Analysis of Depth-Bounded Processes

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Mobile Processes

Why mobile processes ?

- Mobile devices are ubiquitous (e.g. mobile phone).
- Mobility becomes common in PL abstraction (e.g. actor model [Hewitt et al., 1973]).

Interesting features of mobile processes:

- Process creation
- Mobility (communication channels as first class citizens)
- Assume no shared memory

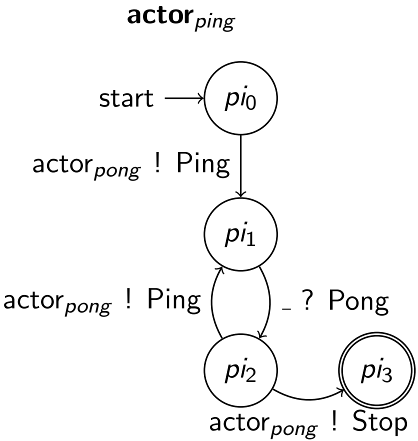
How common is mobility ? What are the use cases ?

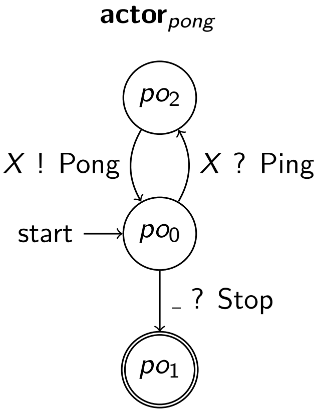
Example (1): scala/docs/examples/actors/pingpong.scala

```
class Ping(count: Int, pong: Actor) extends Actor {
  def act() {
    var pingsLeft = count - 1
    pong ! Ping
    loop {
      react {
        case Pong =>
          if (pingsLeft % 1000 == 0)
            println("Ping: pong")
          if (pingsLeft > 0) {
            pong ! Ping
            pingsLeft -= 1
          } else {
            println("Ping: stop")
            pong ! Stop
            exit()
          }
      }
    }
  }
}
```

```
class Pong extends Actor {
  def act() {
    var pongCount = 0
    loop {
      react {
        case Ping =>
          if (pongCount % 1000 == 0)
            println("Pong: ping "+pongCount)
          sender ! Pong
          pongCount += 1
        case Stop =>
          println("Pong: stop")
          exit()
      }
    }
  }
}
```

Example (2): scala/docs/examples/actors/pingpong.scala





- '?' means receive a message.
- '!' means send a message.

Communication primitives and mobility

In the case of the scala actor library:
(asynchronous communication with mailboxes)

- Send the address of an actor to another (within a message).
- Messages implicitly carry return address to `reply`.
- `forwarding` messages.
- Creating new process (also a new address and mailbox).
- Emulating synch. communication with shared mailboxes.

π-calculus or not ?

Shall I introduce the π-calculus, or can I continue with pictures ?

$A\pi$-calculus: Concepts

The π-calculus [Milner et al., 1992a, Milner et al., 1992b] is a process calculus able to describe concurrent computations whose configuration may change during the computation.
The *asynchronous* π-calculus [Honda and Tokoro, 1991] is a restriction of the π-calculus.

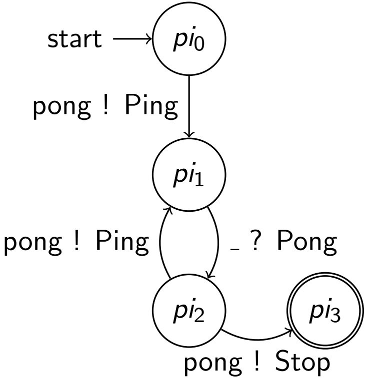
It is build around the notions of

- Names : channels as first class values.
- Threads : concurrent execution of parallel threads: $P \mid Q$.
- i/o prefixes : sending/receiving messages.

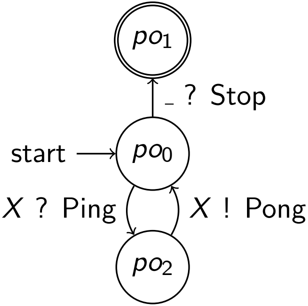
$A\pi$-calculus: Syntax

$$
\begin{array}{lcll}
P & ::= & x(y).P & \text{(input prefix)} \\
 & \mid & \overline{x}\langle y \rangle & \text{(output)} \\
 & \mid & \sum_i a_i(b_i).P_i & \text{(external choice)} \\
 & \mid & P \mid P & \text{(parallel composition)} \\
 & \mid & !P & \text{(replication)} \\
 & \mid & (\nu x)P & \text{(name creation)} \\
 & \mid & 0 & \text{(unit process)}
\end{array}
$$

$$
\begin{aligned}
pi_0 &= \overline{\text{pong}_{Ping}}\langle \text{ping}_{Pong} \rangle | pi_1 \\
pi_1 &= \text{ping}_{Pong}().pi_2 \\
pi_2 &= pi_{2a} \oplus pi_{2b} \\
pi_{2a} &= \overline{\text{pong}_{Ping}}\langle \text{ping}_{Pong} \rangle | pi_1 \\
pi_{2b} &= \overline{\text{pong}_{Stop}}\langle\rangle | pi_3 \\
pi_3 &= 0
\end{aligned}
$$



$$
\begin{array}{rcl}
po_0 & = & \mathrm{pong}_{Stop}().po_1 \\
 & + & \mathrm{pong}_{Ping}(X).po_2(X) \\
po_1 & = & 0 \\
po_2(X) & = & \overline{X}\langle\rangle | po_0
\end{array}
$$



$A\pi$-calculus: Semantics

Evaluating a formula in $A\pi$-calculus reduces to applying the rule:

$$\overline{a}\langle b\rangle \mid \sum_{i\in I} a_i(b_i).Q_i \quad \rightarrow \quad Q_x[b/b_x] \quad \text{where } a_x = a$$

What happens:

- channel a carries b;
- b is sent through a and replace b_x in the continuation Q_x.

