UNIVERSITY OF WISCONSIN–MADISON

STAT371-L5
Midterm 2
Fall 2017

A. You are allowed one piece of paper for notes (both sides), and a hand calculator. Laptops, tablets, and smartphones are not allowed.

B. To receive full credit, you must **show your work**. Partial credit will be awarded when appropriate.

C. If you can't find the exact value you need in the table, state this and use the closest value you can find.

D. Do all your work in the space provided. If you need more space, you may ask for extra paper.

Name: ____________________ Discussion Section Number: ____________________

For instructor's use:

1	10	
2	30	
3	20	
4	20	
Total	80	

1. A researcher is investigating the weights of lizard eggs of a certain species to see if they have decreased in radius during a drought. She wishes to test her hypothesis that the population's radius is below a certain value, μ_0. She decides she will use a t-test to address her hypothesis. Answer the following questions:

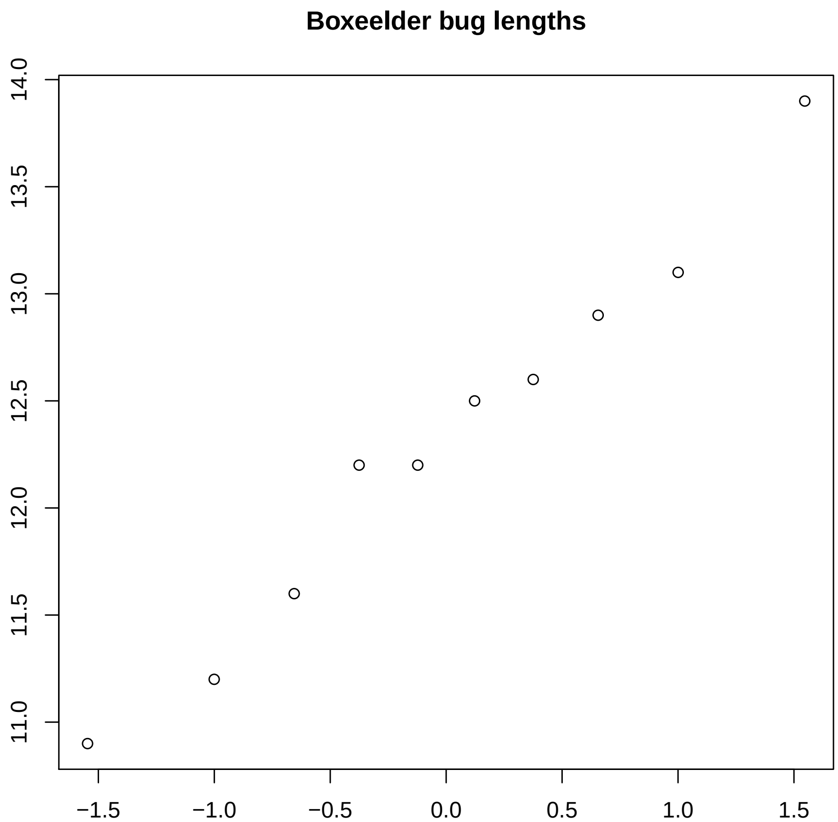
   (a) Explain, **in the context of this problem**, what a **type I error** would be. (5 points)

   The population mean of egg is μ_0, but she rejects the null.

   (b) She calculates a p-value of 0.002 and writes in her report, "**the p-value for this problem is 0.002, from which we conclude the null has only a** 0.2% **chance of being true and we have strong evidence to reject the null.**" Is her analysis correct? Explain. (5 points)

   Her statement that we have evidence against the null is accurate, but the p-value is not the chance that the null is true. It is the chance of having a test statistic as extreme or more extreme than the observed one.

2. The sunny south wall of a house was covered with boxelder bugs. A researcher enclosed the wall in plastic to capture all the bugs. The lengths of a simple random sample of 10 bugs were measured in mm: 10.9, 11.2, 11.6, 12.2, 12.2, 12.5, 12.6, 12.9, 13.1, 13.9. Here $\bar{x} \approx 12.31$ and $s \approx 0.90$. Here is a QQ plot of the 10 lengths:



(a) Is it plausible that the population of lengths is normally distributed? Why or why not?(5 points)

Yes, as the QQ plot looks reasonably linear.

