Math Camp

Justin Grimmer

Professor
Department of Political Science
Stanford University

September 4th, 2018

$< \text{Course} >$

The Systematic Analysis of Politics

Social Science: systematic analysis of society

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- Math Camp: Develop Tools for Analysis

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This class (introduction):

- Math Camp: Develop Tools for Analysis
- Probability theory: systematic model of randomness

Course Goals

First stop in methodology sequence

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Big Goal: prepare you to make discoveries about social world

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Proximate Goals

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1) Mathematical tools to comprehend and use statistical methods
2) Foundation in probability theory/analytic reasoning

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3) Practical Computing Tools: `R`

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1) Mathematical tools to comprehend and use statistical methods
2) Foundation in probability theory/analytic reasoning
3) Practical Computing Tools: `R`
4) Introduction to Logic of Formal Modeling

Course Staff

Me: Justin Grimmer and Avidit Acharya

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Extra Info

- Zuhad Hai and Jesse Yoder
- Github for class: `github/justingrimmer/Math18`

Prerequisites

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- Successful students will know differential and integral calculus

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    - Empirical: calculus and linear algebra
    - Quantitative Methodologist: Real Analysis and Grad level statistics
    - Formal Theory: Real Analysis (through measure theory), Topology

Evaluation

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Grad School Irony Or: How I Learned to Stop Worrying and Love C's

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- Learn as much material as possible
- If you truly only care about learning material, you'll get amazing grades

Homework

Math camp: assigned daily $\rightsquigarrow$ Mechanics of solving problems
Lab Assignment: Twice weekly assignments, help you develop computational and mathematical skills.

Computing/Homeworks

Greatest scientific discovery of 20th Century:

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- Flexible, Cutting Edge Software, great visualization tools

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- If you use start using LaTeX, you'll soon love it

Course Books

1) Simon, Carl and Blume, Lawrence (SB). Mathematics for Economists.
2) Bertsekas, Dimitri P. and Tsitsiklis, John (BT) Introduction to Probability Theory (second edition)

Life in Graduate School/Academy

Three part mixture:

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Three part mixture:

George Strait

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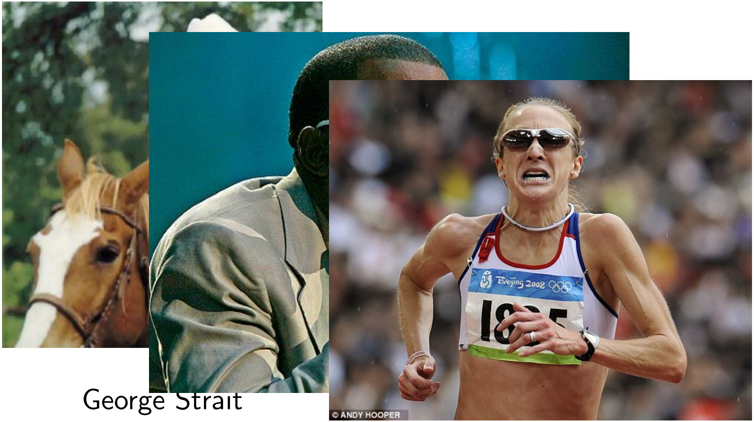

George Strait

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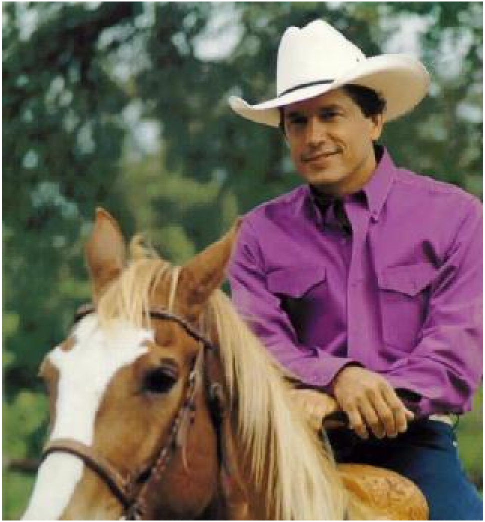

