Homework #5

1. Most penguin species are not sexually dimorphic, which means they lack obvious outward body characteristics which indicate sex. Observation of behavior or a blood test can determine Penguin sex. A penguin researcher is interested in estimating the proportion of females in a large penguin population. She takes a random sample of $n = 20$ penguins and determines the sex of each one using a blood test. She finds 12 males and 8 females. Let π be the proportion of females in the population.

   (a) Compute a numerical point estimate of π.

   ANSWER:

   The sample proportion of females is $p = 8/20 = 0.4$.

   (b) Compute the estimated standard error of your estimate.

   ANSWER:

   The estimated standard error is $\sqrt{\frac{(0.4*(1-0.4)}{20}} = 0.11$.

   (c) Is it reasonable to compute a 95% CI for π using the normal approximation in this case? If it is possible, explain why, and make the CI. If it is not possible, explain why.

   ANSWER:

   Since $20(0.4) = 8 > 5$ and $20(0.6) = 12 > 5$, we can use the CLT to approximate the distribution of P as normal. The C.I. is $0.4 \pm 1.96(0.11)$ or $(0.18, 0.62)$.

2. A spinach producer is testing a new packaging line. They want the mean weight of spinach in each package to be 8 ounces. If the mean is too high, they will lose money, and if the mean is too low, customers will get angry. They run the machine for a few days to get a large population of packages, then select 12 packages at random and weigh the spinach in each. If they find strong evidence the mean is too high or low, they can recalibrate the machine. Here are the sample weight (in ounces):

   7.7, 6.8, 8.0, 7.4, 7.1, 7.4, 7.2, 7.3, 8.3, 7.7, 7.6, 7.0

   (Except for the graph(s), I recommend doing this problem with a calculator and table as practice for exams. You may check your answers with R if you wish.)

   (a) State hypotheses appropriate to the research question.

   ANSWER:

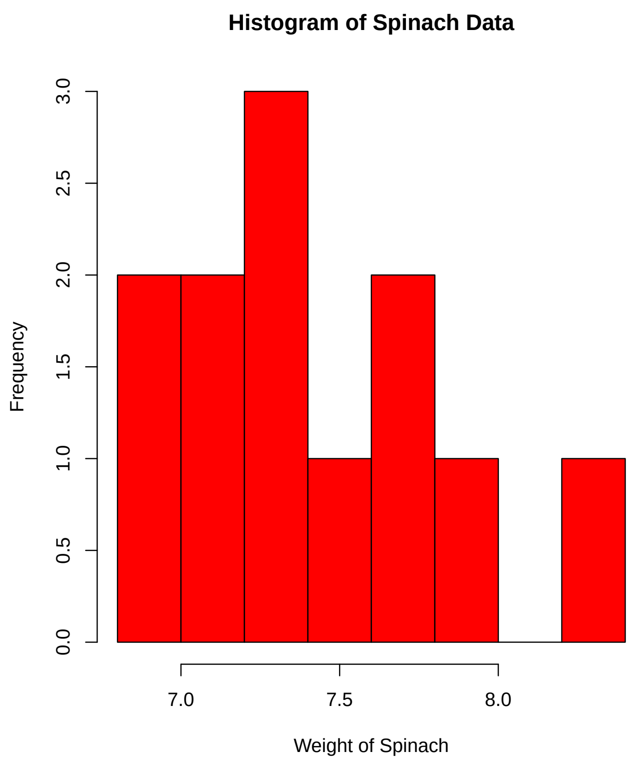
   It is bad if the packages are too heavy or too light, so we need a two-sided alternative. If μ is the population mean package weight, then our hypotheses are:

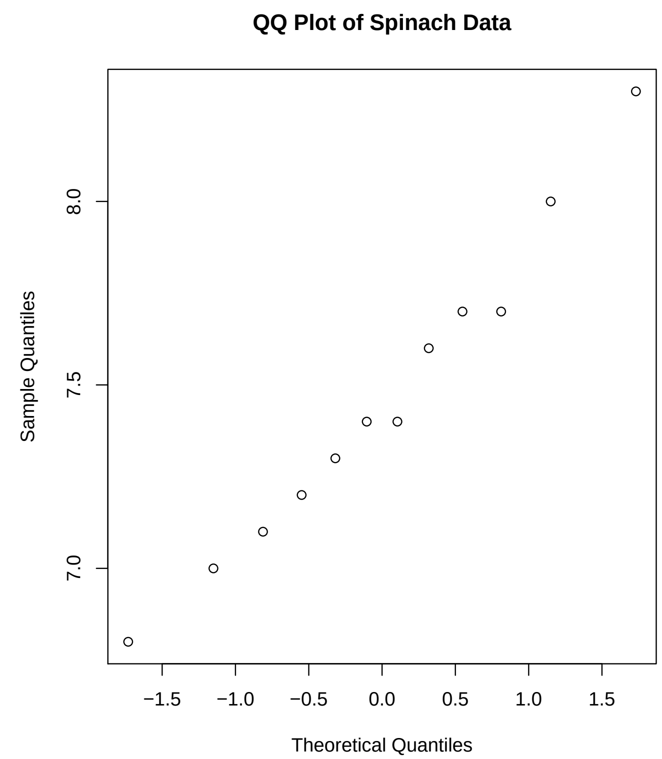
   $$H_0 : \mu = 8\text{oz}$$
   $$H_A : \mu \neq 8\text{oz}$$

