Math Camp - Homework 1

1. Using the sets...

$$\begin{aligned} A &= \{2, 3, 7, 9, 13\} \\ B &= \{x : 4 \leq x \leq 8 \text{ and } x \in \mathbb{Z}\} \\ C &= \{x : 2 < x < 25 \text{ and } x \text{ is prime}\} \\ D &= \{1, 4, 9, 16, 25, \ldots\} \end{aligned}$$

identify the following:

(a) $A \cup B$

(b) $(A \cup B) \cap C$

(c) $C \cap D$

2. Simplify the following:

(a) $k^{x-y} \cdot k^{-x-y}$

(b) $\left(\frac{z^{4v+6}}{z^{v+9}}\right)$

(c) $(a^{b^0} + a^{0^b} - a^{-1} \cdot a^2)^b$

3. Express each of the following as a single logarithm:

(a) $\log(x) + \log(y) - \log(z)$

(b) $2\log(x) + 1$

(c) $\log(x) - 2$

4 (Challenge Problem). Prove that $n! > n^2$ for integers $n \geq 4$. (Hint: try using induction.)

5 (Challenge Problem). A number is *rational* if it can be written as the quotient of two integers — e.g., if $x = \frac{p}{q}$, with $p, q \in \mathbb{Z}$, then x is rational. (The set of rational numbers is often denoted $\mathbb{Q}$.) A number is *irrational* if it is not rational. Prove that $\sqrt{2}$ is irrational. (Hint: try writing a proof by contradiction.)