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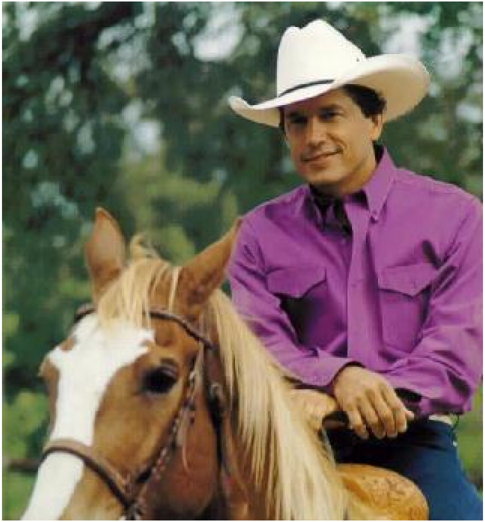

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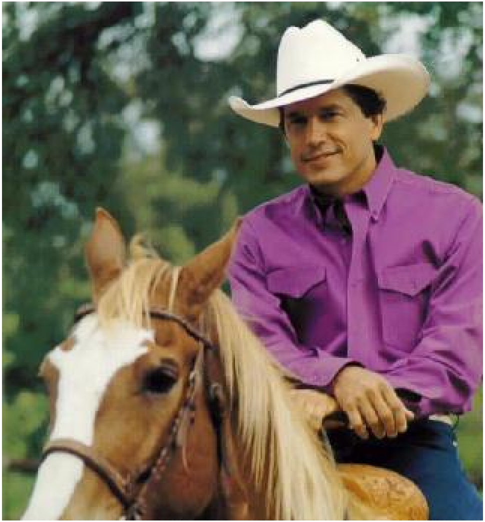

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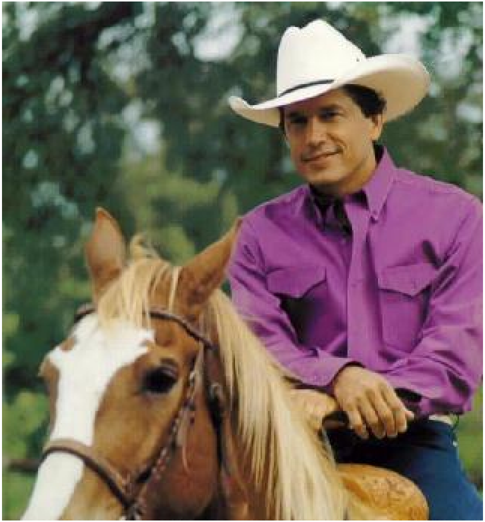

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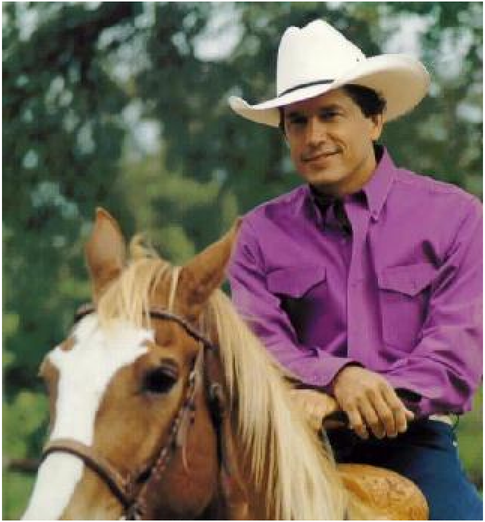

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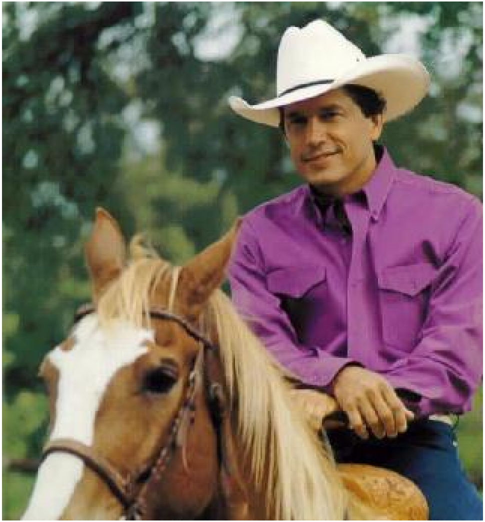

