CPSC 340:
Machine Learning and Data Mining

Decision Trees

Admin

- Assignment 0 due Wednesday 11:59pm.
- Feedback URL: http://128.189.TBD.TBD:6169/test/

Last Time: Data Representation and Exploration

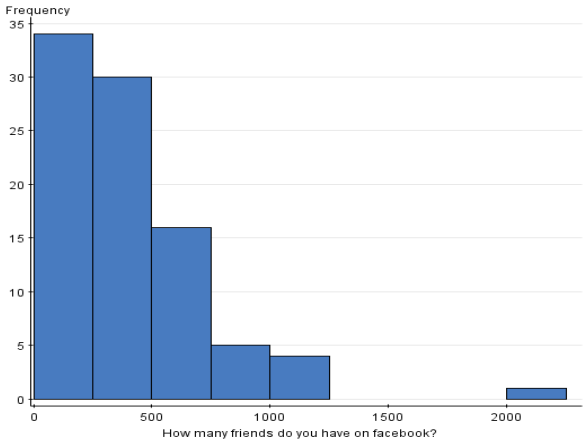
- We discussed object-feature representation:
  - Examples: another name we'll use for objects.

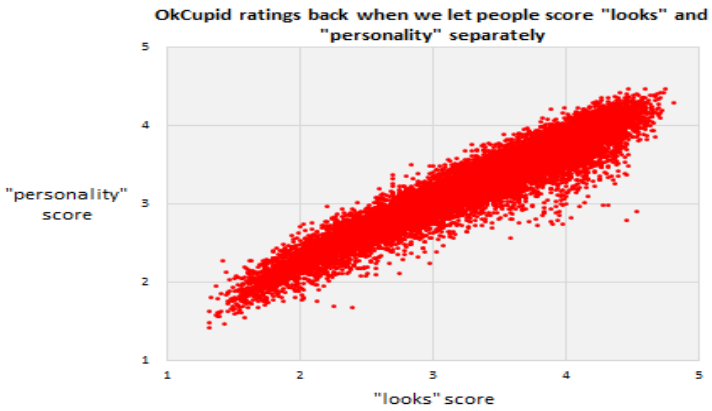
Age	Job?	City	Rating	Income
23	Yes	Van	A	22,000.00
23	Yes	Bur	BBB	21,000.00
22	No	Van	CC	0.00
25	Yes	Sur	AAA	57,000.00

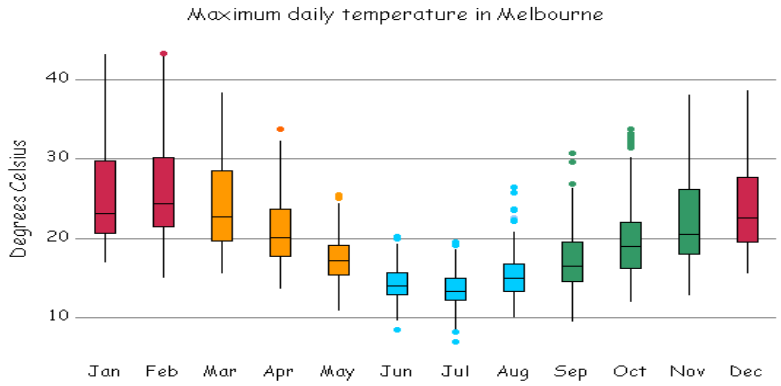
Object

Feature

- We discussed summary statistics and visualizing data.



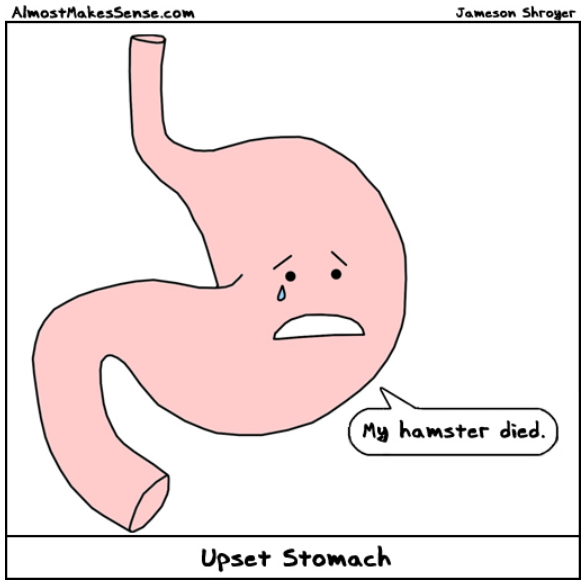




http://www.statcrunch.com/5.0/viewresult.php?resid=1024581
http://cdn.okccdn.com/blog/humanexperiments/looks-v-personality.png
http://www.scc.ms.unimelb.edu.au/whatisstatistics/weather.html

