Homework 4

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1. Marcello the monkey is doing some calculations with his banana hoard. He has 200 bananas, which have a *total* weight of 25 kilograms. Assume the weight of a banana can be represented as a random variable with mean μ and standard deviation σ. Marcello remembers from his freshman course in bananology that σ is equal to 10 grams.

    a. Construct a 95% confidence interval for μ, the average weight of an individual banana. *25,000 / 200 = 125 grams is our sample mean. For a 95% CI, our value of $\alpha/2$ is 0.025, and so that of z from the normal table is 1.96. Thus our confidence interval half width is* $(1.96 * 10)/sqrt(200) = 1.39$. *Our interval is then [125 - 1.39, 125 + 1.39] = [123.61, 126.39].*

    b. Being a precision oriented monkey, Marcello would like to know μ to within a margin of 0.1 grams with 99% certainty. How many bananas would he need in order to accomplish this? *99% certainty gives an $\alpha/2$ of 0.005, and thus a z value of 2.575. We solve for n in the following equation:* $0.1 = (2.575 * 10)/sqrt(n)$ *to give a value of n of 66,306.*

    c. Marcello's banana pudding calls for three bananas and an amount of custard whose weight is exactly equal to that of the bananas. What is the standard deviation of the weight of a recipe of banana pudding? *The weight of three bananas is $X_1 + X_2 + X_3$, where each X_i reperesents 1 banana. Adding the custard doubles the weight to $2X_1 + 2X_2 + 2X_3$. The variance of all this is $2^2 * 3 * Var(X_i) = 1200$, and the standard deviation is the square root of that, which is 34.64.*

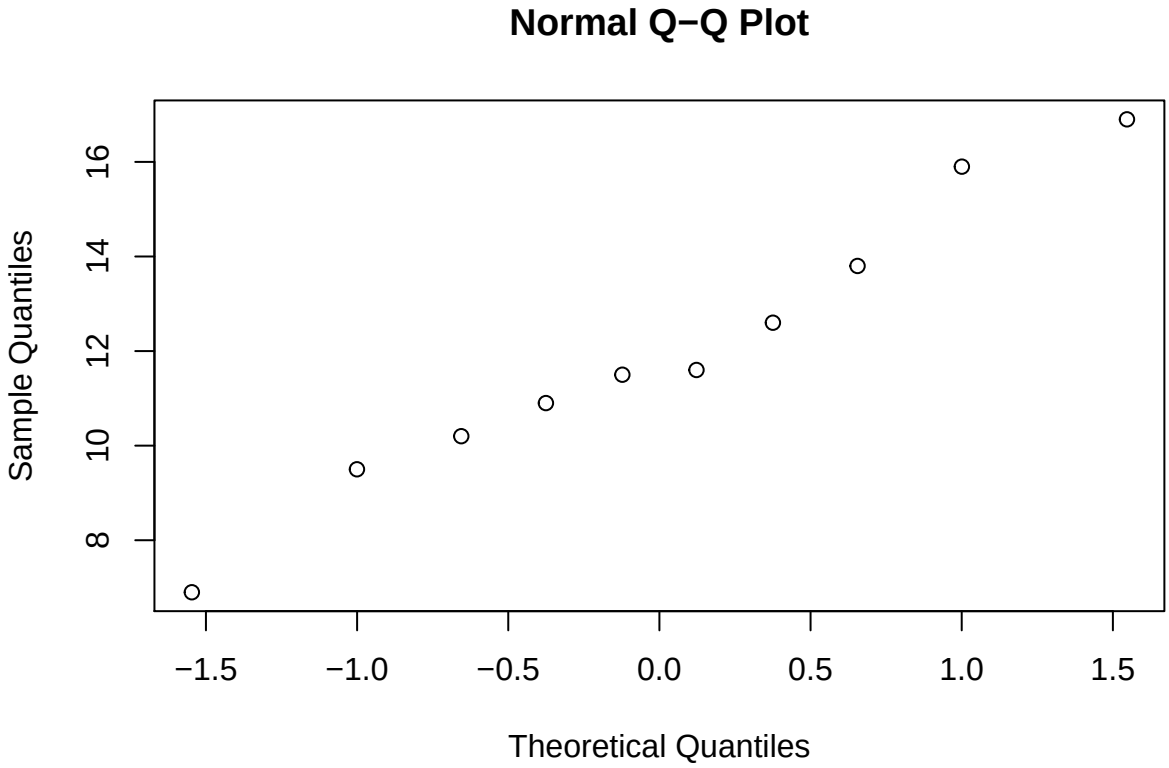
2. A researcher is studying the size of leafcutter ant colonies in a certain region. She measures the radius, in meters, of ten different mounds. Here are her measurements:
10.9, 15.9, 11.6, 9.5, 13.8, 11.5, 12.6, 16.9, 6.9, 10.2

    a. Calculate the sample mean for these values. *Using R: 11.98.*

    b. Calculate the sample standard deviation for these values. *Using R: 2.98*

    c. Construct a 95% confidence interval for leafcutter mound radius. $t_{9,0.025}$ *= 2.262. Our CI half width is thus* $(2.98 * 2.262)/sqrt(10) = 2.13$. *This gives a CI of [11.98 - 2.13, 11.98 + 2.13] = [9.85, 14.11]*

    d. The following is a qqplot of the data. Do you think the assumptions we used to build our confidence interval are appropriate? *Yes, there is a fairly straight line here so it seems likely the data is approximately normal.*



3. Suppose that, in a sample of 600 individuals, 48 have allergic reactions to pollen. Construct a 90% confidence interval for the proportion of individuals from the population who have an allergic reaction to pollen. *$P = 48/600 = 0.08$. A 90% confidence interval gives a value of $\alpha/2$ of 0.05, and $z_{0.05} = 1.645$. Our half width is thus $1.645 * sqrt((0.08)(0.92)/600) = 0.018$, so our CI is [0.08 - 0.018, 0.08 + 0.018] = [0.062, 0.098].*