Problem 1 [Waiting]

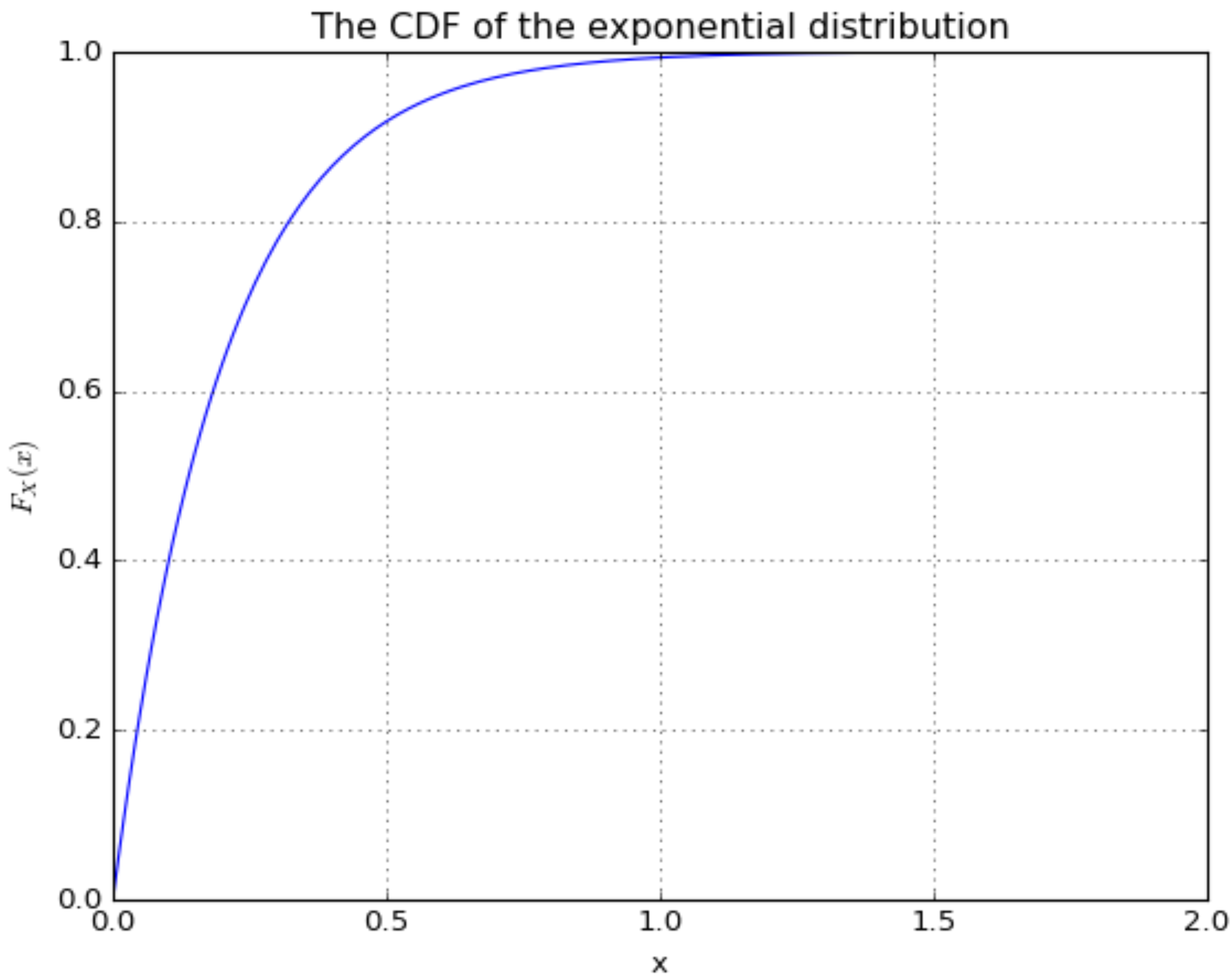
> Use the inverse CDF method to generate independent samples, X_i , of the exponential random variable with average waiting time of 0.2 time units. Evaluate the quality of your generator with goodness of fit tests.

The exponential distribution with $\lambda = \frac{1}{0.2} = 5$ has the cdf:

$$F_X(x) = \begin{cases} 1 - e^{-5x}, & x \geqslant 0 \\ 0, & x < 0 \end{cases}$$

In [1]:

```
%matplotlib notebook
import numpy as np
import matplotlib.pyplot as plt
theta = 0.2
x = np.arange(0,2,0.01)
y = 1-np.exp(-1/theta*x)
plt.figure()
plt.plot(x,y)
plt.title('The CDF of the exponential distribution')
plt.xlabel('x')
plt.ylabel('$F_X(x)$')
plt.grid()
plt.show()
```