Motivating Example: Food Allergies

- You frequently start getting an upset stomach



- You suspect an adult-onset food allergy.

http://www.cliparthut.com/upset-stomach-clipart-cn48e5.html

Motivating Example: Food Allergies

- To solve the mystery, you start a food journal:

Egg	Milk	Fish	Wheat	Shellfish	Peanuts	…	Sick?
0	0.7	0	0.3	0	0		1
0.3	0.7	0	0.6	0	0.01		1
0	0	0	0.8	0	0		0
0.3	0.7	1.2	0	0.10	0.01		1
0.3	0	1.2	0.3	0.10	0.01		1

- But it's hard to find the pattern:
  - You can't isolate and only eat one food at a time.
  - You may be allergic to more than one food.
  - The quantity matters: a small amount may be ok.
  - You may be allergic to specific interactions.

Supervised Learning

- We can formulate this as supervised learning:

Egg	Milk	Fish	Wheat	Shellfish	Peanuts	…	Sick?
0	0.7	0	0.3	0	0		1
0.3	0.7	0	0.6	0	0.01		1
0	0	0	0.8	0	0		0
0.3	0.7	1.2	0	0.10	0.01		1
0.3	0	1.2	0.3	0.10	0.01		1

- Input for an object (day of the week) is a set of features (quantities of food).
- Output is a desired class label (whether or not we got sick).
- Goal of supervised learning:
  - Use data to write a program mapping from features to labels.
  - Program predicts whether foods will make you sick (even with new combinations).

Supervised Learning

- With discrete labels, supervised learning is called classification.
- More generally, we're interested in supervised learning:
  - Take features of objects and corresponding labels as inputs.
  - Output a program that can predict the label of a new object.
- This is the most successful machine learning technique:
  - Spam filtering, optical character recognition, Microsoft Kinect, speech recognition, classifying tumours, etc.
- Most useful when:
  - You don't know how to write a program to do the task.
  - But you have input/output examples.
- Today we will learn about one approach:
  - Decision trees.

But first….

- What types of preprocessing might we do?
  - Data cleaning: check for and fix missing/unreasonable values.
  - Summary statistics:
    - Can help identify "unclean" data.
    - Correlation might reveal an obvious dependence ("sick" ⇔ "peanuts").
  - Data transformations:
    - Convert everything to same scale? (e.g., grams)
    - Add foods from day before? (maybe "sick" depends on multiple days)
    - Add date? (maybe what makes you "sick" changes over time).
  - Data visualization: look at a scatterplot of each feature and the label.
    - Maybe the visualization will show something weird in the features.
    - Maybe the pattern is really obvious!
- What you do might depend on how much data you have:
  - Very little data:
    - Represent food by common allergic ingredients (lactose, gluten, etc.)?
  - Lots of data:
    - Use more fine-grained features (bread from bakery vs. hamburger bun)?

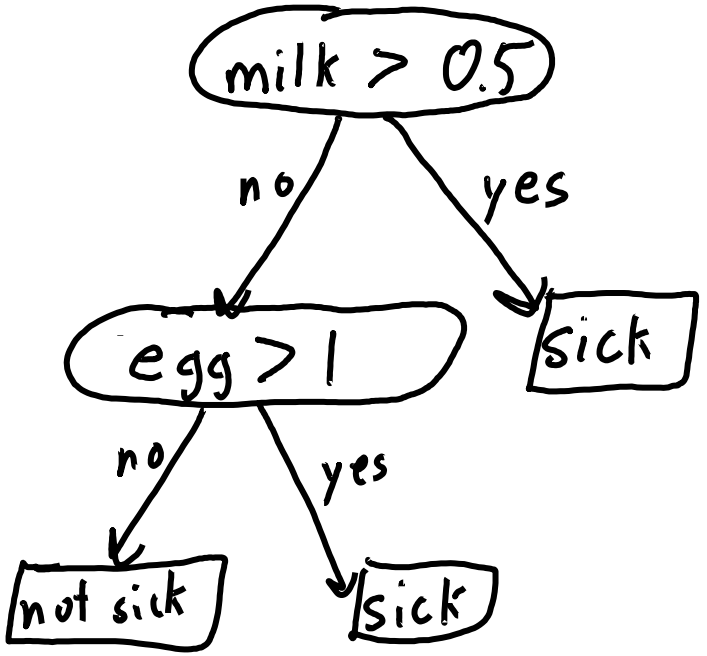
Decision Trees

- Decision trees are simple programs consisting of:
  - A nested sequence of "if-else" decisions based on the features (splitting rules).
  - A class label as a return value at the end of each sequence.

