

Math Camp

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Department of Political Science
Stanford University

September 4th, 2018

< Course >

The Systematic Analysis of Politics

Social Science: systematic analysis of society

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- Math Camp: Develop Tools for Analysis

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This class (introduction):

- Math Camp: Develop Tools for Analysis
- Probability theory: systematic model of randomness

Course Goals

First stop in methodology sequence

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Big Goal: prepare you to make **discoveries** about social world

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- 1) Mathematical tools to comprehend and use statistical methods
- 2) Foundation in probability theory/analytic reasoning

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- 3) Practical Computing Tools: R

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- 1) Mathematical tools to comprehend and use statistical methods
- 2) Foundation in probability theory/analytic reasoning
- 3) Practical Computing Tools: R
- 4) Introduction to Logic of Formal Modeling

Course Staff

Me: Justin Grimmer and Avidit Acharya

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Extra Info

- Zuhad Hai and Jesse Yoder
- Github for class: `github/justingrimmer/Math18`

Prerequisites

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- Successful students will know differential and integral calculus

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 - Empirical: calculus and linear algebra
 - Quantitative Methodologist: Real Analysis and Grad level statistics
 - Formal Theory: Real Analysis (through measure theory), Topology

Evaluation

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- If you truly only care about learning material, you'll get amazing grades

Homework

Math camp: assigned daily \rightsquigarrow Mechanics of solving problems

Lab Assignment: Twice weekly assignments, help you develop computational and mathematical skills.

Computing/Homeworks

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- **If you use start using \LaTeX , you'll soon love it**

Course Books

- 1) Simon, Carl and Blume, Lawrence (SB). Mathematics for Economists.
- 2) Bertsekas, Dimitri P. and Tsitsiklis, John (BT) Introduction to Probability Theory (second edition)

Life in Graduate School/Academy

Three part mixture:

Life in Graduate School/Academy

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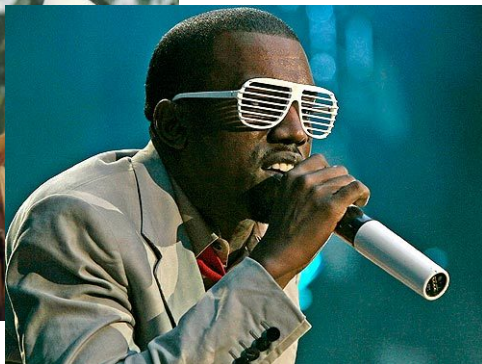
George Strait

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Kanye West

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Paula Radcliffe

$\frac{1}{3}$ George Strait



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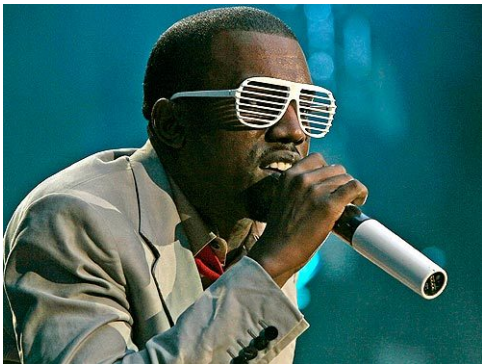
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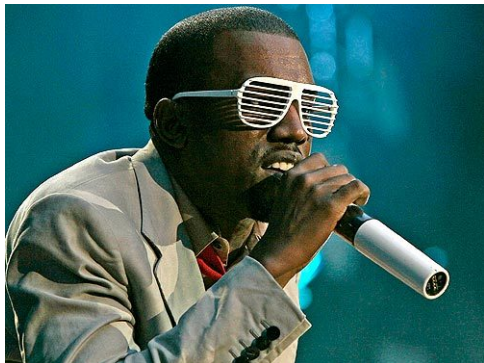
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- But (lord) we’re free
- If you’re good at methods, you’ll be more rich [in expectation] and equally free

