

# Math Camp

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Department of Political Science  
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< Course >

# The Systematic Analysis of Politics

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This class (introduction):

- Math Camp: Develop Tools for Analysis
- Probability theory: systematic model of randomness

# Course Goals

First stop in methodology sequence

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- 1) Mathematical tools to comprehend and use statistical methods
- 2) Foundation in probability theory/analytic reasoning
- 3) Practical Computing Tools: R
- 4) Introduction to Logic of Formal Modeling



# Course Staff

Me: Justin Grimmer and Avidit Acharya

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# Extra Info

- Zuhad Hai and Jesse Yoder
- Github for class: `github/justingrimmer/Math18`

# Prerequisites

No Formal Prerequisites



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  - Political science increasingly requires math
  - Empirical: calculus and linear algebra
  - Quantitative Methodologist: Real Analysis and Grad level statistics
  - Formal Theory: Real Analysis (through measure theory), Topology

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- Learn as much material as possible
- If you truly only care about learning material, you'll get amazing grades

# Homework

Math camp: assigned daily  $\rightsquigarrow$  Mechanics of solving problems

Lab Assignment: Twice weekly assignments, help you develop computational and mathematical skills.

# Computing/Homeworks

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- **If you use start using  $\text{\LaTeX}$ , you'll soon love it**

# Course Books

- 1) Simon, Carl and Blume, Lawrence (SB). Mathematics for Economists.
- 2) Bertsekas, Dimitri P. and Tsitsiklis, John (BT) Introduction to Probability Theory (second edition)

# Life in Graduate School/Academy

Three part mixture:

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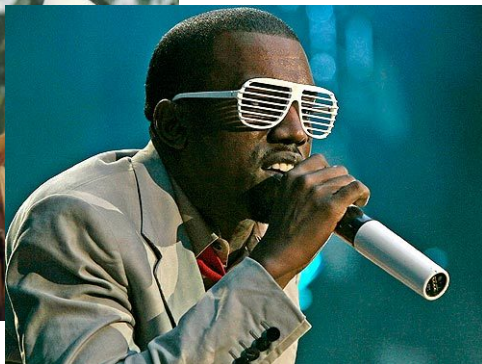


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Paula Radcliffe

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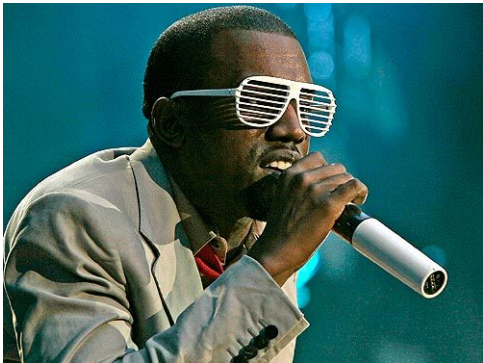
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- If you’re good at methods, you’ll be more rich [in expectation] and equally free