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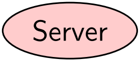
What are Depth-Bounded Processes (DBP) ?

As buzzwords: concurrent/distributed message-passing programs with process creation and mobility. (Warning restrictions may apply.)

For the programmers: some class of programs using the actor model (Erlang, Scala, Akka, ActorFoundry, ...)

For the theoreticians: a fragment of the π-calculus.

Example: client-server communication pattern



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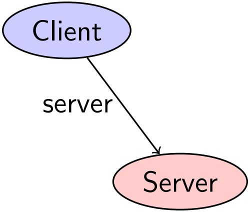

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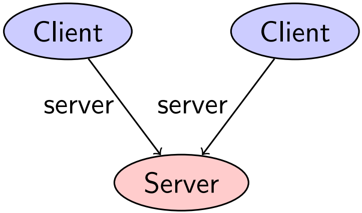
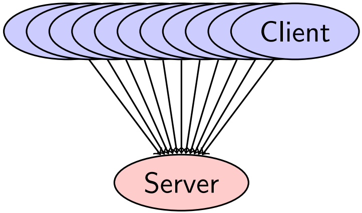
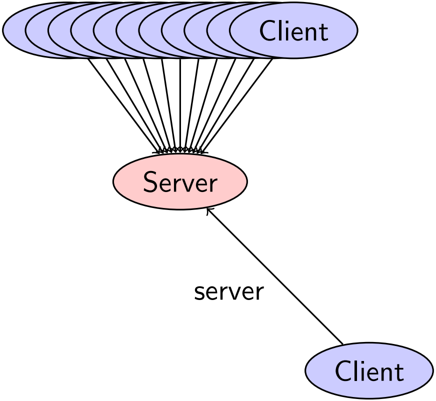



Client
Server

Example: client-server communication pattern



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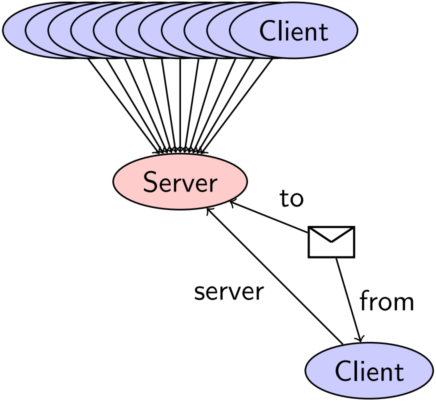

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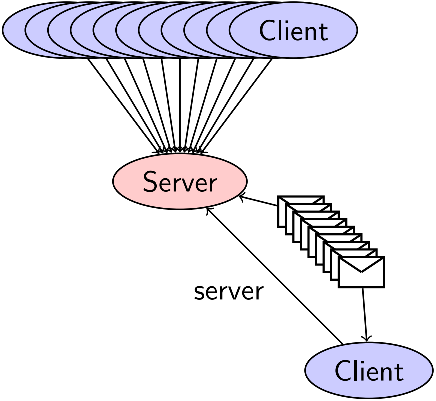
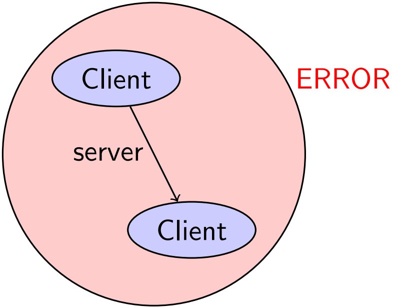


What kind of properties are we looking at ?

Safety properties, more precisely the control-state reachability problem (aka covering problem).



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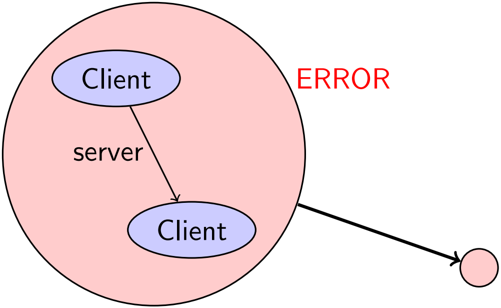

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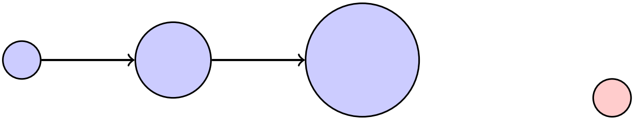

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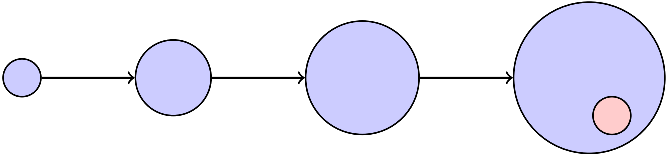

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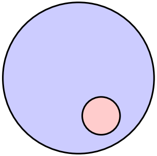

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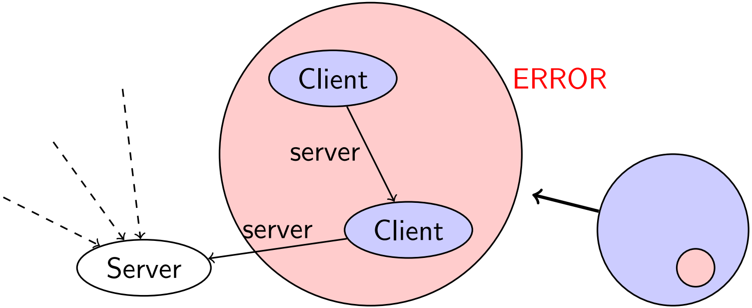


