On Methods for Decoding Non-orthogonal Multiple Access Schemes Based on Polar Codes

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1. Motivation

The amount of information is experiencing exponential growth, but resouces are extremely limited. In case of the growth of IoT and mMTC infrastructures, there is a need for short low-rate codes for multiple access, where the level of interference becomes extremely high. Due to limit of resouces, the non-orthogonal multiple access is used to handle with service time issues, without losing in rate. In this scenario, polar codes, that were proved in [1] to achieve the symmetric capacity of any given B-DMC, are promising candidate as they are good in a short length regime.

2. Problem formulation

Based on results for single-user polar codes usage, the task was to expand decoding system for Polar Codes on 2-user MAC and make one's best to optimize it.

The following problem, risen up in [2], formulation:

- 2-user's code words are represented as binary vectors: $u = [u_1, u_2, ..., u_i, ..., u_N]$ и $v = [v_1, v_2, ..., v_i, ..., v_N]$
- BPSK-modulator's output: $u_1, ..., u_i \in \{-P; +P\}$
- Channel output vector is a ternary vector $y = [y_1, y_2, ..., y_i]$, where every y_i is calculated as elemental sum of 2-users code vectors with AWGN added.
- Users are similar and indistinguishable,there is no use of indentifying which of users transmits.
- The task of joint multi-user decoding is the restoration of all code words for both users, based on the received vector y.

3.1 Proposed solution

The first implementation of decoding systems was based on formulas, generalized in case of 2 users from single user formulas, for probabilities $W_{2N}^{(i)}(y_1^{2N}, u_1^{(i-1)} v_1^{(i-1)} | u_{(i)} v_{(i)})$, where $y_1^N = [y_1, ..., y_N]$.

From this probabilities SC decoder generates estimation $\hat{u}_1^N$ and $\hat{v}_1^N$ of vectors u_1^N and v_1^N respectively. Due to polar coding channel polarization, solution for odd and even bits are calculated separately by recursive formulas, represented below:

$$W_{2N}^{(2i-1)}(y_1^{2N}, u_1^{2i-2}, v_1^{2i-2} | u_{2i-1},\ v_{2i-1}) =$$

$$\sum_{u_{2i}, v_{2i}} \frac{1}{4} W_N^{(i)}(y_1^N, u_{1,e}^{2i-2} \oplus u_{1,o}^{2i-2}, v_{1,e}^{2i-2} \oplus v_{1,o}^{2i-2} | u_{2i-1} \oplus u_{2i}, v_{2i-1} \oplus v_{2i}) W_N^{(i)}(y_{N+1}^{2N}, u_{1,e}^{2i-2}, v_{1,e}^{2i-2} | u_{2i}, v_{2i})$$

$$W_{2N}^{(2i)}(y_1^{2N}, u_1^{2i-1}, v_1^{2i-1} | u_{2i},\ v_{2i}) =$$

$$\frac{1}{4} W_N^{(i)}(y_1^N, u_{1,e}^{2i-2} \oplus u_{1,o}^{2i-2}, v_{1,e}^{2i-2} \oplus v_{1,o}^{2i-2} | u_{2i-1} \oplus u_{2i}, v_{2i-1} \oplus v_{2i}) W_N^{(i)}(y_{N+1}^{2N}, u_{1,e}^{2i-2}, v_{1,e}^{2i-2} | u_{2i}, v_{2i})$$

The solution of decoder is based on Likelyhood Ratio (LR), which equals:

$$L_N(X, Y) = \frac{W(X,\ Y)}{W(0,\ 0)}$$

where:

$$W(X,\ Y) = \begin{cases} W_N^{(2i-1)}(y_1^N, u_1^{2i-2}, v_1^{2i-2} | X,\ Y) & \text{, for odd} \\ W_N^{(2i)}(y_1^N, u_1^{2i-1}, v_1^{2i-1} | X,\ Y) & \text{, for even} \end{cases}$$

The realization of this decoder was done during this research. It has a complexity of $O(N^{m+2})$, where m - number of users, N - length of code.

