Ideal Abstraction for Depth-Bounded Processes

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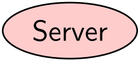
What are Depth-Bounded Processes (DBP) ?

As buzzwords: concurrent/distributed message-passing programs with process creation and mobility. (Warning restrictions may apply.)

For the programmers: some class of programs using the actor model (Erlang, Scala, Akka, ActorFoundry, . . .)

For the theoreticians: a fragment of the π-calculus.

Example: client-server communication pattern



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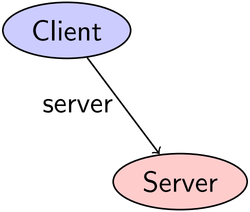

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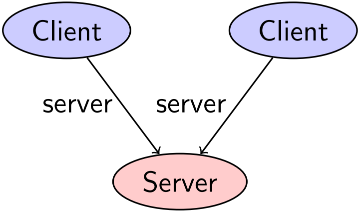

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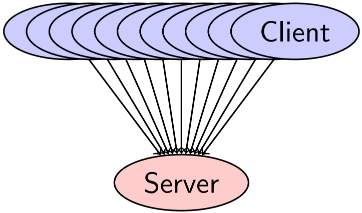

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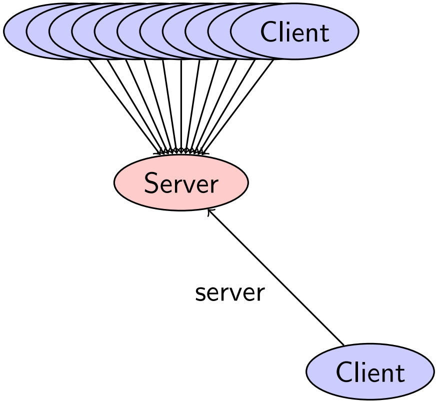

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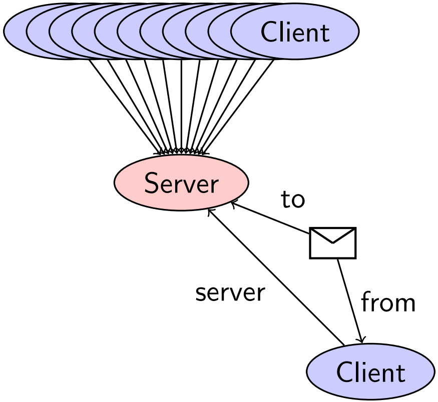

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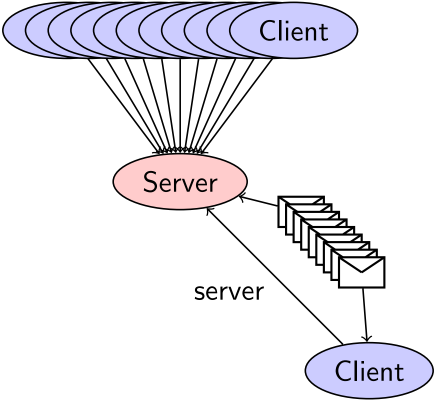
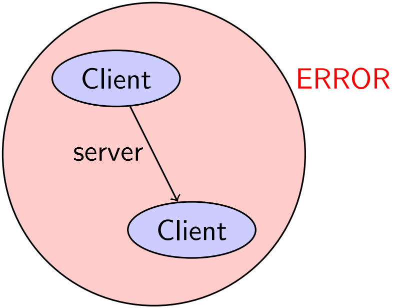


What kind of properties are we looking at ?

Safety properties, more precisely the control-state reachability problem (aka covering problem).



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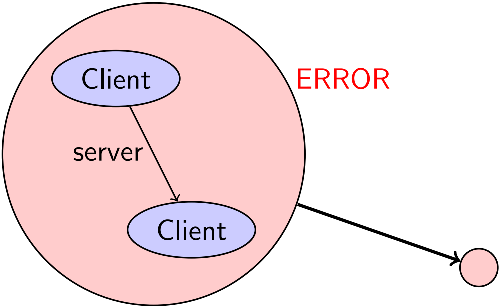

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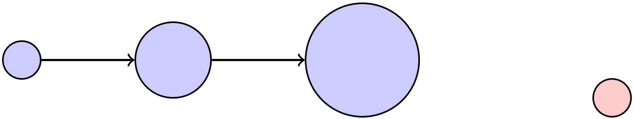

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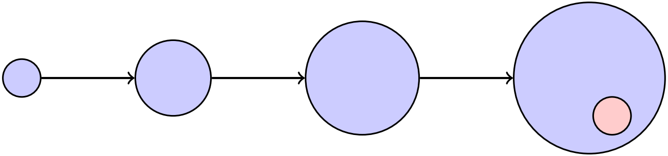

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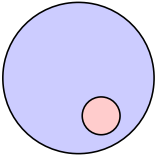

