Forward Analysis of Depth-Bounded Processes

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From the LIFT web framework (using SCALA actors)

```
class DynamicBlogView extends CometActor {
  //...
  override def localSetup {
    //...
    (BlogCache.cache !? AddBlogWatcher(this, this.blogid)) match {
      case BlogUpdate(entries) => this.blog = entries
    }
  }

  override def lowPriority : PartialFunction[Any, Unit] = {
    case BlogUpdate(entries : List[Entry]) => this.blog = entries; reRender(false)
  }
}

class BlogCache extends LiftActor {
  //...
  protected def messageHandler =
    {
      case AddBlogWatcher(me, id) =>
        val blog = cache.getOrElse(id, getEntries(id)).take(20)
        reply(BlogUpdate(blog))
        //...

      case AddEntry(e, id) =>
        cache += (id -> (e :: cache.getOrElse(id, getEntries(id))))
        sessions.getOrElse(id, Nil).foreach(_ ! BlogUpdate(cache.getOrElse(id, Nil)))

      case DeleteEntry(e, id) => //...
      case EditEntry(e, id) => //..
      case _ =>
    }
}
```

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        reply(BlogUpdate(blog))
        //...

      case AddEntry(e, id) =>
        cache += (id -> (e :: cache.getOrElse(id, getEntries(id))))
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      case DeleteEntry(e, id) => //...
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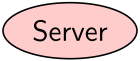
Motivation: Client-Server system (# of clients)



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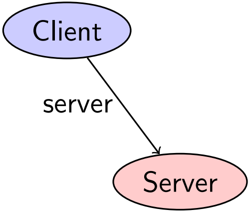

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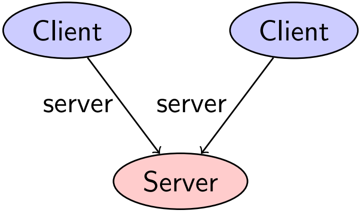

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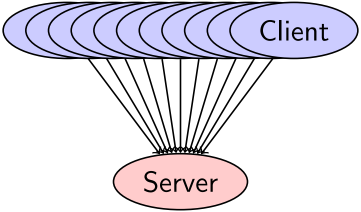

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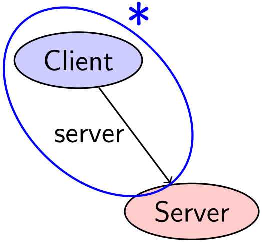

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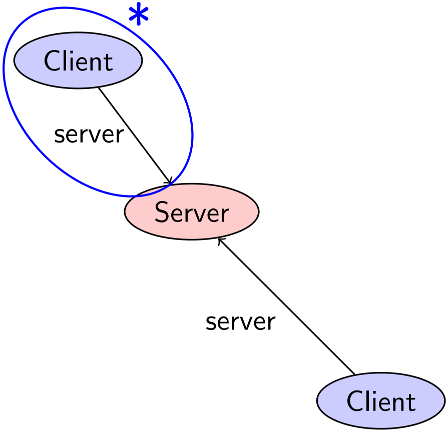

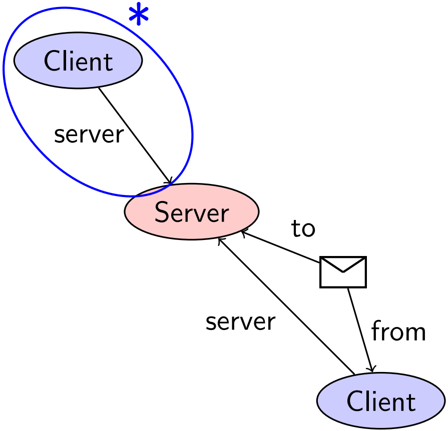
*
Client
server
Server
to
server
from
Client

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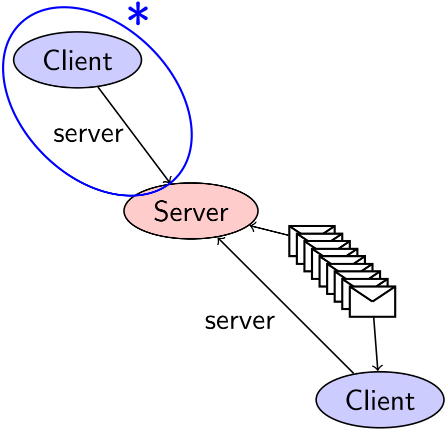

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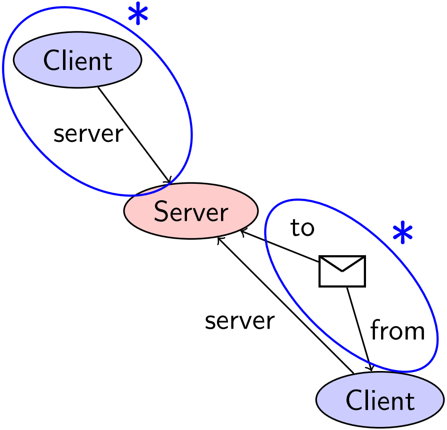

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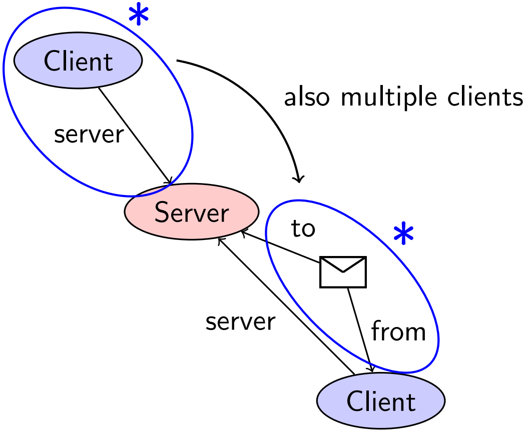

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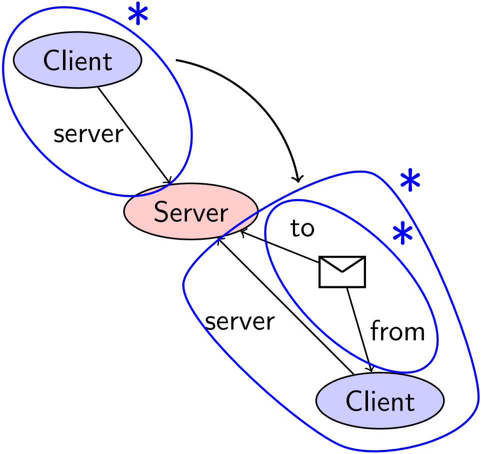

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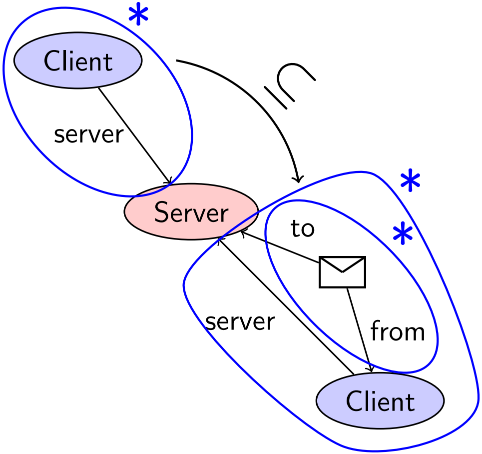

