

Chapter 3 — Probability Summary

1. Probability vs. Statistics

- A **random sample** is a special kind of sample where there is some uncertainty as to which members of the population will be selected.
- **Probability** takes information about a population, and allows us to make statements about what a random sample taken from that population might look like.
- **Statistics** takes information about a random sample, and seeks to make inference about the properties of the population from which the random sample was drawn.
- A numerical property of a population is often called a **population parameter**, or simply a **parameter**.

2. Definitions of Probability

- A **random process** is any process that generates data in which the result has some amount of random chance or uncertainty involved.
- An **outcome** is a distinct result of a random process.
- An **event** is a collection of outcomes.
- There are two widely accepted interpretations of probability:
 - The **long-run frequency** or **frequentist** interpretation says that if a random process is repeated many, many times, the probability of an event is the proportion of times that the event occurs.
 - The **subjective** or **Bayesian** interpretation says that probability of an event is a degree of belief about the chances that the event will occur.
- Here are three properties of frequentist probability:
 - (1) For a random process with a finite set of possible outcomes, the probability of an event is the sum of the probabilities of the outcomes that comprise that event.
 - (2) For a random process with a finite set of possible outcomes, the probability of an event is always between 0 and 1. A probability of exactly 0 indicates the event will never happen no matter how many times the random process is repeated, and a probability of exactly 1 indicates the event will always happen no matter how many times the random process is repeated.

- (3) As a result of properties (1) and (2), the probability that an event does not occur must always be one minus the probability that it does occur.
- Two events are said to be **independent** if the observed outcome of the first event does not change the probability of the second event, or vice versa. If two events are not independent, we call them **dependent**.
 - If two events are independent, then the probability that they both occur is the product of the individual probabilities of the events.