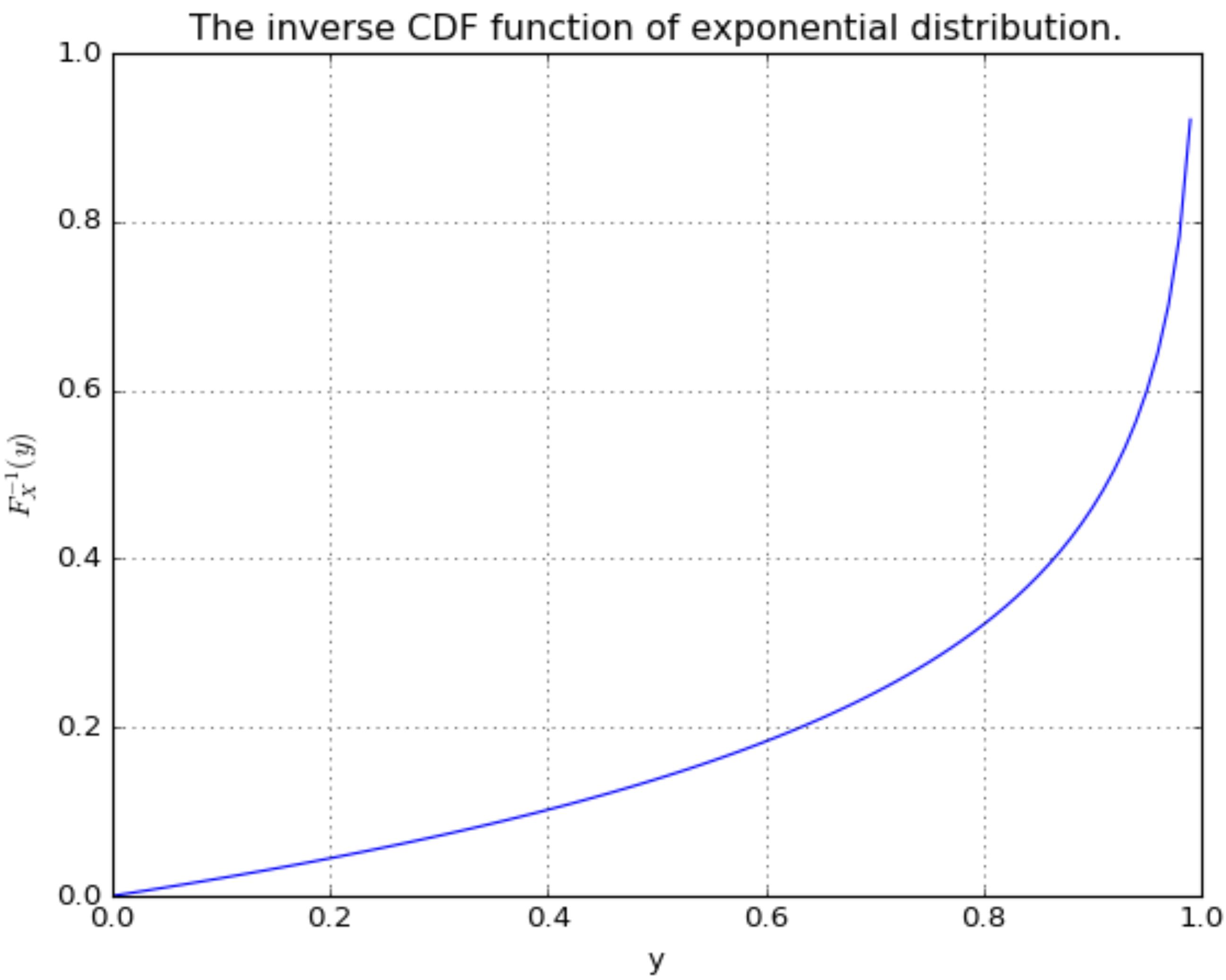


The inverse of it will be,

$$F^{-1}(y) = -0.2log(1 - y),\ 0 \leqslant y \leqslant 1$$

In [2]:

```
x = np.arange(0,1,0.01)
y = -theta*np.log(1-x)
plt.figure()
plt.plot(x,y)
plt.grid()
plt.title('The inverse CDF function of exponential distribution.')
plt.xlabel('y')
plt.ylabel('$F_X^{-1}(y)$')
plt.show()
```