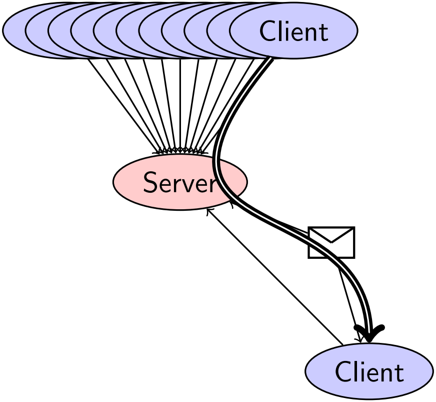
A well-structured transition system (WSTS) is a transition system $\langle S, \rightarrow, \leq \rangle$ such that:

- $\leq$ is a well-quasi-ordering (wqo),
  i.e. well-founded + no infinite antichain.
- compatibility of $\leq$ w.r.t. $\rightarrow$ (simulation)

$$\forall \quad \begin{tikzcd} t \arrow[r, "*"] & t' \\ s \arrow[u, phantom, "\rotatebox{90}{$\leq$}"] \arrow[r] & s' \arrow[u, phantom, "\rotatebox{90}{$\leq$}"] \end{tikzcd} \quad \exists$$

For more detail see:
[Finkel and Schnoebelen, 2001, Abdulla et al., 1996]

System with a bound on the longest acyclic path.
(Concretely: it is not possible to encode an infinite memory.)

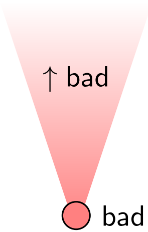


Why DBP are WSTS ?

WQO subgraph isomorphism for graphs of specific shapes (proof by a generalisation of Kruskal tree theorem).

Compatibility The π-calculus has no fairness constraints. Additional processes (greater in the ordering) can simply be ignored.

Backward algorithm for covering



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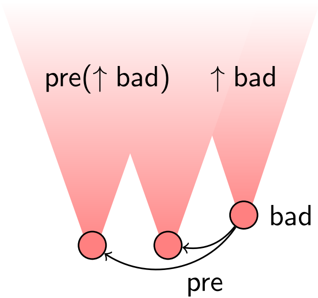

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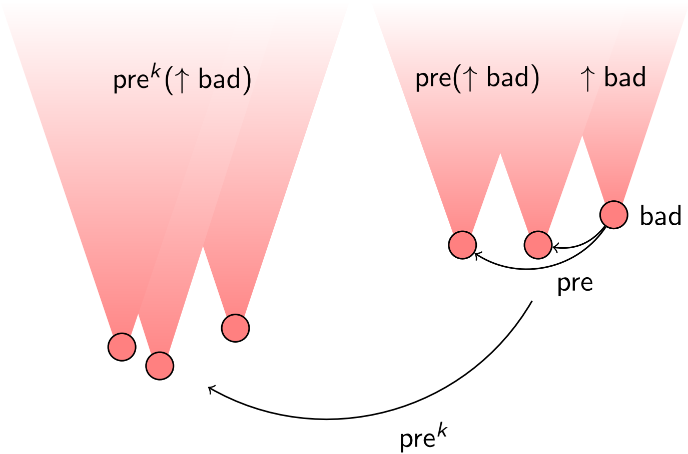

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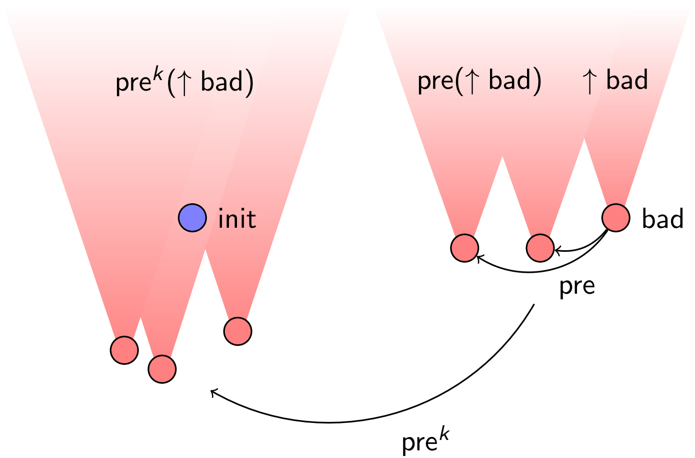

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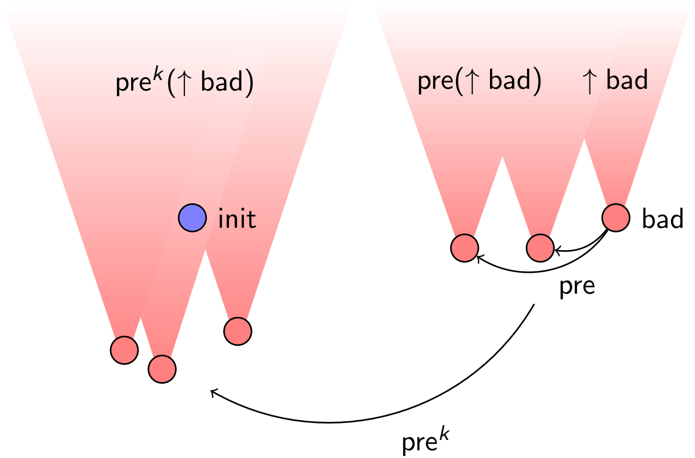
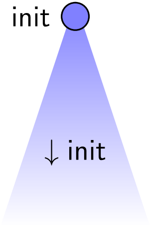


Computing the pre for DBP is not practical (aliasing problem)!
Also the theoretical complexity is terrible.

