Assignment #6 — Solutions

1. A random sample of soil specimens was taken from a large geographic area. The specimens can be assumed to be independent. The amount of organic matter, as a percent, was determined for each specimen. The data are below:

$$0.14, 0.32, 1.17, 1.45, 3.5, 5.02, 5.09, 5.22$$

A soil scientist wants to know whether the population mean percent organic matter is different than 4%. A significance level of $\alpha = 0.05$ is chosen.

(a) State hypotheses appropriate to the research question. Since the question is whether the mean is different than 4%, we want a two-sided alternative, so we test:

$$H_0 : \mu = 4\%$$
$$H_A : \mu \neq 4\%.$$

(b) Graph the data as you see fit. Why did you choose the graph(s) that you did and what does it (do they) tell you? With very little data, a histogram won't tell us much. The most important plot is a QQ plot. The points don't look very lined up, so normality might be questionable. On the other hand, the sample is small, and there's a lot of deviation from straightness in QQ plots from random normal data sets of size 8.

(c) Regardless of your conclusion from part (b), use a T-test to perform a test of the hypotheses you stated in (a). Compute the p-value, and make a reject or not reject conclusion. Then state the conclusion in the context of the problem. In other words, does it seem the mean organic matter level is different than 4%?. The test statistic is identical to that computed in part (c), $t_{obs} = -1.61$. But now the p-value is $P(T_7 < -1.61) + P(T_7 > 1.61) = 2P(t_7 > 1.61) = 2$*(between 0.05 and 0.10) = between 0.10 and 0.20. (R gives 0.151.) This is larger than $\alpha = 0.05$, so we do not reject. We do not have sufficient evidence to say that the mean percent of organic matter is different than 4%.

2. A study is conducted regarding shatterproof glass used in automobiles. Twenty-six glass panes are coated with an anti-shattering film. Then a 5-pound metal ball is fired at 70mph at each pane. Five of the panes shatter. We wish to determine whether, in the population of all such panes, the probability the glass shatters under these conditions is different from $\pi = 0.2$.

(a) State the appropriate null and alternative hypotheses. Hypotheses could be:

$$H_0 : \pi = 0.2$$
$$H_A : \pi \neq 0.2$$

(b) Check the conditions for trusting the conclusion of the test, and calculate the observed value of an appropriate test statistic. The sample size is $n = 26$. The observed sample proportion shattered is $p = \frac{5}{26} = 0.1923$. We assume the data are a simple random sample from the population. To the extent that they are not, our conclusion may be suspect. We also need the expected numbers of successes and failures under H_0 each to be greater than 5 so that we can use the CLT. We have $n\pi_0 = 26(0.2) = 5.2$ and $n(1-\pi_0) = 26(1-0.2) = 20.8$, each of which is greater than 5. The test statistic is $z_{obs} = \frac{p-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.1923-0.2}{\sqrt{\frac{0.2(1-0.2)}{26}}} = -0.098$.

(c) Calculate the rejection region and draw a conclusion, given the significance level $\alpha = 0.05$. Since the alternative is two-sided, the rejection region is reject if $z_{obs} < -z_{\alpha/2}$ or $z_{obs} > z_{\alpha/2}$, where $z_{\alpha/2} = z_{.05/2} = z_{0.025} = 1.96$. That is, the rejection region is reject if $z_{obs} < -1.96$ or $z_{obs} > 1.96$. Our value $z_{obs} = -0.098$ is not in the rejection region, so we do not reject H_0. We don't have sufficient evidence to conclude that the population proportion of panes shattering in response to the ball is different than 0.2.

(d) Calculate the p-value. p-value = P(a test statistic as extreme or more extreme than the one we observed) $= P(Z < -0.098) + P(Z > 0.098) = 2 * P(Z < -0.098) = 2 * P(Z < -0.10) = 2(0.4602) = 0.9204$.