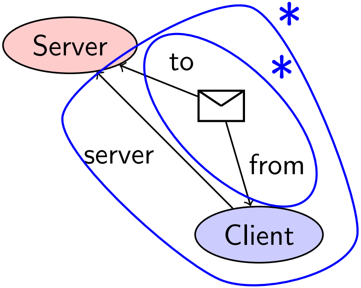
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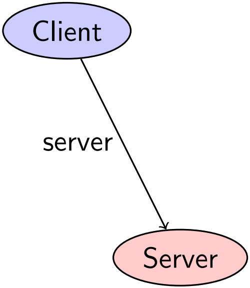

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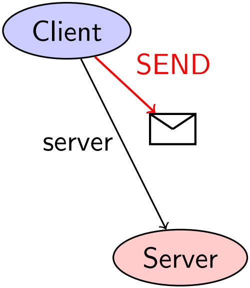

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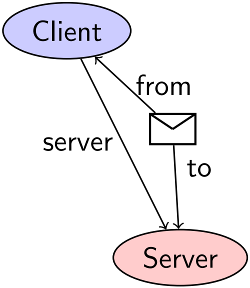

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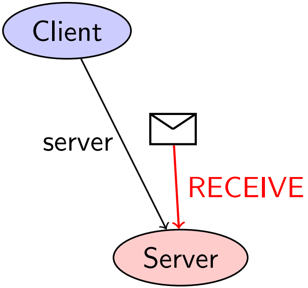

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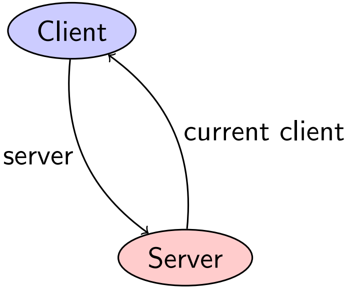
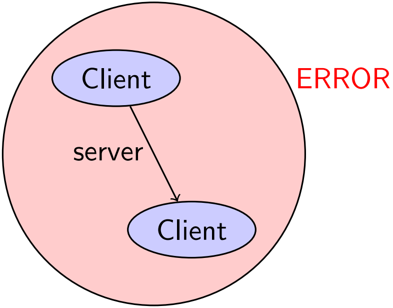


Covering problem



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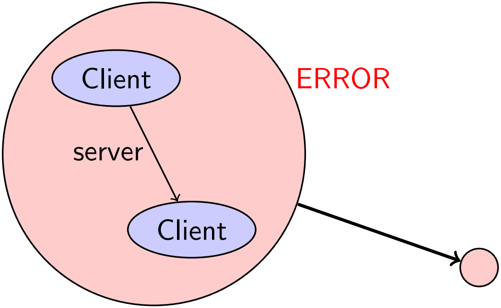

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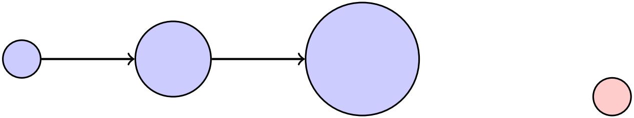

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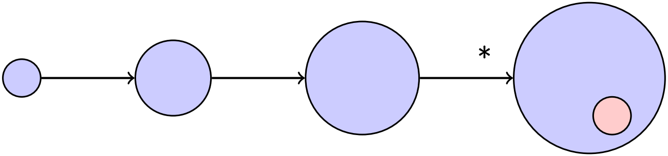

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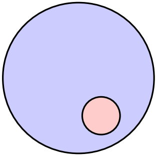

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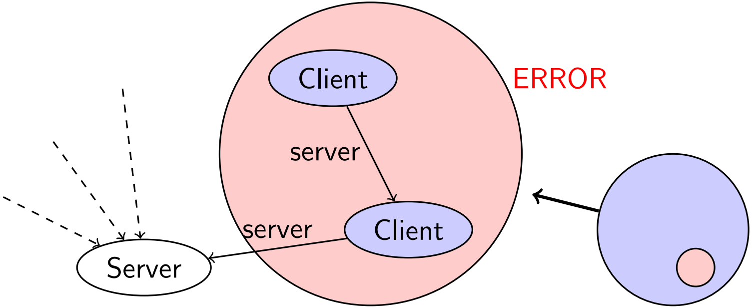
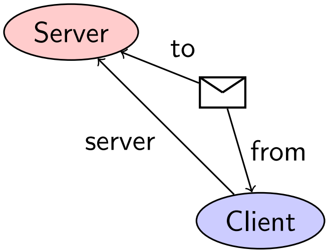


Outline

- π-calculus, depth-bounded systems
- WSTS
- Forward/Backward analysis
- ADL for depth-bounded systems

The π-calculus [Milner et al., 1992a, Milner et al., 1992b] is a process calculus that describes dynamic distributed computations in a message passing-setting.

$$(\nu x)(\mathit{Server}(x) \mid (\nu y)(\mathit{Client}(y,x) \mid \mathit{Messages}(x,y)))$$



π-calculus: Concepts

The π-calculus is build around the notions of

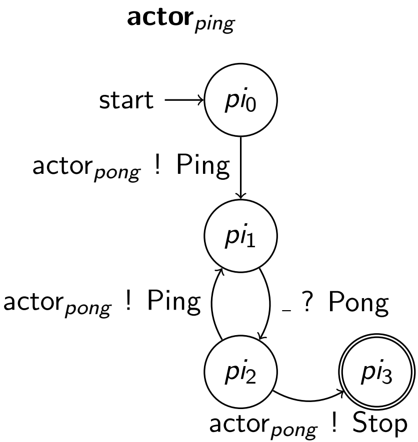
- Names : channels as first class values.
- Threads : concurrent execution of parallel threads: $P \mid Q$.
- i/o prefixes : sending/receiving messages.