$\frac{1}{3}$ Kayne West

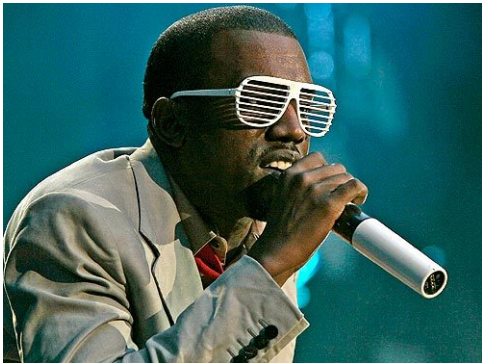


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- Deal with explicit criticism (part of Hip/Hop culture)



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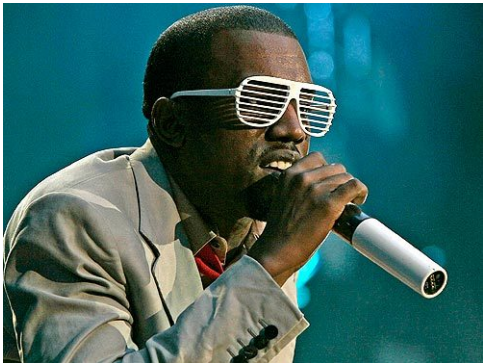
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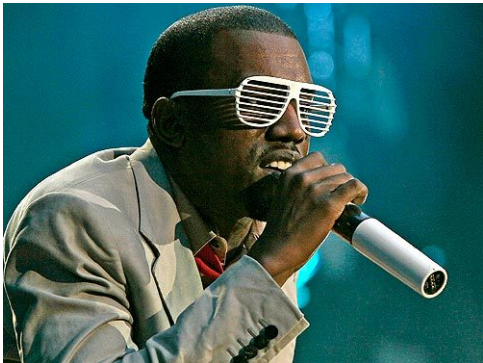
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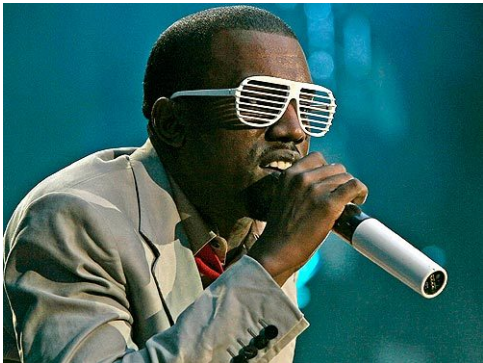
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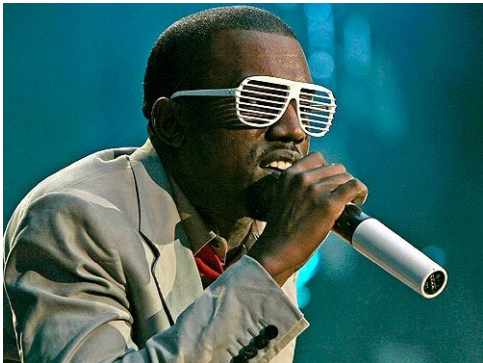
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- Self confidence: believe in work

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"It's not a sprint, it's a marathon".



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“It’s not a sprint, it’s a marathon”.

- World class distance running: it is **hard**

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methods alone

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- Methods \rightsquigarrow skills use for rest of career

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- Time away from lab \rightsquigarrow more productive when back

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Why work so hard?

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Why work so hard?

- You are all smart

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Why work so hard?

- You are all smart Really Smart

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Why work so hard?

- You are all smart Really Smart Mother-in-law brags about you smart

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- Everyone entering graduate school at top programs this fall

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- Success: **work**
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- Who gets ahead? who gets the most work done on the smartest ideas

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What can you learn in a math camp?

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Do not let yourself get lost.

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Do not let yourself get lost.

If at.

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Smartest people ask the most questions!

Let's get to work

Simple Logical Statements

Sets

A **set** is a collection of objects.

$$A = \{1, 2, 3\}$$

$$B = \{4, 5, 6\}$$

$$C = \{\text{First year cohort}\}$$

$$D = \{\text{Stanford University Faculty}\}$$

Definition

If A is a set, we say that x is an element of A by writing, $x \in A$. If x is not an element of A then, we write $x \notin A$.

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- Sets are necessary for probability theory

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Why Care?

- Sets are necessary for probability theory
- Defining **set** is equivalent to choosing population of interest (usually)

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Difference between definitions?

