Automating Separation Logic using SMT

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Motivation: Program with SL Specification

```
procedure concat(a: Node, b: Node) returns (res: Node)
  requires lseg(a, null) * lseg(b, null);
  ensures lseg(res, null);
{
  if (a == null)
    return b;

  Node curr := a;

  while (curr.next != null)
    invariant curr != null * lseg(a, curr) * lseg(curr, null);
    curr := curr.next;

  curr.next := b;
  return a;
}
```

pre / postconditions

loop invariants

Separating Conjunction and Inductive Predicates

```
procedure concat(a: Node, b: Node) returns (res: Node)
  requires lseg(a, null) * lseg(b, null);
  ensures lseg(res, null);
{
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  Node curr := a;

  while (curr.next != null)
    invariant curr != null * lseg(a, curr) * lseg(curr, null);
    curr := curr.next;

  curr.next := b;
  return a;
}
```

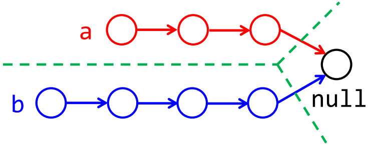

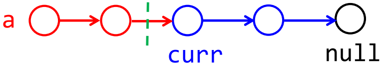

Frame Inference

```
procedure concat(a: Node, b: Node) returns (res: Node)
  requires lseg(a, null) * lseg(b, null);
  ensures lseg(res, null);
{
  if (a == null)
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  Node curr := a;

  while (curr.next != null)
    invariant curr != null * lseg(a, curr) * lseg(curr, null);
    curr := curr.next;

  curr.next := b;
  return a;
}
```
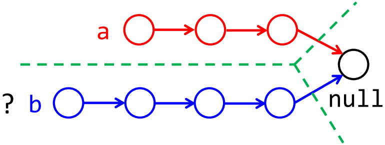
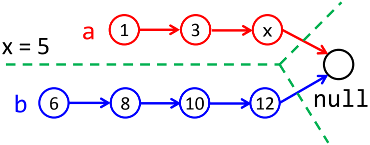
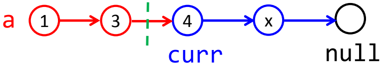



Adding Data

```
procedure concat(a: Node, b: Node) returns (res: Node)
  requires lsleg(a, null, x) * uslseg(b, null, x);
  ensures slseg(res, null);
{
  if (a == null)
    return b;

  Node curr := a;

  while (curr.next != null)
    invariant curr != null;
    invariant lslseg(a, curr, curr.data) * lslseg(curr, null, x);
    curr := curr.next;

  curr.next := b;
  return a;
}
```





Our work

- Reduce a decidable fragment of SL to a decidable FO theory.
- Combining SL with other theories.
- Satisfiability, entailment, frame inference, and abduction problems for SL using SMT solvers.
- Implemented in the GRASShopper tool.

Decidable SL fragment: $\mathsf{SLL}\mathbb{B}$

SLL (separation logic formulas for linked lists) introduced in [Berdine et al., 2004].

SLL

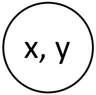
$$\Sigma ::= x = y \mid x \neq y \mid x \mapsto y \mid \mathsf{ls}(x, y) \mid \Sigma * \Sigma$$

With extend SLL to $\mathsf{SLL}\mathbb{B}$ by adding boolean connective on top:

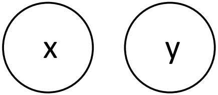
$$H ::= \Sigma \mid \neg H \mid H \wedge H$$

Semantics of SLL$\mathbb{B}$ (1)

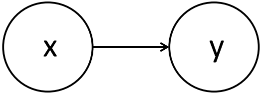
$$\Sigma ::= x = y \mid x \neq y \mid x \mapsto y \mid \mathsf{ls}(x, y) \mid H_1 * H_2$$



footprint = ∅



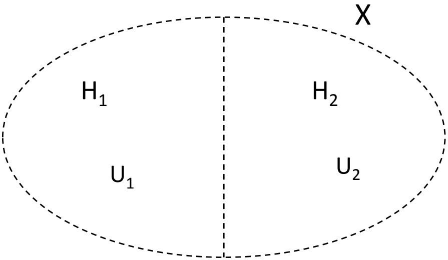
footprint = ∅



footprint = { x }

Semantics of SLL$\mathbb{B}$ (2)