   (b) Graph the data as you see fit. Why did you choose the graph(s) that you did and what does it (do they) tell you?

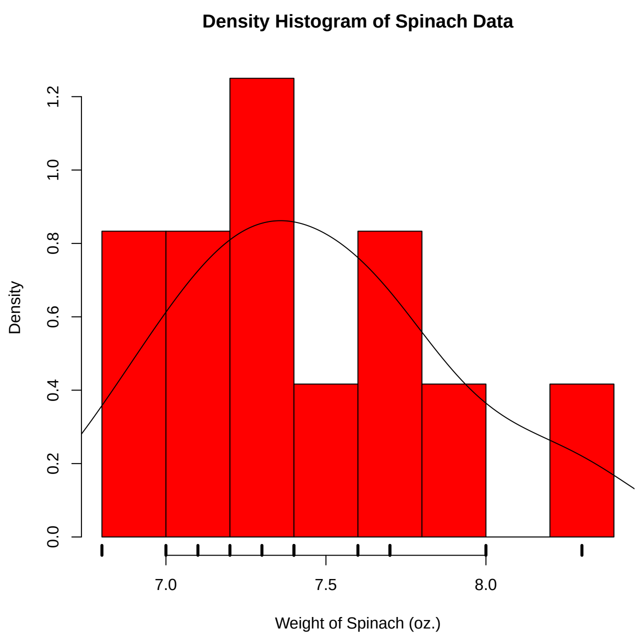
   ANSWER:

   Here are a histogram and QQ plot. Both could be interesting, but the QQ plot is the more important of the two, because we must know whether we can assume normality to decide on an appropriate test.





(Or, converting the histogram to a density histogram and including a density plot, the left graph becomes:



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In light of the departure from linear we see in QQ plots of data sets of size 12 from a normal population (run `qqnorm(rnorm(12))` several times), assuming normality seems reasonable here. This is provided we excuse the two pairs of duplicated values as resulting from rounding measurements of weights that are not duplicated.

(c) Choose an appropriate test statistic for this situation and justify your answer. Then compute the observed value of the test statistic for this data.

ANSWER:

We have assumed normality, but we don't know σ, so a t-test is appropriate. For these data, $\bar{x} = 7.458$ and $s = 0.427$, so $t_{obs} = \frac{(7.458-8)}{\frac{0.427}{\sqrt{12}}} = -4.40$.

(d) Find the rejection region if we desire a test with $\alpha = 0.01$.

ANSWER:

Since we're doing a t-test, we find critical values based on the t distribution. From the table, $t_{n-1,\alpha/2} = t_{(11,0.005)} = 3.106$, so the rejection region is $t < -3.106$ or $t > 3.106$.

(e) Make a reject or not reject decision. Then state your conclusion in the context of the problem. In other words, does it seem the packaging line needs recalibrating, and if so, in which direction?

ANSWER:

Since our observed statistic is in the rejection region, we reject H_0. The observed mean was lower than 8 ounces, so we need to recalibrate the packaging line to add more spinach.

(f) If you calculated a 99% confidence interval for the population mean weight, would you expect it to contain 8? Why or why not?

ANSWER:

No. Since we rejected $H_0 : \mu = 8$ at level $\alpha = 0.01$, a $(1 - \alpha)(100\%) = 99\%$ confidence interval should not contain 8.

(g) Find a 99% confidence interval for the population mean weight.

ANSWER:

We have $n = 12$, $\bar{x} = 7.458$, $s = 0.427$, and $t_{(11,0.005)} = 3.106$, so the interval is $7.458 \pm 3.106\frac{0.427}{\sqrt{12}} = 7.458 \pm 0.383$. (Notice that this interval does not contain 8.)