Forward algorithm for covering using acceleration



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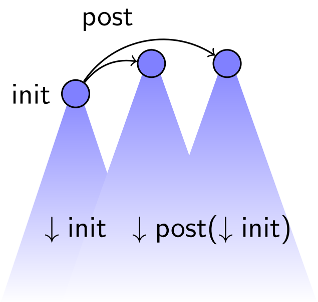

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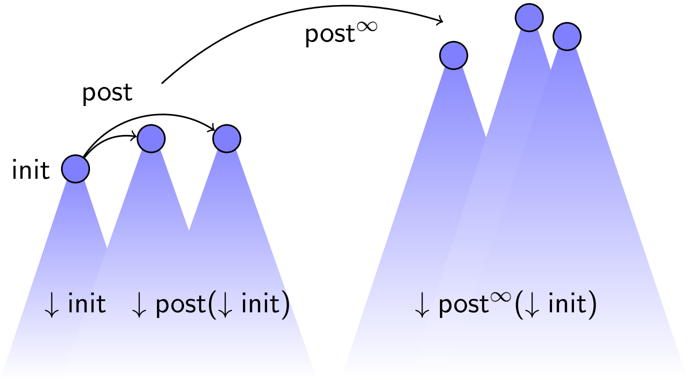

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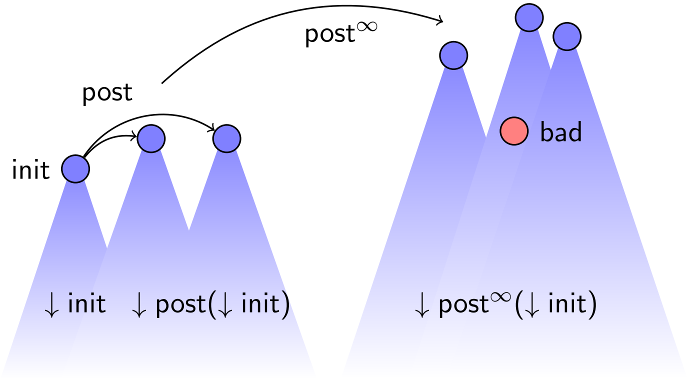

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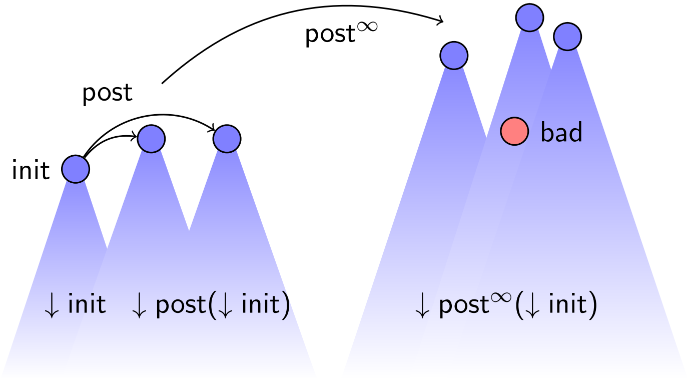
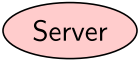


Successfully applied to Petri nets (+extensions) and lossy channels systems. Even though computing the covering set is not decidable [Dufourd et al., 1998].

ADL: [Geeraerts et al., 2006]
Further developed in [Finkel and Goubault-Larrecq, 2009]
Applied to DBP in [Wies et al., 2010]



Adequate domain of limits

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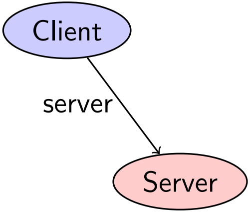

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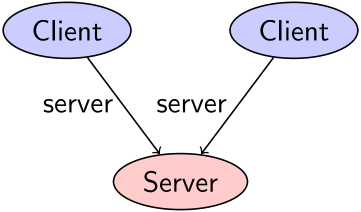

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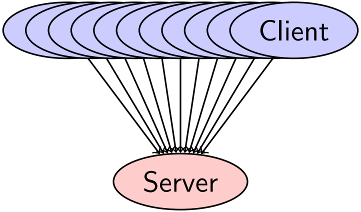

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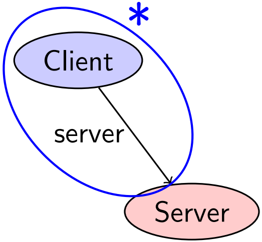

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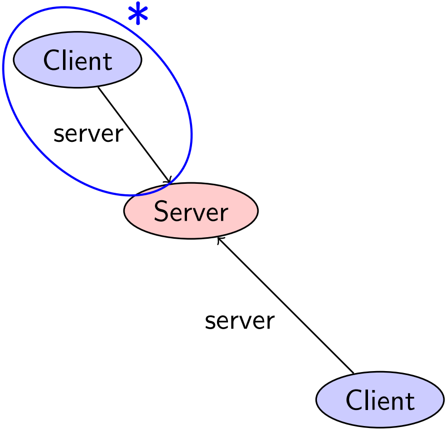

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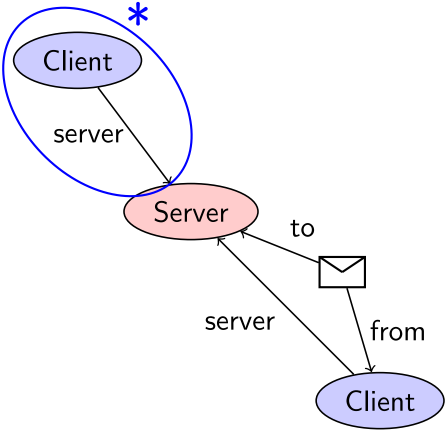

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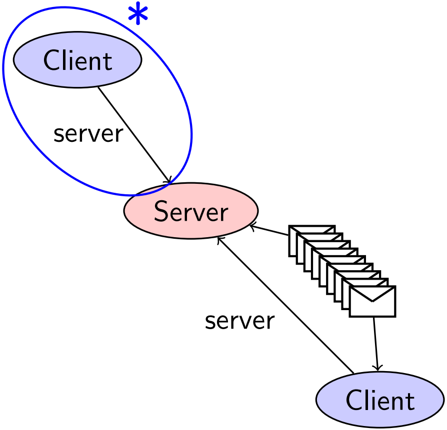