# $\frac{1}{3}$ Kayne West

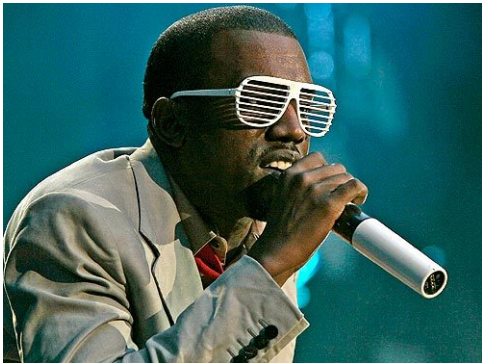


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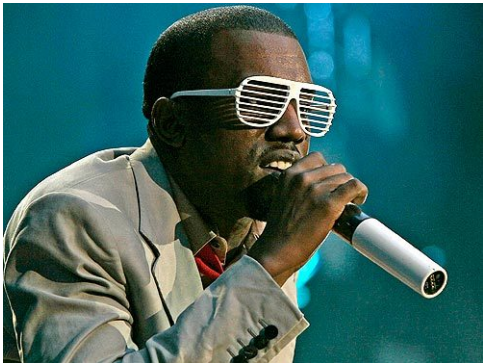
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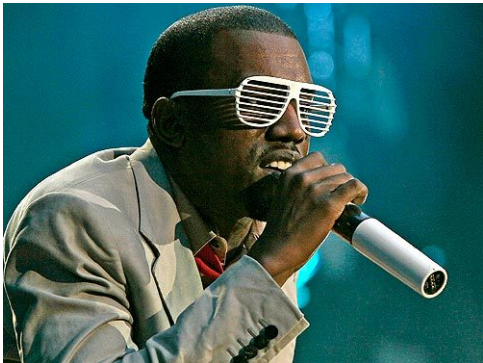
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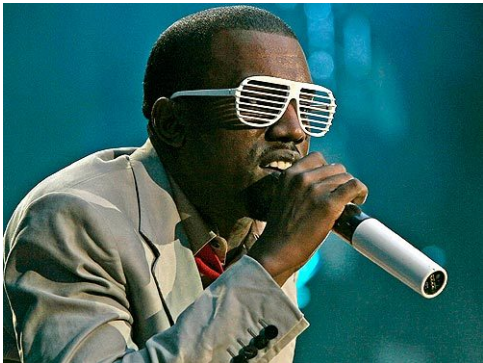
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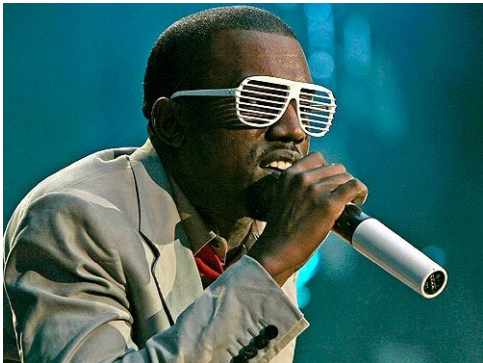


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- Self confidence: believe in work

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"It's not a sprint, it's a marathon".



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- Time away from lab  $\rightsquigarrow$  more productive when back

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Why work so hard?

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- You are all smart

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Why work so hard?

- You are all smart Really Smart

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Why work so hard?

- You are all smart Really Smart Mother-in-law brags about you smart



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- Treat grad school like a job
- Who gets ahead? who gets the most work done on the smartest ideas

# Preliminaries

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- 1) Introduction to more sophisticated mathematics (notation)

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Smartest people ask the most questions!



# Let's get to work

# Simple Logical Statements



# Sets

A **set** is a collection of objects.

$$A = \{1, 2, 3\}$$

$$B = \{4, 5, 6\}$$

$$C = \{\text{First year cohort}\}$$

$$D = \{\text{Stanford University Faculty}\}$$

## Definition

*If  $A$  is a set, we say that  $x$  is an element of  $A$  by writing,  $x \in A$ . If  $x$  is not an element of  $A$  then, we write  $x \notin A$ .*

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- Defining **set** is equivalent to choosing population of interest (usually)

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Difference between definitions?

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$\Leftarrow$  Suppose  $A \subset B$  and that  $B \subset A$ . Now, by way of contradiction, suppose that  $A \neq B$ .  $A \neq B$  only if there is  $x \in A$  and  $x \notin B$  or if  $y \in B$  and  $y \notin A$ . But then, either  $A \not\subset B$  or  $B \not\subset A$ , contradicting our initial assumption. □



# Set Builder Notation

- Some famous sets
  - $J = \{1, 2, 3, \dots\}$
  - $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$
  - $\mathbb{R} = \text{real numbers}$  (more to come about this)
- Use **set builder notation** to identify subsets
  - $[a, b] = \{x : x \in \mathbb{R} \text{ and } a \leq x \leq b\}$
  - $(a, b] = \{x : x \in \mathbb{R} \text{ and } a < x \leq b\}$
  - $[a, b) = \{x : x \in \mathbb{R} \text{ and } a \leq x < b\}$
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  - $\emptyset$

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# Some Facts about Sets (No Venn Diagrams!!!)

$$1) A \cap B = B \cap A$$

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1)  $A \cap B = B \cap A$

Proof.