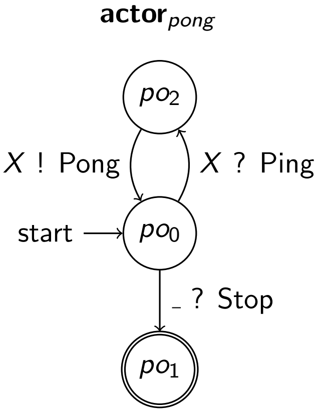
$$
\begin{array}{rcll}
P & ::= & x(y).P & \text{(input prefix)} \\
 & \mid & \overline{x}\langle y\rangle.P & \text{(output prefix)} \\
 & \mid & \sum_i a_i(b_i).P_i & \text{(external choice)} \\
 & \mid & P \mid P & \text{(parallel composition)} \\
 & \mid & !P & \text{(replication)} \\
 & \mid & (\nu x)P & \text{(name creation)} \\
 & \mid & 0 & \text{(unit process)}
\end{array}
$$

```scala
class Ping(count: Int, pong: Actor) extends Actor {
  def act() {
    var pingsLeft = count - 1
    pong ! Ping
    loop {
      react {
        case Pong =>
          if (pingsLeft % 1000 == 0)
            println("Ping: pong")
          if (pingsLeft > 0) {
            pong ! Ping
            pingsLeft -= 1
          } else {
            println("Ping: stop")
            pong ! Stop
            exit()
          }
      }
    }
  }
}
```

```scala
class Pong extends Actor {
  def act() {
    var pongCount = 0
    loop {
      react {
        case Ping =>
          if (pongCount % 1000 == 0)
            println("Pong: ping "+pongCount)
          sender ! Pong
          pongCount += 1
        case Stop =>
          println("Pong: stop")
          exit()
      }
    }
  }
}
```

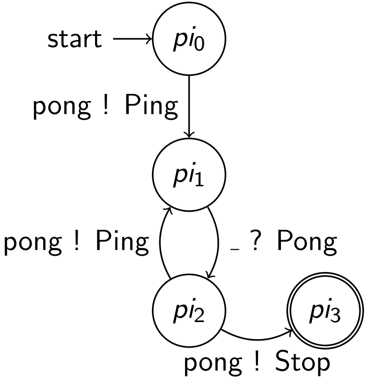
Example (2): scala/docs/examples/actors/pingpong.scala



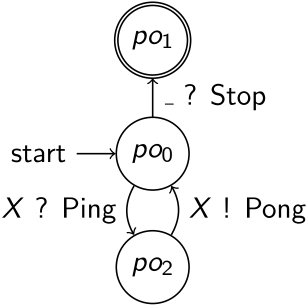


π-calculus: Example (1)

$$
\begin{array}{rcl}
pi_0 & = & \overline{\text{pong}_{Ping}}\langle \text{ping}_{Pong}\rangle | pi_1 \\
pi_1 & = & \text{ping}_{Pong}().pi_2 \\
pi_2 & = & \overline{\text{pong}_{Ping}}\langle \text{ping}_{Pong}\rangle | pi_1 \\
 & \oplus & \overline{\text{pong}_{Stop}}\langle\rangle | pi_3 \\
pi_3 & = & 0
\end{array}
$$



$$
\begin{array}{rcl}
po_0 & = & \text{pong}_{Stop}().po_1 \\
 & + & \text{pong}_{Ping}(X).po_2(X) \\
po_1 & = & 0 \\
po_2(X) & = & \overline{X}\langle\rangle | po_0
\end{array}
$$



Evaluating a formula in π-calculus reduces to applying the rule:

$$\overline{a}\langle b\rangle.P \mid \sum_{i\in I} a_i(b_i).Q_i \quad\rightarrow\quad P \mid Q_x[b/b_x] \quad \text{where } a_x = a$$

What happens:

- channel a carries b;
- b is sent through a and replace b_x in the continuation Q_x.

π-calculus: Semantics

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What happens:

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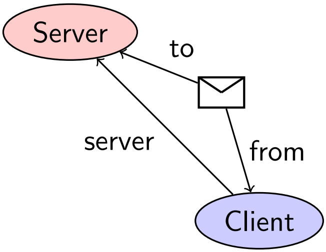
Client-Server: communication topology

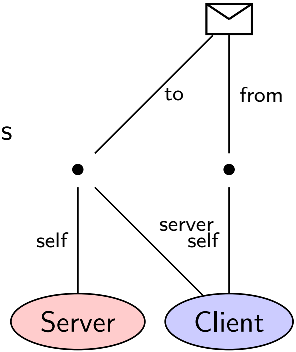
$$(\nu x)(Server(x) \,|\, (\nu y)(Client(y,x) | Messages(x,y)))$$

$$\begin{array}{rcl} Server(self) & = & self(sender).\ldots \\ Client(self, server) & = & self().\ldots \\ Messages(to, from) & = & \overline{to}(from) \end{array}$$

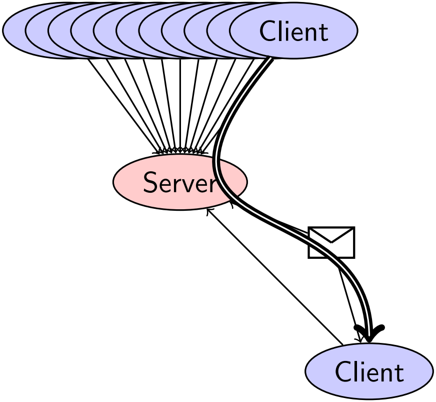


explicit names

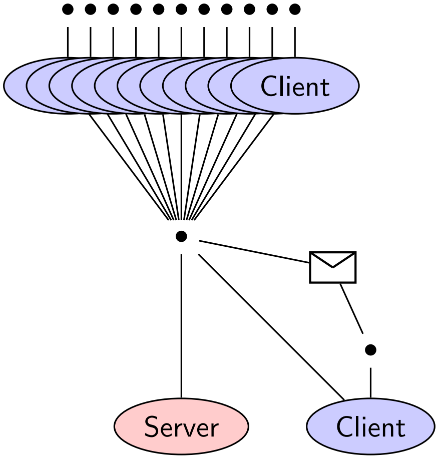
$\longrightarrow$



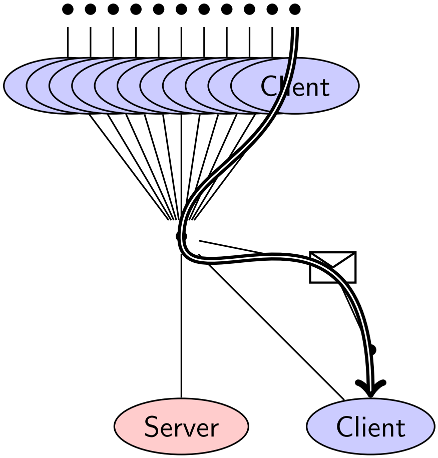
System with a bound on the longest acyclic path.
(Concretely: it is not possible to encode an infinite memory.)



Depth-bounded systems: [Meyer, 2008] (2)



Depth-bounded systems: [Meyer, 2008] (2)



Nesting of names:

$$nest_\nu((\nu x)P) = 1 + nest_\nu(P),$$
$$nest_\nu(P_1 \mid P_2) = \max\{nest_\nu(P_1), nest_\nu(P_2)\},$$
$$\dots$$

The Depth of a configuration:

$$depth(P) = \min\{\, nest_\nu(Q) \mid Q \equiv P \,\}$$

A process $\mathcal{P}$ is *depth-bounded* if:

$$\exists k \in \mathbb{N},\ \forall P \in Reach(\mathcal{P}),\ depth(P) \leq k$$

About Depth-bounded systems

- Depth-bounded systems are well-structured transition systems [Meyer, 2008].
- Reachability is undecidable.
- Termination is decidable.
- Coverability is decidable for system of known depth.
- Coverability for any depth-bounded system was an open problem.

Our contribution:
Coverability is decidable for any depth-bounded system.