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Let A and B be sets. If $A = B$ then $A \subset B$ and $B \subset A$

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\Leftarrow Suppose $A \subset B$ and that $B \subset A$. Now, by way of contradiction, suppose that $A \neq B$. $A \neq B$ only if there is $x \in A$ and $x \notin B$ or if $y \in B$ and $y \notin A$. But then, either $A \not\subset B$ or $B \not\subset A$, contradicting our initial assumption. □

Set Builder Notation

- Some famous sets
 - $J = \{1, 2, 3, \dots\}$
 - $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$
 - $\mathbb{R} = \text{real numbers}$ (more to come about this)
- Use **set builder notation** to identify subsets
 - $[a, b] = \{x : x \in \mathbb{R} \text{ and } a \leq x \leq b\}$
 - $(a, b] = \{x : x \in \mathbb{R} \text{ and } a < x \leq b\}$
 - $[a, b) = \{x : x \in \mathbb{R} \text{ and } a \leq x < b\}$
 - $(a, b) = \{x : x \in \mathbb{R} \text{ and } a < x < b\}$
 - \emptyset

Set Operations

We can build new sets with **set operations**.

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Definition

*Suppose A and B are sets. Define the **Union** of sets A and B as the new set that contains all elements in set A **or** in set B . In notation,*

$$\begin{aligned} C &= A \cup B \\ &= \{x : x \in A \text{ or } x \in B\} \end{aligned}$$

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- $D = \{\text{First Year Cohort}\}, E = \{\text{Me}\}$, then
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Suppose A and B are sets. Define the *Intersection* of sets A and B as the new set that contains all elements in set A *and* set B . In notation,

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Some Facts about Sets (No Venn Diagrams!!!)

$$1) A \cap B = B \cap A$$

Some Facts about Sets (No Venn Diagrams!!!)

1) $A \cap B = B \cap A$

Proof.

This fact (theorem) says that the **set** $A \cap B$ is equal to the set $B \cap A$. We can use the definition of equal sets to test this. Suppose $x \in A \cap B$. Then $x \in A$ and $x \in B$. By definition, then, $x \in B \cap A$. Now, suppose $y \in B \cap A$. Then $y \in B$ and $y \in A$. So, by definition of intersection $y \in A \cap B$. This implies $A \cap B = B \cap A$ □

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Proof.

Suppose $x \in A \cap (B \cup C)$. Then $x \in B$ or $x \in C$ **and** $x \in A$. This implies that $x \in (A \cap B)$ or $x \in (A \cap C)$. Or, $x \in (A \cap B) \cup (A \cap C)$. Now, suppose $y \in (A \cap B) \cup (A \cap C)$. Then, $y \in A$ and $y \in B$ or $y \in C$. Well, this implies $y \in A \cap (B \cup C)$. And we have established equality \square

Some Facts about Sets (No Venn Diagrams!!!)

1) $A \cap B = B \cap A$

2) $A \cup B = B \cup A$

3) $(A \cap B) \cap C = A \cap (B \cap C)$

4) $(A \cup B) \cup C = A \cup (B \cup C)$

5) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Break into groups, derive for the remaining facts

Ordered Pair

You've seen an **ordered pair** before,

$$(a, b)$$

Definition

*Suppose we have two sets, A and B . Define the **Cartesian product** of A and B , $A \times B$ as the set of all ordered pairs (a, b) , where $a \in A$ and $b \in B$. In other words,*

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Example:

$A = \{1, 2\}$ and $B = \{3, 4\}$, then,

$$A \times B = \{(1, 3); (1, 4); (2, 3); (2, 4)\}$$

Function

Start with general and move to specific— (abstract just takes time to get acquainted)

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Definition

A *relation* is a set of ordered pairs. A *function* F is a relation such that,

$$(x, y) \in F \quad ; \quad (x, z) \in F \Rightarrow y = z$$

We will commonly write a function as $F(x)$, where $x \in \text{Domain } F$ and $F(x) \in \text{Codomain } F$. It is common to see people write,

$$F : A \rightarrow B$$

where A is domain and B is codomain

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Examples

- $F(x) = x$
- $F(x) = x^2$
- $F(x) = \sqrt{x}$

R Computing Language

- We're going to use R throughout the course
- R as calculator :

