Downloads
*Images of children synsets are not included. All images shown are thumbnails. Images may be subject to copyright.
Prev 1 2 3 4 5 6 7 8 9 10 ... 39 40 Next

# Image-net


Flying lemur, flying cat, colugo
Arboreal nocturnal mammal of southeast Asia and the Philippines resembling a lemur and having a fold of skin on each side from neck to tail that is used for long gliding leaps
45 pictures
61.09% Popularity Percentile
Wordnet IDs
predator, predatory animal (1)
larva (49)
acrodont (0)
feeder (0)
stunt (0)
chordate (3087)
tunicate, urochordate, uroch
cephalochordate (1)
vertebrate, craniate (3077)
mammal, mammalian (
fossorial mammal (
placental, placental
lagomorph, gnaw
primate (104)
flying lemur, flying
Cynocephalus
tree shrew (1)
proboscidean, pr
aardvark, ant bea
Fissipedia (0)
carnivore (365)
plantigrade mam
unguiculate, ungu
aquatic mammal
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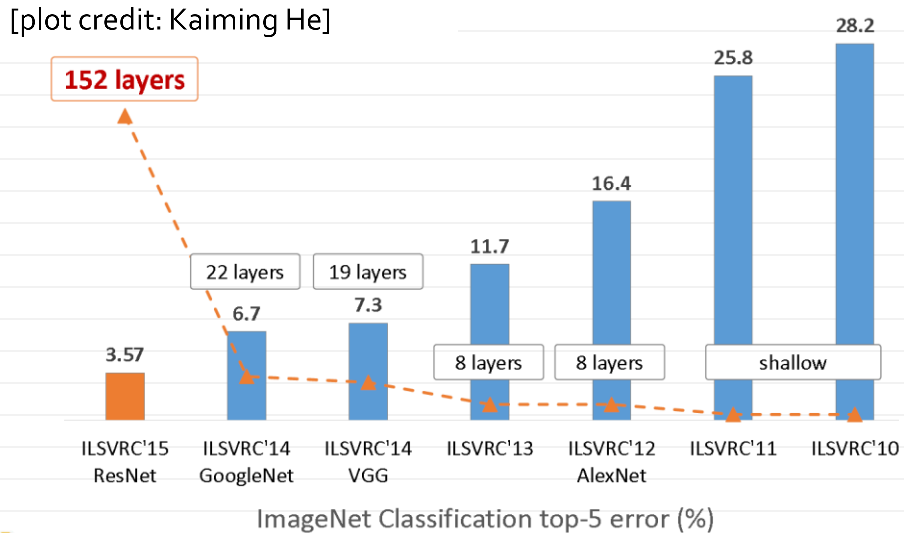
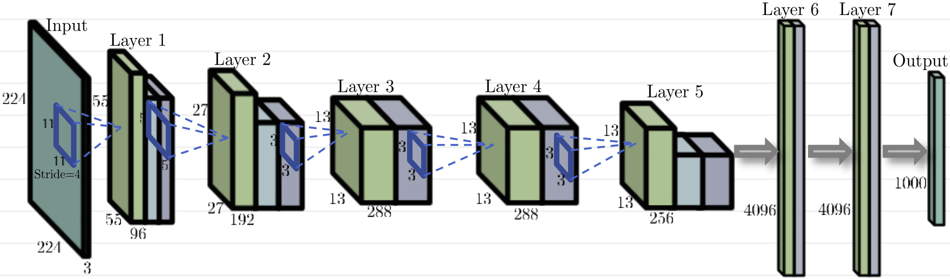
pachyderm (10)
pangolin, scaly ar
Treemap Visualization
Images of the Synset
Downloads
*Images of children synsets are not included. All images shown are thumbnails. Images may be subject to copyright.
Prev
1
2
Next

# Building the best network



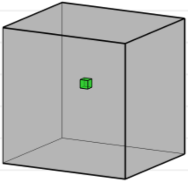
# AlexNet (2012)

