Machine Learning

Quiz 3

Student Name: _____

1. Consider an $D \times N$ matrix $\mathbf{X}$, with $D \ll N$ ("short and wide"), and where **each column sums to zero**.

   (a) (1 point) What are the dimensions of $\mathbf{X}^T\mathbf{X}$ and $\mathbf{X}\mathbf{X}^T$, respectively?

   (b) (1 point) How many **non-zero** eigenvalues do these matrices each have, at most?

   (c) (1 point) How do the non-zero eigenvalues of $\mathbf{X}\mathbf{X}^T$ relate to those of $\mathbf{X}^T\mathbf{X}$?

2. (1 point) Let $\mathbf{x}$ be a D-dimensional random variable with Gaussian distribution $\mathcal{N}(\mathbf{x} \mid \mu, \Sigma)$, be $\mathbf{A}$ a non-singular $D \times D$ matrix, and $\mathbf{b} \in \mathbb{R}^D$. What is the expected value of the random variable $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{b}$?

$$\mathbb{E}[\mathbf{y}] =$$