

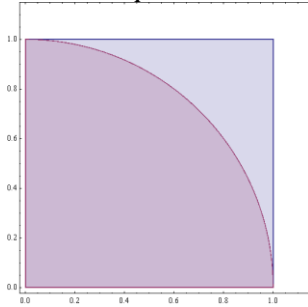
EE 511 Simulation Methods for Stochastic Systems

Project #5: MC & MCMC

[Due 2-Dec-2016]

Problem 1 [Monte Carlo]

Generate $n=100$ samples of i.i.d 2-dimensional uniform random variables in the unit-square. Count how many of these samples fall within the quarter unit-circle centered at the origin. This quarter circle inscribes the unit square as shown below:



- Use these random samples to estimate the area of the inscribed quarter circle. Use this area estimate to estimate the value of π . Do $k=50$ runs of these π -estimations. Plot the histogram of the 50 π -estimates.
- Repeat the experiment with different numbers of uniform samples, n (using $k=50$ for all these runs). Plot the sample variance of the π -estimates for these different values of n . What is the relationship the estimate variance and Monte Carlo sample size?
- Adapt your Monte Carlo solution to provide integral and **error estimates** for the function:
$$g(x, y) = |4x - 2| \times |4y - 2| \quad x, y \text{ in } [0, 1]$$

Problem 2 [Variance Reduction Methods for Monte Carlo]

Use a total sample budget of $n=1000$ to obtain Monte Carlo estimates and sample MC estimate variances for the definite integrals in 2 dimensions (x_1, x_2) :

- $\exp(\sum_{i=1}^2 5|x_i - 0.5|)$ for x_i in $[0,1]$
- $\cos(\pi + \sum_{i=1}^2 5x_i)$ for x_i in $[-1,1]$
- $|4x - 2| \times |4y - 2|$ for x, y in $[0,1]$

Implement *stratification* and *importance sampling* (separately) in the Monte Carlo estimation procedures using the same sample budget $n=1000$. Compare the 3 different Monte Carlo integral estimates and their sample variances. Discuss the quality of the Monte Carlo estimates from each method.

Problem 3 [MCMC for Optimization]

The n -dimensional Schwefel function

$$f(\vec{x}) = 418.9829n - \sum_{i=1}^n x_i \sin \sqrt{|x_i|}$$

x_i in $[-500, 500]$

is a very bumpy surface with many local critical points and one global maximum. We will explore the surface for the case $n=2$ dimensions.

- Plot a contour plot of the surface for the 2-D surface
- Implement a simulated annealing procedure to find the global minimum of this surface
- Explore the behavior of the procedure starting from the origin with an exponential, a polynomial, and a logarithmic cooling schedule. Run the procedure for $t=\{20, 50, 100, 1000\}$ iterations for $k=100$ runs each. Plot a histogram of the function minima your procedure converges to.
- Choose your best run and overlay your 2-D sample path on the contour plot of the Schwefel function to visualize the locations your optimization routine explored.

Turn in:

- Analyses & discussions of your findings
 - A summary of your experiments including histograms of your data, sample path plots, and other relevant numerical results
 - A print out of your code.
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