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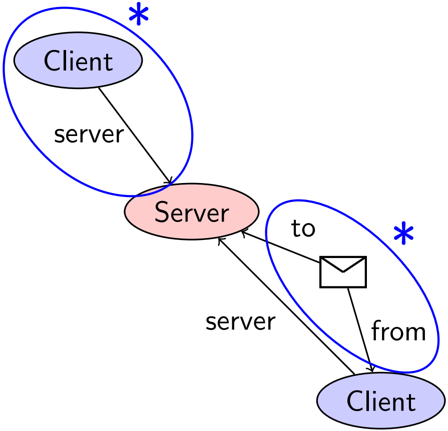

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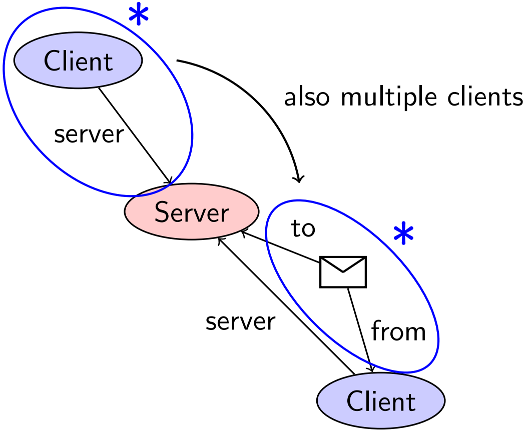

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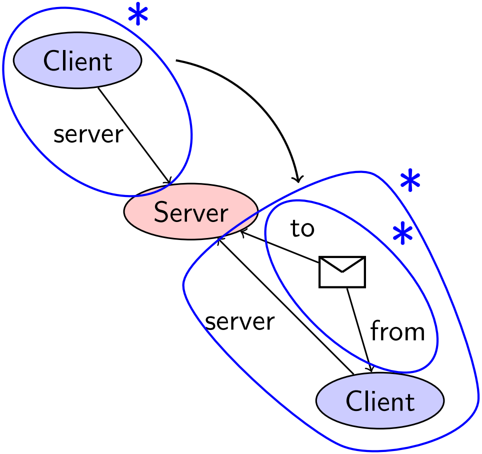

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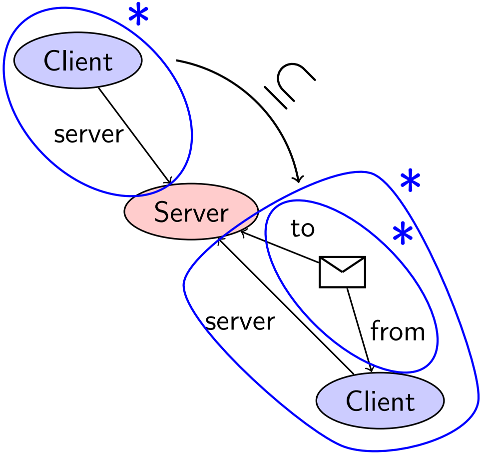

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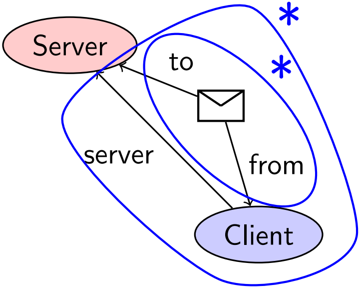

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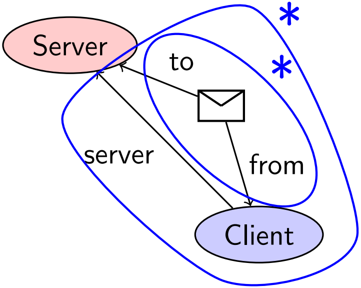
$$(\nu x)(Server(x) \mid !(\nu y)(Client(y, x) | !Messages(x, y)))$$



When does acceleration work ? (flat systems)

Usually forward algorithms are based on acceleration. By acceleration we mean computing the result of executing a loop infinitely many time.

We can see this as computing the result of execution traces of length $< \omega^2$. Concretely, it means that the algorithm can saturate the covering set by executing only simple loops (see [Bardin et al., 2005]). This condition is known as flattability.

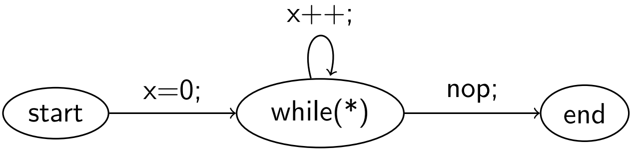
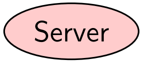


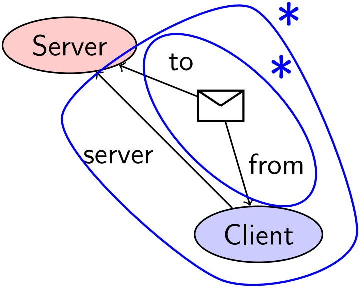
Figure: Example of a flat program

DBP are intrinsically not flat.

initial configuration:



covering set:

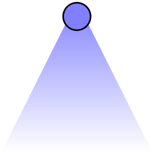


How many steps are there between the initial configuration and the final configuration ?
ω^2 steps

Hence, we need to consider nested loops if we want to compute the covering set.

From acceleration to widening

Acceleration considers transitions. Widening only states.



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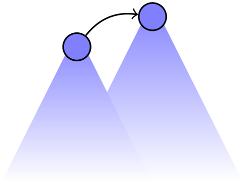
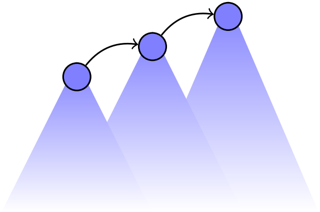


Acceleration considers transitions. Widening only states.