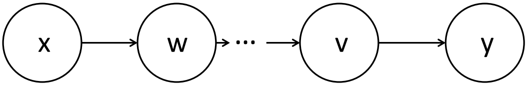
$$\Sigma ::= x = y \mid x \neq y \mid x \mapsto y \mid \mathsf{ls}(x, y) \mid H_1 * H_2$$



important: $\exists U_1, U_2$

Semantics of SLL$\mathbb{B}$ (3)

$$\Sigma ::= x = y \mid x \neq y \mid x \mapsto y \mid \mathsf{ls}(x, y) \mid H_1 * H_2$$



	footprint
$\varnothing$	$\mathsf{ls}^0(y,y)$
	$\mathsf{ls}^1(v,y)$
...	...
	$\mathsf{ls}^{n-1}(w,y)$
	$\mathsf{ls}^n(x,y)$

SLL$\mathbb{B}$ → GRASS

Translate SLL$\mathbb{B}$ to a decidable FO theory.

Requirements:

- easy automation with SMT solvers
- well-behaved under theory combination
- no increase in complexity

GRASS: combination of two theories

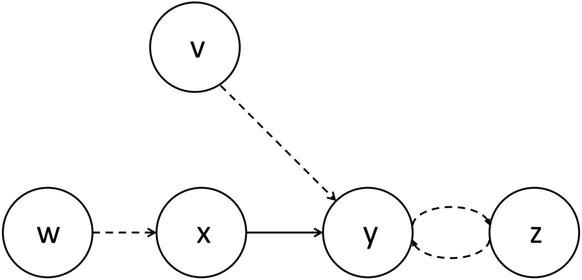
- structure: *functional graph reachability* ($\mathcal{T}_\mathsf{G}$)
  to encode the shape of the heap (pointers)
- footprint: *stratified sets* ($\mathcal{T}_\mathsf{S}$)
  to encode the part of the heap used by a formula

GRASS: graph reachability and stratified sets

$$
\begin{array}{rcll}
 & & \text{graph reachability} & \\
T & ::= & x \mid h(T) & \\
A & ::= & T = T \mid T \xrightarrow{h\setminus T} T & \\
R & ::= & A \mid \neg R \mid R \wedge R \mid R \vee R & \\
 & & & \\
 & & \text{stratified sets} & \\
S & ::= & X \mid \emptyset \mid S \setminus S \mid S \cap S \mid S \cup S \mid \{x.\, R\} & x \text{ not below } h \text{ in } R \\
B & ::= & S = S \mid T \in S & \\
 & & & \\
 & & \text{top level boolean combination} & \\
F & ::= & A \mid B \mid \neg F \mid F \wedge F \mid F \vee F &
\end{array}
$$

$\mathcal{T}_G$: theory of function graphs

$t_1 \xrightarrow{h \backslash t_3} t_2$ is true if there exists a path in the graph of h that connects t_1 and t_2 without going through t_3.



$w \xrightarrow{h} w$ (reflexivity)

$x \xrightarrow{h} y$ (induced by h)

$\neg v \xrightarrow{h} w$ (no path)

$\neg x \xrightarrow{h \backslash y} z$ (y is before z)

$$BtwnWO(w, z) = \{y.w \xrightarrow{h \backslash z} y \wedge z \neq y\} = \{w, x, y\}$$

SLL$\mathbb{B}$ $\rightarrow$ GRASS (1)

Usual way of translating SL to FO:

- structure: $\mathcal{T}_G$ to encode the shape of the heap (pointers)
- footprint: $\mathcal{T}_S$ to encode the part of the heap used by a formula

Negation (entailment check, frame) $\Rightarrow$ more complicated

- structure: uses $\mathcal{T}_G$ and $\mathcal{T}_S$ to encode the shape of the heap (pointers) and disjointness
- set definition: uses $\mathcal{T}_S$ for keep track of the sets that will make the footprint

SLL$\mathbb{B}$ $\rightarrow$ GRASS: interesting cases

$$Tr_X(H) = \text{let } (F, G) = tr_X(H) \text{ in } F \wedge G$$

F is the structure

G is the set definitions.