- Example decision tree:

```
if (milk > 0.5)
        {
                return 'sick'
        }
        else
        {
                if (egg > 1)
                        return 'sick'
                else
                        return 'not sick'
        }
```

Can draw sequences of decisions as a tree:



Decision Tree Learning

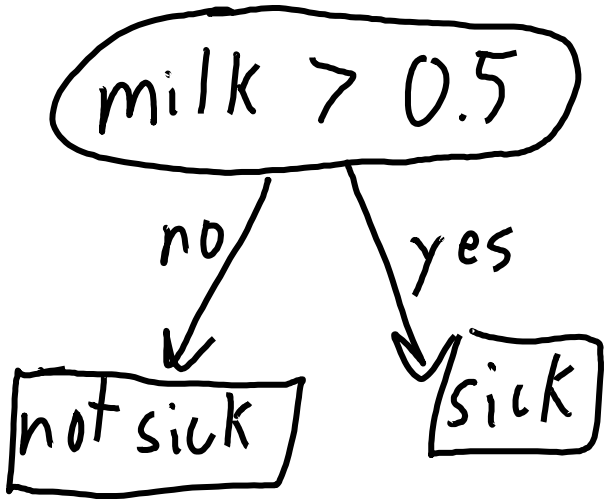
- It might be hard to find a good decision tree by hand.
  - There could be a huge number of variables.
  - Sequences of rules might be hard to find.

- Decision tree learning:
  - Use the data to automatically write the decision tree program.

- Basic idea: search over trees for the “best” tree.

Learning A Decision Stump

- We'll start decision stumps:
  - Simple decision tree with 1 splitting rule based on thresholding 1 feature.



- How do we find the best "rule" (i.e., the feature and threshold)?
  1. Define a 'score' for the rule.
  2. Search for the rule with the best score.

Decision Stump: Accuracy Score

- Most intuitive score: classification accuracy.
  - "If we use this rule, how many objects do we label correctly?"
- Computing classification accuracy for (egg > 1):
  - Find most common labels if we use this rule:
    - When (egg > 1), we were "sick" both times.
    - When (egg <= 1), we were "not sick" three out of four times.
  - Compute accuracy:
    - Rule (egg > 1) is correct on 5/6 objects.
- Scores of other rules:
  - (milk > 0.5) obtains lower accuracy of 4/6 .
  - (egg > 0) obtains optimal accuracy of 6/6.
  - () obtains "baseline" accuracy of 3/6, as does (egg > 2).

Egg	Milk	Fish	…	Sick?
1	0.7	0		1
2	0.7	0		1
0	0	0		0
0	0.7	1.2		0
2	0	1.2		1
0	0	0		0

Decision Stump: Rule Search (Attempt 1)

- Accuracy "score" evaluates quality of a rule.
  - Find the best rule by maximizing score.
- Attempt 1 (exhaustive search):

Compute score of (egg > 0)	Compute score of (milk > 0)	…
Compute score of (egg > 0.01)	Compute score of (milk > 0.01)	…
Compute score of (egg > 0.02)	Compute score of (milk > 0.02)	…
Compute score of (egg > 0.03)	Compute score of (milk > 0.03)	…
…	…	…
Compute score of (egg > 99.99)	Compute score of (milk > 0.99)	…

- As you go, keep track of the highest score.
- Return highest-scoring rule.

Cost of Decision Stumps (Attempt 1)

- How much does this cost?
- Assume we have:
  - 'n' objects (days that we measured).
  - 'd' features (foods that we measured).
  - 't' thresholds (>0, >0.01, >0.02,…)
- Computing the score of one rule costs O(n):
  - We need to go through all 'n' examples.
  - If you are not familiar with "O(n)" see notes on webpage.
- To compute scores for d*t rules, total cost is O(ndt).
- Can we do better?