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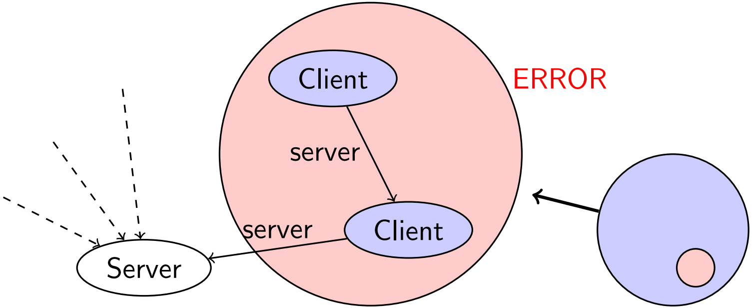


Formal model: WSTS

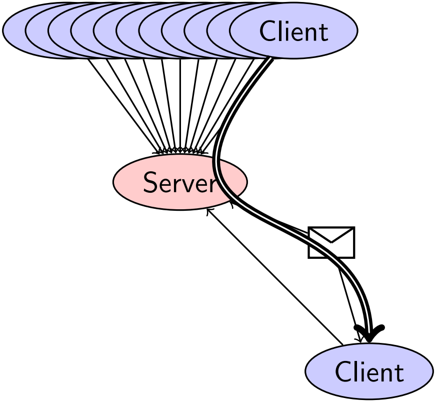
A well-structured transition system (WSTS) is a transition system $\langle S, \rightarrow, \leq \rangle$ such that:

- $\leq$ is a well-quasi-ordering (wqo), i.e. well-founded + no infinite antichain.
- compatibility of $\leq$ w.r.t. $\rightarrow$

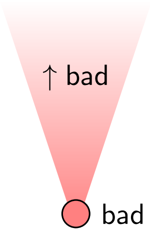
$$\forall \qquad \begin{array}{ccc} t & \xrightarrow{*} & t' \\ \rotatebox[origin=c]{90}{$\leq$} & & \rotatebox[origin=c]{90}{$\leq$} \\ s & \longrightarrow & s' \end{array} \qquad \exists$$

For more detail see:
[Finkel and Schnoebelen, 2001, Abdulla et al., 1996]

System with a bound on the longest acyclic path.
(Concretely: it is not possible to encode an infinite memory.)



Backward algorithm for covering



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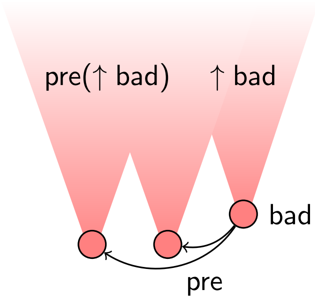

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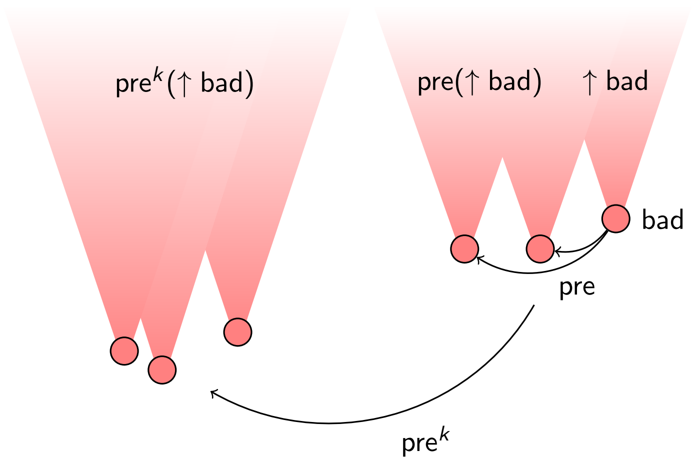

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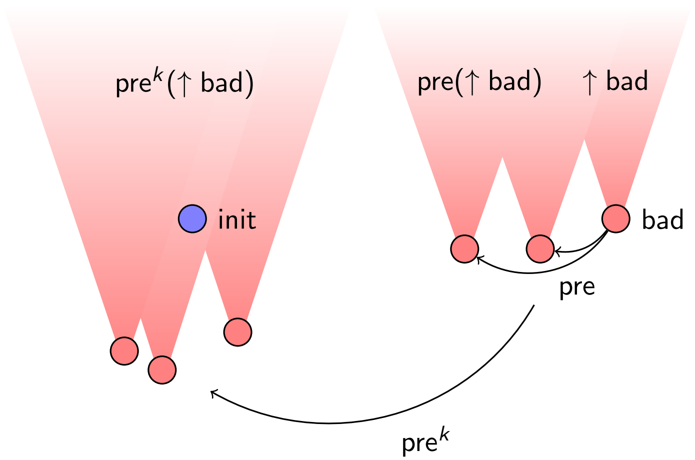