$$tr_X(\mathsf{ls}(x, y)) = (x \xrightarrow{h} y,\ X = BtwnWO(x, y))$$

$$\begin{aligned} tr_X(\Sigma_1 * \Sigma_2) = \text{let } & Y_1, Y_2 \in \mathcal{X} \text{ fresh} \\ & \text{and } (F_1, G_1) = tr_{Y_1}(\Sigma_1) \\ & \text{and } (F_2, G_2) = tr_{Y_2}(\Sigma_2) \\ & \text{in } (F_1 \wedge F_2 \wedge Y_1 \cap Y_2 = \emptyset,\ X = Y_1 \cup Y_2 \wedge G_1 \wedge G_2) \end{aligned}$$

$$tr_X(\neg H) = \text{let } (F, G) = tr_X(H) \text{ in } (\neg F,\ G)$$

Example: without negation

a non-empty acyclic list segment from x to z

$$x \neq z * x \mapsto y * \mathsf{ls}(y, z)$$

translate to

$$x \neq z \wedge h(x) = y \wedge y \xrightarrow{h} z \wedge Y_2 \cap Y_3 = \emptyset \wedge Y_4 \cap Y_5 = \emptyset \wedge X = Y_1 \wedge Y_1 = Y_2 \cup Y_3 \wedge Y_2 = \emptyset \wedge Y_3 = Y_4 \cup Y_5 \wedge Y_4 = \{x\} \wedge Y_5 = BtwnWO(y, z)$$

Example: with negation

a non-empty acyclic list segment from x to z

$$\neg(x \neq z * x \mapsto y * \mathsf{ls}(y, z))$$

with negation

structure (negated)

$$x = z \vee h(x) \neq y \vee \neg y \xrightarrow{h} z \vee Y_2 \cap Y_3 \neq \emptyset \vee Y_4 \cap Y_5 \neq \emptyset \vee X \neq Y_1$$

set definitions (unchanged)

$$Y_1 = Y_2 \cup Y_3 \wedge Y_2 = \emptyset \wedge Y_3 = Y_4 \cup Y_5 \wedge Y_4 = \{x\} \wedge Y_5 = BtwnWO(y, z)$$

Why is that correct ?

Translation: $Tr_X(H) = \text{ let } (F, G) = tr_X(H) \text{ in } F \wedge G$

the auxiliary variables Y_i (in G) are existentially quantified

below negation, the existential quantifiers should become universal

the Y_i are defined as finite unions of set comprehensions
→ satisfiable in any given heap interpretation $\mathcal{A}$

Due to the precise semantics of SLL$\mathbb{B}$
→ exists exactly one assignment of the Y_i that makes G true in $\mathcal{A}$

$\exists Y_1, \ldots, Y_n.\ F \wedge G$ and
$\forall Y_1, \ldots, Y_n.\ G \Rightarrow F$ are equivalent.

Where are we now ?

With the SLL$\mathbb{B}$ to GRASS translation we can

- Check for satisfiability
- Check entailment (reduces to satisfiability of $H_1 \wedge \neg H_2$)

We also have a translation from GRASS to SLL$\mathbb{B}$:

- compute F in $A \models_{\mathsf{SL}} B * F$ (frame)
- compute F in $A * F \models_{\mathsf{SL}} B$ (antiframe)

Done by model ennumeration $\rightarrow$ not practical
Therefore, we will see another way of doing compositional reasoning.

Implicit frame inference

Idea: let the solver do the frame inference:

$$\forall x.\, x \in Frame \Rightarrow h'(x) = h(x)$$

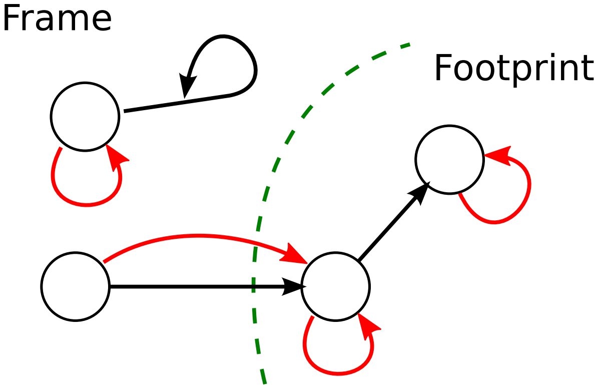
Not that easy: our decision procedure works on partial model.

We need to tell the solver how to update $x \xrightarrow{h} y$:

All *paths* in the frame are preserved.

We add entry points to interface the frame and the footprint.