Further research was devoted to decoder's structure optimization. Continuing to transform formulas, we can write: for even:

$$L_N^e(X,\ Y) = \frac{W_N^{(2i)}(y_1^N, u_1^{2i-1}, v_1^{2i-1} | X,\ Y)}{W_N^{(2i)}(y_1^N, u_1^{2i-1}, v_1^{2i-1} | 0,\ 0)}$$

Let's introduce the notation:

$$W_{N/2}^{(i)}(y_1^{N/2}, u_{1,e}^{2i-2} \oplus u_{1,o}^{2i-2}, v_{1,e}^{2i-2} \oplus v_{1,o}^{2i-2} | X,\ Y) = W_{N_1}^e(X,\ Y)$$

another notation is:

$$W_{N/2}^{(i)}(y_{N/2+1}^N, u_{1,e}^{2i-2}, v_{1,e}^{2i-2} | X,\ Y) = W_{N_2}^e(X,\ Y)$$

then (same for L_{N_2}):

$$L_{N_1}^e(X,\ Y) = \frac{W_{N_1}^e(X,\ Y)}{W_{N_1}^e(0,\ 0)}$$

Final expression for even:

$$L_N^e(X,\ Y) = L_{N_2}^e(X,\ Y) \frac{L_{N_1}^e(u_{2i-1} \oplus X,\ v_{2i-1} \oplus Y)}{L_{N_1}^e(u_{2i-1}, v_{2i-1})}$$

Continuing transformations, we make same calculations for odd situation, assuming that:

$$L_N^o(X,\ Y) = \frac{\sum_{u_{2i}, v_{2i}} W_{2N}^{(2i-1)}(y_1^{2N}, u_1^{2i-1}, v_1^{2i-1} | X,\ Y)}{\sum_{u_{2i}, v_{2i}} W_{2N}^{(2i-1)}(y_1^{2N}, u_1^{2i-1}, v_1^{2i-1} | 0,\ 0)}$$

Let the notation be introduced:

$$W_{N_1}^o(X,\ Y) = W_N^{(i)}(y_1^N, u_{1,e}^{2i-2} \oplus u_{1,o}^{2i-2}, v_{1,e}^{2i-2} \oplus v_{1,o}^{2i-2} | X \oplus u_{2i}, Y \oplus v_{2i})$$

$$W_{N_2}^o(X,\ Y) = W_N^{(i)}(y_{N+1}^{2N}, u_{1,e}^{2i-2}, v_{1,e}^{2i-2} | X,\ Y)$$

Same as for even, we denote $L_{N_1}^o$ and $L_{N_2}^o$:

$$L_{N_1}^o = \frac{W_{N_1}^o(X,\ Y)}{W_{N_1}^o(0,\ 0)}$$

3.2 Proposed solution

Final expression for even situation:

$$L_N^o(X,\ Y) = \frac{L_{N_1}(0 \oplus X,\ 0 \oplus Y) + L_{N_1}(0 \oplus X,\ 1 \oplus Y) L_{N_2}(0,\ 1) + L_{N_1}(1 \oplus X,\ 0 \oplus Y) L_{N_2}(1,\ 0)}{1 + L_{N_1}(0,\ 1) L_{N_2}(0,\ 1) + L_{N_1}(1,\ 0) L_{N_2}(1,\ 0) + L_{N_1}(1,\ 1) L_{N_2}(1,\ 1)} +$$
$$+ \frac{L_{N_1}(1 \oplus X,\ 1 \oplus Y) L_{N_2}(1,\ 1)}{1 + L_{N_1}(0,\ 1) L_{N_2}(0,\ 1) + L_{N_1}(1,\ 0) L_{N_2}(1,\ 0) + L_{N_1}(1,\ 1) L_{N_2}(1,\ 1)}$$