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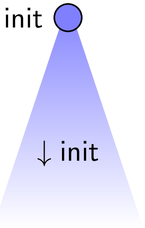

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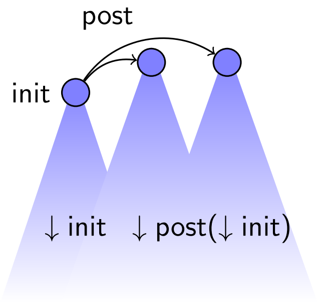

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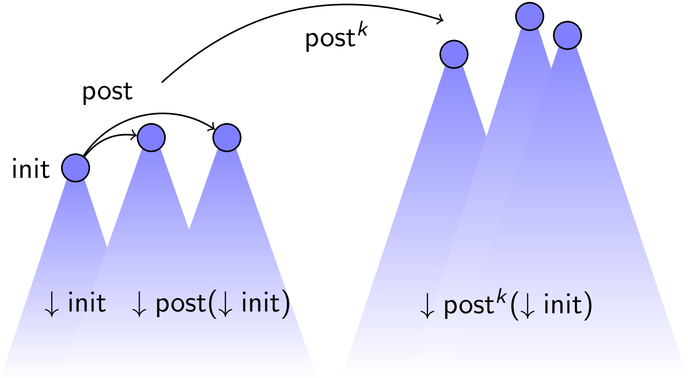

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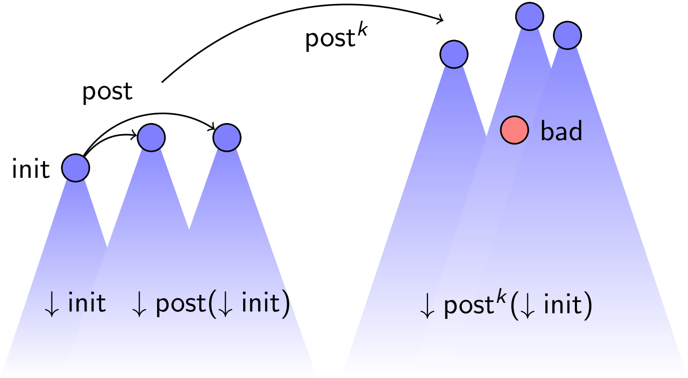
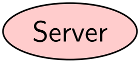


Adequate domain of limits

ADL: [Geeraerts et al., 2006]
Further developed in [Finkel and Goubault-Larrecq, 2009]
Applied to DBP in [Wies et al., 2010]



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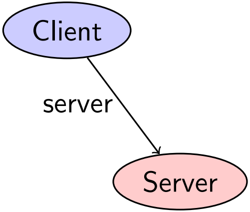

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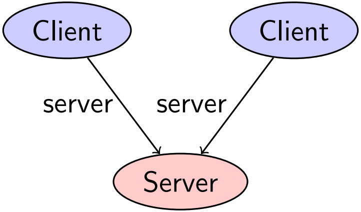

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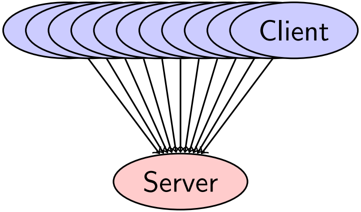

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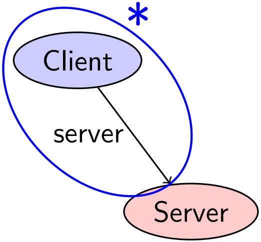

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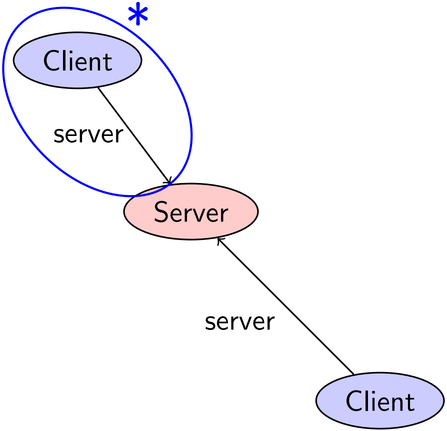

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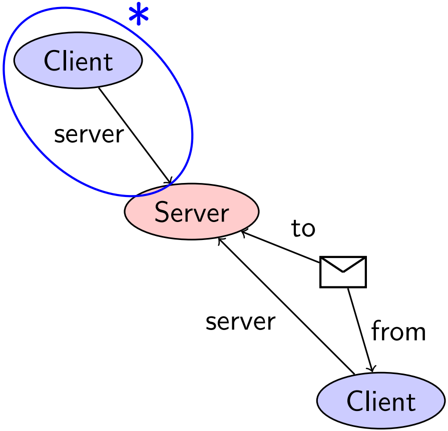