Speeding up Rule Search

- We can ignore rules outside feature ranges:
  - E.g., we never have (egg > 50) in our data.
  - These rules can never improve accuracy.
  - Restrict thresholds to range of features.

- Most of the thresholds give the same score.
  - If we never have (0.5 < egg < 1) in the data,
    - then (egg < 0.6) and (egg < 0.9) have the same score.
  - Restrict thresholds to values in data.

Decision Stump: Rule Search (Attempt 2)

- Attempt 2 (search only over features in data):

Compute score of (eggs > 0)	Compute score of (milk > 0.5)	…
Compute score of (eggs > 1)	Compute score of (milk > 0.7)	…
Compute score of (eggs > 2)	Compute score of (milk > 1)	…
Compute score of (eggs > 3)	Compute score of (milk > 1.25)	…
Compute score of (eggs > 4)		…

- Now at most 'n' thresholds for each feature.
- We only consider O(nd) rules instead of O(dt) rules:
  - Total cost changes from O(ndt) to $O(n^2d)$.

Supervised Learning Notation

$X =$

Egg	Milk	Fish	Wheat	Shellfish	Peanuts
0	0.7	0	0.3	0	0
0.3	0.7	0	0.6	0	0.01
0	0	0	0.8	0	0
0.3	0.7	1.2	0	0.10	0.01
0.3	0	1.2	0.3	0.10	0.01

'd'

$y =$

Sick?
1
1
0
1
1

'n'

- Feature matrix 'X' has rows as objects, columns as features.
  - X_{ij} is feature 'j' for object 'i'.
  - E.g., X_{ij} is quantity of food 'j' on day 'i'.
- Label vector 'y' contains the labels of the objects.
  - y_i is the label of object 'i'.

Decision Stump Learning Pseudo-Code

Input: feature matrix X and label vector y

for each feature 'j'

  for each example 'i'

    set threshold to feature 'j' in example 'i'.

    find mode of label vector when feature 'j' is above threshold

    find mode of label vector when feature 'j' is below threshold.

    classify all examples based on threshold

    count the number of errors.

    store this rule if it has the lowest error so far.

Output: an optimal decision stump rule

Input: feature matrix X and label vector y

```
[n, d] = size(X)
minError = sum(y ~= mode(y))
minRule = []
for j = 1:d
    for i = 1:n
        t = X(i,j)
        y_above = mode(y(X(:,j) > t))
        y_below = mode(y(X(:,j) <= t))
        yhat = y_above * ones(n,1)
        yhat(X(:,j) <= t) = y_below
        error = sum(yhat ~= y)
        if error < minError
            minError = error
            minRule = [j t]
```

"^" "y"

compute error if you don't split.

for each feature 'j'

for each example 'i'

set threshold to feature 'j' in

find mode of label vector when f

find mode of label vector when t

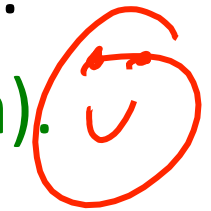
classify all examples based on thresh

count the number of errors.

store this rule if it has the lowest

Decision Stump: Rule Search (Attempt 3)

- Do we have to compute score from scratch?
  - Rule (egg > 1) and (egg > 2) have same decisions, except when (egg == 2).
  - Sort the examples based on 'egg'.
  - Go through the rules in order, updating the score.
- Sorting costs O(n log n) per feature.
- You do at most O(n) score updates per feature.
- Total cost is reduced from $O(n^2d)$ to $O(nd \log n)$.
- This is a good runtime:
  - O(nd) is the size of data, same as runtime up to a log factor.
  - We can apply this algorithm to huge datasets.



Decision Tree Learning

- Decision stumps have only 1 rule based on only 1 feature.
  - Very limited class of models: usually not very accurate for most tasks.

- Decision trees allow sequences of splits based on multiple features.
  - Very general class of models: can get very high accuracy.
  - However, it's computationally infeasible to find the best decision tree.

- Most common decision tree learning algorithm in practice:
  - Greedy recursive splitting.

