Normalization – Shadows1

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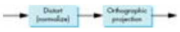
Viewing – Projection
Repeated & extended

Projection normalization



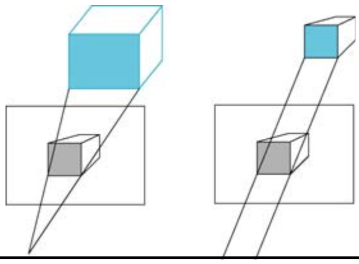
- Canonical View Volume

$x \pm 1$

$y \pm 1$

$z \pm 1$

- Normalization matrix
- Projection matrix

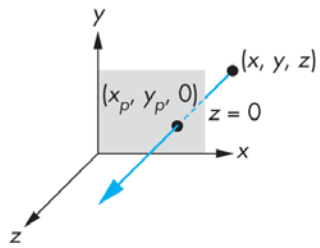


Orthographics

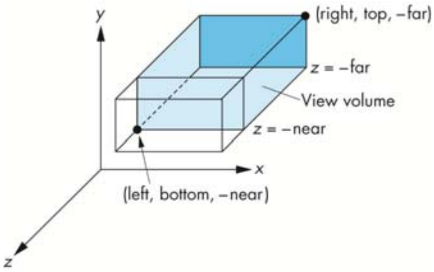
$x_s = x_e$

$y_s = y_e$

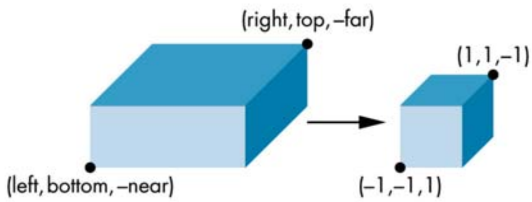
$$\mathbf{P}_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

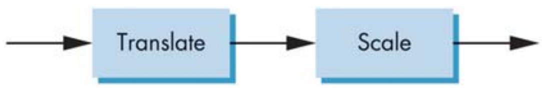


Orthographic - View Volume



Normalisation - Orthographic





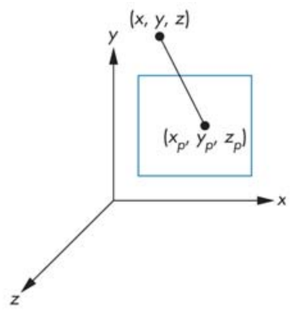
$$\mathbf{T}\left(-\frac{\text{right}+\text{left}}{2}, -\frac{\text{top}+\text{bottom}}{2}, -\frac{\text{far}+\text{near}}{2}\right)$$

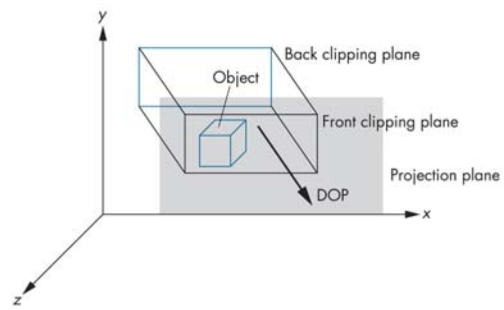
$$\mathbf{S}\left(\frac{2}{\text{right - left}}, \frac{2}{\text{top - bottom}}, -\frac{2}{\text{far - near}}\right)$$

$$\mathbf{ST} = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & -\frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & \frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oblique parallel projection



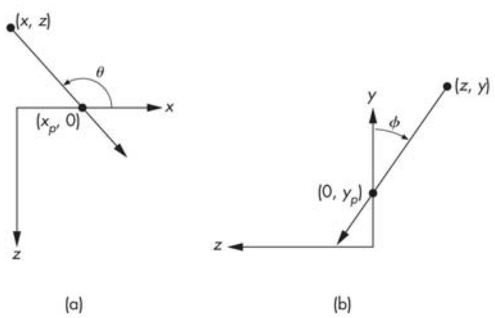


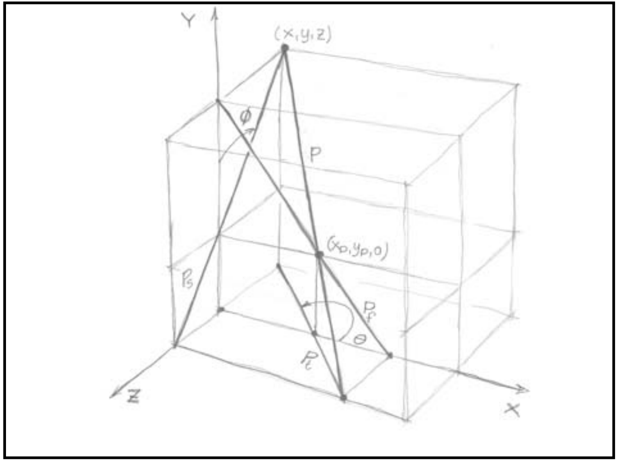
$$\mathbf{M}_{orth} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

