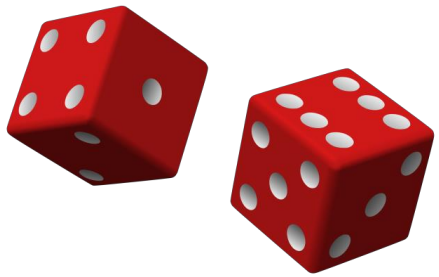
Digression to Review Probabilities…

- Dungeons & Dragons scenario:
  - You roll dice 1:
    - Roll 5 or 6 you sneak past monster.
    - Otherwise, you are eaten.
  - If you survive, you roll dice 2:
    - Roll 4-6, find pizza.
    - Otherwise, you find nothing.



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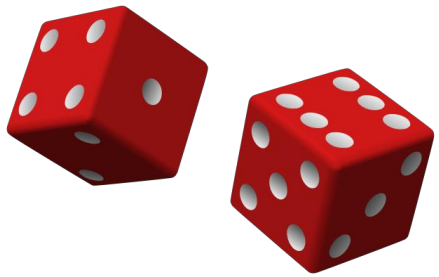

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- Probabilities defined on ‘event space’:

D1\D2	1	2	3	4	5	6
1						
2						
3		D_1=3,D_2=2				
4						
5						
6						

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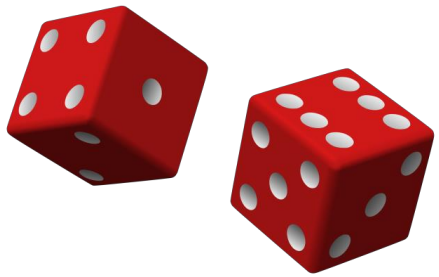

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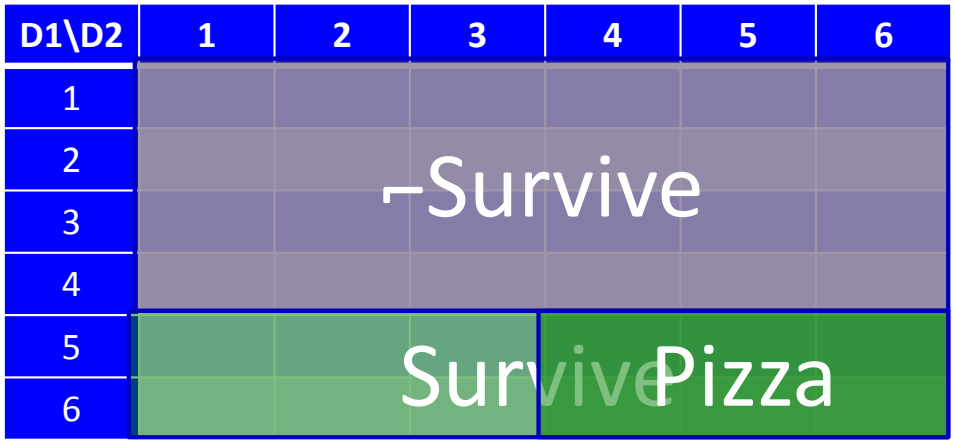
Calculating Basic Probabilities

- Probability of event ‘A’ is ratio:
  - p(A) = Area(A)/TotalArea.
  - “Likelihood” that ‘A’ happens.
- Examples:
  - p(Survive) = 12/36 = 1/3.
  - p(Pizza) = 6/36 = 1/6.
  - p(¬Survive) = 1 – p(Survive) = 2/3.



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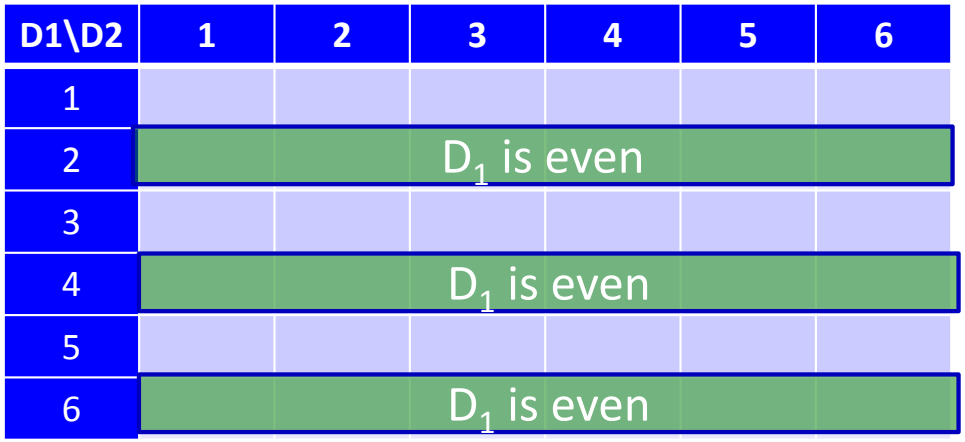
  - p(D_1 is even) = 18/36 = ½.



