Assignment #7 — Solutions

1. A dairy scientist is testing a new feed additive. She chooses 13 cows at random from a large population of cows. She randomly assigns $n_{old} = 8$ to get the old diet, and $n_{new} = 5$ to get the new diet including the additive. The cows are housed in 13 separated pens and each gets separate feed, with or without additive as appropriate. After two weeks, she picks a day and milks each cow using standard procedures and records the milk produced in pounds. The data are below:

Old Diet: 43, 51, 44, 47, 38, 46, 40, 35
New Diet: 47, 75, 85, 100, 58

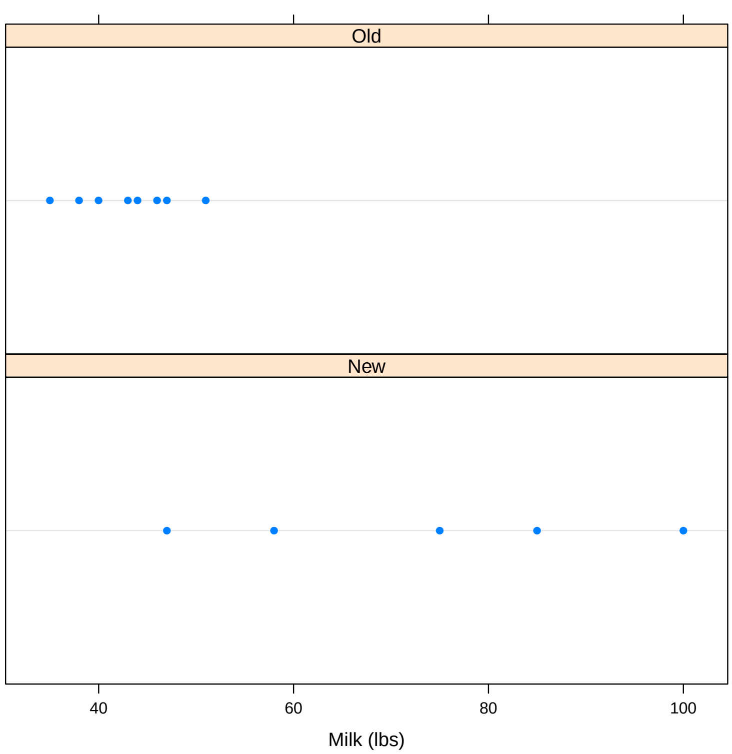
Let μ_{new} and μ_{old} be the population mean milk productions for the new and old diets, respectively. She wishes to test:

$$H_0 : \mu_{new} - \mu_{old} = 0$$
vs.
$$H_A : \mu_{new} - \mu_{old} \neq 0$$

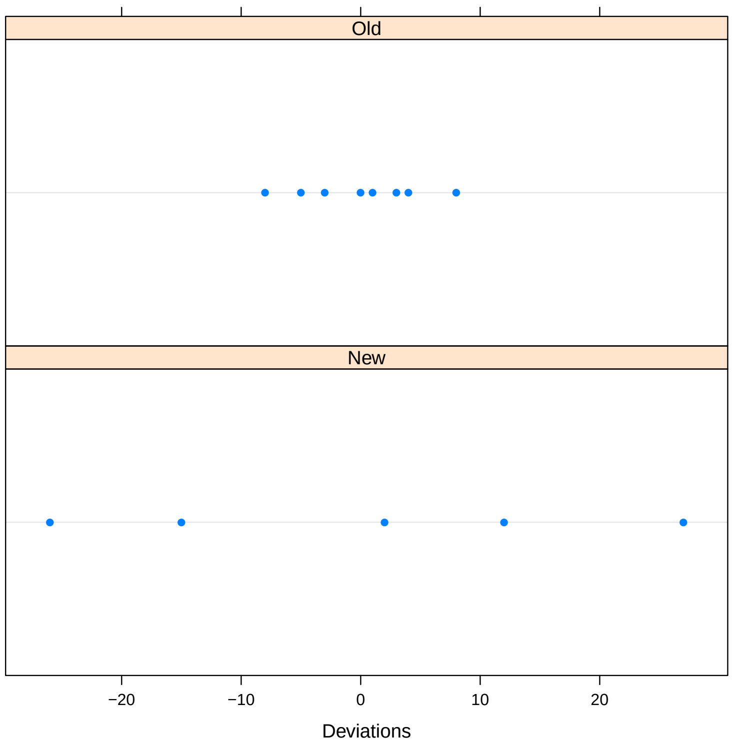
using $\alpha = 0.05$.

(a) Comment on the assumption that the cows are independent both within and between treatments. The assumption seems met. Cows were randomly assigned to treatment groups, and were housed in separate pens.