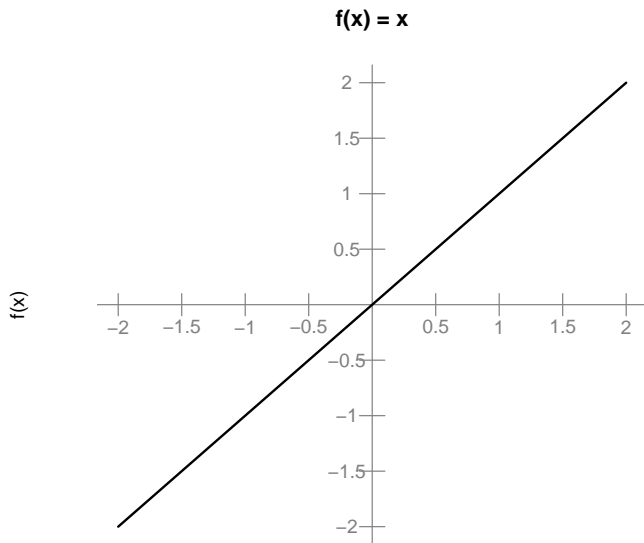
```
> 1 + 1  
[1] 2  
> 'Hello World'  
[1] "Hello World"
```

- `object<- 2` ## assign numbers to objects
- R has functions defined, we can define them to objects as well

```
first.func<- function(x) {  
  out<- 2*x  
  return(out) }
```

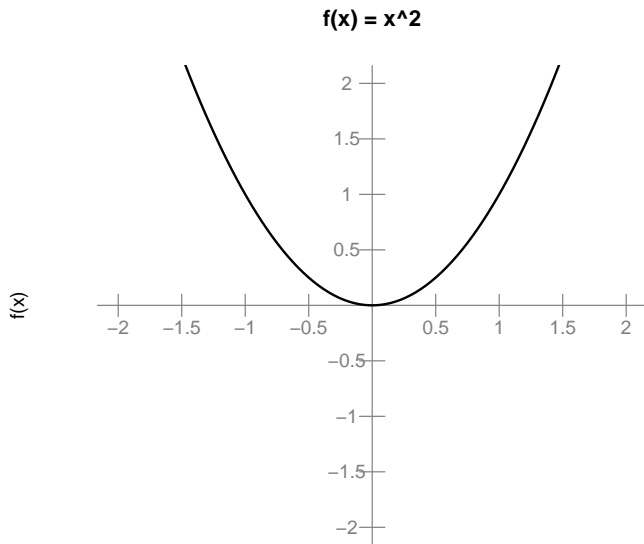
```
first.func(2)  
[1] 4
```

Plotting Functions



```
x<- seq(-2, 2,  
len=1000)  
plot(x~x) ##  
Results may vary
```

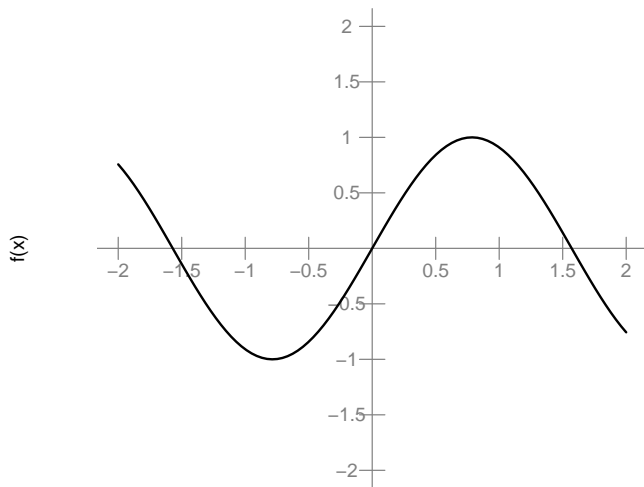
Plotting Functions



```
x<- seq(-2, 2,  
len=1000)  
x.2<- x*x  
plot(x.2~x)
```