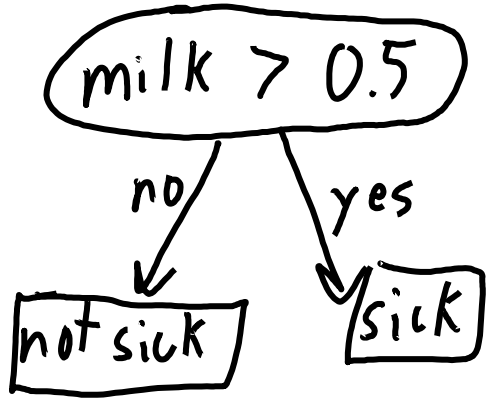
Example of Greedy Recursive Splitting

- Start with the full dataset:

Egg	Milk	…
0	0.7	
1	0.7	
0	0	
1	0.6	
1	0	
2	0.6	
0	1	
2	0	
0	0.3	
1	0.6	
2	0	

Sick?
1
1
0
1
0
1
1
1
0
0
1

Find the decision stump with the best score:



Split into two smaller datasets based on stump:

Egg	Milk	…
0	0	
1	0	
2	0	
0	0.3	
2	0	

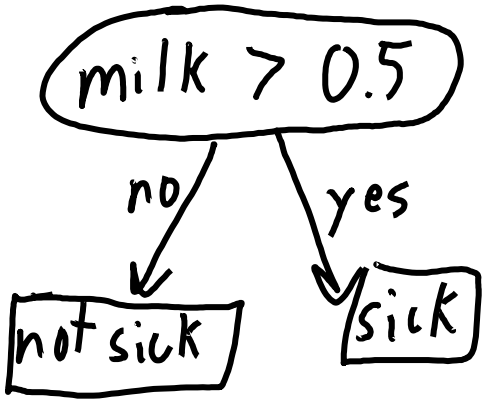
Sick?
0
0
1
0
1

Egg	Milk	…
0	0.7	
1	0.7	
1	0.6	
2	0.6	
0	1	
1	0.6	

Sick?
1
1
1
1
1
0

Greedy Recursive Splitting

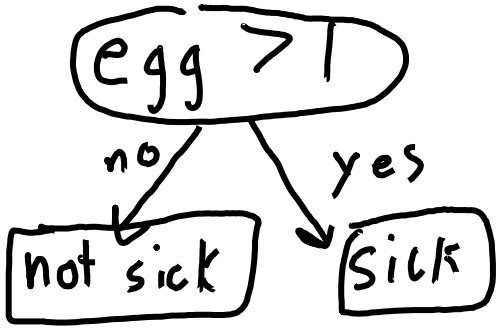
We now have a decision stump and two datasets:

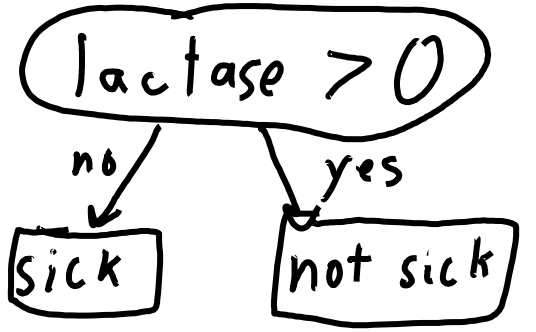


Egg	Milk	…	Sick?
0	0		0
1	0		0
2	0		1
0	0.3		0
2	0		1

Egg	Milk	…	Sick?
0	0.7		1
1	0.7		1
1	0.6		1
2	0.6		1
0	1		1
1	0.6		0

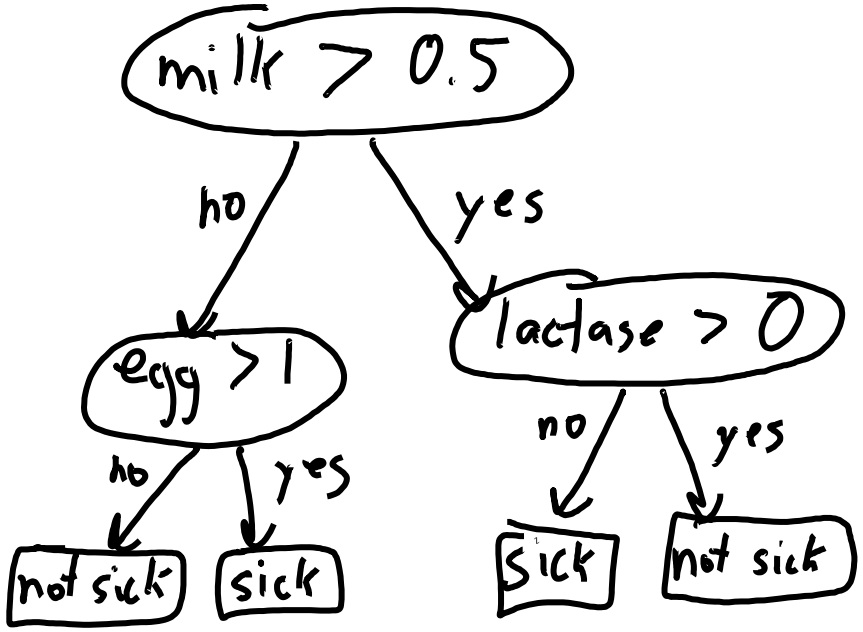
Split the leaves by fitting a decision stump to each dataset:





Greedy Recursive Splitting

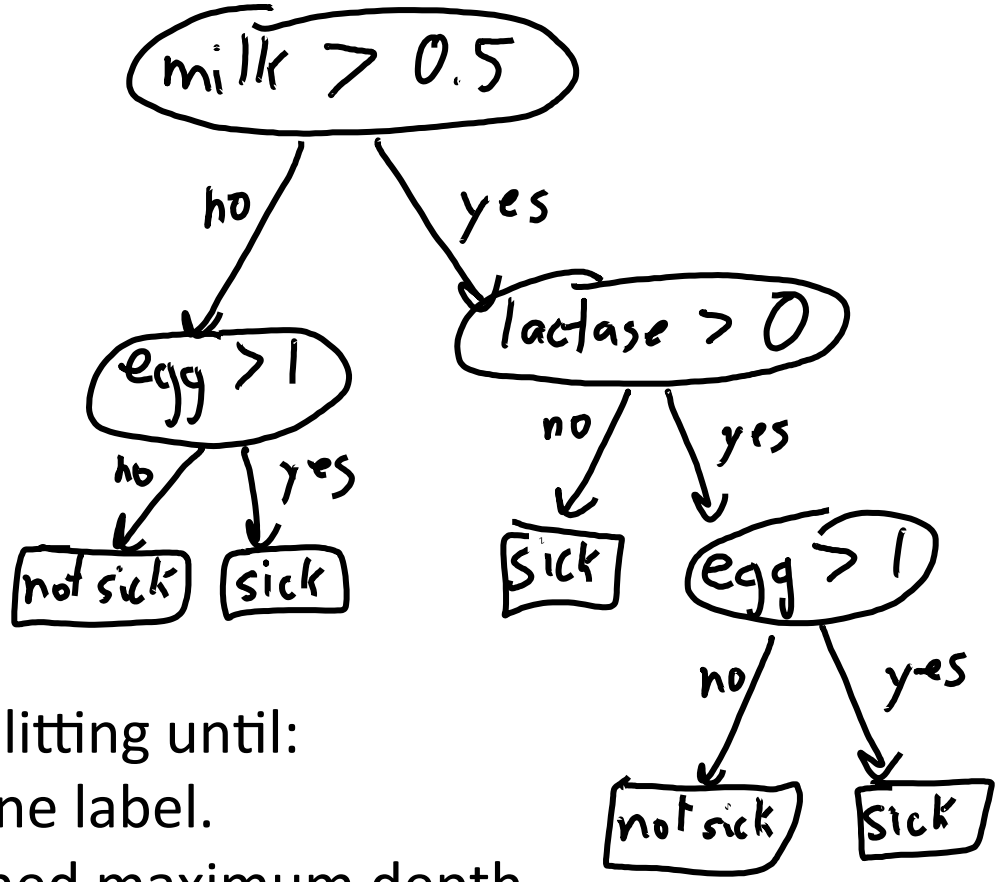
Splitting the leaves gives a “depth 2” decision tree:



We can then split the training examples into 4 datasets, and recurse on these…

Greedy Recursive Splitting

A “depth 3” decision tree:



Typically we continue splitting until:

- The leaves only has one label.
- We reach a user-defined maximum depth.

Discussion of Decision Tree Learning

- Advantages:
  - Interpretable.
  - Fast to learn.
  - Very fast to classify
- Disadvantages:
  - Hard to find optimal set of rules.
  - Greedy splitting uses very simple rules.
  - Unless very deep, greedy splitting often not accurate.
- Issues:
  - Can you revisit a feature?
    - Yes, knowing other information could make feature relevant again.
  - More complicated rules?
    - Yes, but searching for the best rule gets much more expensive.
  - Is accuracy the best score?
    - No, there may no split that increase accuracy. Alternative: information gain.
  - What depth?