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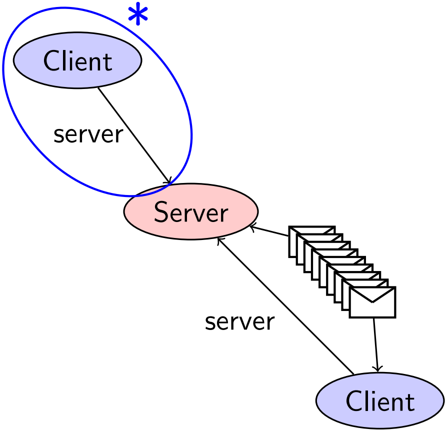

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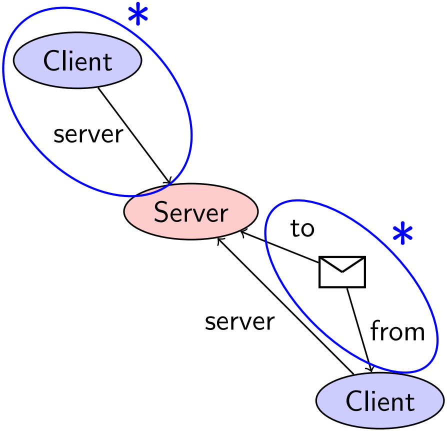

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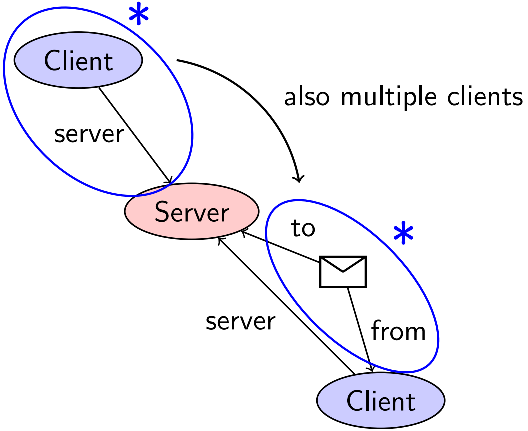

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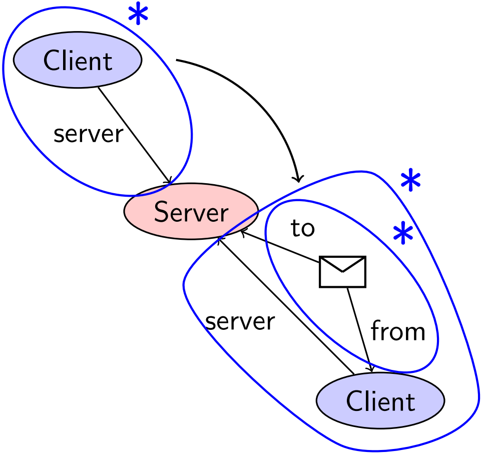

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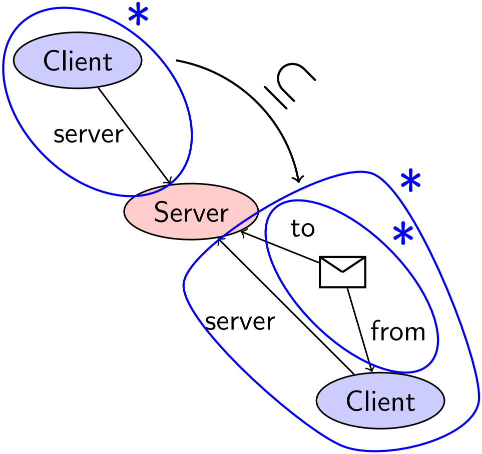

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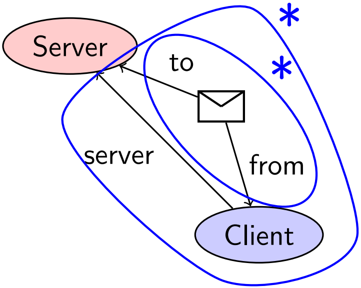

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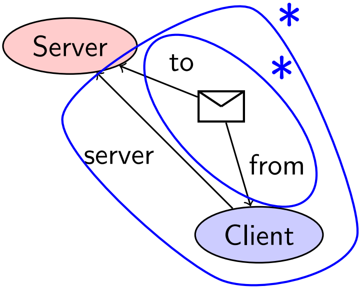
$$(\nu x)(\mathit{Server}(x) \mid !(\nu y)(\mathit{Client}(y, x) | !\mathit{Messages}(x, y)))$$



When does acceleration work ? (flat systems)

Usually forward algorithms are based on acceleration. By acceleration we mean computing the result of executing a loop infinitely many time.

We can see this as computing the result of execution traces of length $< \omega^2$. Concretely, it means that the algorithm can saturate the covering set by executing only simple loops (see [Bardin et al., 2005]). This condition is known as flattability.