Well-structured transition system

A well-structured transition system (WSTS) is a transition system $\langle S, \rightarrow, \leq \rangle$ such that:

- $\leq$ is a well-quasi-ordering (wqo),
  i.e. well-founded + no infinite antichain.
- compatibility of $\leq$ w.r.t. $\rightarrow$

$$\forall \quad \begin{array}{ccc} t & \xrightarrow{*} & t' \\ \rotatebox{90}{$\leq$} & & \rotatebox{90}{$\leq$} \\ s & \longrightarrow & s' \end{array} \quad \exists$$

for more detail see:
[Finkel and Schnoebelen, 2001, Abdulla et al., 1996]

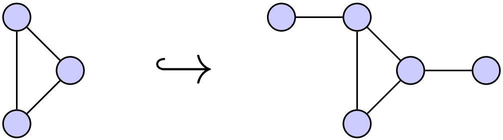
A better-quasi-ordering is a wqo closed under the powerset construction.

$\uparrow x = \{x' \in S \mid x \leq x'\}$ is an upward-closed set.
$\downarrow x = \{x' \in S \mid x' \leq x\}$ is a downward-closed set.

Depth bounded systems as WSTS

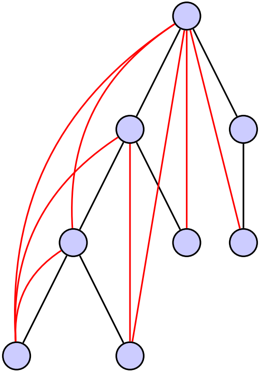
[Meyer, 2008] showed that depth-bounded processes are WSTS for

- their *reachable* configurations and
- the quasi-ordering $\hookrightarrow$ induced by *subgraph isomorphism*.



[Meyer, 2008] showed that $\hookrightarrow$ is a well-quasi-ordering on the reachable configurations.
We show that it is a better-quasi-ordering.

Closure of a tree



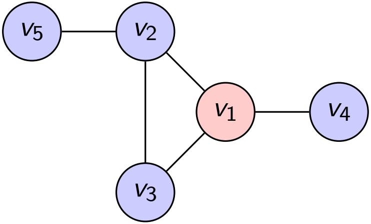
Add edges according to the transitive closure of the parent relation.

Every (undirected) graph is included in the closure of some tree.

Tree-Depth of a graph

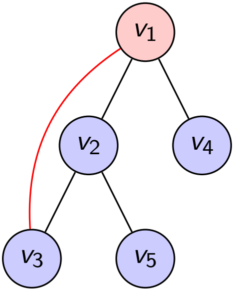
Tree-Depth

The tree-depth td(G) of a graph G is the minimal height of all trees whose closure contains G.



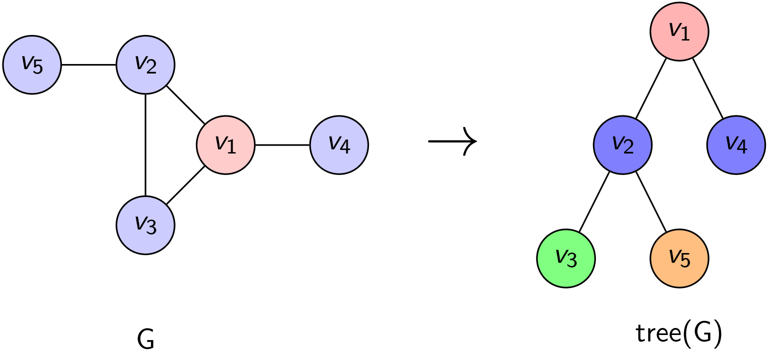
tree-depth $= 2$

$\equiv$



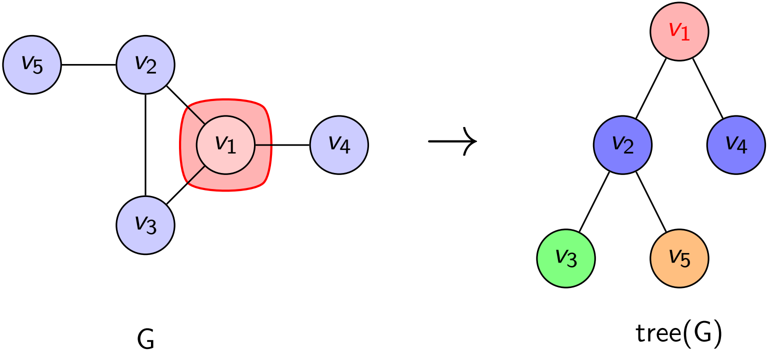
height $= 2$

Tree encoding of depth bounded graph



The labels of tree(G) are graphs.
The number of labels used in the encodings is finite.

Tree encoding of depth bounded graph



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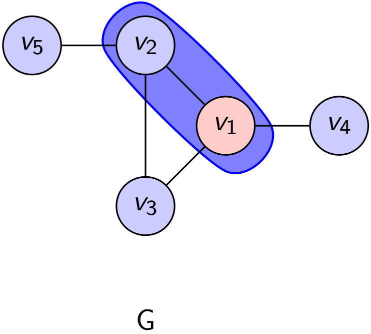
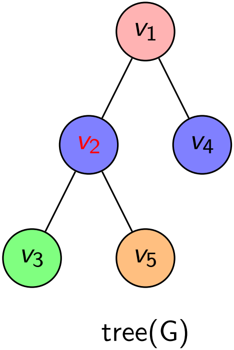


$\rightarrow$



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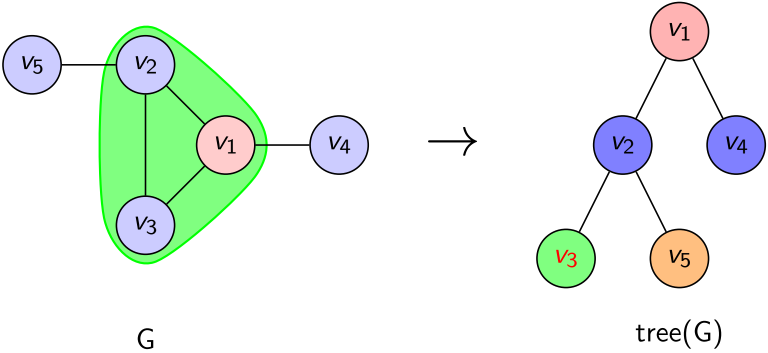

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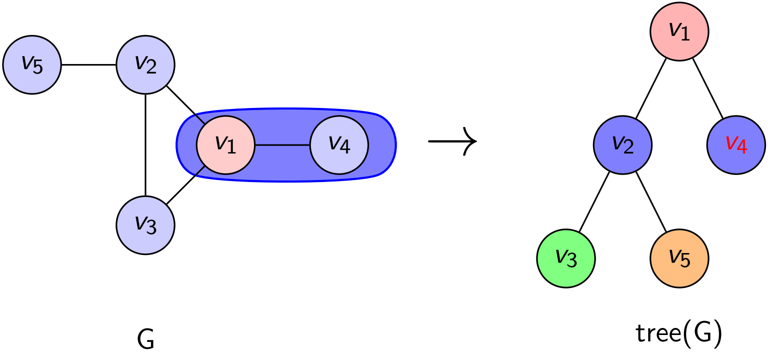