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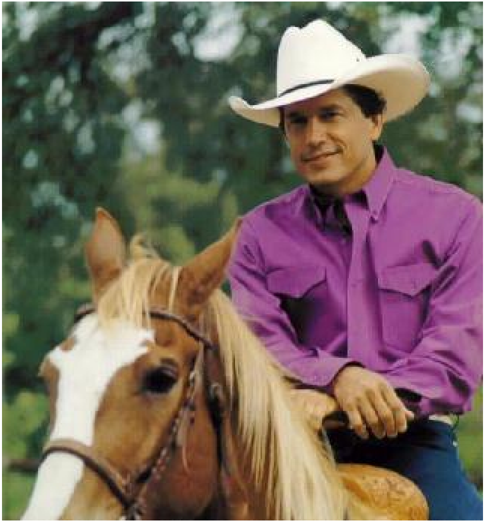

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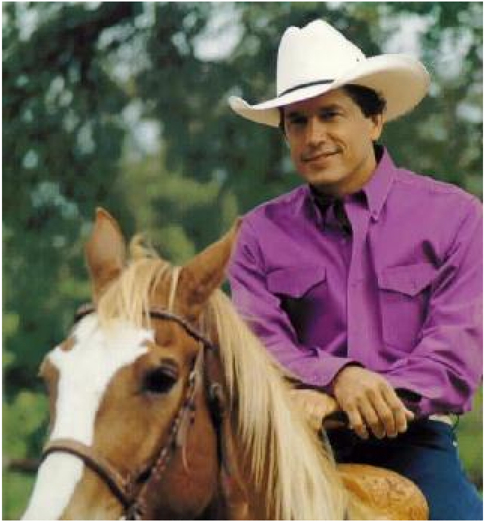

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- If you're good at methods, you'll be more rich [in expectation] and equally free

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- Self confidence: believe in work

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- World class distance running: it is hard

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- Mantra: sustained agony

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- Time away from lab $\rightsquigarrow$ more productive when back

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- Success: work
- Treat grad school like a job
- Who gets ahead? who gets the most work done on the smartest ideas

Preliminaries

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1) Introduction to more sophisticated mathematics (notation)

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2) Getting acquainted with proof techniques and proofs

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3) I'm going to introduce ideas/example problems common in research that will help with your seminar

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Smartest people ask the most questions!

Let's get to work

Simple Logical Statements

Sets

A set is a collection of objects.

$$\begin{aligned} A &= \{1, 2, 3\} \\ B &= \{4, 5, 6\} \\ C &= \{\text{First year cohort}\} \\ D &= \{\text{Stanford University Faculty}\} \end{aligned}$$

Definition

If A is a set, we say that x is an element of A by writing, $x \in A$. If x is not an element of A then, we write $x \notin A$.

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- Sets are necessary for probability theory

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Why Care?

- Sets are necessary for probability theory
- Defining set is equivalent ot choosing population of interest (usually)

Definition

If A and B are sets, then we say that $A = B$ *if, for all* $x \in A$ *then* $x \in B$ *and for all* $y \in B$ *then* $y \in A$.

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Difference between definitions?