Summary

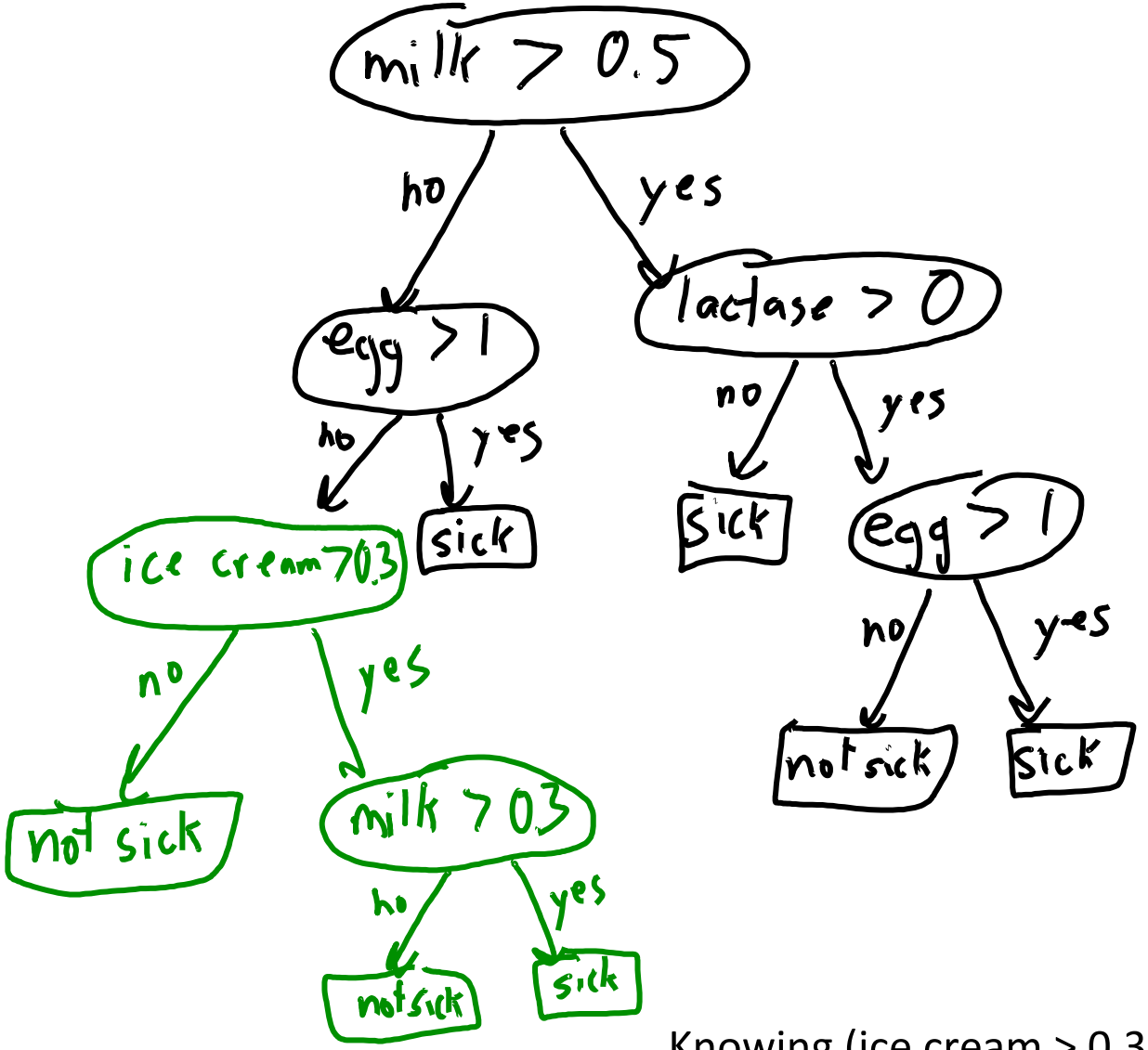
- Supervised learning:
  - using data to write a program based on input/output examples.
- Decision trees: predicting a label using a sequence of simple rules.
- Decision stumps: simple decision tree that is very fast to fit.
- Greedy recursive splitting: uses a sequence of stumps to fit a tree.
  - Very fast and interpretable, but not always the most accurate.

- Next time: the most important ideas in machine learning.

