

Math Camp - Homework 4

1: Evaluate the following integrals, or explain why they do not exist:

(a)

$$\int_0^1 x^{\frac{3}{7}} dx$$

(c)

$$\int_8^9 2^x dx$$

(b)

$$\int_1^2 \left(\frac{3}{x^4} + 2 \right) dx$$

(d)

$$\int_3^3 \sqrt{x^5 + 2} dx$$

Question 2: A group of three unidentified first-year political science students at Stanford University are worn out after a week of math camp. Wanting to unwind, the students agree to not talk about math and decide to chat over some casual drinks in downtown Palo Alto.¹

After five shots of tequila each, two pitchers of beer, a bottle of wine, and a large Chicago-style pizza, the three students have had enough fun and decide to start the trip back home.

Student *A* gets on a bike and starts pedalling away at a velocity of $v_A(t) = 2t^4 + t$, where t represents minutes. However, the student crashes into the side of a Marguerite shuttle and ends the journey after only 2 minutes.

Student *B* has no bike, so starts running at a velocity of $v_B(t) = 4\sqrt{t}$. Sadly, after only 4 minutes, the student's legs give out and the student decides to sing a song, instead.

Student *C* can't even stand up, so has no choice but to slowly crawl at a velocity of $v_C(t) = 2e^{-t}$. Student *C* steadily plods along for 20 minutes before falling asleep on the sidewalk.

Generally, if an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$. The Fundamental Theorem of Calculus then tells us that

$$\begin{aligned} \text{Total distance traveled} &= \int_{t_1}^{t_2} v(t) dt \\ s(t_2) - s(t_1) &= \int_{t_1}^{t_2} v(t) dt \end{aligned}$$

Without using a calculator, use this formula to find the distance traveled by Students *A*, *B*, and *C*. (Assume, however unrealistic in may be, that all three students traveled in a straight line.) Who traveled the farthest? The least far?

¹Protip: Try not to talk about math and/or political science at bars, *especially* with your cohort-mates.

Question 3: Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a)

$$\int_1^{\infty} \left(\frac{1}{3x}\right)^2 dx$$

(c)

$$\int_0^{\infty} e^{-x} dx$$

(b)

$$\int_0^{\infty} \cos(x) dx$$

(d)

$$\int_{-\infty}^0 x^3 dx$$

Question 4: Evaluate the following indefinite integrals showing all your work:

(a)

$$\int \cos^3 x \sin x dx$$

(b)

$$\int \frac{\log(x)}{x} dx$$

(c)

$$\int x^3 \log(5x) dx$$