This fact (theorem) says that the **set**  $A \cap B$  is equal to the set  $B \cap A$ . We can use the definition of equal sets to test this. Suppose  $x \in A \cap B$ . Then  $x \in A$  and  $x \in B$ . By definition, then,  $x \in B \cap A$ . Now, suppose  $y \in B \cap A$ . Then  $y \in B$  and  $y \in A$ . So, by definition of intersection  $y \in A \cap B$ . This implies  $A \cap B = B \cap A$  □

# Some Facts about Sets (No Venn Diagrams!!!)

$$1) A \cap B = B \cap A$$

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Suppose  $x \in A \cap (B \cup C)$ . Then  $x \in B$  or  $x \in C$  **and**  $x \in A$ . This implies that  $x \in (A \cap B)$  or  $x \in (A \cap C)$ . Or,  $x \in (A \cap B) \cup (A \cap C)$ . Now, suppose  $y \in (A \cap B) \cup (A \cap C)$ . Then,  $y \in A$  and  $y \in B$  or  $y \in C$ . Well, this implies  $y \in A \cap (B \cup C)$ . And we have established equality  $\square$

# Some Facts about Sets (No Venn Diagrams!!!)

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2)  $A \cup B = B \cup A$

3)  $(A \cap B) \cap C = A \cap (B \cap C)$

4)  $(A \cup B) \cup C = A \cup (B \cup C)$

5)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Break into groups, derive for the remaining facts

# Ordered Pair

You've seen an **ordered pair** before,

$$(a, b)$$

## Definition

*Suppose we have two sets,  $A$  and  $B$ . Define the **Cartesian product** of  $A$  and  $B$ ,  $A \times B$  as the set of all ordered pairs  $(a, b)$ , where  $a \in A$  and  $b \in B$ . In other words,*

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Example:

$A = \{1, 2\}$  and  $B = \{3, 4\}$ , then,

$$A \times B = \{(1, 3); (1, 4); (2, 3); (2, 4)\}$$

# Function

Start with general and move to specific— (abstract just takes time to get acquainted)

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## Definition

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$$(x, y) \in F \quad ; \quad (x, z) \in F \Rightarrow y = z$$

We will commonly write a function as  $F(x)$ , where  $x \in \text{Domain } F$  and  $F(x) \in \text{Codomain } F$ . It is common to see people write,

$$F : A \rightarrow B$$

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- $F(x) = \sqrt{x}$

# R Computing Language

- We're going to use R throughout the course
- R as calculator :

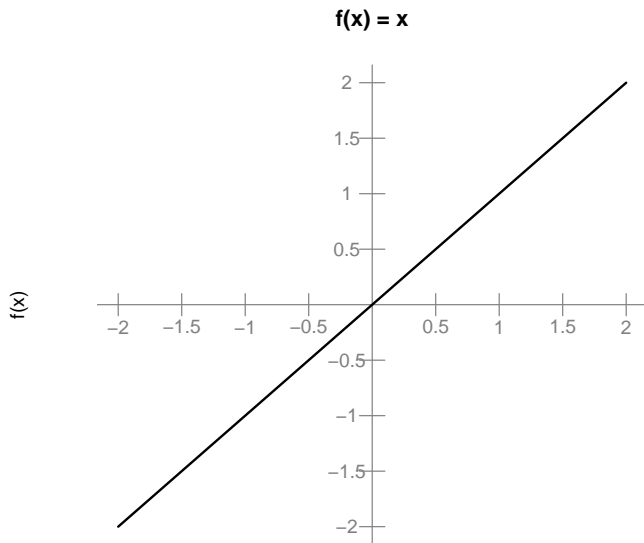
```
> 1 + 1  
[1] 2  
> 'Hello World'  
[1] "Hello World"
```