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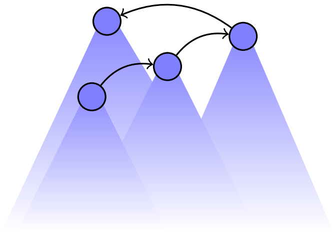

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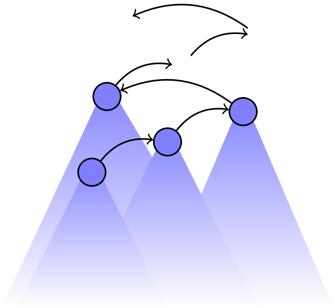

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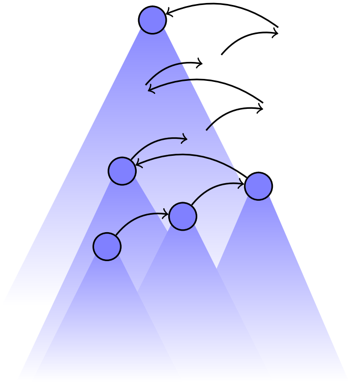

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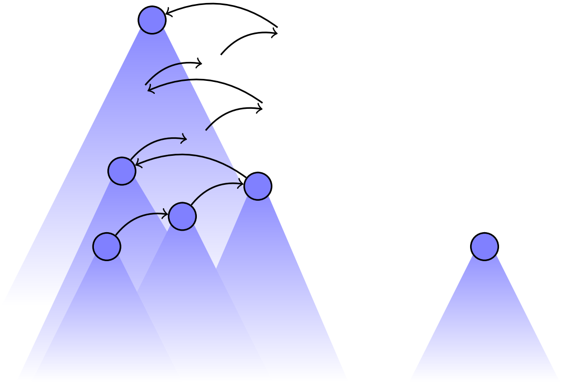

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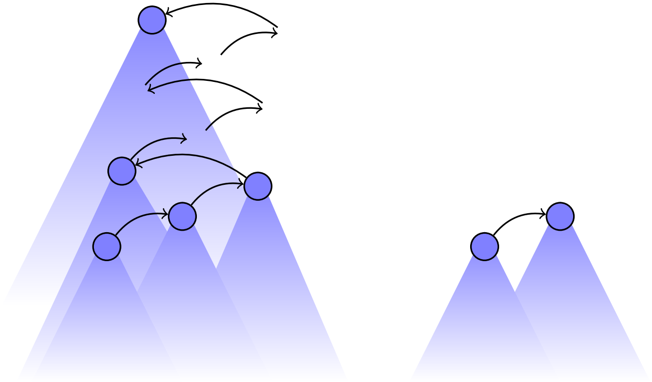
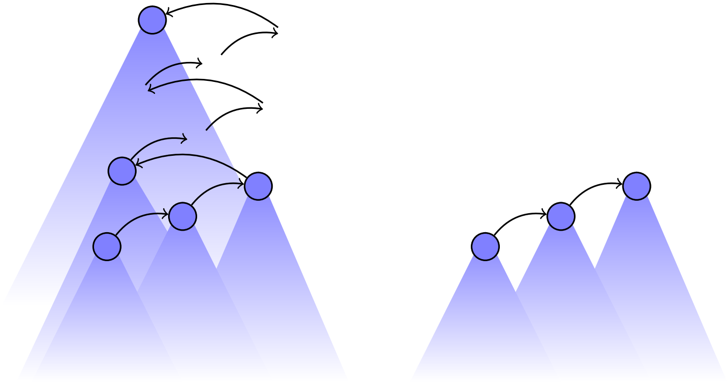


Acceleration considers transitions. Widening only states.



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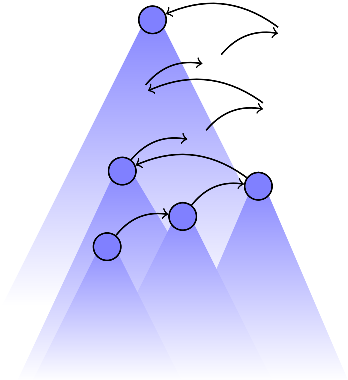

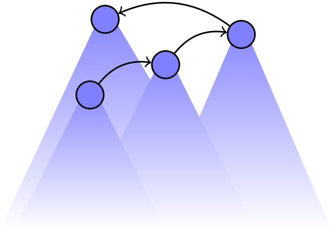

From acceleration to widening

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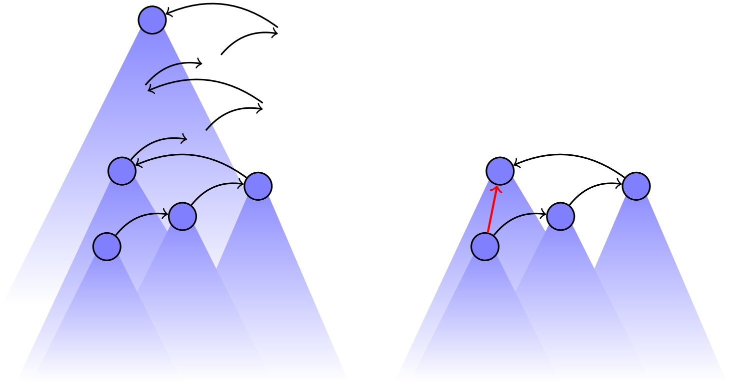

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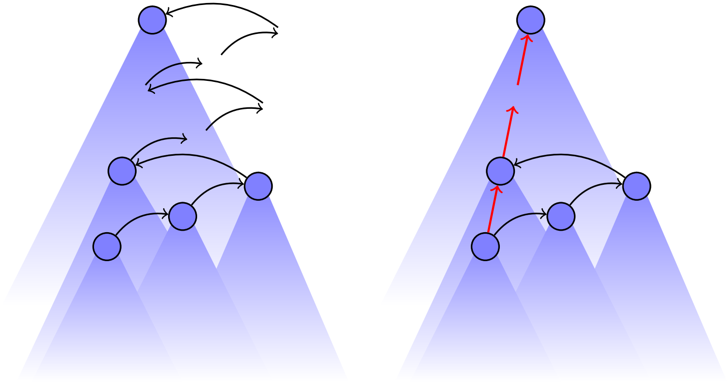


Abstract interpretation: Domains

- Concrete domain: $D = \mathcal{P}(S)$
- Abstract domain: $D_{\downarrow} = \{ \downarrow X \mid X \subseteq S \}$

The abstract domain can be further refined from the set of downward-closed set to the set of ideals (downward-closed and *directed*).