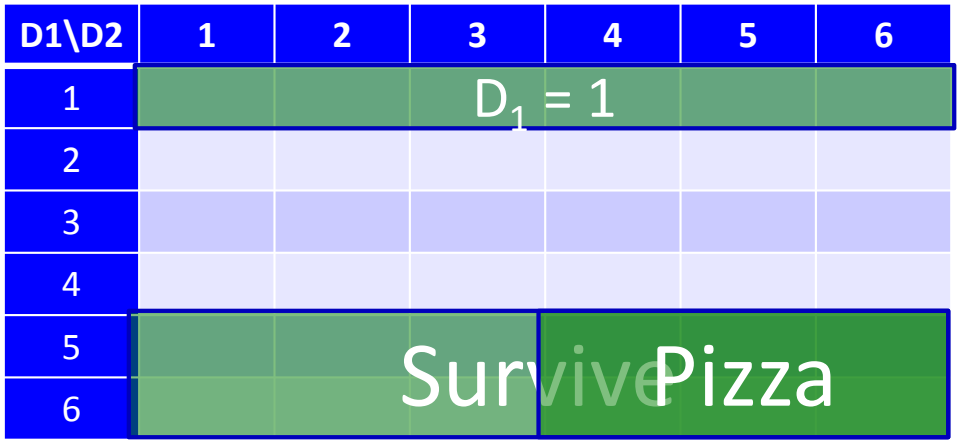
Random Variables and 'Sum to 1' Property

- Random variable: variable whose value depends on probability.
- Example: event (D_1 = x) depends on random variable D_1.
- Convention:
  - We'll use p(x) to mean p(X = x), when random variable X is obvious.
- Sum of probabilities of random variable over entire domain is 1:
  - $\sum_x p(x) = 1$.
  - E.g, $\sum_i p(D_1 = i)$ = 1/6+1/6 + …
    = 1.

D1\D2	1	2	3	4	5	6
1	D_1 =1					
2	D_1 =2					
3	D_1 = 3					
4	D_1 = 4					
5	D_1 = 5					
6	D_1 = 6					

Joint Probability

- Joint probability: probability that A and B happen, written 'p(A,B)'.
  - Intersection of Area(A) and Area(B).
- Examples:
  - $p(D_1 = 1, \text{Survive}) = 0$.
  - $p(\text{Survive}, \text{Pizza}) = 6/36 = 1/6$.



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- Examples:
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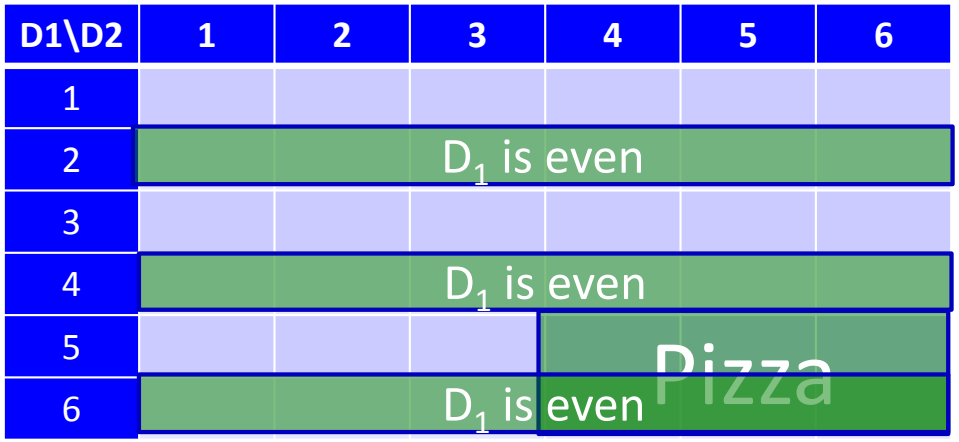
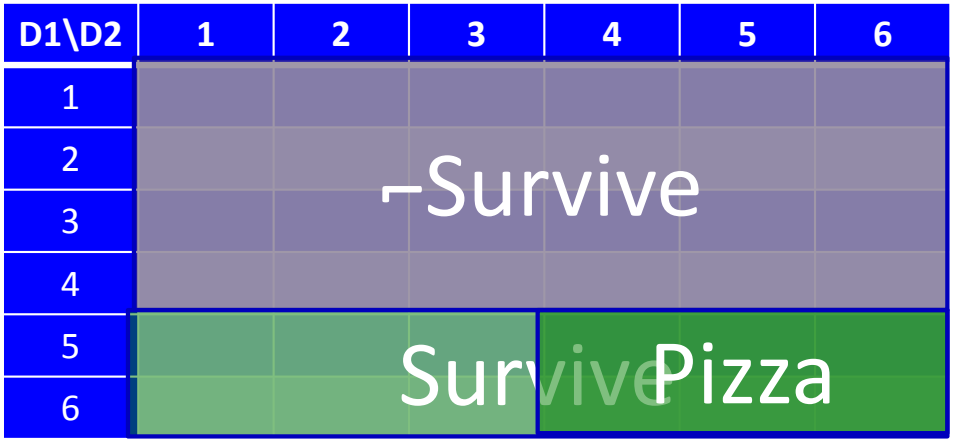
  - $p(\text{Survive}, \text{Pizza}) = 6/36 = 1/6$.
  - $p(D_1 \text{ even}, \text{Pizza}) = 3/36 = 1/12$.