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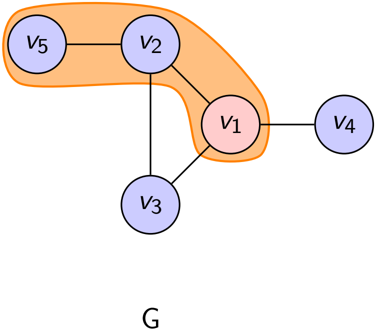
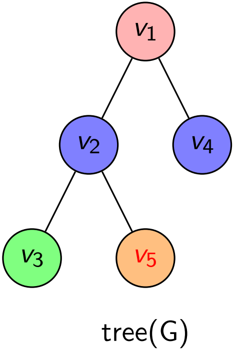
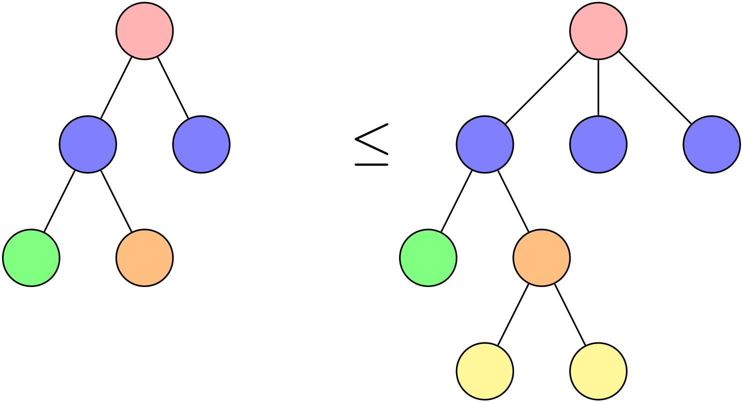


$\rightarrow$



The labels of tree(G) are graphs.
The number of labels used in the encodings is finite.

Homeomorphic tree embedding



We can show for all graphs G_1, G_2:

$$\mathrm{tree}(G_1) \leq \mathrm{tree}(G_2) \text{ implies } G_1 \hookrightarrow G_2$$

Kruskal's tree theorem

Extension of Kruskal's tree theorem [Laver, 1971]

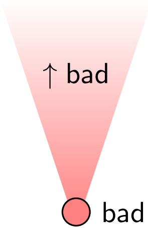
Homeomorphic tree embedding is a better-quasi-ordering on finite trees, where the labels are better-quasi-ordered.

Proposition

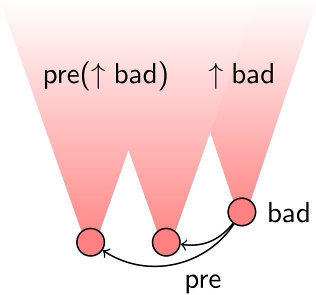
Labelled graphs of bounded tree-depth are better-quasi-orderered by the relation induced by subgraph isomorphisms.

$\Rightarrow$ Subgraph isomorphisms induce a better-quasi-ordering on the reachable configurations of a depth-bounded system.

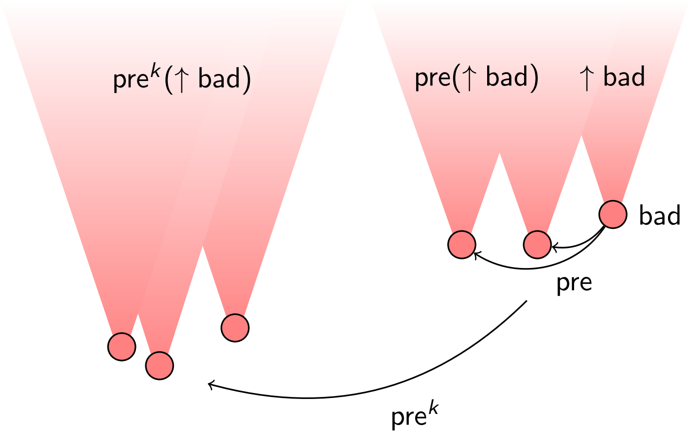
Backward analysis



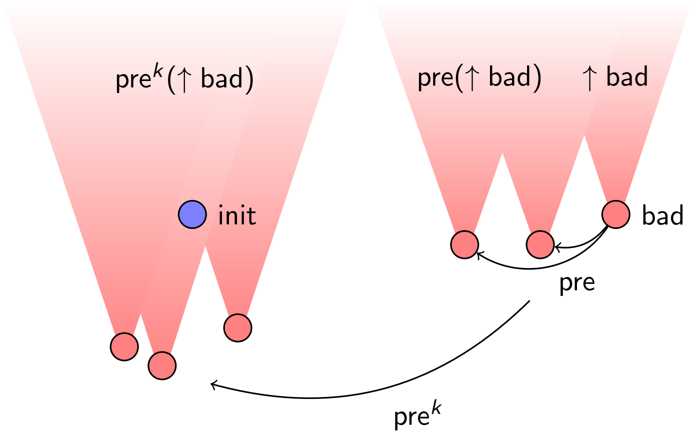
Backward analysis



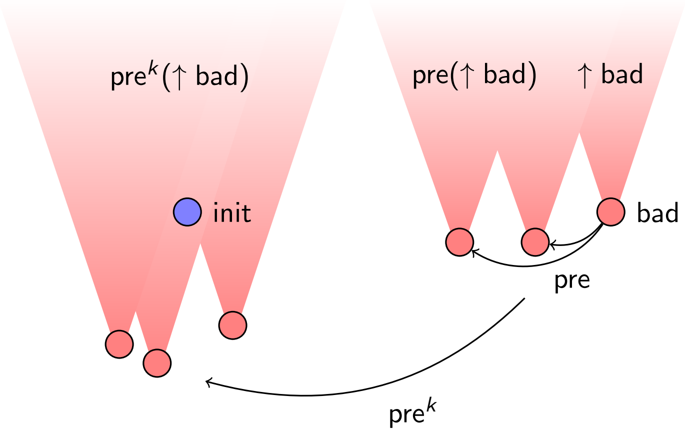
Backward analysis



Backward analysis



Backward analysis



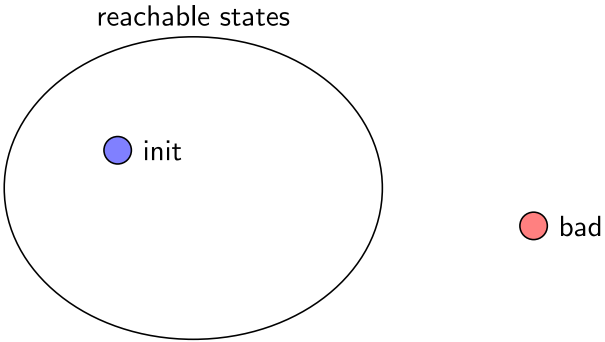
Termination

Let $I_n = \bigcup_{i=0}^{n} pre^i(\mathrm{bad})$ then $I_0 \subseteq I_1 \subseteq \ldots \subseteq I_n \subseteq I_{n+1} \ldots$
This sequence stabilizes because a wqo has no infinite antichain.