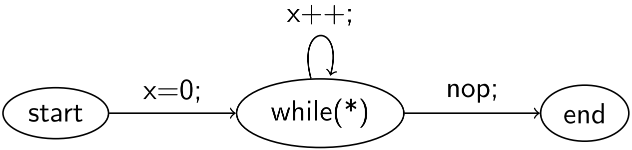
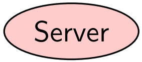
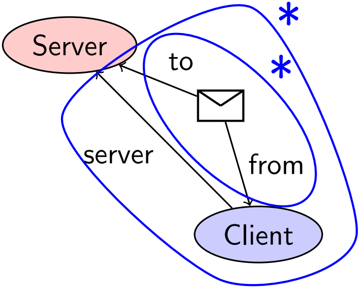


Figure: Example of a flat program

initial configuration:



covering set:

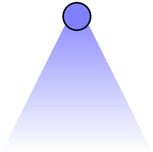


How many steps are there between the initial configuration and the final configuration ?
ω^2 steps

Hence, we need to consider nested loops if we want to compute the covering set.

From acceleration to widening

Acceleration considers transitions. Widening only states.



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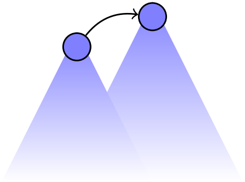

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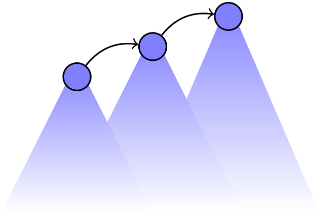

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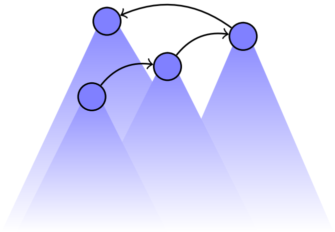

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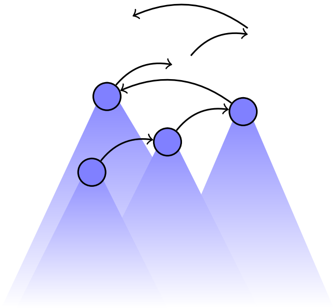

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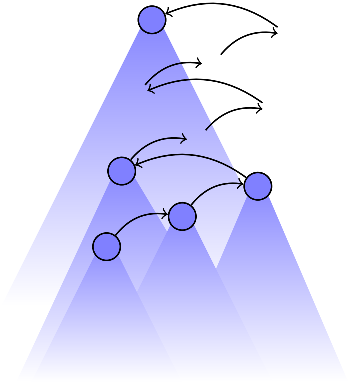

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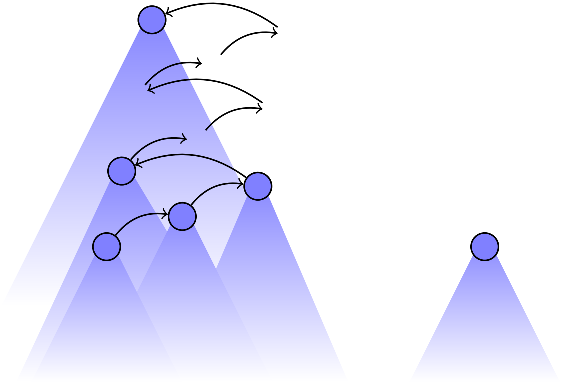

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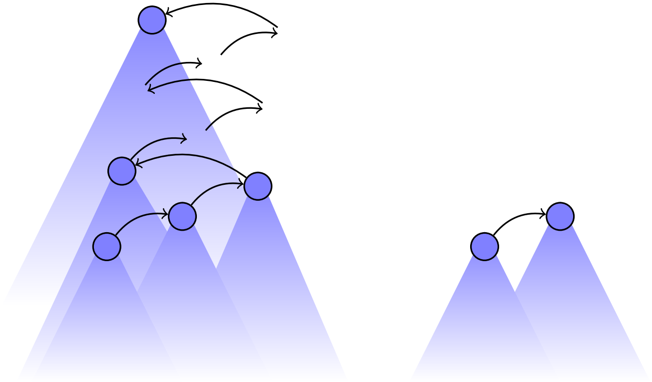

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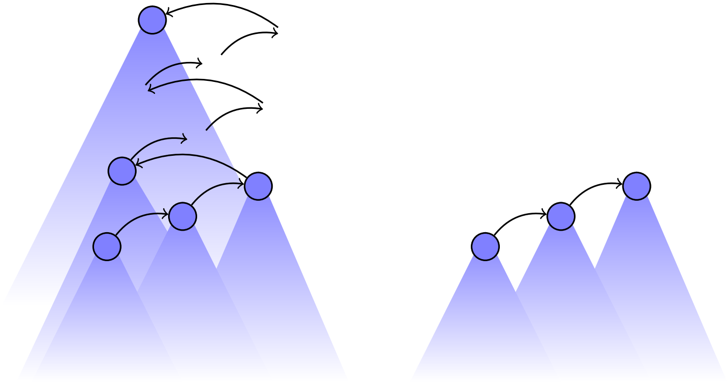