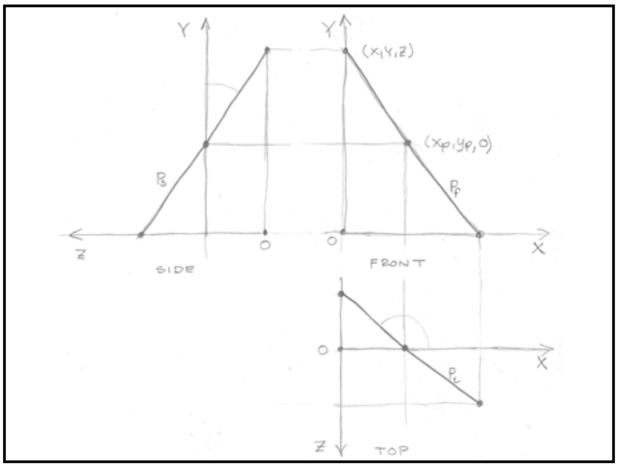
$$\mathbf{P} = \mathbf{M}_{orth}\mathbf{H}(\theta, \varphi)$$

$$\mathbf{P} = \mathbf{M}_{orth}\mathbf{STH}(\theta, \varphi)$$ - with normalization

$\mathbf{H}(\theta, \varphi)$







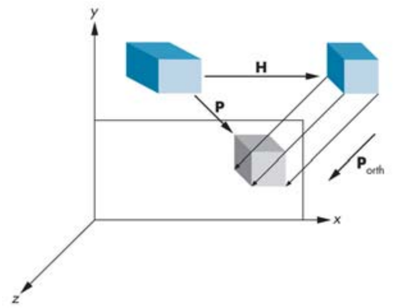
$$\tan\theta = \frac{z}{x_p - x} \Rightarrow$$

$$x_p = x + z\cot\theta$$

$$y_p = y + z\cot\phi$$

$$z_p = 0$$

$$\mathbf{P}_{oblique} = \begin{bmatrix} 1 & 0 & \cot\theta & 0 \\ 0 & 1 & \cot\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\mathbf{P} = \mathbf{M}_{orth}\mathbf{H}(\theta,\phi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & \cot\theta & 0 \\ 0 & 1 & \cot\phi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & \cot\theta & 0 \\ 0 & 1 & \cot\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P} = \mathbf{M}_{orth}\mathbf{H}(\theta,\phi)$$

$$\mathbf{P} = \mathbf{M}_{orth}\mathbf{S}\,\mathbf{T}\,\mathbf{H}(\theta,\phi)$$

Oblique

$$\mathbf{H}(\theta,\varphi) = \begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\varphi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

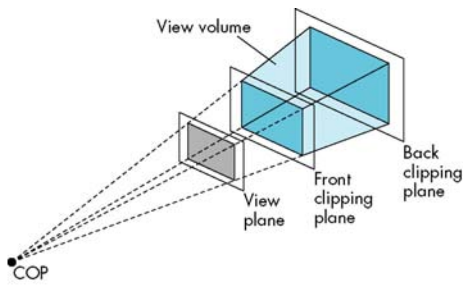
$$\mathbf{P} = \mathbf{P}_{orth}\mathbf{H}(\theta,\varphi)$$

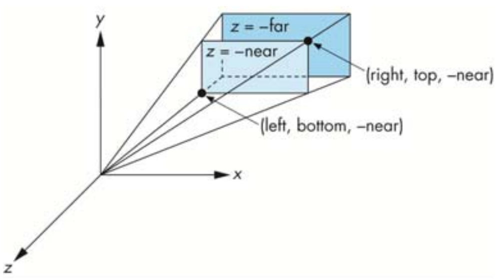
$$\mathbf{P} = \mathbf{P}_{orth}\mathbf{STH}(\theta,\varphi) \quad -\text{also normaliset}$$

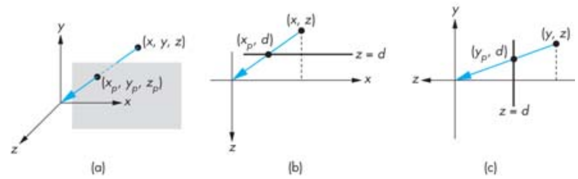
$$\mathbf{ST} = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & -\frac{2}{far-near} & \frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

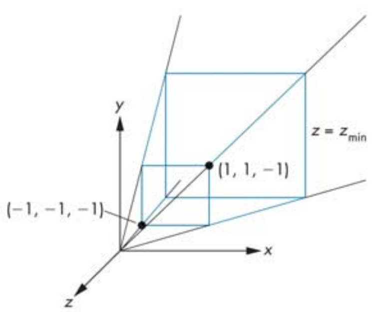
$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & \frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Central projection









