Tests of Location for a Single Population

1. When the data is drawn from a population that has a normal distribution and σ is unknown, use a t-test. To test:

$$H_0 : \mu = \mu_0$$
$$H_A : \mu \neq \mu_0$$

at the $100 * \alpha\%$ level based on a sample of size n, use one of the following methods:

- Using the rejection region method, determine the value $t_{(n-1,\alpha/2)}$ so that:

$$P(-t_{(n-1,\alpha/2)} \leq t \leq t_{(n-1,\alpha/2)}) = 1 - \alpha.$$

Then compute $t_{obs} = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$. Reject the null if $t_{obs} < -t_{(n-1,\alpha/2)}$ or $t_{obs} > t_{(n-1,\alpha/2)}$.

- Using the p-value method, compute

$$p - value = P(t_{(n-1)} < -|t_{obs}|) + P(t_{(n-1)} > |t_{obs}|).$$

Reject if p-value$< \alpha$.

2. When the data is drawn from a population that has a normal distribution and σ is known, the sample size n required to achieve power $1 - \beta$ for a test of $H_0 : \mu = \mu_0$ vs. $H_A : \mu \neq \mu_0$ when the real μ is μ_A at level α is approximately:

$$n = (\frac{\sigma(z_{\alpha/2}+z_\beta)}{\mu_0-\mu_A})^2.$$

3. When the data is not normal and n is too small to use the CLT, use sign test to test the population median. If M is the population median, test:

$$H_0 : M = M_0$$
$$H_A : M > M_0$$

by computing $b =$ the number of observations strictly larger than M_0. If any observations are equal to M_0, remove them. The p-value is then $P(B \geq b)$, where $B \sim Bin(n, 0.5)$.

4. When making a test about population proportion π based on a sample of size n, if $n(\pi_0) > 5$ and $n(1-\pi_0) > 5$, then test:

$$H_0 : \pi = \pi_0$$
$$H_A : \pi \neq \pi_0.$$

by computing the sample proportion p, and then finding:

$$z_{obs} = \frac{(p - \pi_0)}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}.$$

Then the p-value is $P(Z < -|z_{obs}|) + P(Z > |z_{obs}|)$. Reject if p-value $< \alpha$.