- `object<- 2` ## assign numbers to objects
- R has functions defined, we can define them to objects as well

```
first.func<- function(x) {  
  out<- 2*x  
  return(out) }
```

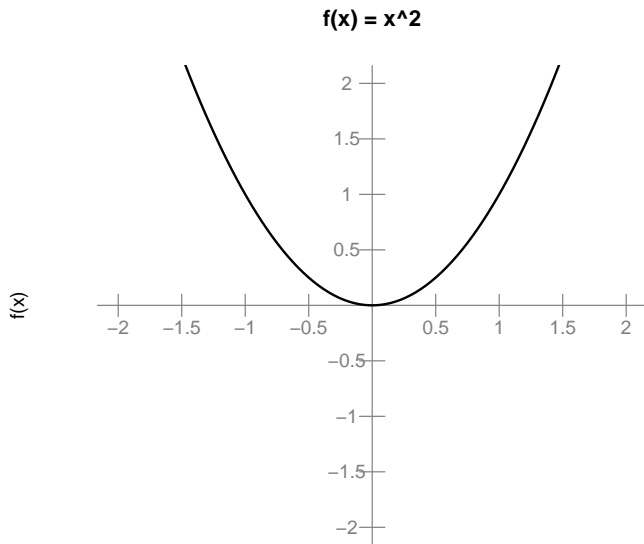
```
first.func(2)  
[1] 4
```

# Plotting Functions



```
x<- seq(-2, 2,  
len=1000)  
plot(x~x) ##  
Results may vary
```

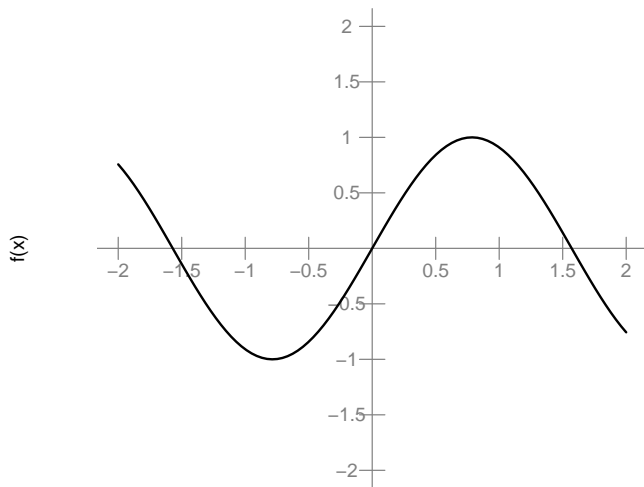
# Plotting Functions



```
x<- seq(-2, 2,  
len=1000)  
x.2<- x*x  
plot(x.2~x)
```

# Plotting Functions

$$f(x) = \sin(2 \cdot x)$$

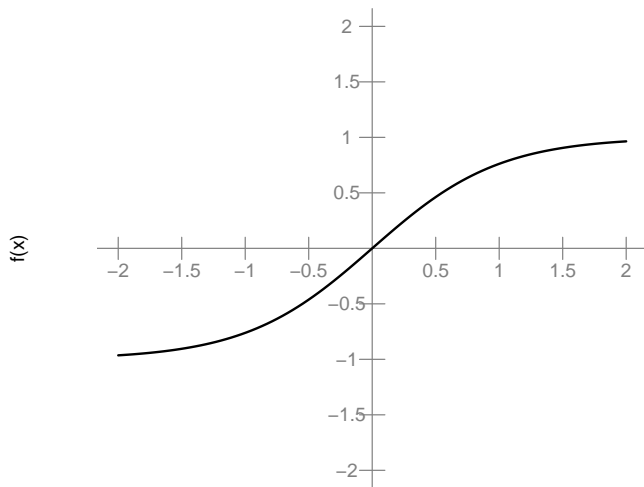


```
x<- seq(-2, 2,  
len=1000)  
sin.2x<- sin(2*x)  
plot(sin.2x~x)
```