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Let A and B be sets. If $A = B$ then $A \subset B$ and $B \subset A$

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$\Leftarrow$ Suppose $A \subset B$ and that $B \subset A$. Now, by way of contradiction, suppose that $A \neq B$. $A \neq B$ only if there is $x \in A$ and $x \notin B$ or if $y \in B$ and $y \notin A$. But then, either $A \not\subset B$ or $B \not\subset A$, contradicting our initial assumption. $\square$

Set Builder Notation

- Some famous sets
    - $J = \{1, 2, 3, \ldots\}$
    - $Z = \{\ldots, -2, -1, 0, 1, 2, \ldots, \}$
    - $\Re$ = real numbers (more to come about this)
- Use set builder notation to identify subsets
    - $[a, b] = \{x : x \in \Re \text{ and } a \leq x \leq b\}$
    - $(a, b] = \{x : x \in \Re \text{ and } a < x \leq b\}$
    - $[a, b) = \{x : x \in \Re \text{ and } a \leq x < b\}$
    - $(a, b) = \{x : x \in \Re \text{ and } a < x < b\}$
    - $\emptyset$

Set Operations

We can build new sets with set operations.

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Definition

Suppose A and B are sets. Define the Union of sets A and B as the new set that contains all elements in set A or in set B. In notation,

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- $D = \{\text{First Year Cohort}\}, E = \{\text{Me}\}$, then $F = D \cup E = \{\text{First Year Cohort, ME}\}$

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Some Facts about Sets (No Venn Diagrams!!!)

1) $A \cap B = B \cap A$

Some Facts about Sets (No Venn Diagrams!!!)

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Proof.

This fact (theorem) says that the set $A \cap B$ is equal to the set $B \cap A$. We can use the definition of equal sets to test this. Suppose $x \in A \cap B$. Then $x \in A$ and $x \in B$. By definition, then, $x \in B \cap A$. Now, suppose $y \in B \cap A$. Then $y \in B$ and $y \in A$. So, by definition of intersection $y \in A \cap B$. This implies $A \cap B = B \cap A$ □

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Proof.

Suppose $x \in A \cap (B \cup C)$. Then $x \in B$ or $x \in C$ and $x \in A$. This implies that $x \in (A \cap B)$ or $x \in (A \cap C)$. Or, $x \in (A \cap B) \cup (A \cap C)$. Now, suppose $y \in (A \cap B) \cup (A \cap C)$. Then, $y \in A$ and $y \in B$ or $y \in C$. Well, this implies $y \in A \cap (B \cup C)$. And we have established equality $\square$

Some Facts about Sets (No Venn Diagrams!!!)

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6) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Break into groups, derive for the remaining facts

Ordered Pair

You've seen an ordered pair before,

$$(a, b)$$

Definition

Suppose we have two sets, A and B. Define the Cartesian product of A and B, $A \times B$ as the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. In other words,

$$A \times B \quad = \quad \{(a, b) : a \in A \text{ and } b \in B\}$$

Example:
$A = \{1, 2\}$ and $B = \{3, 4\}$, then,
$A \times B = \{(1, 3); (1, 4); (2, 3); (2, 4)\}$

Function

Start with general and move to specific— (abstract just takes time to get acquainted)

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Definition

A relation is a set of ordered pairs. A function F is a relation such that,

$$(x, y) \in F \quad ; \quad (x, z) \in F \Rightarrow y = z$$

We will commonly write a function as $F(x)$, where $x \in$ Domain F and $F(x) \in$ Codomain F. It is common to see people write,

$$F : A \rightarrow B$$

where A is domain and B is codomain

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Examples

- $F(x) = x$
- $F(x) = x^2$
- $F(x) = \sqrt{x}$

R Computing Language

- We're going to use R throughout the course
- R as calculator :

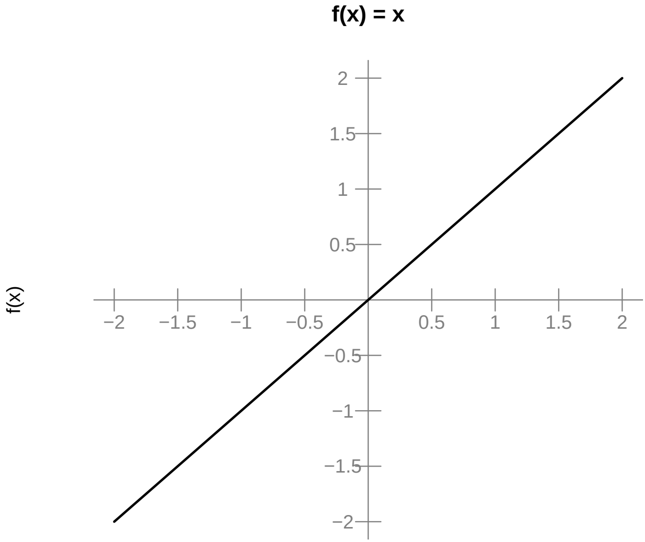
```
> 1 + 1
[1] 2
> 'Hello World'
[1] ''Hello World"
```

