Problem 1 **4 Points**

Derive the following difference-formulas by differentiating a interpolation parabola.

E a) $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$.

E b) $f''(x) \approx \frac{f(x-h)-2f(x)+f(x+h)}{h^2}$.

E c) $f'(x) \approx \frac{1}{h}\left(\frac{1}{2}\,f(x-2h) - 2\,f(x-h) + \frac{3}{2}\,f(x)\right)$.

H) $f'(x) \approx \frac{1}{h}\left(-\frac{3}{2}\,f(x) + 2\,f(x+h) - \frac{1}{2}\,f(x+2h)\right)$

E d) Estimate the error between the derivative and the difference-formula for (E a) and (E b).

Problem 2 **4 Points**

E) Consider the following quadrature formula

$$\int_a^b f(x)\,dx \;\approx\; (b-a)\left(\gamma_1\,f(a) + \gamma_2\,f(\frac{a+b}{2}) + \gamma_3\,f(b)\right). \qquad (*)$$

Determine the weights $\gamma_1, \gamma_2, \gamma_3$ such that the quadrature formula is exact for all polynomials of degree ≤ 2.

H a) Check that the formula $(*)$ with the weights computed in (E) is even exact for all polynomials of degree ≤ 3.

H b) Consider the following quadrature formula

$$\int_0^1 f(x)\,dx \;\approx\; \gamma_1\,f(0) + \gamma_2\,f(\frac{1}{3}).$$

Determine the weights γ_1, γ_2 such that the quadrature formula is exact for all polynomials of degree ≤ 1. Is it also exact for polynomials of degree 2?

Programming Assignment 1

Write a function `drivativeplot(f,a,b,n,h)`, which computes and plots the graphs of f, f', f'' in the interval $[a, b]$. Here n is the number of interpolation points. Use the difference formulas to compute the derivatives

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}, \qquad f''(x) \approx \frac{f(x-h)-2f(x)+f(x+h)}{h^2},$$

and test your program with the functions $f(x) = \sin(x)$ and $f(x) = x^3$.

Programming Assignment 2

Write a function `[T,S]=integral(f,a,b,n)`, which computes the integral $\int_a^b f(x)\,dx$ with the trapezoidal sum rule (returned in `T`) and with the Simpson sum rule (returned in `S`). Here `n` is the number of control points, i. e. the length of the interval is `h=(b-a)/(n-1)`.