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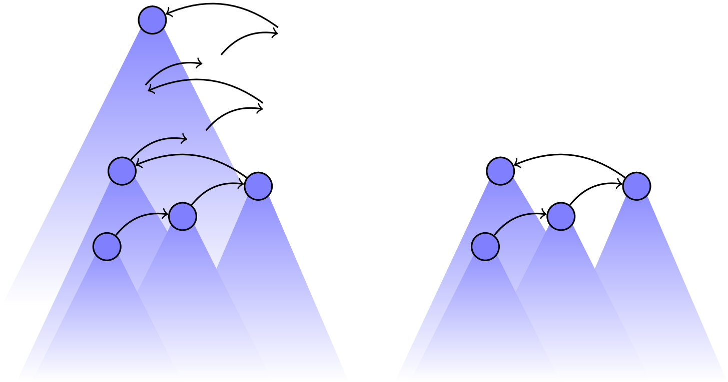

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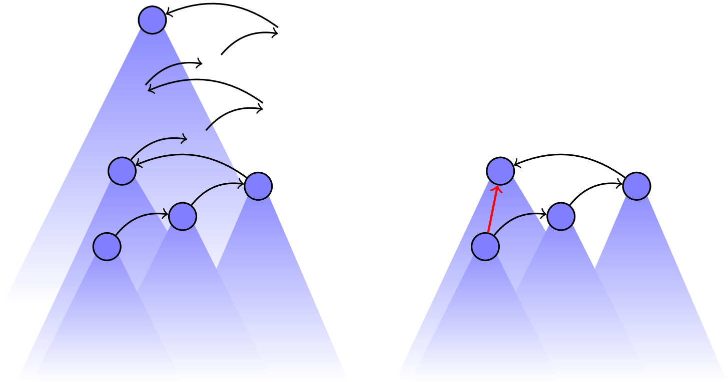

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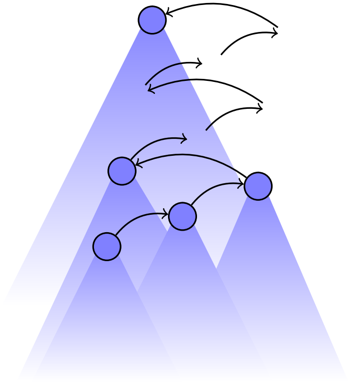


Abstract interpretation: Domains

- Concrete domain: $D = \mathcal{P}(S)$
- Abstract domain: $D_{\downarrow} = \{ \downarrow X \mid X \subseteq S \}$

The abstract domain can be further refined from the set of downward-closed set to the set of ideals (downward-closed and *directed*).

- Abstract domain 2: D_{Idl}

An arbitray downward-closed set can be represented as the finite union of ideals.

Widening (1)

Goal: try to mimic acceleration (when possible), and force termination

A set-widening operator (∇) for a poset X is partial function ($\mathcal{P}(X) \to X$) that satisfies:

Covering : for all $Y \subseteq X$, $y \in Y \Rightarrow y \leq \nabla(Y)$;

Termination : widening of any ascending chain stabilizes.

Reason of using a set-widening operator: we need the history.

Widening (2)

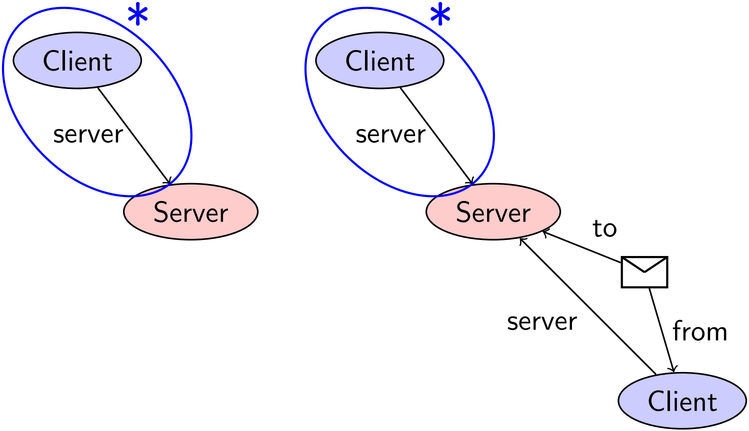
Lifting a widening operators from $Idl(S)$ to D_{Idl}: going from elements of the domain to finite powerset is non-trivial. We assume that the ordering is a *bqo*. Thus $Idl(S)$ is also a bqo.