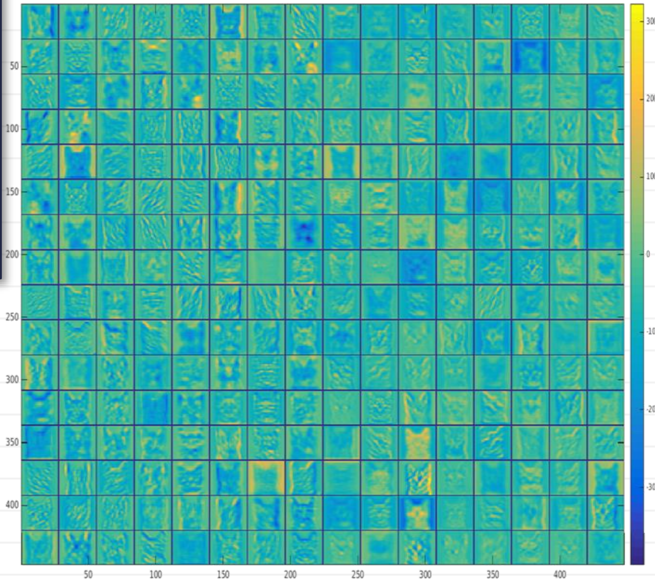



- .."CaffeNet" variant
- 5 conv layers (11x11,5x5,3x3,3x3,3x3)
- 60M parameters
- Learns in 3-5 days on a GPU
- Faster than subsequent architectures

[Krizhevsky et al. 2012]

# Idea 1: neuron maps





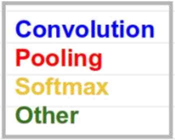
# VGGNet (2014)

- upto 16 conv. layers
- All filters are 3x3
- balances load between layers
- ~140M params
- Stagewise training (before batch norms)
- The highest performance among chain-like models

[Simonyan & Zisserman, 2014]

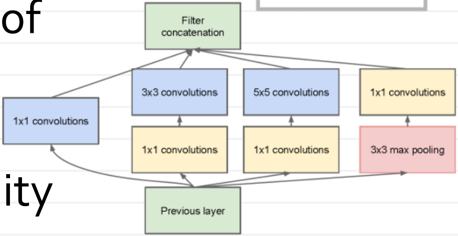
| B | C | D | E |
|---|---|---|---|
| weight ayers | 16 weight layers | 16 weight layers | 19 weight layers |
| (224 × 224 RGB image) | | | |
| nv3-64<br>**nv3-64** | conv3-64<br>conv3-64 | conv3-64<br>conv3-64 | conv3-64<br>conv3-64 |
| maxpool | | | |
| nv3-128<br>**nv3-128** | conv3-128<br>conv3-128 | conv3-128<br>conv3-128 | conv3-128<br>conv3-128 |
| maxpool | | | |
| nv3-256<br>nv3-256 | conv3-256<br>conv3-256<br>**conv1-256** | conv3-256<br>conv3-256<br>**conv3-256** | conv3-256<br>conv3-256<br>conv3-256<br>**conv3-256** |
| maxpool | | | |
| nv3-512<br>nv3-512 | conv3-512<br>conv3-512<br>**conv1-512** | conv3-512<br>conv3-512<br>**conv3-512** | conv3-512<br>conv3-512<br>conv3-512<br>**conv3-512** |
| maxpool | | | |
| nv3-512<br>nv3-512 | conv3-512<br>conv3-512<br>**conv1-512** | conv3-512<br>conv3-512<br>**conv3-512** | conv3-512<br>conv3-512<br>conv3-512<br>**conv3-512** |
| maxpool | | | |
| FC-4096 | | | |
| FC-4096 | | | |
| FC-1000 | | | |
| soft-max | | | |

# GoogleNet (2014)




"Large but fast":

- Keep large number of maps
- apply convolutions only to dimensionality reduced stacks