Perspective Normalization - Frustum 45º

$x = \pm z$

$y = \pm z$

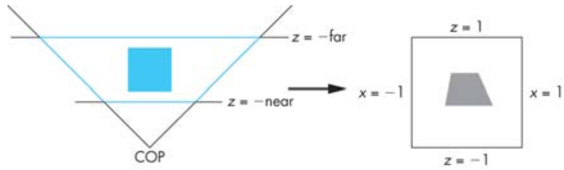
$z = -1$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

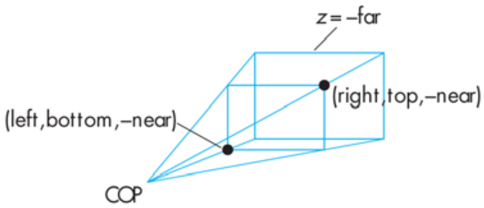
Perspective normalization matrix

$$
\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}
\qquad
\alpha = \frac{near + far}{near - far}
\qquad
\beta = \frac{2 \cdot near \cdot far}{near - far}
$$

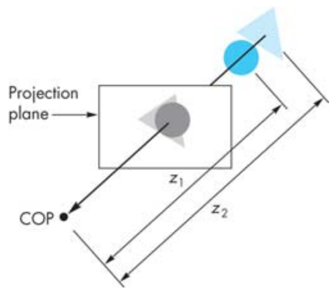
Normalization - perspective



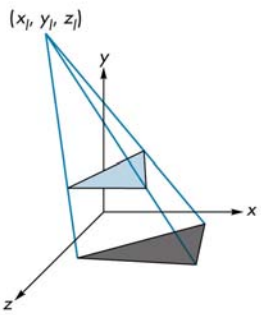
OpenGL - notation



Hidden surface removal
Z-buffer



Shadows – point light



Shadows - projection

$T(-x_l, -y_l, -z_l)$

$$
\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{-y_l} & 0 & 0 \end{bmatrix}
$$

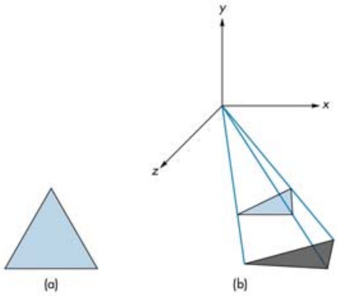
$T(x_l, y_l, z_l)$

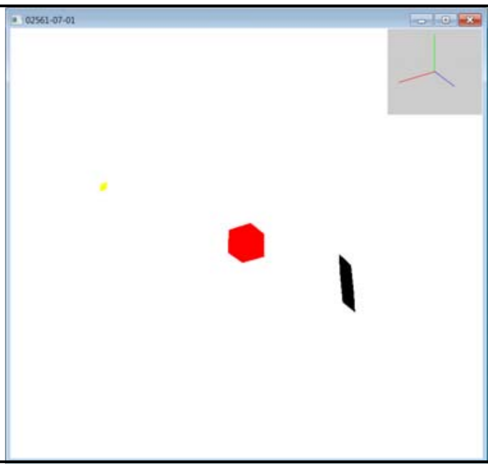
$$x_p = x_l - \frac{x - x_l}{(y - y_l)/y_l}$$

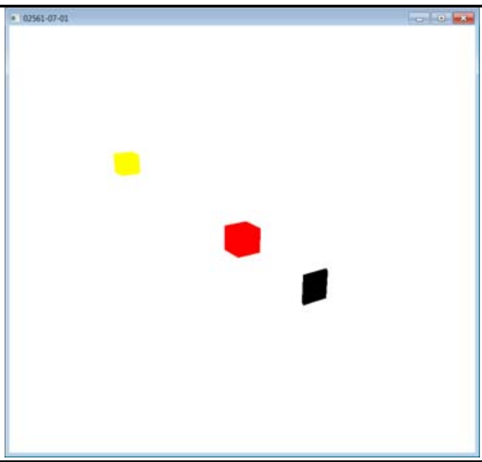
$$y_p = 0$$

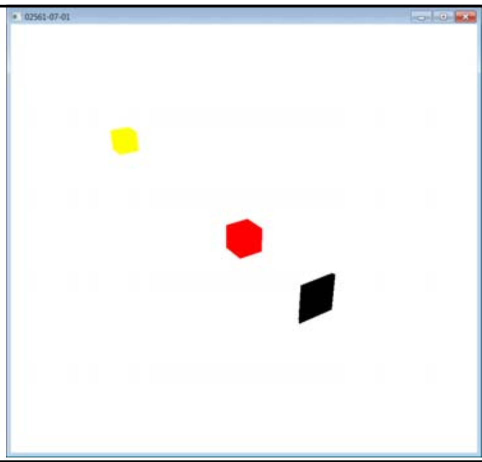
$$z_p = z_l - \frac{z - z_l}{(y - y_l)/y_l}$$

Lightsource in origo









Code - exemple

- Angel p. 281-282