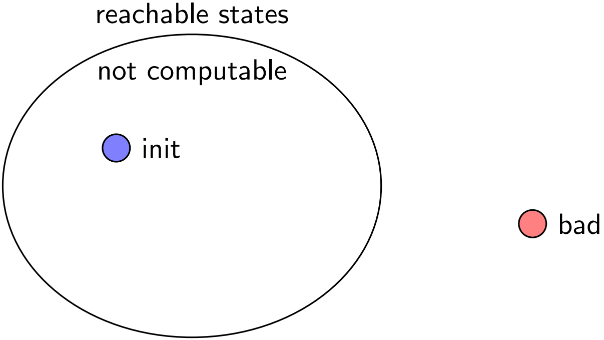
Analysis of depth-bounded systems: Backward analysis

- Backward analysis requires *pre* to be computable.
- The WSTS of a depth-bounded system is defined wrt. the *forward-reachable* configurations.
- pre generates unreachable configurations.
- The set of reachable configurations is not computable
- We need to known the depth to preserve the wqo.
- Backward algorithms for coverability works only with processes of *known depth*.

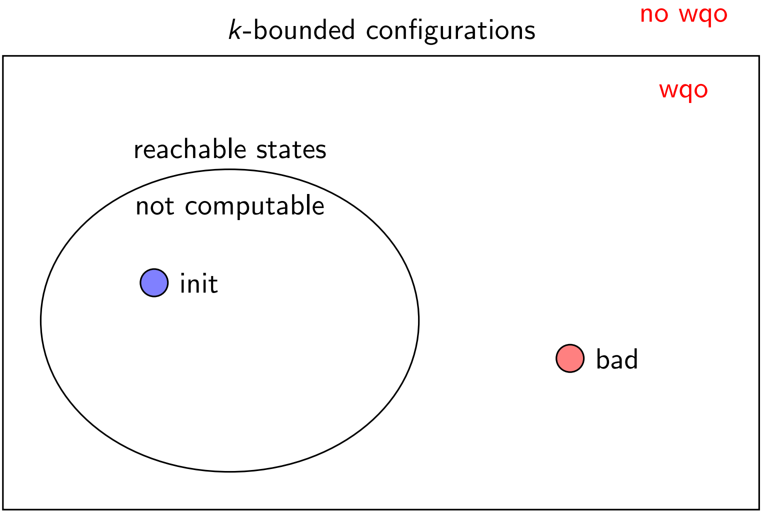
Analysis of depth-bounded systems: computing *pre*



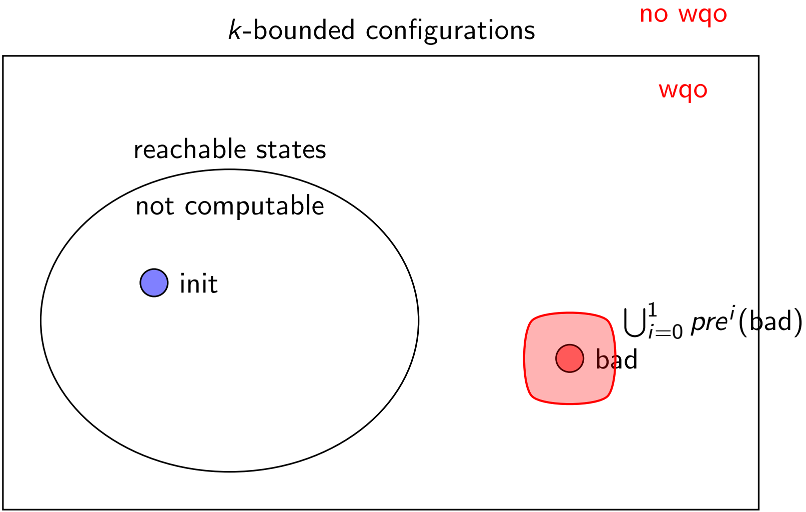
Analysis of depth-bounded systems: computing *pre*



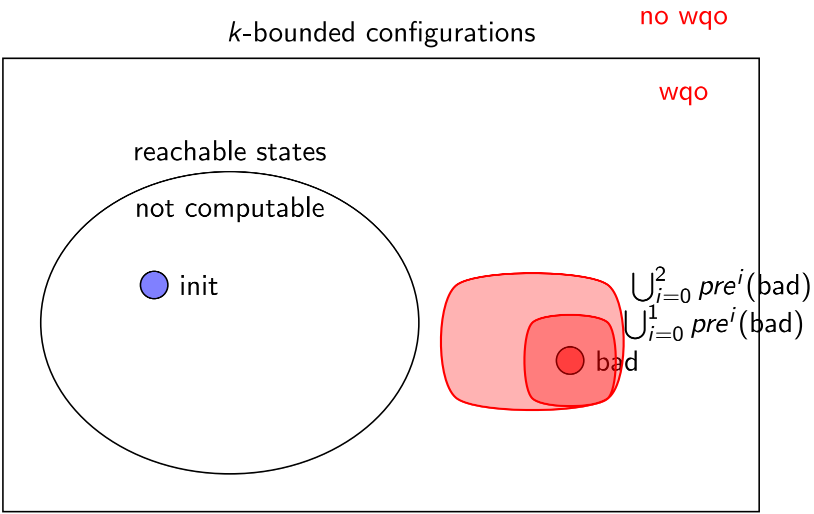
Analysis of depth-bounded systems: computing *pre*

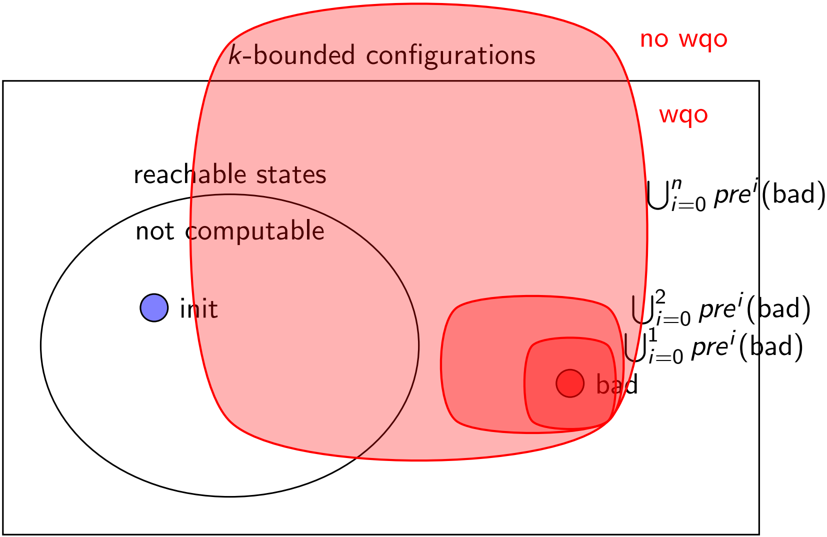


Analysis of depth-bounded systems: computing *pre*



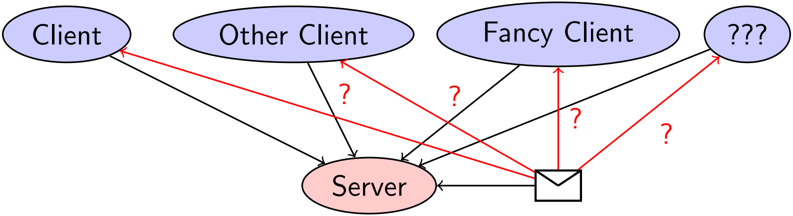
Analysis of depth-bounded systems: computing *pre*