- Abstract domain 2: D_{Idl}

An arbitray downward-closed set can be represented as the finite union of ideals.

Widening (1)

Goal: try to mimic acceleration (when possible), and force termination

A set-widening operator (∇) for a poset X is partial function ($\mathcal{P}(X) \to X$) that satisfies:

- Covering : for all $Y \subseteq X$, $y \in Y \Rightarrow y \leq \nabla(Y)$;
- Termination : widening of any ascending chain stabilizes.

Reason of using a set-widening operator: we need the history.

Widening (2)

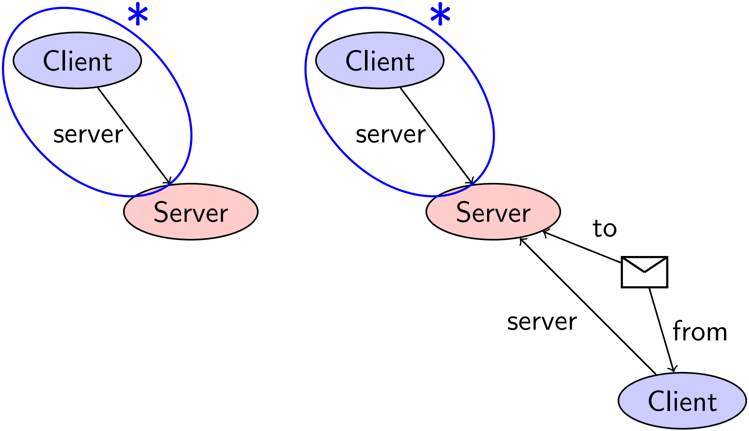
Lifting a widening operators from $Idl(S)$ to D_{Idl}: going from elements of the domain to finite powerset is non-trivial. We assume that the ordering is a *bqo*. Thus $Idl(S)$ is also a bqo.

Given an ascending chain: $C = \{L_i\}_{0 \leq i \leq n}, C \subseteq D_{Idl}$ (history)

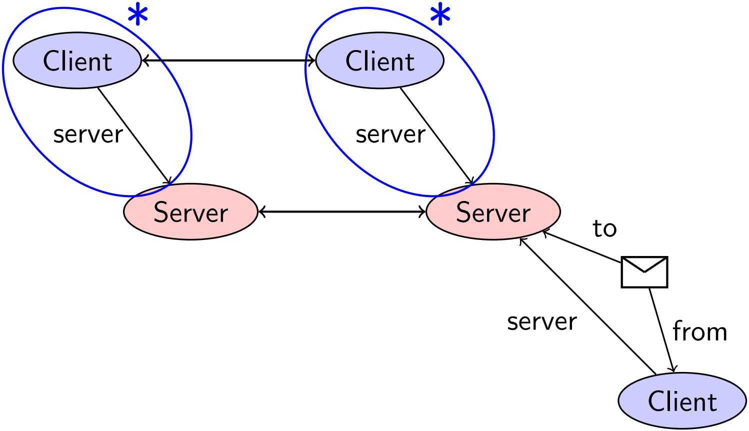
- $\nabla(\{L_0\}) = \{L_0\}$
- $\nabla(\{L_0, \ldots, L_i\}) = \nabla(\{L_0, \ldots, L_{i-1}\}) \sqcup \{\nabla_S(\mathcal{I}) \ \mid \mathcal{I} \text{ max ascending chain in} \nabla(\{L_0, \ldots, L_{i-1}\})\}$

Why a *bqo* ? To avoid having an infinite antichain in $Idl(S)$.

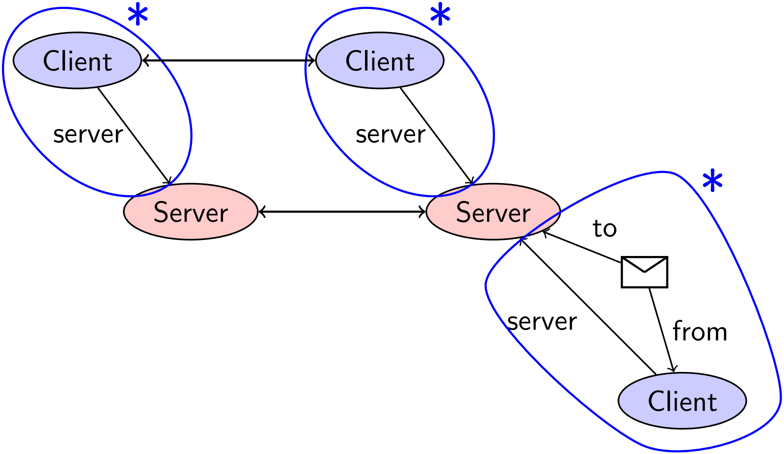
Set-widening for DBP (1)

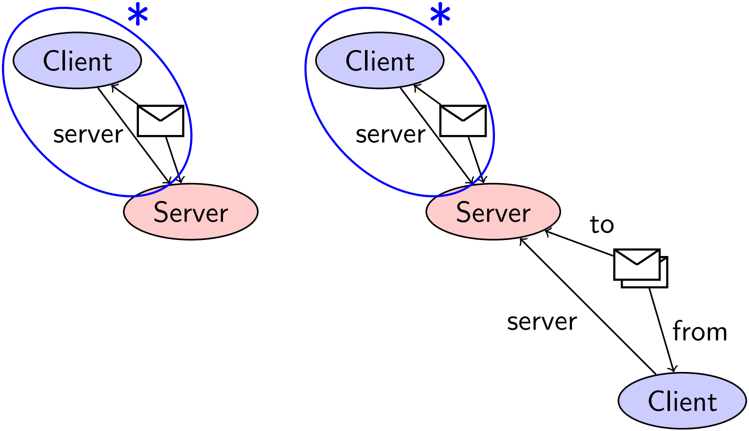


Set-widening for DBP (1)