- Note: order of A and B does not matter

Marginalization Rule

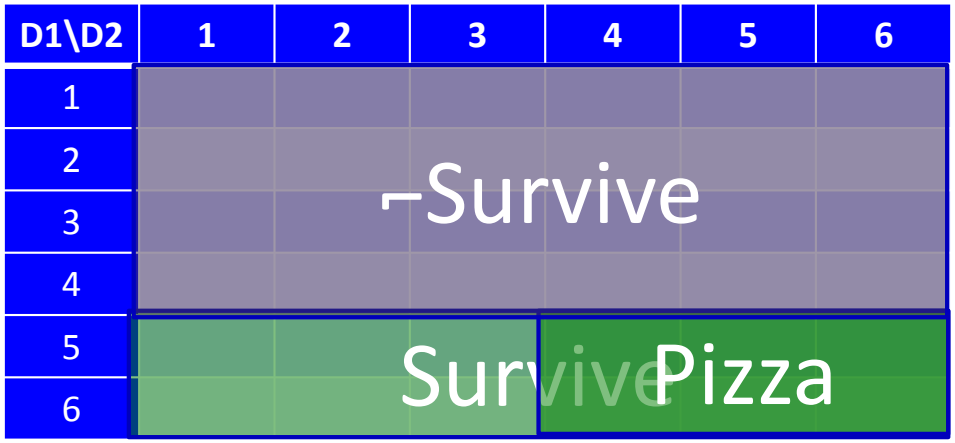
- Marginalization rule:
  - $P(A) = \sum_x P(A, X = x)$.
  - Summing joint over all values of one variable gives probability of the other.
  - Example: $P(Pizza) = P(Pizza, Survive) + P(Pizza, \neg Survive) = \frac{1}{6}$.



  - Applying rule twice: $\sum_x \sum_y p(Y = y, X = x) = 1$.

Conditional Probability

- Conditional probability:
  - probability that A will happen *if we know* that B happens.
  - "probability of A *restricted* to scenarios where B happens".
  - Written p(A|B), said "probability of A given B".
- Calculation:
  - Within area of B:
    - Compute Area(A)/TotalArea.
  - p(Pizza | Survive) =



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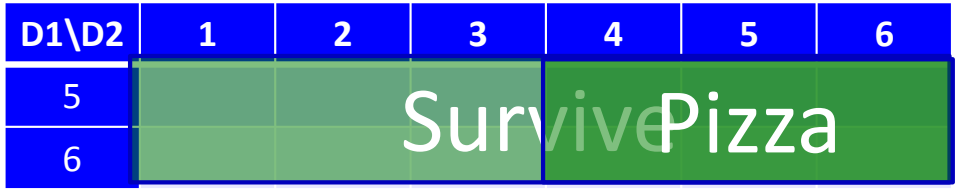
- Calculation:
  - Within area of B:
    - Compute Area(A)/TotalArea.
  - p(Pizza | Survive) =
    p(Pizza, Survive)/p(Survive) = 6/12 = ½.
  - Higher than p(Pizza, Survive) = 6/36 = 1/6.
  - More generally, p(A | B) = p(A,B)/p(B).

Geometrically: compute area of A on new space where B happened.



‘Sum to 1’ Properties and Bayes Rule.

- Conditional probability P(A | B) sums to one over all A:
  - $\sum_x P(x \mid B) = 1$.
  - P(Pizza | Survive) + P(⌐ Pizza | Survive) = 1.
  - P(Pizza | Survive) + P(Pizza | ⌐Survive) ≠ 1.
- Product rule: p(A,B) = p(A | B)p(B).
- Bayes Rule: $$p(A \mid B) = \frac{p(B \mid A)p(A)}{p(B)}$$
  - Allows you to “reverse” the conditional probability.
- Example:
  - P(Pizza | Survive) = P(Survive | Pizza)P(Pizza)/P(Survive)
    = (1) * (1/6) / (1/3) = ½.
  - http://setosa.io/ev/conditional-probability