Given an ascending chain: $C = \{L_i\}_{0 \leq i \leq n}, C \subseteq D_{Idl}$ (history)

- $\nabla(\{L_0\}) = \{L_0\}$
- $\nabla(\{L_0, \ldots, L_i\}) = \nabla(\{L_0, \ldots, L_{i-1}\}) \sqcup$
  $\{\nabla_S(\mathcal{I}) \ \mid \mathcal{I}$ max ascending chain in$\nabla(\{L_0, \ldots, L_{i-1}\})\}$

Why a *bqo* ? To avoid having an infinite antichain in $Idl(S)$.

Set-widening for DBP (1)



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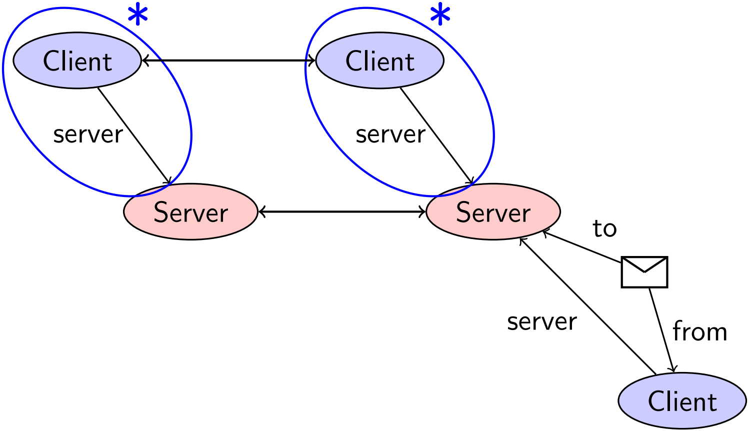

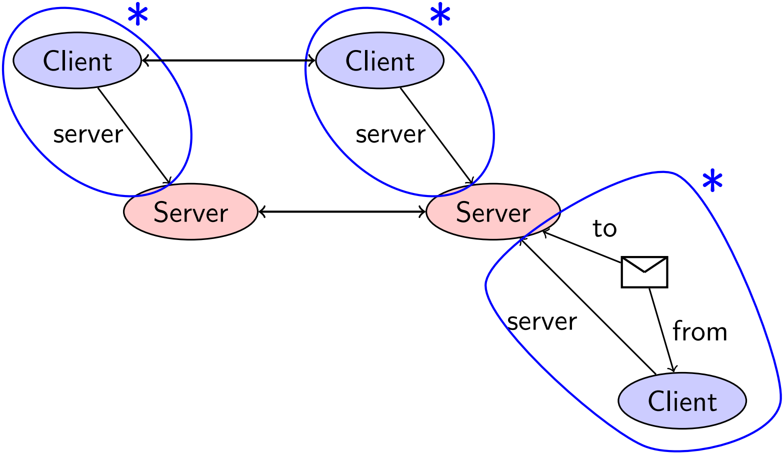
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Client
server
Server
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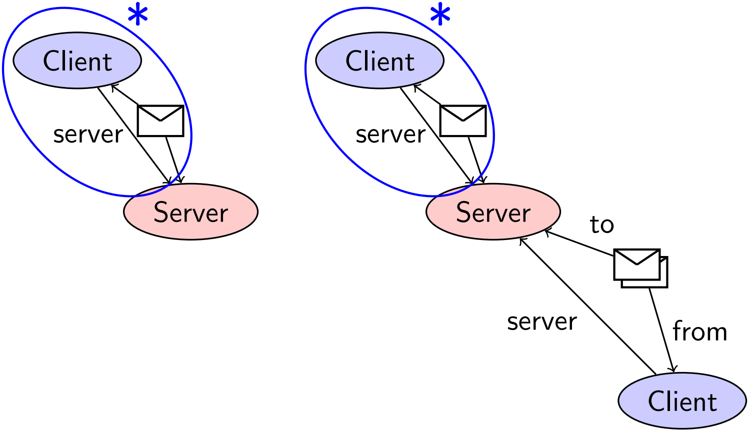
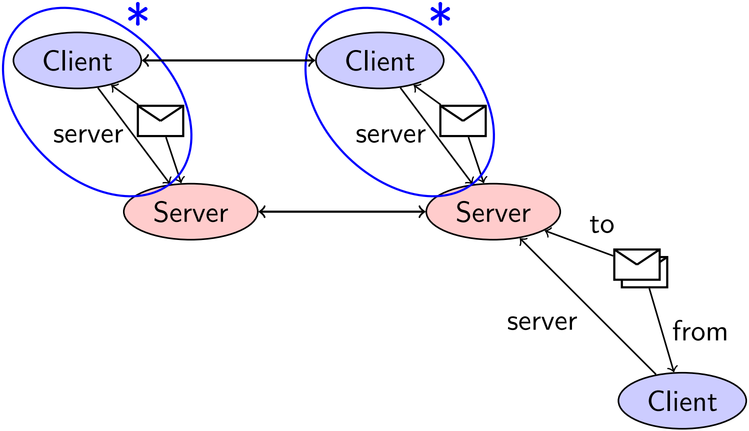
Client
server
Server
*
to
server
from
Client