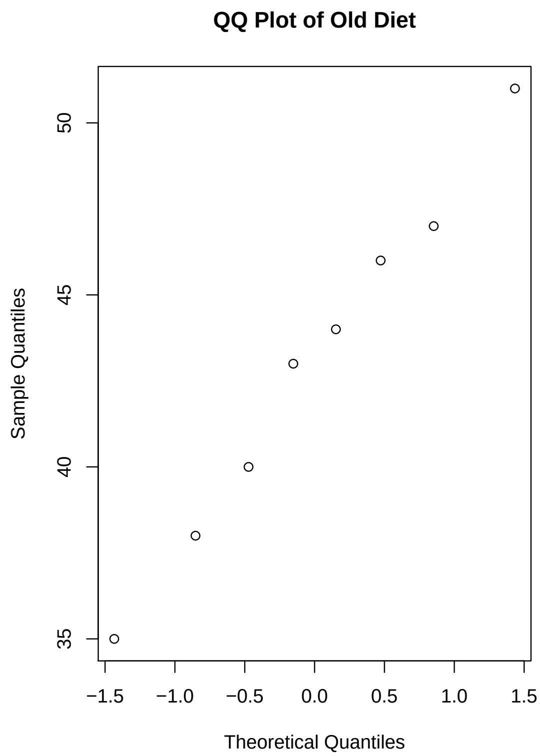
(b) Graph the data as you see fit. Why did you choose the graph(s) that you did and what does it (do they) tell you? We might start with a dotplot of the original data so we can get a sense for both location and spread. This is probably better than a histogram or boxplot since the sample sizes are small.

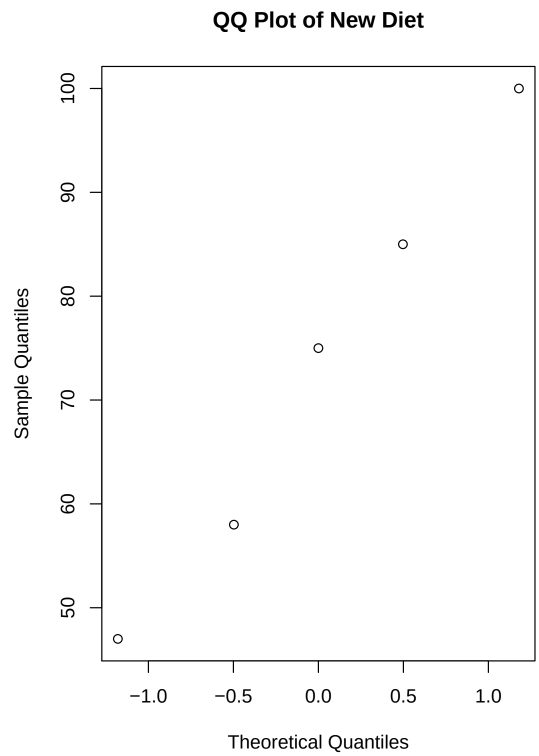


The new diet looks a lot higher but also seems to have more spread. To look at spread even closer, we could do a dotplot of deviations:



There seems to be a lot more spread for the new diet. This is also confirmed by checking our guideline, since the ratio of the SDs is $5.18/21.08 = 0.24$ which is less than 0.5. For normality we must make separate QQ plots since the variances are not equal:





They look fairly straight, so normality can be assumed.

(c) Choose a test appropriate for the hypotheses above, and justify your choice based on your answers to parts (a) and (b). Then perform the test by computing a p-value, and making a reject or not reject decision. You may use R to compute means and SDs for the groups, but do not use `t.test()` to perform the test. Show your work. State your conclusion in the context of the problem. If a difference exists, could you infer that the additive caused the change? Based on parts (a) and (b), since the data is independent and normal, and the variances are likely different, we will use a Welch T-test. The observed test statistic is $\frac{(43-73)-0}{\sqrt{26.86/8+444.5/5}} = -3.12$, and the approximate df by the formula from class works out to 4.3, round down to 4. Thus the p-value, comparing to a T_4, is $2 * 0.018 = 0.036$. We would reject, and conclude that the new diet seems to lead to more milk. We can infer causation here since the experiment is randomized and not observational.

2. A shoe manufacturer compared two new materials for the soles of shoes, call them A and B. Twelve adult volunteers, from locations spread around the USA, each got two shoes. One was randomly assigned to be made with material A, and the other was made with material B. On both shoes, the material was exactly 1 inch thick. They were instructed to wear the shoes as they would normal shoes, and ship them back to the manufacturer after 2 months. Technicians then re-measured the thickness of the soles, and recorded the amount of wear (in microns). The data is below:

Participant	1	2	3	4	5	6	7	8	9	10	11	12
Sole A	379	378	328	372	325	304	356	309	354	318	355	392
Sole B	372	376	328	368	283	252	369	321	379	303	328	411