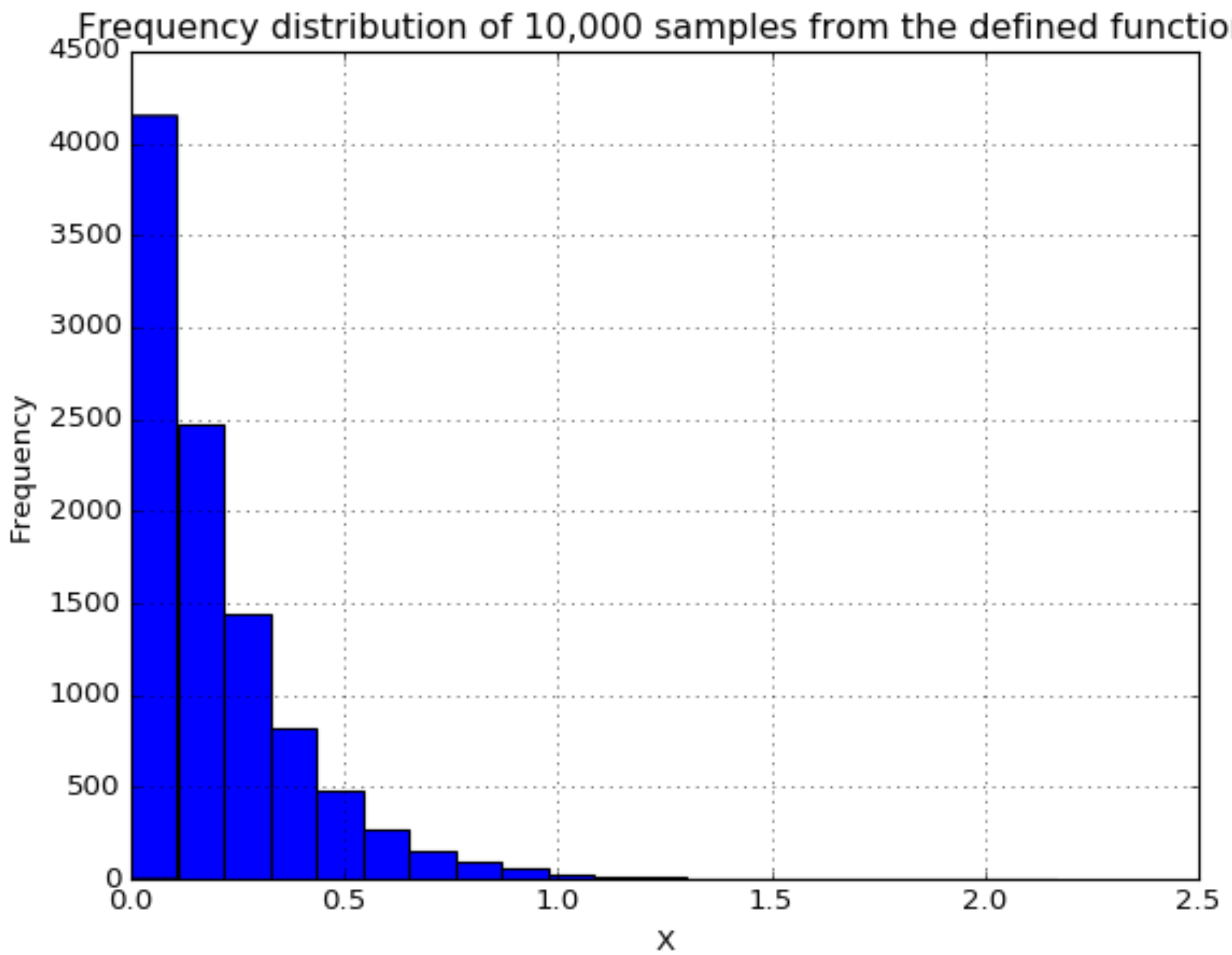
The inverse CDF method has the following procedures,

1. Generate the random variable $Y \sim U(0, 1)$
2. Use the inverse CDF function $F_X^{-1}(y)$ to get the output value.

In [3]:

```
def my_exp_rv(row = 1,column = 1):
    y = np.random.rand(row, column)
    X = -theta*np.log(1-y)
    return X

x = my_exp_rv(10000,1)
plt.figure()
plt.hist(x, bins = 20)
plt.title('Frequency distribution of 10,000 samples from the defined function'
)
plt.xlabel('X')
plt.ylabel('Frequency')
plt.grid()
plt.show()
```



We define the funtion 'my_exp_rv' to generate the RV that we want for 10,000 times. Next, we use the chi-square to test the goodness of fit. We set the 11 intervals as the following:

$$[0, 0.1), [0.1, 0.2), [0.2, 0.3)\ldots[0.8, 0.9), [0.9, 1.0), [1.0, +\infty]$$

In [4]:

```
from scipy.stats import chisquare
# Generate the expected frequency.
x_e_1 = np.arange(0,1,0.1)
x_e_2 = np.arange(0.1,1.1,0.1)
p1 = 1-np.exp(-1/theta*x_e_1)
p2 = 1-np.exp(-1/theta*x_e_2)
p_int = p2 - p1
p_int = np.append(p_int, [1-p2[-1]])
ne = p_int * 10000

# Count the number of our experiment.
n, bins = np.histogram(x, bins = np.append(np.arange(0,1.1,0.1),[float('inf')]
))


# Combine the group with sample count smaller than 5 into the smallest group w
ith count > 5.
com_index = np.logical_or(n < 5,ne < 5)
into_index = n[com_index == False].argmin()
new_on = n[com_index==False]
new_en = ne[com_index==False]
new_on[into_index] += sum(n[com_index])
new_en[into_index] += sum(ne[com_index])

# Perform chisquare test.
print(chisquare(new_on, f_exp=new_en))
```

```
Power_divergenceResult(statistic=6.7155531835469127, pvalue=0.7519
9782866514941)
```

As we can see in the above result, the pvalue is greater than the significant level that we often use $\alpha = 0.05$, which means a very good fit to the exponential distribution.