Plotting Functions

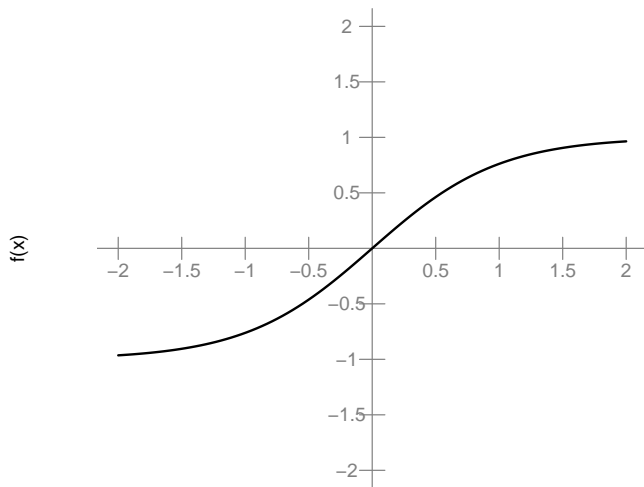
$$f(x) = \sin(2 \cdot x)$$



```
x<- seq(-2, 2,  
len=1000)  
sin.2x<- sin(2*x)  
plot(sin.2x~x)
```


Plotting Functions

$$f(x) = \tanh(x)$$



```
x<- seq(-2, 2,  
len=1000)  
tanhx<- tanh(x)  
plot(tanhx~x)
```

Exponents, Logarithms, and All That

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A function $f : A \rightarrow B$ is 1-1 (one-to-one, or injective) if for all $y \in A$ and $z \in A$ in Domain, $f(y) = f(z)$ implies $y = z$. In other words, preserves distinctiveness.

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- $f : \mathbb{R} \rightarrow \mathbb{R} f(x) = x$. Onto and 1-1, **bijjective**

Composite Functions

Definition

Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$. Then, define,

$$g \circ f = g(f(x))$$

- $f(x) = x$, $g(x) = x^2$. Then $g \circ f = x^2$.
- $f(x) = \sqrt{x}$, $g(x) = e^x$. Then $g \circ f = e^{\sqrt{x}}$.
- $f(x) = \sin(x)$, $g(x) = |x|$. Then $g \circ f = |\sin(x)|$.

Inverse Function

Definition

*Suppose a function f is 1-1. Then we'll define f^{-1} as its **inverse** if,*

$$f^{-1}(f(x)) = x$$

Why do we need 1-1?

Induction

Well Ordering Principle Every non-empty set J has a smallest number

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Theorem

If $P(n)$ is a statement containing the variable n such that

- i. $P(1)$ is a true statement, and*
 - ii. for each $k \in 1, 2, 3, 4, \dots, n, \dots$ if $P(k)$ is true then $P(k + 1)$ is true*
- then $P(n)$ is true for all $n \in 1, 2, 3, 4, \dots, n, \dots$*

Induction and Contradiction

We'll use **contradiction** and well ordering to prove that induction works.



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Summing N numbers

Induction is a useful proof technique.

Theorem

$$\sum_{i=1}^N i = 1 + 2 + 3 + 4 + \dots + N = \frac{N(N+1)}{2}$$

Two conditions to show:

i. $\sum_{i=1}^1 i = 1$ and $\frac{1(1+1)}{2} = 1$

Summing N numbers

ii. Suppose true N . Then, for $N + 1$ we have,

$$\begin{aligned}\sum_{i=1}^{N+1} i &= \sum_{i=1}^N i + (N + 1) \\ &= \frac{N(N + 1)}{2} + \frac{2(N + 1)}{2} \\ &= \frac{(N + 1)(N + 2)}{2} \\ &= \frac{(N + 1)((N + 1) + 1)}{2}\end{aligned}$$

Conditions of induction met. Therefore, proof complete

Very Simple R Code

Finite, Countable, and Uncountable

Three sizes of sets

- 1) A set, X is finite if there is a bijective function from $\{1, 2, 3, \dots, n\}$ to X .
- 2) A set X is **countably infinite** if there is a bijective function from $\{1, 2, 3, 4, \dots, \}$ to X .
- 3) A set X is **uncountably infinite** if it is not countable

The **Real numbers** are **uncountably infinite**

Recap

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- 1) Sets + Operations
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- 3) Contradiction, Induction, and direct proofs

Tomorrow:

- Convergence of sequences
- Limits
- Continuity
- Derivatives