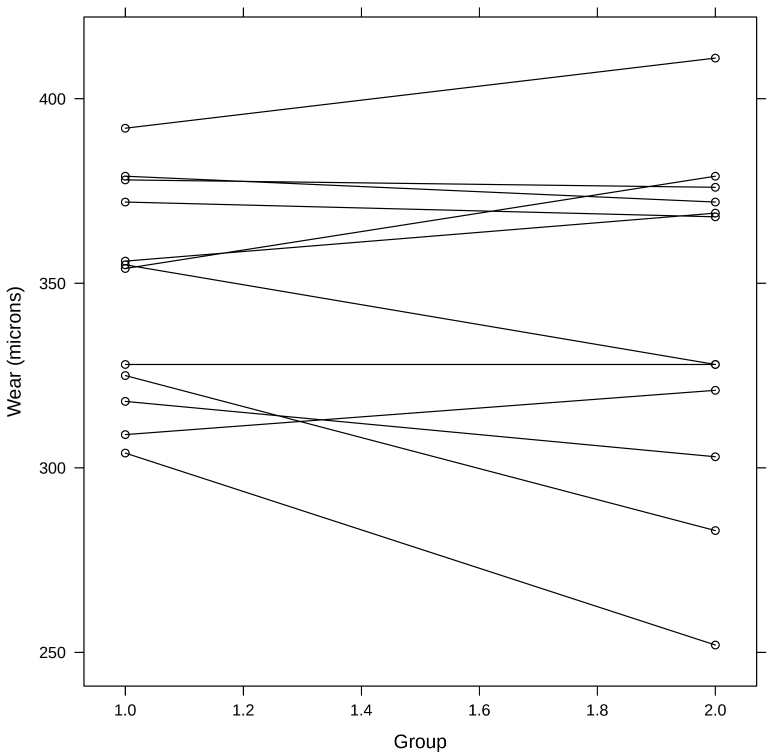
They wish to test:

$$H_0 : \mu_A - \mu_B = 0$$
$$\text{vs.}$$
$$H_A : \mu_A - \mu_B \neq 0,$$

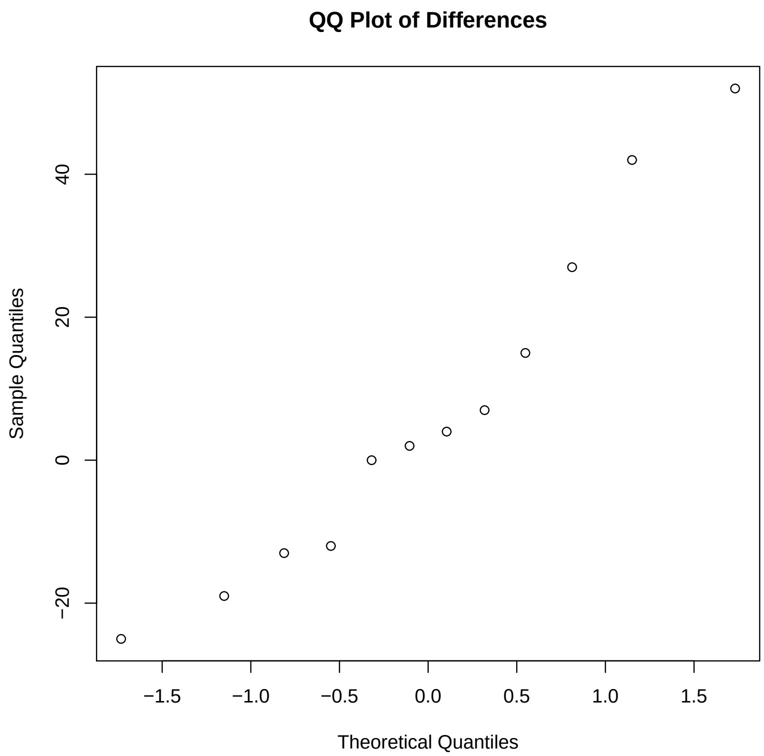
using $\alpha = 0.05$.

(a) Are the two populations paired or independent? Explain your answer. Each person in the study wears a shoe of each sole type. So there is a natural pairing. However, since the volunteers are widely spread out geographically, the pairs themselves can be considered independent.

(b) Graph the data as you see fit. Why did you choose the graph(s) that you did and what does it (do they) tell you? We can start with a graph that preserves the pairing:



There seems to be about as many where B is lower than A than vice versa. We can also check normality on the differences:



It looks fine, so normality can be assumed.

(c) Choose a test appropriate for the hypotheses above, and justify your choice based on your answers to parts (a) and (b). Then perform the test by computing a p-value, and making a reject or not reject decision. You may use R to compute means and SDs, but do not use `t.test()` to perform the test. Show your work. Finally, state your conclusion in the context of the problem. Since the data is paired and the differences are normal, we'll use a paired T-test. The mean difference is 6.67, and the sd of differences is 23.85, so the observed stat is: $t_{obs} = \frac{6.67-0}{\frac{23.85}{\sqrt{12}}} = 0.968$. The df are $12 - 1 = 11$, so the p-value is about $2 * 0.17 = 0.34$. This is large, so we would not reject. There is not enough evidence to say that the sole materials differ.