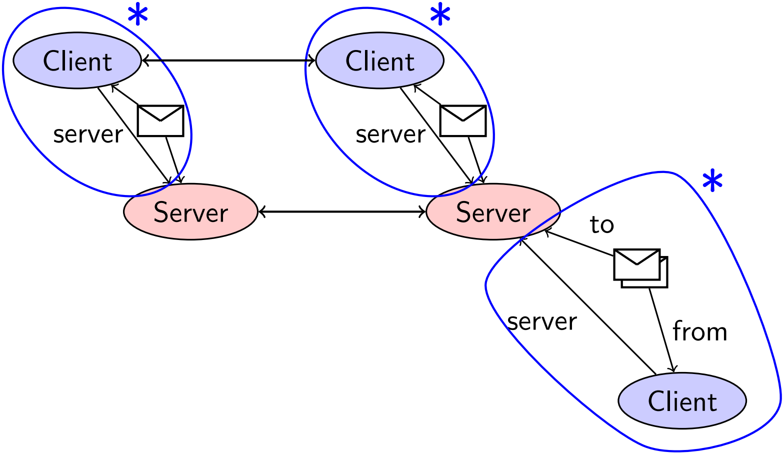
Set-widening for DBP (2)



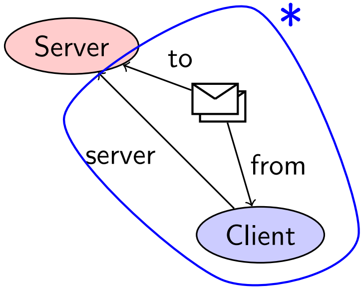
Set-widening for DBP (2)



Set-widening for DBP (2)



Set-widening for DBP (3)



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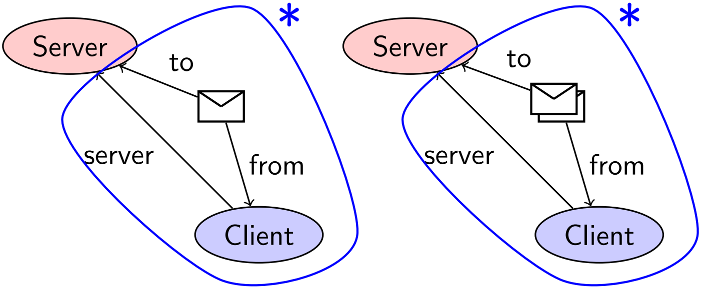

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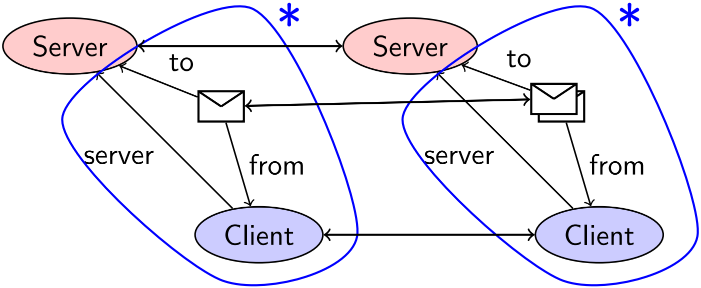

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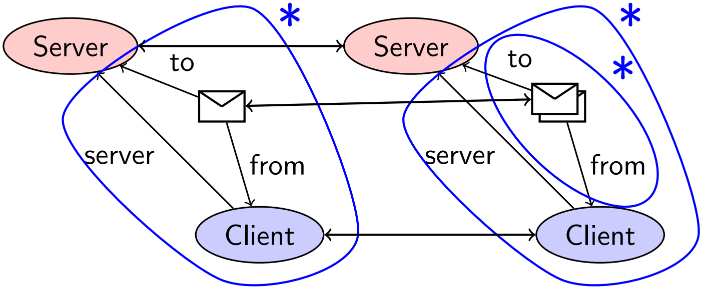

What about the precision ?

- Acceleration and widening seems like the *extreme* ends of some spectrum.
- Is there a class of nested loops for which we can compute exactly the result ?
- Can we get a good characterisation of the programs for which this kind of widening matches acceleration ?

Recap

- DBP is one of the largest fragment of the π-calculus for which interesting verification questions are still decidable.
- Not yet clear what is the right way of handling features such as process creation and mobility.
- WSTS approach gives decidability a result, now we are working on an efficient analysis.

Questions ?

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