# Семинар

$f : [a; \omega) \to \mathbb{R}$

$\forall b \in [a, \omega) \ [f \in \mathcal{R}[a, b]]$

$\Rightarrow \int_a^\omega f(x)\,dx := \lim_{b \to \omega - 0} \underbrace{\int_a^b f(x)\,dx}_{\mathcal{R}}$

$$\int_{\omega_1}^{\omega_2} := \int_{\omega_1}^{e} + \int_{e}^{\omega_2}$$

$e \in (\omega_1, \omega_2)$

$\exists \int_1^{\boxed{+\infty}} \frac{1}{x^p}\,dx = \lim_{b \to +\infty} \int_1^b \frac{1}{x^p}\,dx = \lim_{b \to +\infty} \begin{cases} -\frac{1}{p-1} \cdot \frac{1}{x^{p-1}} \Big|_1^b, & p \neq 1 \Rightarrow \frac{1}{p-1}, \ p > 1 \\ \ln x \Big|_1^b; \ p = 1 & \text{не существ.} \end{cases}$

не сущ. $p < 1$

$\ln x \Big|_1^b = \ln b - \ln 1 = \ln b$

$\Updownarrow$

$p > 1$

$\exists \int_{\boxed{0}}^1 \frac{1}{x^p}\,dx = \lim_{a \to 0+0} \int_a^1 \frac{1}{x^p}\,dx$

$\Updownarrow$

$p < 1$

**Th.** $f, g : [a, \omega) \to \mathbb{R} \quad \forall b \in [a; \omega) \ [f, g \in \mathcal{R}[a, b]]$

$\forall x \in [a, \omega) \ [0 \leq f(x) \leq g(x)]$

$\Rightarrow \exists \int_a^\omega g\,dx \Rightarrow \exists \int_a^\omega f\,dx$

$\nexists \int_a^\omega f\,dx \Rightarrow \nexists \int_a^\omega g\,dx$

Задача не вычислить, а опред. сходимость

$$I=\int_1^{+\infty}\frac{\ln x}{x^{1,1}}\,dx=\underbrace{\int_1^{c}\frac{\ln x}{x^{1,1}}\,dx}_{\in\mathbb{R}}+\int_c^{+\infty}\frac{1}{x^{1,05}}\cdot\frac{\ln x}{x^{0.05}}\,dx=$$

$$\frac{\ln x}{x^{1,1}}=\frac{1}{x^{1,05}}\cdot\frac{\ln x}{x^{0.05}}$$

$$\lim_{x\to+\infty}\frac{\ln x}{x^{0.05}}=0\Rightarrow\exists\, c\in[1,+\infty):\left[\forall x\in[c;+\infty)\left[0\le\frac{\ln x}{x^{0.05}}<1\right]\right]$$

$$I\overset{\text{сход.}}{\sim}\int_c^{+\infty}\underbrace{\frac{1}{x^{1.05}}\cdot\frac{\ln x}{x^{0.05}}}_{=f}\,dx,\quad g=\frac{1}{x^{1.05}}$$

$$\forall x\in[c;+\infty)\left[0\le\frac{1}{x^{0.05}}\cdot\underbrace{\frac{\ln x}{x^{0.05}}}_{\in[0,1)}\le\frac{1}{x^{1.05}}\right]$$

$$\exists\int_c^{+\infty}\frac{1}{x^{1.05}}\,dx\overset{\text{пр. Вейерш.}}{\Longrightarrow}\exists\int_c^{+\infty}\frac{\ln x}{x^{1.1}}\,dx\underset{\text{сход.}}{\sim}I$$

Th. Второй признак

$\exists$ пр. Вейерш. выполнено; $\forall x\in[a,\omega)\,[f(x)\ge0,\ g(x)\ge0]$

$$\lim_{x\to\omega-0}\frac{f(x)}{g(x)}=k\in(0;+\infty)$$

$$\Downarrow$$

$$\int_a^{\omega}f\,dx\overset{\text{сход.}}{\sim}\int_a^{\omega}g\,dx$$

$$\int_{\boxed{0}}^{\boxed{+\infty}} \frac{\operatorname{arctg} x}{x^p}\,dx = \left(\int_0^{c=1} + \int_{c=1}^{+\infty}\right)\frac{\operatorname{arctg} x}{x^p}\,dx$$

$p \in \mathbb{R}$

$c \in (0; +\infty)$

* $c=1$ выбрали сами

$\int_1^{+\infty} \frac{\operatorname{arctg} x}{x^p}\,dx$, $g = \frac{1}{x^p}$ $\quad f(x) > 0,\ g(x) > 0\ \ \forall x \in [1; +\infty)$

$$\lim_{x\to+\infty}\left\{\frac{f(x)}{g(x)} = \operatorname{arctg} x\right\} = \frac{\pi}{2} = k \in (0; +\infty)$$ (выполн. усл. 2 пр.)

$$\int_0^{+\infty} \frac{\operatorname{arctg} x}{x^p}\,dx = I_1 + I_2$$

$p \in \mathbb{R}$

сх. $\int_0^1 \frac{1}{x^{p-1}}\,dx \Leftrightarrow p < 2$

сход. $\int_1^{+\infty} \frac{1}{x^p}\,dx \Leftrightarrow p > 1$

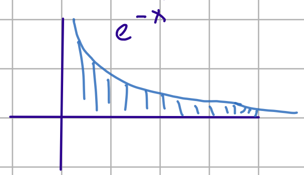
$\int_0^1 \frac{\operatorname{arctg} x}{x^p}\,dx \qquad \lim_{x\to 0+0}\left\{\frac{f(x)}{g(x)} = \frac{\operatorname{arctg} x}{x} = \frac{\frac{1}{1+x^2}}{1}\right\} = 1 = k \in (0; +\infty)$

$\Rightarrow$ выполн. 2 пр.

$g(x) = \frac{1}{x^{p-1}}$

$$\Rightarrow \exists \int_0^{+\infty} \frac{\operatorname{arctg} x}{x^p}\,dx \Leftrightarrow p \in (1; 2)$$

$$\int_0^{\boxed{+\infty}} e^{-x^2}\,dx \overset{\text{сх.}}{\sim} \int_1^{+\infty} e^{-x^2}\,dx$$




$\frac{1}{x^2} > e^{-x^2}\ \ \forall x \geq 1$

$\frac{e^{-x^2}}{\frac{1}{x^2}} \to 0$

$$\exists \int_1^{+\infty} \frac{1}{x^2}\,dx \Rightarrow \exists \int_1^{+\infty} e^{-x^2}\,dx$$

$\nexists \int_0^{+\infty} \frac{1}{x^2}\,dx$ - расходится (т.к. $\int_0^1$ - не сход.)