$ep(FP, x)$ in picture



$ep(FP, x)$

Updating the paths (roughly):

$$\forall x, y, z \in Frame.\, x \xrightarrow{h \setminus ep(FP,x)} y \Rightarrow (Btwn(x, z, y) \Leftrightarrow Btwn'(x, z, y))$$
$$\forall x, y, z.\, x \in Frame \wedge x = ep(FP, x) \Rightarrow (Btwn(x, y, z) \Leftrightarrow Btwn'(x, y, z))$$

Axioms defining the entry point function:

$$\begin{aligned}
&\forall x.\, Btwn(x, ep(FP, x), ep(FP, x)) \\
&\forall x.\, ep(FP, x) \in FP \vee ep(FP, x) = x \\
&\forall x, y.\, Btwn(x, y, y) \wedge y \in FP \Rightarrow \\
&\qquad\qquad ep(FP, x) \in FP \wedge Btwn(x, ep(FP, x), y)
\end{aligned}$$

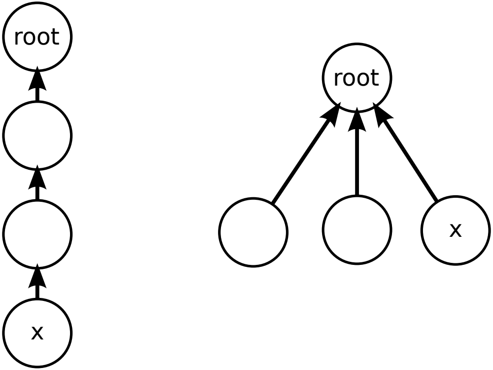
$ep(FP, \_)$ is idempotent (still decidable).

Combination with other theories and extensions

- The theories $\mathcal{T}_\mathsf{G}$ and $\mathcal{T}_\mathsf{S}$ are stably infinite. (Nelson-Oppen)
- Data: we can add data with constraints (see paper for details).
- More pointers: we can extend the signature with fields and use $\bullet \xrightarrow{\bullet \setminus \bullet} \bullet$ with different fields (array theory).
- More complex data structures, e.g. doubly linked lists, ...

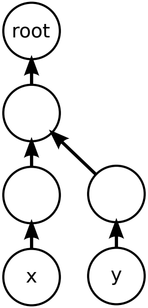
Mixed specifications: union-find

procedure find(x: Node, **ghost** root_x: Node, **implicit ghost** X: set<Node>)
  returns (res: Node)
  requires lseg_set(x, root_x, X) $*$ root_x.next $\mapsto$ null;
  ensures res $=$ root_x $*$ acc(X) $*$ ($\forall$ z $\in$ X :: z.next $=$ res) $*$ res.next $\mapsto$ null;



Mixed specifications: union-find

procedure union(x: Node, y: Node, **ghost** root_x: Node, **ghost** root_y: Node,
implicit ghost X: set<Node>, **implicit ghost** Y: set<Node>)
requires lseg_set(x, root_x, X) + lseg_set(y, root_y, Y);
requires root_x.next $\mapsto$ null + root_y.next $\mapsto$ null;
ensures (acc(X) + acc(Y)) $*$ (root_y.next $\mapsto$ null + acc(root_x));
ensures ($\forall$ z $\in$ X :: z.next = root_x) $*$ ($\forall$ z $\in$ Y :: z.next = root_y);
ensures root_x = root_y $\vee$ root_x.next = root_y;



Experimental results

Implementation: GRASShopper available at
https://cs.nyu.edu/wies/software/grasshopper/

Benchmarks	# VCs	time in s
SLL (loop)	56	1.9
SLL (rec.)	70	3.1
sorted SLL	55	6.6
DLL	59	11
sorting algorithms	98	15
union-find	8	4.8
SLL.filter (deref. null pointer)	7	0.4
DLL.insert (missing update)	8	3.1
quicksort (underspec. split)	12	0.9
union-find (bug in postcond.)	4	12.8

Conclusion

- Reduce a decidable fragment of SL to a decidable FO theory.
- Combining SL with other theories.
- Satisfiability, entailment, frame inference, and abduction problems for SL using SMT solvers.
- Implemented in the GRASShopper tool.

Related work

- decidable fragments of SL: linked lists [Berdine et al., 2004], decidable in polynomial time [Cook et al., 2011] (graph-based).
- SL $\rightarrow$ FO: [Calcagno and Hague, 2005] (no inductive predicate) and [Bobot and Filliâtre, 2012] (not a decidable fragment).
- Alternatives to SL: (implicit) dynamic frames [Kassios, 2011] and region logic [Banerjee et al., 2008, Rosenberg et al., 2012].
- The connection between SL and implicit dynamic frames has been studied in [Parkinson and Summers, 2012].
- SMT-based decision procedures for reachability in graphs [Lahiri and Qadeer, 2008, Wies et al., 2011, Totla and Wies, 2013] , decision procedures for theories of stratified sets [Zarba, 2004].
- Entry points for modular reasoning [Itzhaky et al., 2014]

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