THE UNIVERSITY OF CHICAGO
Department of Economics
Econ 30200 Problem Set 1

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Due: Friday January 11

1. Prove that if $\succsim$ is a preference relation on $\mathbb{R}^l_+$, then

   (a) $\sim$ and $\succ$ are transitive

   (b) $x \succ y$ and $y \sim z$ imply $x \succ z$

   (c) for all $x, y \in \mathbb{R}^l_+$, exactly one of $x \succ y$, $x \sim y$, $x \prec y$ holds.

2. There are two commodities.

   (a) Sketch some preferences which have a bliss point and which can be represented by a continuous utility function. (A bliss point is a point which is preferred or indifferent to any other point.)

   (b) No matter what his income, Jones will demand fifteen units of commodity one provided he can afford it and provided the price of commodity 2 exceeds that of commodity one. Is this consistent with Axioms 1-5 and also Axioms $4'$ and $5'$?

3. Let the utility function $U(\cdot)$ represent the preference relation $\succsim$ on $\mathbb{R}^l_+$, and let $\phi : \mathbb{R}^1 \to \mathbb{R}^1$ be strictly increasing. Prove that $\phi(U(\cdot))$ represents $\succsim$. Show that this is false if the word "strictly" is deleted.

4. Give an example of preferences on $\mathbb{R}^l_+$ for which there exists a utility function, but no continuous utility function.

5. Let $u(x, y) = \sqrt{xy}$. Show that u_x is strictly decreasing in x; i.e. that u exhibits strictly diminishing marginal utility for x. Exhibit a utility function representing the same preferences, but not satisfying strictly diminishing marginal utility for x.

6. Show that if X is any finite set, and $\succsim$ satisfies Axioms 1 and 2 on X, then $\succsim$ can be represented by a utility function. Provide an example showing what can go wrong if Axiom 1 is not satisfied.

7. Suppose that $\{x^n\}$ and $\{y^n\}$ are two sequences of consumption bundles in $\mathbb{R}^l_+$ converging to x and y respectively. Prove that if $x^n \sim y^n$ for all n and $\succsim$ is complete, transitive, and continuous on $\mathbb{R}^l_+$, then $x \sim y$.

8. Show that if $\succsim$ satisfies Axiom 2 on $\mathbb{R}^l_+$, then for every $x, y \in \mathbb{R}^l_+$, the sets $\sim (x)$ and $\sim (y)$ are either disjoint or equal. (i.e. Distinct indifference curves do not cross.)

9. Consider the preferences on $\mathbb{R}^2_+$ defined by the *discontinuous* utility function

$$u(x,y) = \begin{cases} 2 + xy, & \text{if } xy > 1 \\ 1 + xy, & \text{if } xy = 1 \\ xy, & \text{if } xy < 1 \end{cases}.$$

Are these preferences continuous? If not, why not? If so, display a continuous utility function representing them.

10. Consider two consumers' preferences over bundles in $\mathbb{R}^2_+$. Consumer 1's preferences are represented by the Cobb-Douglas utility function $u(x,y) = xy$. Consumer 2's preferences are identical except for bundles lying along the ray $x = y$. Bundles on this ray are strictly preferred by consumer 2 to distinct bundles on the Cobb-Douglas indifference curve that passes through them, and are also strictly less desirable than all bundles above that indifference curve.

    (a) Assuming that consumer 2's preferences are transitive, prove that they are complete and strictly monotonic.

    (b) Assuming that consumer 2's preferences are complete and transitive, can they be represented by a continuous utility function?

11. Suppose that $\succsim$ is a preference relation on $\mathbb{R}^l_+$. Show that the continuity axiom implies that for all $x, y, z \in \mathbb{R}^l_+$ such that $x \succ y \succ z$, the line segment joining x and z must contain a point that is indifferent to y.