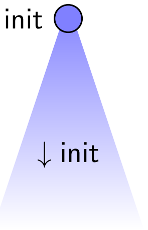


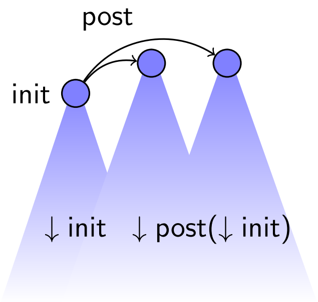
no wqo
k-bounded configurations
wqo
reachable states
not computable
$\bigcup_{i=0}^{n} pre^i(\text{bad})$
init
$\bigcup_{i=0}^{2} pre^i(\text{bad})$
$\bigcup_{i=0}^{1} pre^i(\text{bad})$
bad

Backward analysis has to guess the exchanged names of each reduction step.
$\rightarrow$ explosion in the nondeterminism.

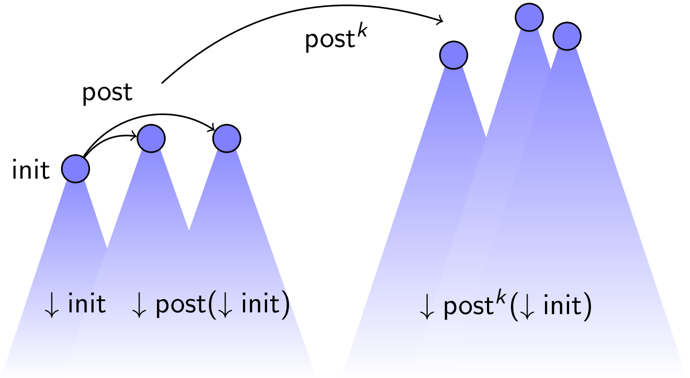


Forward analysis

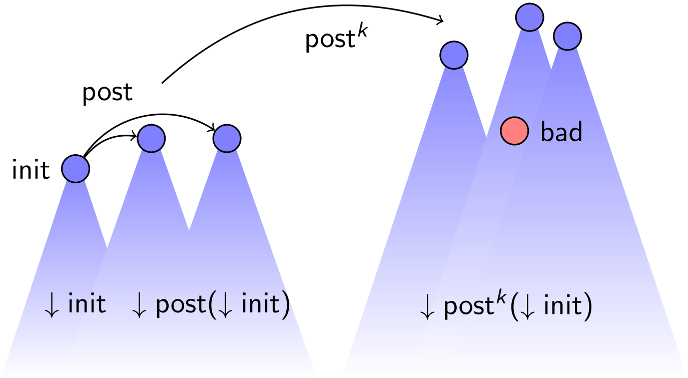



post
init
↓ init
↓ post(↓ init)

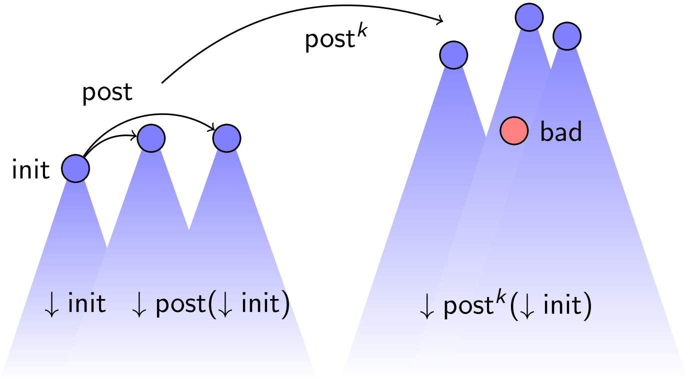
Forward analysis



Forward analysis



Forward analysis



Problem

How to represents downward-closed sets ?

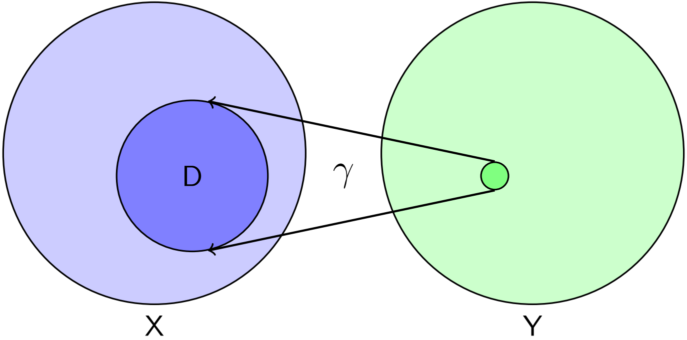
Analysis of depth-bounded systems: Forward analysis

- Forward algorithms terminate even if the *bound is not known*.
- The algorithm is an instance of the expand enlarge check algorithm [Geeraerts et al., 2006] that uses adequate domain of limits (ADL).
- [Finkel and Goubault-Larrecq, 2009b] provides a theoretical framework for the manipulation of downward-closed sets and the construction of ADL.
- We build such an ADL by extending configurations with '!'.

$\Rightarrow$ coverability is decidable for the entire class of depth-bounded systems.

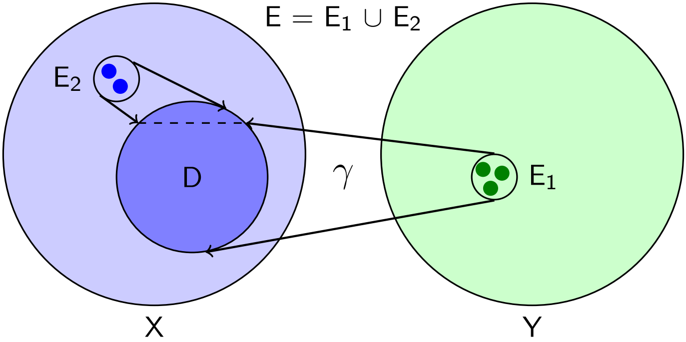
ADL: [Geeraerts et al., 2006]
let Y an ADL for wqo set X:

For every $z \in \mathrm{X} \cup \mathrm{Y}, \gamma(z)$ is a downward-closed subset of X.

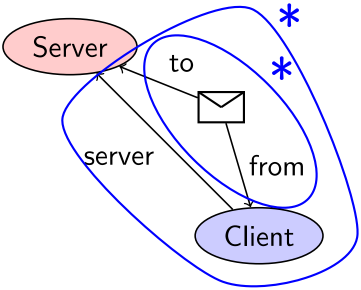


Adequate Domain of Limit (2)

Every downward-closed subset D of X is generated by a finite subset E of $Y \cup X$.



$$(\nu x)(Server(x) \mid !(\nu y)(Client(y, x) | !Messages(x, y)))$$



Limits configuration for depth-bounded systems

We use '!' not as a recursion operator but as a mean to represent infinite sets of configurations.

