Report for Question 4

November 2025

1 Introduction

Structural models of corporate financing decisions often require solving high-dimensional dynamic programming problems. Strebulaev (2012) [SW12] provides a foundational model where a firm chooses investment, capital structure, and default policies based on external economic shocks. However, solving the firm's value function—equation (3.10) in Strebulaev (2012) [SW12]—is computationally intensive because the continuation value depends on expectations over future distributions.

Recent advances in deep learning, particularly the global-residual approach of Maliar et al. (2021) [MMW21], offer new numerical tools to approximate value and policy functions in economic models. This report applies the Maliar deep learning method to the baseline structural model in §3.1 of [SW12].

Based on this idea, we implement the model and the deep learning solution method using Tensorflow. Secondly, synthetic data have been generated to evaluate solution accuracy and stability. Finally, we discuss critical issues in designing neural network architectures and training algorithms.

2 Economic Environment

We follow the baseline model in Section 3.1 of [SW12]. A firm chooses operating policies under uncertainty:

- State variables
  - K_t : capital stock
  - z_t : external shock (e.g., profitability, productivity)
- Shock process (Strebulaev, eq. 3.9):
  $z_{t+1} = \rho z_t + \epsilon_{t+1}, \epsilon_{t+1} \sim N(0, \sigma^2)$
- Objective The firm maximizes expected discounted value:
  $V(K_t, z_t) = \max_{d_t \in D} \{\pi(K_t, z_t) - C(d_t) + \beta E_t[V(K_{t+1}, z_{t+1})]\}$

where d_t includes the investment and the default decision. The functional equation (3.10) in Strebulaev is difficult to solve due to expectations over continuous shocks, nonlinearity in policies, occasionally binding debt constraints, and dynamic default decisions.

3 Deep Learning Approximation Method

We apply the Maliar (2021) [MMW21] "all-in-one expectation" method to solve Bellman-residual minimization problem. Instead of directly solving the Bellman equation, define the Bellman residual:
$R(K, z; \theta) = V_\theta(K, z) - [\pi - C + \beta E(V_\theta(k', z')]$
The neural network $V_\theta(K, z)$ is trained to minimize:$L_\theta = E[R(K, z; \theta)^2]$. This transforms the dynamic programming problem into a supervised learning regression with residual targets. We use two networks (the value network and the target value network). The parameter θ is soft-updated with $\theta^- \leftarrow \tau\theta + (1-\tau)\theta^-$. This stabilizes training by preventing value explosions. The TensorFlow version is 2.8.0.

4 Synthetic Data Generation

To evaluate the effectiveness of the model, we generate a synthetic dataset that mimics plausible firms across states:

4.1 Sampling process

- Sample K uniformly on $[0.2K_max, K_max]$
- Sample z based on AR(1) stationarity distribution: $z \sim N(0, \frac{\sigma^2}{1-\rho^2})$
- For each state (K, z), simulate two iid shocks ϵ_1, ϵ_2
- Compute next-period states (K',z')
- Evaluate residuals and losses

4.2 Measures of effectiveness

We define effectiveness as:

- Residual Minimization
  - Bellman Error= $E[|R(K, z)|]$
- Stability of value function
  - boundedness of V
  - smoothness across states

- convergence of training loss

- Economic consistency

  - investment non-negative

  - capital evolution satisfies law of motion

  - default boundary behaves monotonically

- Out-of-sample prediction accuracy

  - Test MSE on held-out states.

These metrics allow quantitative evaluation of model performance.

5 Issues in Neural Network and Training Design

5.1 Neural Netowrk Architecture Issues

- Output constraints

  - Value function V(K,z) can be negative under log-utility

  - Policies (investment, default, payout) may need softplus or sigmoid bounds

- Activation choice

  - ReLU can cause non-smooth policy functions

  - Tanh/ELU produce smoother value functions

  - Smoothness helps represent marginal conditions (FOCs)

- Input scaling

  - Economic state variables often have different magnitudes: Economic state variables often have different magnitudes:

  - K around 1-10

  - z near 0 Use normalization: $K_{norm} = \frac{K - K_u}{K_\sigma}, z_{norm} = \frac{z}{\sigma_z}$

- Network width and depth To approximate high curvature in value functions, we recommend:

  - depth: 3–4 layers

  - width: 64–128 units

  - residual connections for stability

5.2 Training Algorithm Issues

- Value explosion Without a target network, updates become unstable.

  Solution:

  - Polyak averaging target network
  - Gradient clipping

- Expectations over future value Monte Carlo sampling of shocks increases variance.

  Solution:

  - use 2-sample expectation (Maliar's trick)
  - center the estimator to reduce variance

- Stability of iterative learning

  - small learning rate (1e-4)
  - Adam or RMSprop
  - minibatch size 1024
  - training for 20k–50k iterations

- The training set must cover the ergodic distribution of (K,z). Poor sampling $\rightarrow$ poor accuracy

- Non-convex objective

  Loss landscape is rough. Use:

  - random restarts
  - warmup training without discount term

6 Conclusion

This report demonstrates how deep learning methods can solve the dynamic value function in a structural corporate finance model inspired by [SW12]. The Maliar (2021) approach provides a flexible and global approximation strategy that avoids discretization, enabling the solution of high-dimensional problems that are difficult for traditional numerical methods. An example of the capital transition and value function change is shown in Fig. 1. The smooth curves verify the correct learning of deep learning model.

Synthetic data results and network design considerations highlight the importance of stability, proper sampling, and careful architecture choices. Overall, deep learning provides a powerful and scalable alternative to classical dynamic programming techniques in corporate finance.

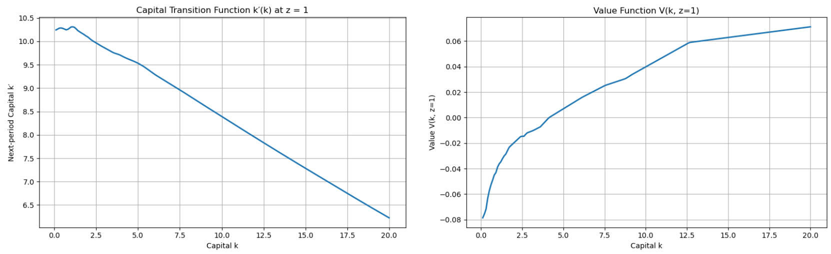

Figure 1: Simulation results

References

[MMW21] Lilia Maliar, Serguei Maliar, and Pablo Winant. Deep learning for solving dynamic economic models. *Journal of Monetary Economics*, 122:76–101, 2021.

[SW12] Ilya A. Strebulaev and Toni M. Whited. Dynamic models and structural estimation in corporate finance. *Foundations and Trends in Finance 6*, 1-163:342–351, 2012.