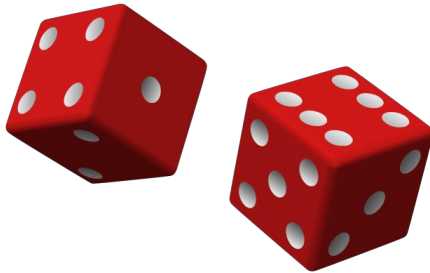


Digression to Review Probabilities...

- Dungeons & Dragons scenario:
 - You roll dice 1:
 - Roll 5 or 6 you sneak past monster.
 - Otherwise, you are eaten.
 - If you survive, you roll dice 2:
 - Roll 4-6, find pizza.
 - Otherwise, you find nothing.



Digression to Review Probabilities...

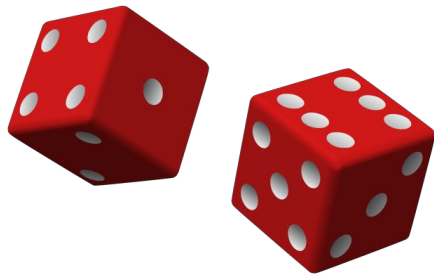
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- Probabilities defined on 'event space':

D1\D2	1	2	3	4	5	6
1						
2						
3		D ₁ =3, D ₂ =2				
4						
5						
6						

Digression to Review Probabilities...

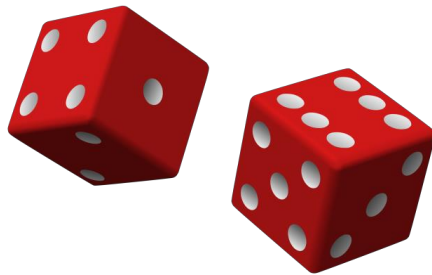
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1						
2						
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4						
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6						

–Survive

Survive Pizza

Calculating Basic Probabilities

- Probability of event 'A' is ratio:
 - $p(A) = \text{Area}(A) / \text{TotalArea}$.
 - “Likelihood” that 'A' happens.
- Examples:
 - $p(\text{Survive}) = 12/36 = 1/3$.
 - $p(\text{Pizza}) = 6/36 = 1/6$.
 - $p(\neg\text{Survive}) = 1 - p(\text{Survive}) = 2/3$.

D1\D2	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Diagram illustrating a 6x6 grid representing a sample space of 36 outcomes. The grid is divided into two main regions:

- Survive (Gray Region):** The top 4 rows (rows 1-4) are shaded gray, representing 24 outcomes. The text "Survive" is written across this region.
- Pizza (Green Region):** The bottom 2 rows (rows 5-6) are shaded green, representing 12 outcomes. The text "Pizza" is written across this region.

The text "Survive" is also written across the gray region, and "Pizza" is written across the green region.

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 - $p(\text{Pizza}) = 6/36 = 1/6$.
 - $p(\neg\text{Survive}) = 1 - p(\text{Survive}) = 2/3$.
 - $p(D_1 \text{ is even}) = 18/36 = 1/2$.

D1\D2	1	2	3	4	5	6
1						
2	D ₁ is even					
3						
4	D ₁ is even					
5						
6	D ₁ is even					

Random Variables and ‘Sum to 1’ Property

- **Random variable**: variable whose value depends on probability.
- Example: event ($D_1 = x$) depends on random variable D_1 .
- Convention:
 - We’ll use $p(x)$ to mean $p(X = x)$, when random variable X is obvious.
- Sum of probabilities of random variable over entire domain is 1:
 - $\sum_x p(x) = 1$.
 - E.g, $\sum_i p(D_1 = i) = 1/6 + 1/6 + \dots = 1$.

D1\D2	1	2	3	4	5	6
1			$D_1 = 1$			
2			$D_1 = 2$			
3			$D_1 = 3$			
4			$D_1 = 4$			
5			$D_1 = 5$			
6			$D_1 = 6$			

Joint Probability

- **Joint probability:** probability that A **and** B happen, written ' $p(A,B)$ '.
 - Intersection of Area(A) and Area(B).
- Examples:
 - $p(D_1 = 1, \text{Survive}) = 0$.
 - $p(\text{Survive}, \text{Pizza}) = 6/36 = 1/6$.

D1\D2	1	2	3	4	5	6
1	D ₁ = 1					
2						
3						
4						
5	Survive			Pizza		
6						

Joint Probability

- **Joint probability:** probability that A **and** B happen, written ' $p(A,B)$ '.
 - Intersection of Area(A) and Area(B).

- **Examples:**

- $p(D_1 = 1, \text{Survive}) = 0$.
- $p(\text{Survive}, \text{Pizza}) = 6/36 = 1/6$.
- $p(D_1 \text{ even}, \text{Pizza}) = 3/36 = 1/12$.

D1\D2	1	2	3	4	5	6
1						
2	D ₁ is even					
3						
4	D ₁ is even					
5				Pizza		
6	D ₁ is even					

- Note: order of A and B does not matter

Marginalization Rule

- Marginalization rule:

- $P(A) = \sum_x P(A, X = x)$.
- Summing joint over all values of one variable gives probability of the other.
- Example: $P(Pizza) = P(Pizza, Survive) + P(Pizza, \neg Survive) = \frac{1}{6}$.

D1\D2	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

¬Survive
Survive
Pizza

- Applying rule twice: $\sum_x \sum_y p(Y = y, X = x) = 1$.

Conditional Probability

- Conditional probability:
 - probability that A will happen *if we know* that B happens.
 - “probability of A *restricted* to scenarios where B happens”.
 - Written $p(A | B)$, said “probability of A given B”.
- Calculation:
 - Within area of B:
 - Compute $\text{Area}(A)/\text{TotalArea}$.
 - $p(\text{Pizza} | \text{Survive}) =$

D1\D2	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

¬Survive

Survive Pizza

Conditional Probability

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- Written $p(A | B)$, said “probability of A given B”.

- Calculation:

- Within area of B:

- Compute $\text{Area}(A)/\text{TotalArea}$.

- $p(\text{Pizza} | \text{Survive}) =$

$$p(\text{Pizza}, \text{Survive})/p(\text{Survive}) = 6/12 = \frac{1}{2}.$$

- Higher than $p(\text{Pizza}, \text{Survive}) = 6/36 = 1/6$.

- More generally, $p(A | B) = p(A, B)/p(B)$.

Geometrically: compute area of A on new space where B happened.

D1\D2	1	2	3	4	5	6
5						
6						

Survive Pizza

'Sum to 1' Properties and Bayes Rule.

- Conditional probability $P(A | B)$ sums to one over all A:

- $\sum_x P(x | B) = 1.$
- $P(\text{Pizza} | \text{Survive}) + P(\neg \text{Pizza} | \text{Survive}) = 1.$
- $P(\text{Pizza} | \text{Survive}) + P(\text{Pizza} | \neg \text{Survive}) \neq 1.$

- Product rule: $p(A, B) = p(A | B)p(B).$

- Bayes Rule:

$$p(A | B) = \frac{p(B | A)p(A)}{p(B)}$$

- Allows you to “reverse” the conditional probability.

- Example:

- $P(\text{Pizza} | \text{Survive}) = \frac{P(\text{Survive} | \text{Pizza})P(\text{Pizza})}{P(\text{Survive})}$
 $= \frac{(1) * (1/6)}{(1/3)} = \frac{1}{2}.$

- <http://setosa.io/ev/conditional-probability>