[Szegedy et al. 2014]

# ResNet (2015)

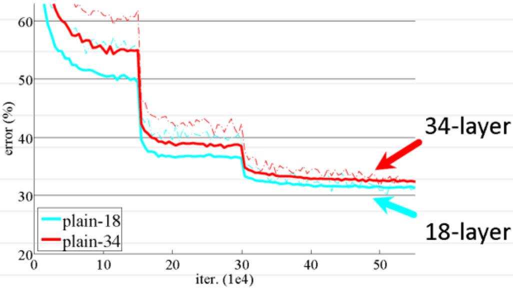
AlexNet, 8 layers
(ILSVRC 2012)

VGG, 19 layers
(ILSVRC 2014)

Simply deepening does not work:

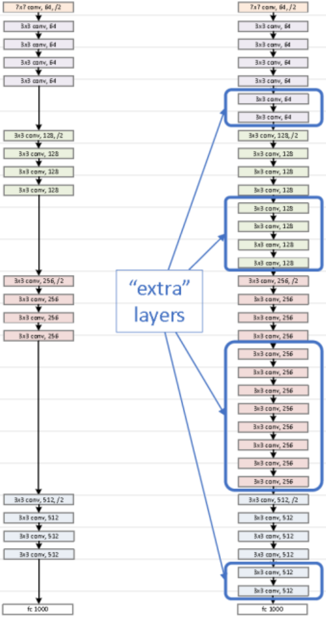
ResNet, 152 layers
(ILSVRC 2015)

ImageNet-1000



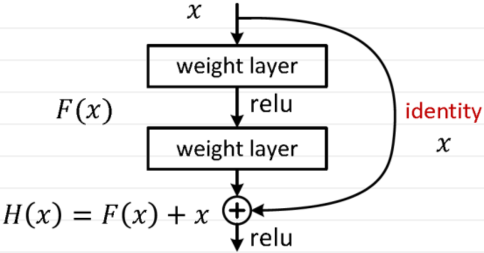

[He et al. 2015]

# ResNet (2015)




Q: How to ensure that the training error does not go up?

A: shortcuts

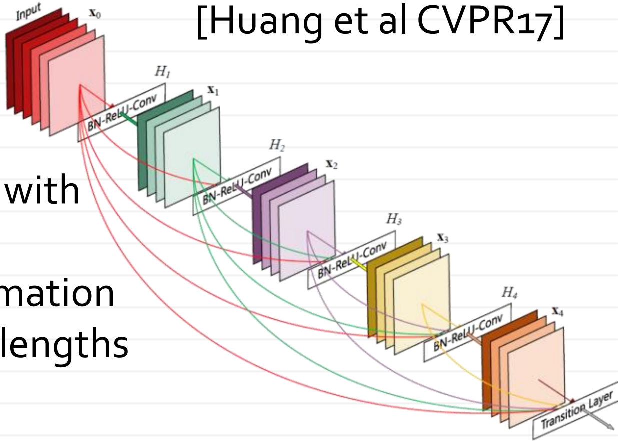



Learn F(x) instead of H(x)

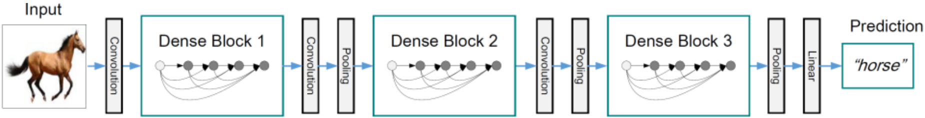
[He et al. 2015]

# DenseNet

[Huang et al CVPR17]




- Matches SOTA with smaller depth
- Gives the information variety of path lengths to flow along



# Recap

- Convolutional networks are the most popular and influential model in modern deep learning
- Convolutional layer is a special type of a multiplicative one with greatly reduced number of parameters
- Different ways to compute convolutions exist
- Good ConvNet architectures have been discovered

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