# Plotting Functions

$$f(x) = \tanh(x)$$



```
x<- seq(-2, 2,  
len=1000)  
tanhx<- tanh(x)  
plot(tanhx~x)
```

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- $f : \mathbb{R} \rightarrow \mathbb{R} f(x) = x$ . Onto and 1-1, **bijjective**

# Composite Functions

## Definition

*Suppose  $f : A \rightarrow B$  and  $g : B \rightarrow C$ . Then, define,*

$$g \circ f = g(f(x))$$

- $f(x) = x$ ,  $g(x) = x^2$ . Then  $g \circ f = x^2$ .
- $f(x) = \sqrt{x}$ ,  $g(x) = e^x$ . Then  $g \circ f = e^{\sqrt{x}}$ .
- $f(x) = \sin(x)$ ,  $g(x) = |x|$ . Then  $g \circ f = |\sin(x)|$ .



# Inverse Function

## Definition

*Suppose a function  $f$  is 1-1. Then we'll define  $f^{-1}$  as its **inverse** if,*

$$f^{-1}(f(x)) = x$$

Why do we need 1-1?

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## Theorem

*If  $P(n)$  is a statement containing the variable  $n$  such that*

- i.  $P(1)$  is a true statement, and*
  - ii. for each  $k \in 1, 2, 3, 4, \dots, n, \dots$  if  $P(k)$  is true then  $P(k + 1)$  is true*
- then  $P(n)$  is true for all  $n \in 1, 2, 3, 4, \dots, n, \dots$*

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- ii. If  $P(k)$  is true then  $P(k + 1)$  is true.

Now suppose, **by way of contradiction** that there exists  $N$  such that  $P(N)$  is false. This implies that

$$S = \{x : P(x) \text{ is not true} \}$$

By well ordering principle, there is smallest member of  $S$ , call it  $n_0$ . By *i*. we know that  $n_0 > 1$ . Further, because  $n_0$  is smallest member of  $S$ , then  $P(n_0)$  is false, but  $P(n_0 - 1)$  is true. But now we have a problem, because if  $P(n_0 - 1)$  is true, then  $P(n_0)$  is also true. This implies that there is no smallest element of  $S$ . **CONTRADICTION** □

# Summing $N$ numbers

Induction is a useful proof technique.

Theorem

$$\sum_{i=1}^N i = 1 + 2 + 3 + 4 + \dots + N = \frac{N(N+1)}{2}$$

Two conditions to show:

i.  $\sum_{i=1}^1 i = 1$  and  $\frac{1(1+1)}{2} = 1$

# Summing $N$ numbers

ii. Suppose true  $N$ . Then, for  $N + 1$  we have,

$$\begin{aligned}\sum_{i=1}^{N+1} i &= \sum_{i=1}^N i + (N + 1) \\ &= \frac{N(N + 1)}{2} + \frac{2(N + 1)}{2} \\ &= \frac{(N + 1)(N + 2)}{2} \\ &= \frac{(N + 1)((N + 1) + 1)}{2}\end{aligned}$$

Conditions of induction met. Therefore, proof complete

## Very Simple R Code



# Finite, Countable, and Uncountable

## Three sizes of sets

- 1) A set,  $X$  is finite if there is a bijective function from  $\{1, 2, 3, \dots, n\}$  to  $X$ .
- 2) A set  $X$  is **countably infinite** if there is a bijective function from  $\{1, 2, 3, 4, \dots, \}$  to  $X$ .
- 3) A set  $X$  is **uncountably infinite** if it is not countable

The **Real numbers** are **uncountably infinite**

# Recap

We've covered a lot.

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**PLEASE** don't worry—we're here to help!

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1) Sets + Operations

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- 1) Sets + Operations
- 2) Functions

# Recap

We've covered a lot.

**PLEASE** don't worry—we're here to help!

- 1) Sets + Operations
- 2) Functions
- 3) Contradiction, Induction, and direct proofs

Tomorrow:

- Convergence of sequences
- Limits
- Continuity
- Derivatives