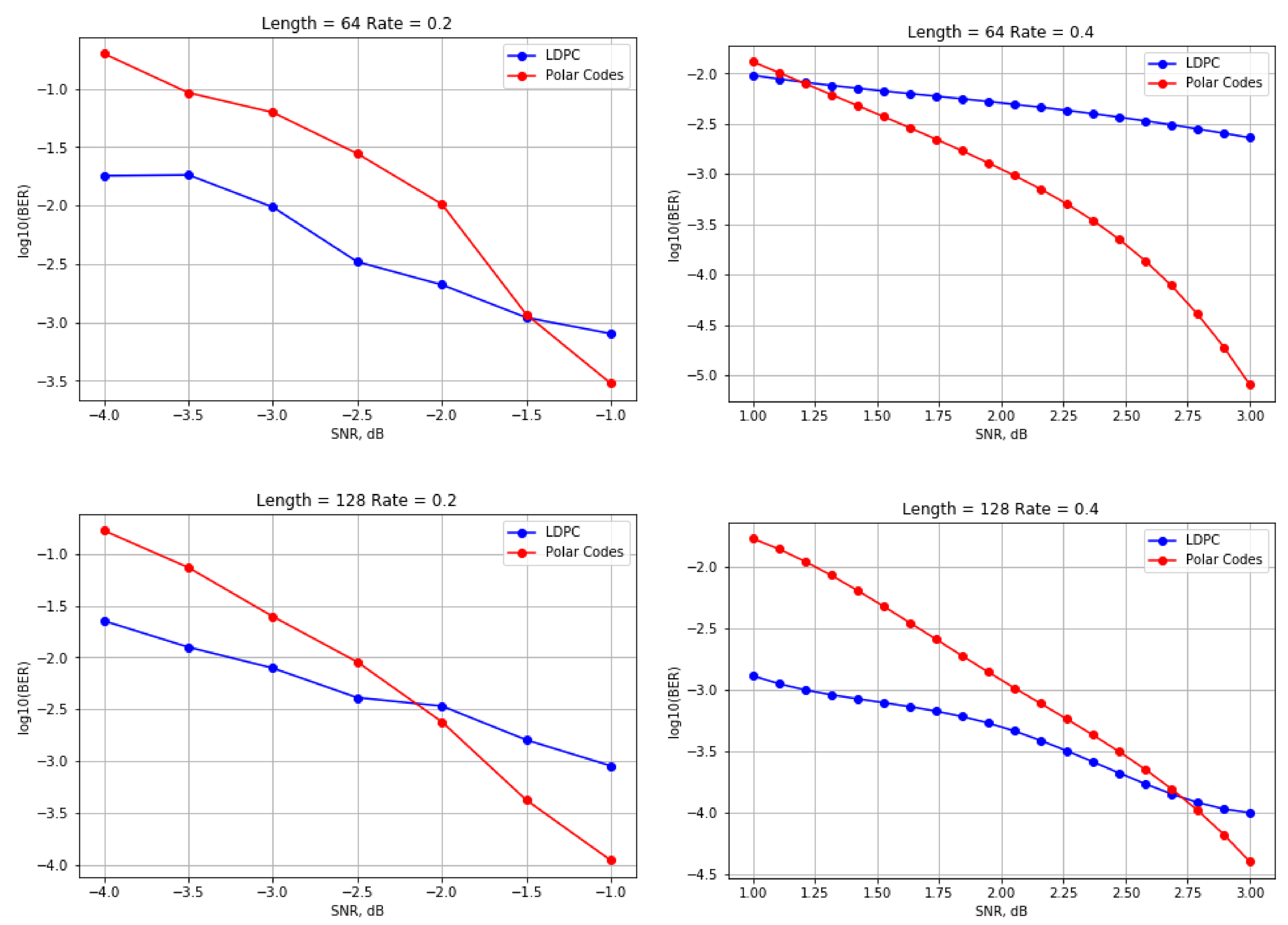
This received type for LR can be transformed to LLR.The second implementation of decoder was based on transformed to LLRs formulas. This realization has a complexity of $O((2^m - 1) N \log N)$. This complexity was achieved by the fact, that while calculating LLR, we can skip some calculations, because of some LLR duplications occur, during the recursive algorithm.

3.3 Decoding algorithm

```
if 1 + L_N^i(0,1) > L_N^i(1,0) + L_N^i(1,1) then
    u_i = 0
    if 1 > L_N(0,1) then
        v_i = 0;
    else
        v_i = 1;
    end if
else
    u_i = 1;
    if L_N^i(1,0) > L_N^i(1,1) then
        v_i = 0;
    else
        v_i = 1;
    end if
end if
```

4. Results

The following decoding system based on polar codes, was compared to NOMA LDPC constructions. A comparison on lengths of 64 and 128 at 0.2 and 0.4 (for single person) rate is given below.



6. Further research

Field of futher research will be done in ways, represented below:

- Scalability research: to find out the way to expand decoding system to m-users and (if it is possible) execute and optimize the realisation of that type of decoder.
- Optimization: to find out new ways to improve optimization, to implement list decoding to to the current decoder,

7. References

[1] E. Arıkan, "Channel polarization: A method for constructing capacityachieving codes for symmetric binary-input memoryless channels," IEEE Trans. Inf. Theory, vol. 55, pp. 3051 –3073, July 2009.

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[3] E. Arıkan, "Polar coding for the slepian-wolf problem based on monotone chain rules," in Proc. IEEE Int'l Symp. Inform. Theory (ISIT'2012), (Cambridge, MA, U.S.A.), pp. 566 –570, July 2012.

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