- `object<- 2 ## assign numbers to objects`
- R has functions defined, we can define them to objects as well

```
first.func<- function(x) {
out<- 2*x
return(out) }
```

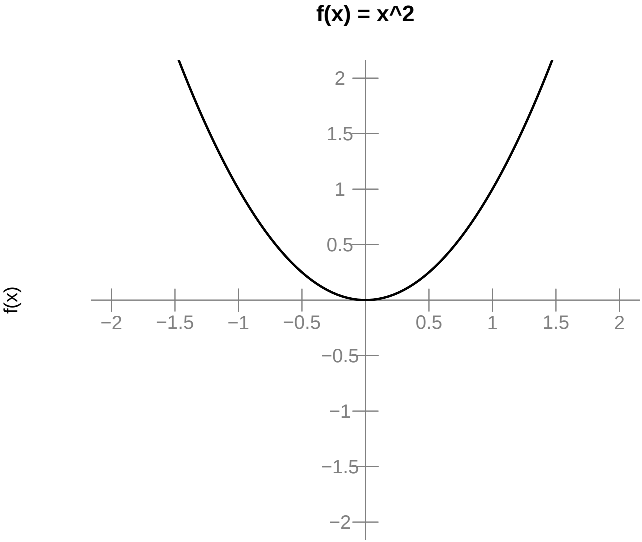
```
first.func(2)
[1] 4
```

Plotting Functions



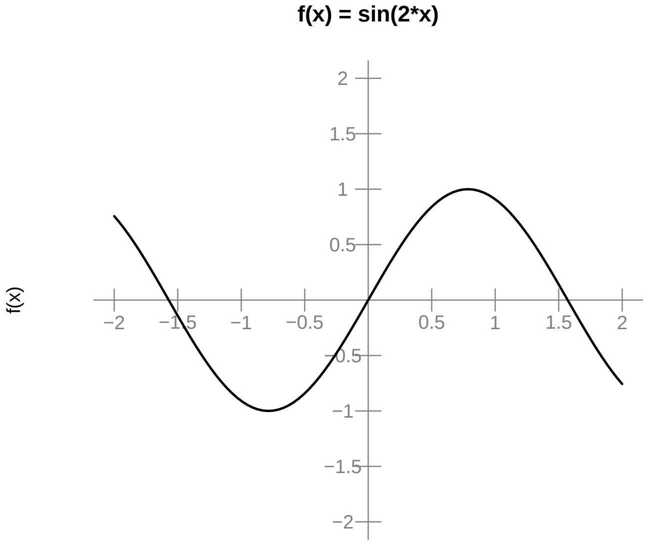

```
x<- seq(-2, 2,
len=1000)
plot(x~x) ##
Results may vary
```

Plotting Functions



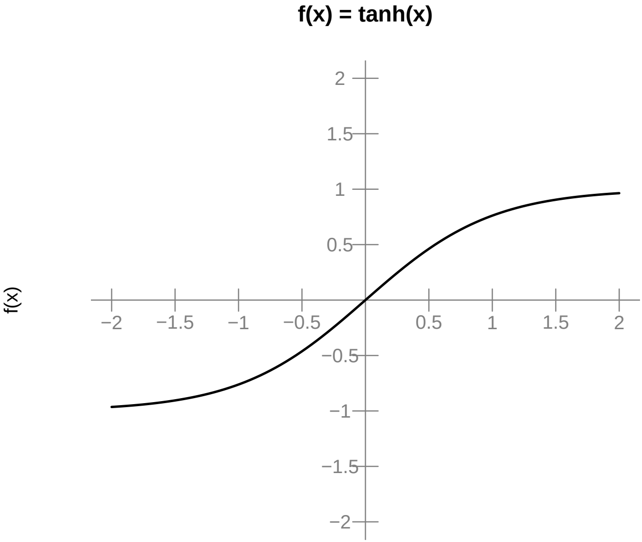

```
x<- seq(-2, 2,
len=1000)
x.2<- x*x
plot(x.2~x)
```