3. A nutritionist in a large company's cafeteria has a guideline saying employees' daily zinc intake should be about 14 mg/day. She selects a simple random sample of 70 employees and measures their zinc intake for one day. She finds their average intake is 13.8 mg. Earlier studies suggest that the population standard deviation of intakes is about 0.9 mg.

   (a) Run a test, using significance level $\alpha = 0.05$, to decide whether these data are stong evidence that the whole company population of employees took in too little zinc that day.

   - Hypotheses:
   - Assumptions:
   - Test statistic:
   - p-value:
   - Conclusion:

   ANSWER:

   - Hypotheses: $H_0 : \mu = 14$ against $H_A : \mu < 14$
   - Assumptions: The problem statement says that the sample is a simple random sample. $n = 70$ is large enough (by our $n > 30$ rule-of-thumb) for the CLT to apply, so $\bar{X}$ should be normal.
   - Test statistic: We know σ, so use a z test. $z = \frac{13.8-14}{0.9/\sqrt{70}} \approx -1.86$.
   - p-value: $= P(Z < z) = P(Z < -1.86) = 0.0314$
   - Conclusion: Since our p-value, 0.0314, is less than $\alpha = 0.05$, we reject H_0. The data are strong evidence that the company's employees took in too little zinc.

   (b) Suppose the population mean really was 14. Before sampling, what was the probability the test would reject $H_0 : \mu = 14$ even though it is true? Which type of error is this?
   ANSWER:
   $P(\text{reject } H_0 | H_0 \text{ is true}) = \alpha = 0.05$. This is a type I error.

4. A random sample of size $n = 10$ is taken from a large population. Let μ be the unknown population mean. A test is planned of $H_0 : \mu = 12$ vs. $H_A : \mu \neq 12$ using $\alpha = 0.1$. A QQ plot indicates it it is reasonable to assume a normal population. From the sample, $\bar{x} = 14.2$ and $s = 4.88$.

   (I suggest doing this problem with a calculator and table as practice for exams. You may check your answers with R if you wish.)

   (a) Since the data leave it plausible that the population is normal, and the population standard deviation σ is unknown, a t-test is appropriate. Compute the p-value of the test. Do you reject or not reject H_0?
   ANSWER:
   $t_{obs} = \frac{14.2-12}{\frac{4.88}{\sqrt{10}}} = 1.425$. Since the alternative is two sided, the p-value is $P(t_9 < -1.425) + P(t_9 > 1.425) = 2P(t_9 > 1.425)$. In the t-table's row 9, 1.425 is between table values 1.383 and 1.833, which correspond to right tail areas .10 and .05, so $P(t_9 > 1.425)$ is between .05 and .10. The p-value is therefore 2*(between .05 and .10) = between .10 and .20. (A more precise p-value from R is 0.1878.) Since this is larger than $\alpha = 0.1$, we do not reject H_0.

   (b) Using $s = 4.88$ as our best guess of σ, compute the power of the test if the true population mean is $\mu_A = 15$.
   ANSWER:
   The critical values $\pm z_{\alpha/2}$ for $\alpha = 0.1$ are $\pm z_{.05} = \pm 1.645$ from the Z table (or the same number with more digits using R). Un-standardizing (from $z = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$) with $\mu_0 = 12$) gives the corresponding critical

$\bar{x}$ values of $\bar{x} = -z_{\alpha/2}\frac{s}{\sqrt{n}} + \mu_0$ and $\bar{x} = z_{\alpha/2}\frac{s}{\sqrt{n}} + \mu_0$, which are about 9.46 and 14.54. The power is then $P(\bar{X} < 9.46|\mu_A = 15) + P(\bar{X} > 14.54|\mu_A = 15) = P(Z < -3.59) + P(Z > -0.30) =.0002 + .6179 = .6181$.

Or, using the formula, $\text{power}_{\mu_A} \approx P\left(Z < \frac{|\mu_0 - \mu_A|}{\sigma/\sqrt{n}} - z_{\alpha/2}\right)$, we have $\text{power}_{\mu_A=15} \approx P\left(Z < \frac{|12-15|}{4.88/\sqrt{10}} - z_{.05}\right)$

$P\left(Z < \frac{|12-15|}{4.88/\sqrt{10}} - z_{.05}\right) = P(Z < 1.94 - 1.645) \approx P(Z < 0.30 = .6179$

(c) Using $s = 4.88$ as our best guess of σ, approximately what sample size would be required to achieve a power of 0.8 if the true population mean is $\mu_A = 15$? Give your answer as the smallest whole number that meets the criterion.

ANSWER:

Using the formula, $n = (\frac{4.88(1.645+0.84)}{12-15})^2 = 16.34$; round up to 17.