Chapter 7: One sample tests

(Ott & Longnecker Sections: 10.2)

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Part 3

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What do we study

Key concepts: test for population proportion

Review of sign test

- **Assumption.** Let $X_1, \ldots, X_n$ be an i.i.d. sample from some population with population median m.
- **Hypothesis.** We wish to test $H_0 : m = m_0$ vs. H_A. Possible alternative hypotheses: $H_A : m > m_0$(one-tailed), $H_A : m < m_0$(one-tailed), or $H_A : m \neq m_0$(two-tailed).
- **Test statistic.** $B =$ number of data values greater than m_0. (Ignore values tied with m_0.) Note that if H_0 is true, $B \sim \text{Binomial}(n^*, 0.5)$, where n^* is the number of data points not equal to m_0.
- **p-value**. Let b be the observation of B. If:
  - $H_A : m > m_0$: p-value is $P(B \geq b) = P(B = b) + P(B = b+1) + \ldots + P(B = n^*)$.
  - $H_A : m < m_0$: p-value is $P(B \leq b) = P(B = b) + P(B = b-1) + \ldots + P(B = 1) + P(B = 0)$.
  - $H_A : m \neq m_0$: p-value is $2\min\{P(B \geq b), P(B \leq b)\}$.

Example

An accounting firm has a large list of clients (the population), and each client has a file with information about that client. The firm has noticed errors in some of these files, and has decided that it would be worthwhile to know the proportion of files that contain an error. Call the population proportion of files in error π. It was decided to take a simple random sample of size $n = 50$, and use the results of the sample to estimate π. Each selected file was thoroughly reviewed, and classified as either containing an error (call this 1), or not (call this 0). The results are as follows:

Files with an error: 10; Files without any errors: 40.

Example continued

The company CEO decides that if π is greater than 0.1, then it will be worthwhile to review and fix every file. Therefore we wish to test:

$$H_0 : \pi = 0.1$$
$$H_A : \pi > 0.1.$$

Example continued

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- When $n\pi$ and $n(1-\pi)$ are greater than 5, by CLT, sample proportion P is approximately distributed as a normal: $P \dot{\sim} N(\pi, \frac{\pi(1-\pi)}{n})$.

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- Under H_0, $0.1(50) = 5$ and $0.9(50) = 45 > 5$, so we should be able to use the CLT. Thus, if H_0 were true:

$$P \dot{\sim} N(0.1, \frac{0.1(1-0.1)}{50}),$$

which means that:

$$Z = \frac{P - 0.1}{\sqrt{\frac{0.1(1-0.1)}{50}}} \dot{\sim} N(0, 1).$$

So we use Z test statistic.

Example continued

- Our observed statistic is $z_{obs} = \frac{0.2-0.1}{\sqrt{\frac{0.1(1-0.1)}{50}}} = 2.357$, so the p-value is $P(Z > 2.357) = 0.009$.
- At the 5% level($\alpha = 0.05$), we would reject the null, and conclude that too high of a proportion of files are in error. All files should be checked and fixed.

Test for population proportion

- **Assumption.** $X_1, \dots, X_n$ are i.i.d. Ber(π), and n is large.
- **Hypotheses.** $H_0 : \pi = \pi_0$.
- **Test statistic.** Let $P = \frac{\sum_{i=1}^n X_i}{n}$ be the proportion of successes. Check that $n\pi_0 > 5$ and $n(1-\pi_0) > 5$ so that the CLT holds under H_0. Then the test statistic is a Z-statistic:

$$Z = \frac{P - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}.$$

- **P-value.** Let P_{obs} be the observed proportion of successes in the data. If:
  - $H_A : \pi > \pi_0$: calculate $z_{obs} = \frac{P_{obs} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$ and calculate $P(Z > z_{obs})$.
  - $H_A : \pi < \pi_0$: calculate $P(Z < z_{obs})$.
  - $H_A : \pi \neq \pi_0$: calculate $2 * P(Z > |z_{obs}|)$.