# MAT 460 – Numerical Differential Equations – Solutions to Assignment 4 (E)

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**Problem 1**

E a) Hornerschema for $p(x) = 5x^4 + 3x^3 - 30x^2 + 7x + 8, \quad x_0 = 2$:

| | | | | |
|---|---|---|---|---|
| 5 | 3 | -30 | 7 | 8 |
| 0 | 10 | 26 | -8 | -2 |
| 5 | 13 | -4 | -1 | 6 |

$\Rightarrow \qquad p(x) = (5x^3 + 13x^2 - 4x - 1)(x-2) + 6$, $p(2) = 6$.

**Problem 2**

1) General solution: We write the system of equations

$$f_j = p(x_j) = a_3 x_j^3 + a_2 x_j^2 + a_1 x_j + a_0.$$

in matrix-vector form:

$$\begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}.$$

This matrix is called Vandermonde-matrix.

The Lagrange-representation is

$$p(x) = f_1\, L_1(x) + f_2\, L_2(x) + f_3\, L_3(x) + f_4\, L_4(x)$$

with the Lagrange-basis polynomials

$$\begin{aligned} L_1(x) &= \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} \\ L_2(x) &= \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} \\ L_3(x) &= \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} \\ L_4(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} \end{aligned}$$

**Problem 3**

E a) From the table of the divided differences

| $x_j$ | $f_j$ | | | |
|---|---|---|---|---|
| 3 | 6 | 2 | 5 | $-7$ |
| 1 | 2 | $-13$ | 40 | |
| 0 | 15 | $-133$ | | |
| $-2$ | 281 | | | |

$\Rightarrow$ we obtain the interpolation polynomial

$$p(x) = 6 + 2(x-3) + 5(x-3)(x-1) - 7(x-3)(x-1)x\,.$$

E b) The only difference between this problem and problem E a) is that we have one additional interpolation point. So we add entries on the diagonal below the table from E a) (see the lecture slides):

$$\begin{array}{|c|c|c|c|c|c|}
x_j & f_j & & & & \\
3 & 6 & 2 & 5 & -7 & 3/2 \\
1 & 2 & -13 & 40 & -4 & \\
0 & 15 & -133 & 24 & & \\
-2 & 281 & -13 & & & \\
5 & 190 & & & &
\end{array}$$

$\Rightarrow$ and obtain the interpolation polynomial

$$p(x) = 6 + 2(x-3) + 5(x-3)(x-1) - 7(x-3)(x-1)x + \frac{3}{2}(x-3)(x-1)x(x+2)\,.$$

E c) From the table of the divided differences

$$\begin{array}{|c|c|c|c|c|}
x_j & f_j & & & \\
-2 & 281 & -133 & 40 & -7 \\
0 & 15 & -13 & 5 & \\
1 & 2 & 2 & & \\
3 & 6 & & &
\end{array}$$

$\Rightarrow$ we obtain the interpolation polynomial

$$p(x) = 281 - 133(x+2) + 40(x+2)x - 7(x+2)x(x-1).$$

This polynomial is the same as in E a), in a different notation.