Assignment #2 — Solutions

1. A certain circuit board consists of two resistors, green and red. The circuit board manufacturer has two huge bins filled with the resistors, one for each color. Based on several years of data, it is known that 90% of the red resistors are functional, and 75% of the green resistors are functional. When creating a circuit board, the technician selects one red and one green resistor at random. **Hints for the two problems below: It may help to write out the list of all possible outcomes of this random process. Also, remember that the probabilities of outcomes add, and that independent probabilities multiply.**

   (a) The circuit board as a whole is only functional if both resistors are functional. What is the probability that the circuit board is functional? The circuit board as a whole is only functional if both resistors are functional. Since the draws out of each bin are independent, we multiply the probabilities that each resistor is functional and find the probability that the board is functional is $0.9 * 0.75 = 0.675$.

   (b) What is the probability that EXACTLY one of the resistors chosen is functional? One outcome is that the red is functional, but not the green. The probability that green is not functional is $1 - 0.75 = 0.25$. These are independent events, so we multiply the probabilities, $0.9*0.25 = 0.225$. The other outcome is that red is not functional, but green is. The probability that red is not functional is $1-0.9 = 0.1$. Again these are independent events, so we multiply, $0.1 * 0.75 = 0.075$. Both of these outcomes comprise the event that exactly one resistor is functional, so we add the probabilites, and find that the probability that exactly one resistor is functional is $0.225 + 0.075 = 0.3$.

2. A nefarious gambler has developed a weighted six-sided die, with sides marked 1,2,3,4,5, and 6. The probability that a 6 is rolled is 0.4, and all the other sides are equally likely. Define the random variable X = the value on the die when it is rolled once.

   (a) Write out the pmf of X. The probability for 6 is given as 0.4. Since the probabilities of all possible outcomes must add to 1, the probabilities of the other outcomes must add to $1 - 0.4 = 0.6$. Since the other outcomes are equally likely, they must each have probability $0.6/5 = 0.12$. The pmf is thus:

x	$p(x)$
1	0.12
2	0.12
3	0.12
4	0.12
5	0.12
6	0.40

   (b) Compute the probability that an even number is rolled. The probability of the event that the roll is even is the sum of the probabilites of the three outcomes where the roll is even, 2, 4, or 6. Thus the probability is $p(2) + p(4) + p(6) = 0.12 + 0.12 + 0.4 = 0.64$.

   (c) Compute the expectation and variance of X. The expectation is $1*0.12+2*0.12+3*0.12+4*0.12+5*0.12+6*0.4 = 4.2$. The variance is $0.12*(1-4.2)^2+0.12*(2-4.2)^2+0.12*(3-4.2)^2+0.12*(4-4.2)^2+0.12*(5-4.2)^2+0.4*(6-4.2)^2 = 3.36$.

3. For each of the following questions, say whether the random process is a binomial process or not, and explain your answer. As part of your explanation, you will want to comment on the potential validity of each of the four things that must be true for a process to be a binomial process.

    (a) One basketball player attempts 10 free throws and the number of successful attempts is totalled. The process does consist of trials (each attempt), and the result has two outcomes, success and failure. However, it is unlikely that the trials are independent. The performance on the previous attempt might affect the probability of making the next attempt. The player might, for example, concentrate harder after a miss. Or they might get discouraged after a miss and concentrate less, etc. This is not a good approximation to a binomial process.

    (b) Ten different basketball players each attempt 1 free throw and the total number of successful attempts is totalled. The process does consist of trials (each attempt), the result has two outcomes, success and failure, and as long as the players do not influence one another, they are probably approximately independent. However, it is unlikely that every player has the same chance of making a free throw, therefore the probability of success on each trial is not constant. This is not a good approximation to a binomial process.

4. Let $B \sim Bin(20, 0.2)$. Compute the following probabilities. I would suggest computing these with a hand calculator using the formula provided in class , but you can check your answers using R if you wish.

    (a) $P(B = 4)$. $P(B = 4) = \binom{20}{4} 0.2^4 (1 - 0.2)^{16} = 0.2182$. In R this could be done with `dbinom(4,20,0.2)`.

    (b) $P(B \leq 1)$. $P(B \leq 1) = \binom{20}{0} 0.2^0 (1 - 0.2)^{20} + \binom{20}{1} 0.2^1 (1 - 0.2)^{19} = 0.0692$. In R this could be done with `dbinom(0,20,0.2) + dbinom(1,20,0.2)` or `pbinom(1,20,0.2)`.

    (c) $P(B > 1)$. $P(B > 1) = 1 - P(B \leq 1) = 1 - 0.0692 = 0.9308$.