$\mathcal{C}(PI, k)$ is the set of configurations.
$\mathcal{L}(PI, k)$ in the set of limit configurations.

Theorem

Let $k \in \mathbb{N}$ and let PI be a finite set of process identifiers. Then $(\mathcal{L}(PI, k), \sqsubseteq, \gamma)$ is a weak adequate domain of limits for the well-quasi-ordered set $(\mathcal{C}(PI, k), \leq)$.

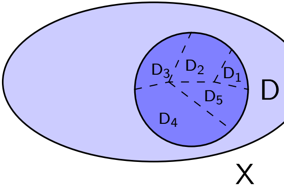
Corollary

Coverability is decidable for the entire class of depth-bounded systems.

Limits configuration for depth-bounded systems

Theorem [Finkel and Goubault-Larrecq, 2009b]

The downward-closed directed subsets of a wqo set X form an ADL for X.

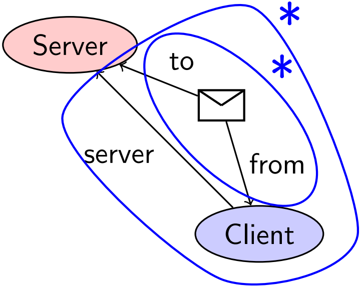


Proposition

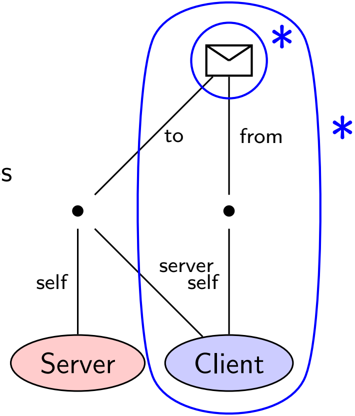
The directed downward-closed sets of depth-bounded configurations are exactly the denotations of limit configurations.

We characterize the tree encodings of downward-closed sets of configurations in terms of the languages of *hedge automata*.

$$(\nu x)(\mathit{Server}(x) \mid !(\nu y)(\mathit{Client}(y,x) | !\mathit{Messages}(x,y)))$$

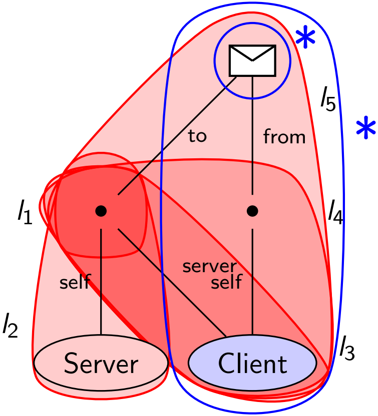


explicit names
$\longrightarrow$

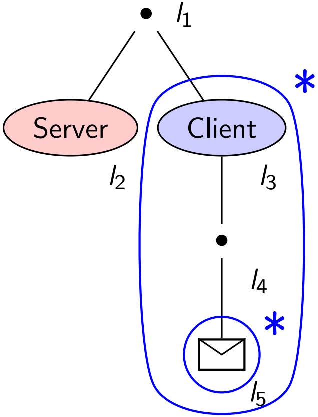


Regular language of unranked trees for Client-Server (2)

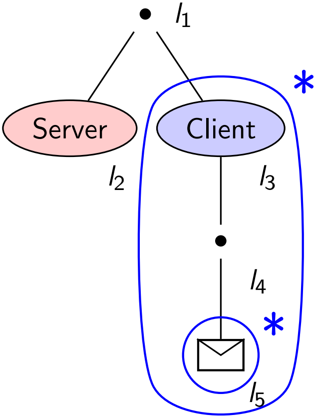
$$(\nu x)(Server(x) \mid !(\nu y)(Client(y,x) | !Messages(x,y)))$$



as tree
$\longrightarrow$



$$(\nu x)(Server(x) \mid !(\nu y)(Client(y,x) | !Messages(x,y)))$$



tree language
$\longrightarrow$

$$l_1(q_1 q_2^*) \to q_{final}$$
$$l_2(\epsilon) \to q_1$$
$$l_3(q_3) \to q_2$$
$$l_4(q_4^*) \to q_3$$
$$l_5(\epsilon) \to q_4$$

Further Work

We started an implemention to compute (an over-approximation of) the cover using [Finkel and Goubault-Larrecq, 2009a].

```
Equations:
----------------------------------------
client1(A, B) = (A().(client1(A, B) |
                     request1(B, A)))
answer1(A) = (A<>.0)
new1(A) = (A<>.0)
request1(A, B) = (A<B>.0)
server(A, B) = (A(C).(answer1(C) |
                       server(A, B)) +
                B().(ny D)
                      (client1(D, A) |
                       answer1(D) |
                       new1(B) |
                       server(A, B)))

Initial configuration:
----------------------------------------
(ny A, B)
    (new1(A) |
     server(B, A))
```

```
Computed cover:

(ny A, B)
    (!((ny C)
         (answer1(C) |
          client1(C, B))) |
    !((ny D)
         (client1(D, B) |
          request1(B, D))) |
    new1(A) |
    server(B, A))
```

Recap

Coverability is decidable for depth-bounded processes.

- We provide an ADL for depth-bounded processes;
- prepared the ground for a spectrum of forward algorithms for depth-bounded processes.

Questions ?

Abdulla, P. A., Cerans, K., Jonsson, B. and Tsay, Y.-K. (1996).
General Decidability Theorems for Infinite-State Systems.
In LICS pp. 313–321,.

Finkel, A. and Goubault-Larrecq, J. (2009a).
Forward Analysis for WSTS, Part II: Complete WSTS.
In ICALP (2) pp. 188–199,.

Finkel, A. and Goubault-Larrecq, J. (2009b).
Forward Analysis for WSTS, Part I: Completions.
In STACS vol. 09001, of Dagstuhl Sem. Proc. pp. 433–444,.

Finkel, A. and Schnoebelen, P. (2001).
Well-structured transition systems everywhere!
Theor. Comput. Sci. *256*, 63–92.

Geeraerts, G., Raskin, J.-F. and Van Begin, L. (2006).
Expand, Enlarge and Check: New algorithms for the coverability problem of WSTS.
J. Comput. Syst. Sci. *72*, 180–203.

Laver, R. (1971).
On Fraïssé's Order Type Conjecture.
Ann. of Math. *93*, 89–111.

Meyer, R. (2008).
On Boundedness in Depth in the pi-Calculus.
In IFIP TCS vol. 273, of IFIP pp. 477–489, Springer.

References II

Milner, R., Parrow, J. and Walker, D. (1992a).
A Calculus of Mobile Processes, I.
Inf. Comput. *100*, 1–40.

Milner, R., Parrow, J. and Walker, D. (1992b).
A Calculus of Mobile Processes, II.
Inf. Comput. *100*, 41–77.