Chapter 11: Simple linear regression

(Ott & Longnecker Sections: 11.1-11.5)

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Part 1

https://dzwang91.github.io/stat371/

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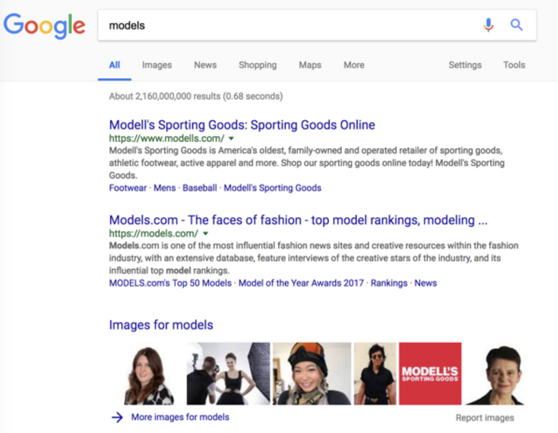
Models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Models

Non-math/statistics Models

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- A statistics model describes relationship between different variables.
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- We'll introduce one of the most important and popular probabilistic models: regression model.

Example

Sir Francis Galton (1822-1911) was interested in how children resemble their parents. One simple measure of this is height. So Galton (actually his disciple, Karl Pearson) measured the heights of father son pairs (in inches) at maturity. In the actual study, 1078 pairs were measured. For convenience, we will use a small subsample of $n = 14$ pairs:

Example continued

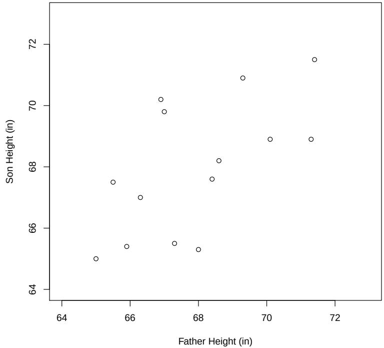
Family	Father's Height	Son's Height
1	71.3	68.9
2	65.5	67.5
3	65.9	65.4
4	68.6	68.2
5	71.4	71.5
6	68.4	67.6
7	65.0	65.0
8	66.3	67.0
9	68.0	65.3
10	67.3	65.5
11	67.0	69.8
12	69.3	70.9
13	70.1	68.9
14	66.9	70.2

Predict sons' heights from father's heights. Deterministic or Probabilistic?

Example continued

- Scatterplot: for each father-son pair, put a point in the two-dimensional plane whose x-coordinate is the father's height and whose y-coordinate is the son's height.
- The response variable is the variable we'd like to predict. By convention, in regression we put the response variable on the vertical axis.
- The predictor variable is the variable we will use to make the prediction.
- The statistical technique of estimating and/or inferring a relationship between two variables is called regression.

Example continued



- It seems that as father's height increases, so does son's height. On a genetic basis, we expect this.
- The nature of the relationship seems approximately linear. However, we also see that sometimes short fathers have tall sons, and vice versa. The relationship is not perfect.

Example continued

- A linear model: an equation that captures that the expected son's heights are linear functions of father's heights.

$$\text{Son's height} = \beta_0 + \beta_1 * \text{Father's height} + \text{Random error}$$

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- β_0 is the intercept.
- β_1 is the slope.
- The Random error term picks up sources of variation in an individual son's height that are not due to his father's height (mother's genetics, environmental factors, etc.) and which cause the points to be "off line."
- Our hope is that the random error term is truly random, so there are no other systemic/structural sources of variation explaining a son's height (if there were, we should try and find them and put them in the model!).

Example continued

- Denote the height of son i by y_i, the height of father i by x_i, and the random error by ϵ_i, so that the model becomes:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i.$$

- Our goal is to estimate the values of β_0 and β_1 from data.