(b) Suppose the population of lengths is normal. Find a 95% confidence interval for the unknown population mean length. **Keep two digits after decimal**. (5 points)

For 95% confidence we have $1-\alpha = .95 \implies \alpha = .05$ and we need $t_{n-1,\alpha/2} = t_{10-1,.05/2} = t_{9,.025} = 2.262$. Our interval is $\bar{X} \pm t_{n-1,\alpha/2}\frac{S}{\sqrt{n}} = 12.31 \pm 2.262\frac{0.90}{\sqrt{10}} = 12.31 \pm 0.64$.

(c) Suppose the population of lengths is normal. **Find the test statistic** and **p-value** for a test to decide whether the population mean length **is different than** 12.5. **Draw a conclusion** using significance level 0.05. (10 points)

$t = \frac{\bar{X}-\mu_0}{\frac{S}{\sqrt{n}}} = \frac{12.31-12.5}{\frac{0.90}{\sqrt{10}}} \approx -0.67$. The p-value is $P(T_{10-1} < -0.67) + P(T_{10-1} > 0.67) = 2P(T_{10-1} > 0.67) = 2(> .25) > 0.50$, so we do not reject H_0. The data are not strong evidence the population mean length is different from 12.5.

(d) **Find the test statistic** and **p-value** for a test of $H_0: M = 11$ vs. $H_A: M > 11$, where M is the population median length. **Draw a conclusion** using significance level 0.05. (10 points)

For a sign test, use test statistic B = the number of differences, length minus median, that are positive (i.e. the number of lengths greater than 11). If H_0 is true, then $B \sim \text{Bin}(n = 11, \pi = .5)$. The p-value is $P(B \geq 9) = P(B = 9) + P(B = 10) = (10 + 1).5^{10} \approx 0.01 \implies$ reject H_0. The data are strong evidence the median bug length is greater than 11.

3. The Wisconsin State Patrol are worried their fleet of vehicles–which includes 500 police cruisers, motorcycles, SUVs, and so forth–is aging. They are interested in the proportion of their vehicles which have traveled over 100,000 miles. Call this proportion π. The State patrol hires you, a statistician, to make inferences about π.

   (a) Suppose you collect a simple random sample of size $n = 35$ Wisconsin State Patrol vehicles. In the sample, 10 vehicles have traveled more than 100,000 miles. Calculate the **point estimate** of π and its **estimated standard error**. **Keep two digits after decimal.** (10 points)

   We have $\hat{\pi} = p = 10/35 = 0.286$. This yields estimated SE $\hat{SE}(p) = \sqrt{\frac{0.286*0.714}{35}} = 0.076$.

   (b) Calculate a 90% confidence interval for π. **Interpret the interval in context. Keep two digits after decimal.** (10 points)

   In our sample we have 10 successes and 25 failures, so we can compute a confidence interval based on the CLT. We have $z_{0.05} = 1.65$ which yields the interval

   $$0.286 \pm 1.65 * 0.076 = [0.16, 0.41].$$

   So we are 95% *confident* the proportion of State Patrol vehicles with over 100,000 miles is between 0.161 and 0.411.

4. For a certain experiment, a neuroscientist has gathered a sample of 80 Drosophila Melanogaster(fruitflies) and found that 55 of the flies reacted when prodded with a needle heated at $41°C$. He knows that if **over** 62% of flies react, he will need to recalibrate his heated stimulus. (10 points)

   (a) What are the hypotheses? (5 points)

   $H_0 : \pi = 0.62$
   $H_a : \pi > 0.62$

   (b) Choose an appropriate test by **checking assumptions**, calculate the test statistic and solve for the associated p-value. **Keep two digits after decimal.** (10 points)

   First we check CLT conditions:
   $\pi_0(n) = .62(80) = 49.6 > 5$ ✓
   $(1 - \pi_0)n = .38(80) = 30.4 > 5$✓

   From the problem, we find our $\hat{pi} = p = \frac{55}{80} = .6875$

   Now we can solve for our test statistic:

   $t.s = \frac{.6875-.62}{\sqrt{\frac{.62(.38)}{80}}} = 1.24$

   $p.val = P(Z > 1.24)$=.11

   (c) Given $\alpha = 0.05$, make a reject or not reject decision **in the context of the problem**. (5 points)
   At the $\alpha = 0.05$-level we would fail to reject the null hypothesis. This means that he does not appear to need to recalibrate his stimulus tool.