Set-widening for DBP (1)

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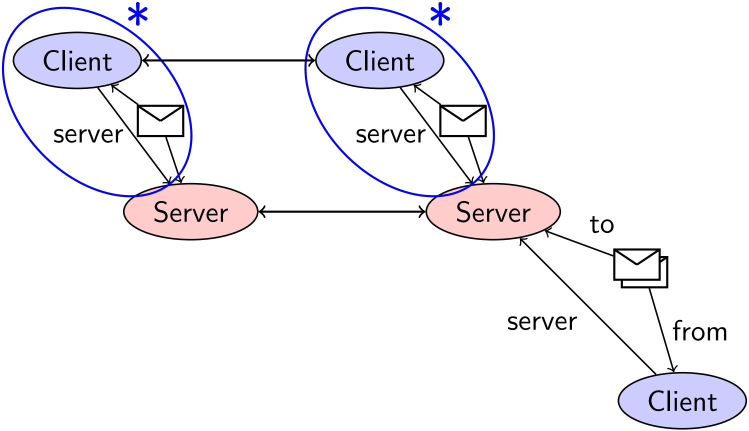
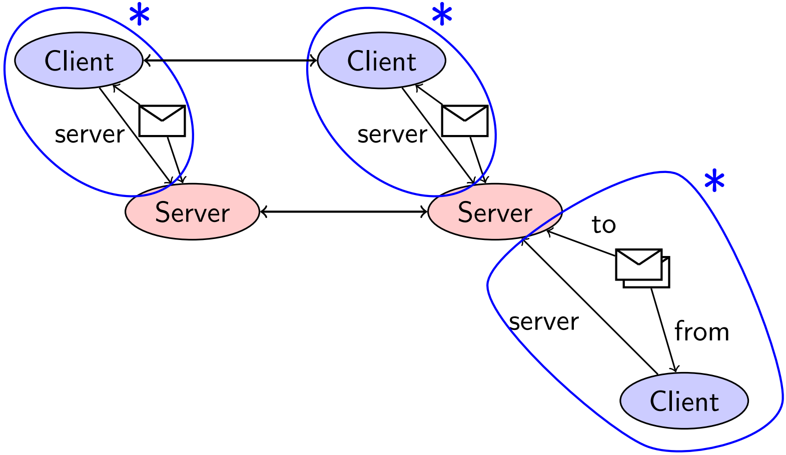
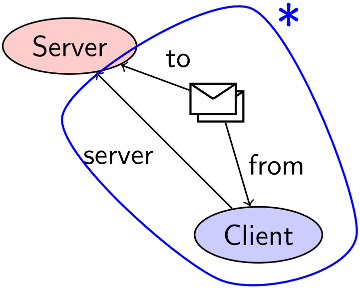
Set-widening for DBP (2)



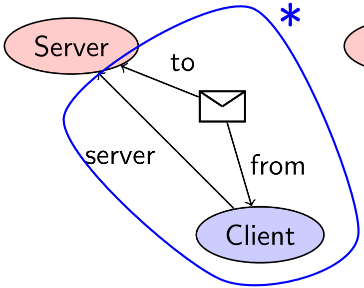
Set-widening for DBP (2)

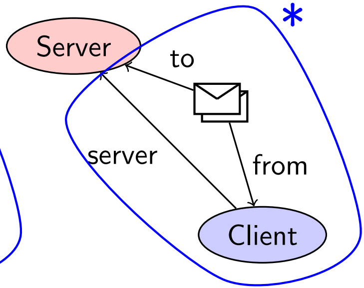


Set-widening for DBP (3)

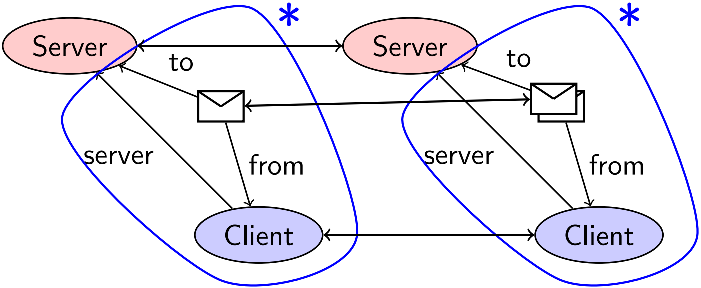


Set-widening for DBP (3)





Set-widening for DBP (3)



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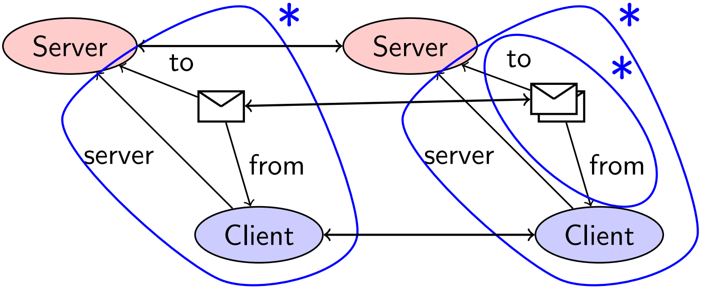

What about the precision ?

- Acceleration and widening seems like the *extreme* ends of some spectrum.
- Is there a class of nested loops for which we can compute exactly the result ?
- Can we get a good characterisation of the programs for which this kind of widening matches acceleration ?

- DBP is one of the largest fragment of the π-calculus for which interesting verification questions are still decidable.
- Not yet clear what is the right way of handling features such as process creation and mobility.
- WSTS approach gives decidability a result, now we are working on an efficient analysis.

Questions ?

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