Plotting Functions




```
x<- seq(-2, 2,
len=1000)
sin.2x<- sin(2*x)
plot(sin.2x~x)
```

Plotting Functions




```
x<- seq(-2, 2,
len=1000)
tanhx<- tanh(x)
plot(tanhx~x)
```

Exponents, Logarithms, and All That

Exponents, Logarithms, and All That

$$f(x) = 2^x$$

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Some rules of exponents remember a could equal e

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Properties of Functions

Two important properties of functions

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A function $f : A \rightarrow B$ is 1-1 (one-to-one, or injective) if for all $y \in A$ and $z \in A$ in Domain, $f(y) = f(z)$ implies $y = z$. In other words, preserves distinctiveness.

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- $f : R \rightarrow R$ $f(x) = x$. Onto and 1-1, bijective

Composite Functions

Definition

Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$. Then, define,

$$g \circ f \quad = \quad g(f(x))$$

- $f(x) = x$, $g(x) = x^2$. Then $g \circ f = x^2$.
- $f(x) = \sqrt{x}$, $g(x) = e^x$. Then $g \circ f = e^{\sqrt{x}}$.
- $f(x) = sin(x)$, $g(x) = |x|$. Then $g \circ f = |sin(x)|$.

Inverse Function

Definition

Suppose a function f is 1-1. Then we'll define f^{-1} as its inverse if,

$$f^{-1}(f(x)) = x$$

Why do we need 1-1?

Induction

Well Ordering Principle Every non-empty set J has a smallest number

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Theorem

If $P(n)$ is a statement containing the variable n such that

i. *$P(1)$ is a true statement, and*

ii. *for each $k \in 1, 2, 3, 4, \ldots, n, \ldots$ if $P(k)$ is true then $P(k+1)$ is true*

then $P(n)$ is true for all $n \in 1, 2, 3, 4, \ldots, n, \ldots$

Induction and Contradiction

We'll use contradiction and well ordering to prove that induction works.

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□

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Summing N numbers

Induction is a useful proof technique.

Theorem

$\sum_{i=1}^{N} i = 1 + 2 + 3 + 4 + \ldots + N = \frac{N(N+1)}{2}$

Two conditions to show:

i. $\sum_{i=1}^{1} i = 1$ and $\frac{1(1+1)}{2} = 1$

Summing N numbers

ii. Suppose true N. Then, for $N+1$ we have,

$$\begin{aligned}
\sum_{i=1}^{N+1} i &= \sum_{i=1}^{N} i + (N+1) \\
&= \frac{N(N+1)}{2} + \frac{2(N+1)}{2} \\
&= \frac{(N+1)(N+2)}{2} \\
&= \frac{(N+1)((N+1)+1)}{2}
\end{aligned}$$

Conditions of induction met. Therefore, proof complete

Very Simple R Code

Finite, Countable, and Uncountable

Three sizes of sets

1) A set, X is finite if there is a bijective function from $\{1, 2, 3, \ldots, n\}$ to X.
2) A set X is countably infinite if there is a bijective function from $\{1, 2, 3, 4, \ldots, \}$ to X.
3) A set X is uncountably infinite if it is not countable

The Real numbers are uncountably infinite

Recap

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PLEASE don't worry—we're here to help!

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1) Sets + Operations
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3) Contradiction, Induction, and direct proofs

Tomorrow:

- Convergence of sequences
- Limits
- Continuity
- Derivatives