- All the remaining slides are “bonus”.
- We may go through them briefly, if time permits.

Bonus Slide: Can you re-visit a feature?

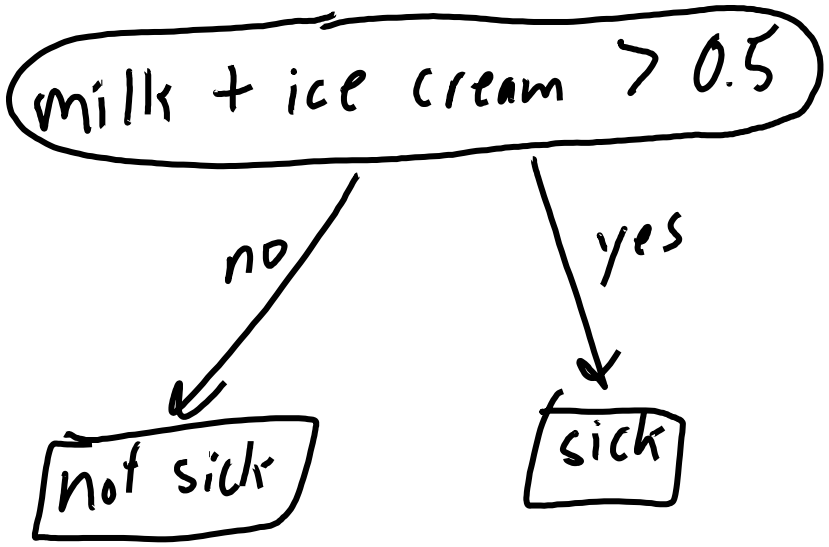
- Yes.



Knowing (ice cream > 0.3) makes small milk quantities

Bonus Slide: Can you have more complicated rules?

- Yes:



- But searching for best rule can get expensive.

Bonus Slide: Which Score Function?

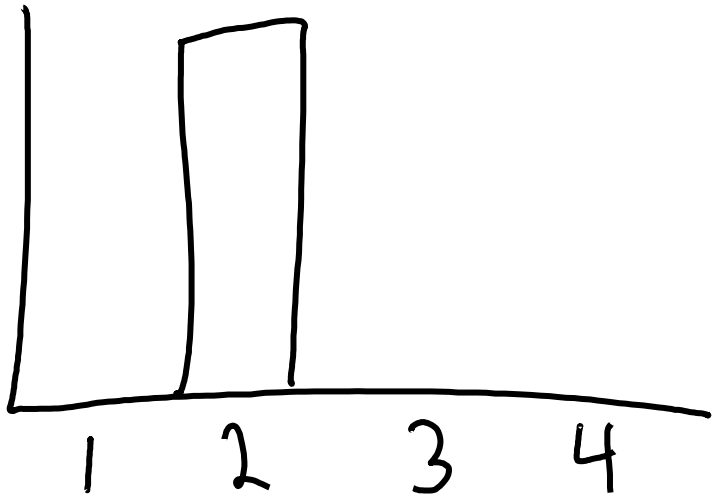
- Shouldn't we just use accuracy score?
  - For leafs: yes, just maximize accuracy.
  - For internal nodes: maybe not.
    - There may be no simple rule like (egg > 0.5) that improves accuracy.

- Most common score in practice: information gain.
  - Choose split that decreases entropy ("randomness") of labels the most.
  - Basic idea: easier to find good rules on "less random" labels.

Bonus Slide: Entropy as Measure of Randomness

- Entropy is measure of “randomness” of a set of variables.

Low entropy means “very predictable”



High entropy means “very random”

- For discrete data, the uniform distribution has the highest entropy.
- So information gain tries to make labels “more predictable”.

Bonus Slide: Probabilistic Predictions

- Often, we'll have multiple 'y' values at each leaf node.
- In these cases, we might return probabilities instead of a label.

- E.g., if in the leaf node we 5 have "sick" objects and 1 "not sick":
  - Return $p(y = \text{"sick"} \mid x_i) = 5/6$ and $p(y = \text{"not sick"} \mid x_i) = 1/6$.

- In general, a natural estimate of the probabilities at the leaf nodes:
  - Let 'n_k' be the number of objects that arrive to leaf node 'k'.
  - Let 'n_{kc}' be the number of times (y == c) in the objects at leaf node 'k'.
  - Maximum likelihood estimate for this leaft is $p(y = c \mid x_i) = n_{kc}/n_k$.