# CPSC 340: Machine Learning and Data Mining

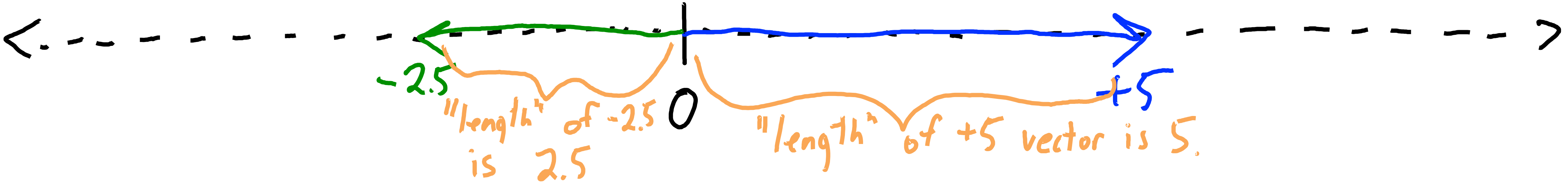
Density-Based Clustering

# Admin

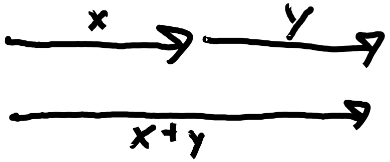
- Assignment 1
  - Due last night
  - Solutions coming Thursday
- Assignment 2: coming soon.
  - If you want to work with a partner, open the issue ASAP (ideally today!)
- Private posts
  - Disabled to avoid repeated questions
  - But you need a way of asking about code
  - For hw2 onwards, open an issue in your hw repo
  - Make sure to tag @cpsc340/staff
  - We can't guarantee we'll get to all of them, depending on volume
- End of class today: informal course evaluation

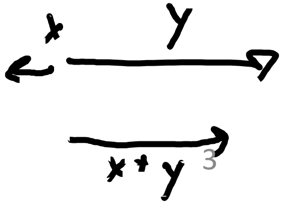
# Norms in 1-Dimension

- We can view absolute value, $|x|$, as 'size' or 'length' of a number:




- It satisfies three intuitive properties of 'length':
  1. Only '0' has a 'length' of zero.
  2. Multiplying 'x' by constant 'α' multiplies length by $|\alpha|$:
     - "Absolute homogeneity": $|\alpha x| = |\alpha||x|$.
     - "If will twice as long if you multiply by 2".
  3. Length of 'x+y' is not more than length of 'x' plus length of 'y':
     - "Triangle" inequality: $|x + y| <= |x| + |y|$.
     - Think of "how far you travel".



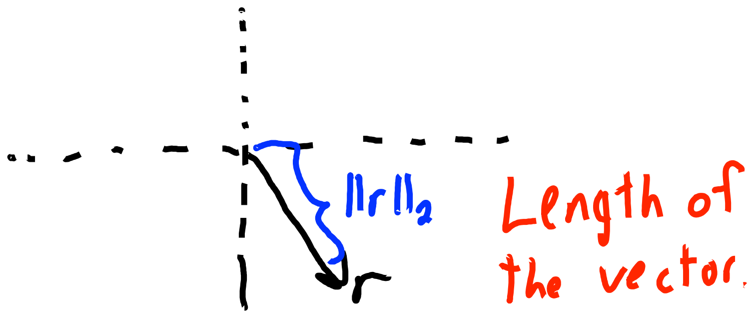



# Norms in 2-Dimensions

- In 1-dimension, only scaled absolute values satisfy the 3 properties.
- In 2-dimensions, there is no unique function satisfying them.
- We call any function satisfying them a norm:
  - Measures of “size” or “length” in 2-dimensions.
- Three most common examples:

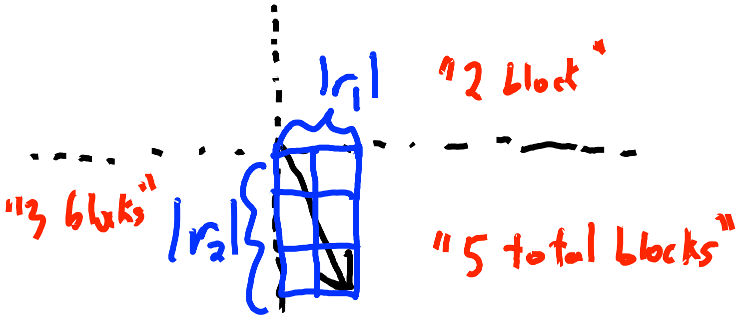
$L_2$ or "Euclidean" norm.

$$\|r\|_2 = \sqrt{r_1^2 + r_2^2}$$



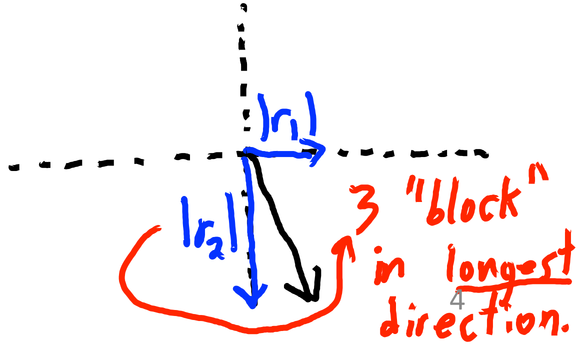

$L_1$ or "Manhattan" norm:

$$\|r\|_1 = |r_1| + |r_2|$$




$L_\infty$ or "max" norm:

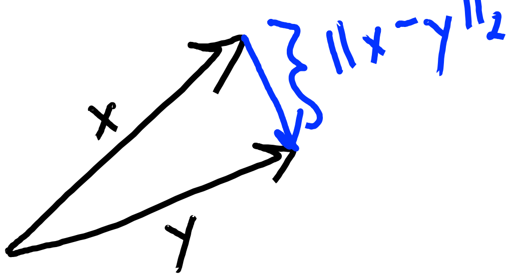
$$\|r\|_\infty = \max\{|r_1|, |r_2|\}$$



# Norms as Measures of Distance

- By taking norm of difference, we get a "distance" between vectors:

$$\|x-y\|_2 = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$$




$$\|x-y\|_1 = |x_1-y_1| + |x_2-y_2|$$

"Number of blocks you need to walk to get from x to y."

$$\|x-y\|_\infty = \max\{|x_1-y_1|, |x_2-y_2|\}$$

"Most number of blocks in any direction you would have to walk."

# Norms in d-Dimensions

- We can generalize these common norms to d-dimensional vectors:

$L_2$: $\|r\|_2 = \sqrt{\sum_{j=1}^{d} r_j^2}$ $L_1$: $\|r\|_1 = \sum_{j=1}^{d} |r_j|$ $L_\infty$: $\max_j \{|r_j|\}$

E.g., in 3-dimensions:

$\|r\|_2 = \sqrt{r_1^2 + r_2^2 + r_3^2}$

in 4-dimensions:

$\|r\|_2 = \sqrt{r_1^2 + r_2^2 + r_3^2 + r_4^2}$

Notation: $\|r\|_2^2 = (\|r\|_2)^2$

$= \left(\sqrt{\sum_{j=1}^{d} r_j^2}\right)^2$

$= \sum_{j=1}^{d} r_j^2$

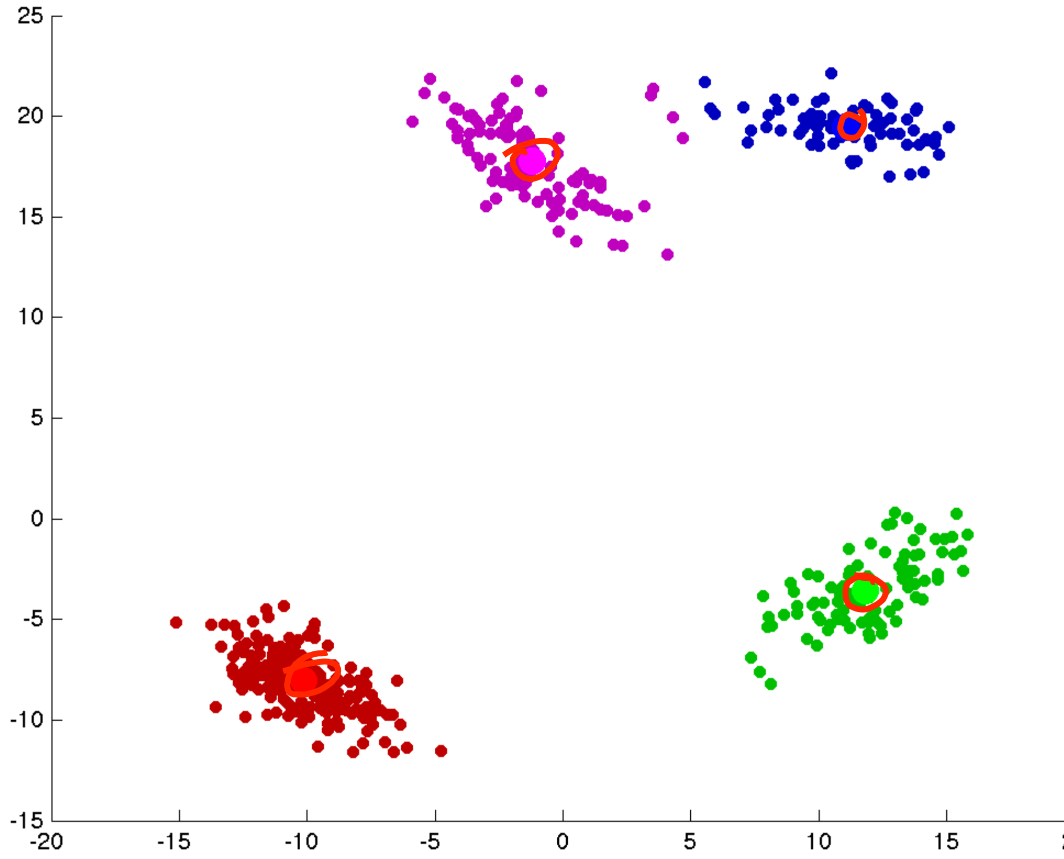
$= r^T r$

Different ways to write the same thing

- These norms place different "weights" on large values:
  - $L_1$: all values are equal.
  - $L_2$: bigger values are more important (because of squaring).
  - $L_\infty$: only biggest value is important.

# Last Time: K-Means Clustering

- We want to cluster data:
  - Assign objects to groups.
- K-means clustering:
  - Define groups by “means”
  - Assign objects to nearest mean.
    (Then update means during training.)
- Also used for vector quantization:
  - Use means as prototypes of groups.



# K-Means Initialization

- K-means is fast but sensitive to initialization.

- Classic approach to initialization: random restarts.
  - Run to convergence using different random initializations.
  - Choose the one that minimizes average squared distance of data to means.

- Newer approach: k-means++
  - Random initialization that prefers means that are far apart.
  - Yields provable bounds on expected approximation ratio.

# K-Means++

- Steps of k-means++:
  1. Select initial mean $w_1$ as a random $x_i$.
  2. Compute distance $d_{ic}$ of each object $x_i$ to each mean $w_c$.

     $$d_{ic} = \sqrt{\sum_{j=1}^{d} (x_{ij} - w_{cj})^2} = \|x_i - w_c\|_2$$

  3. For each object 'i' set $d_i$ to the distance to the closest mean.

     $$d_i = \min_c \{d_{ic}\}$$

  4. Choose next mean by sampling an example 'i' proportional to $(d_i)^2$.

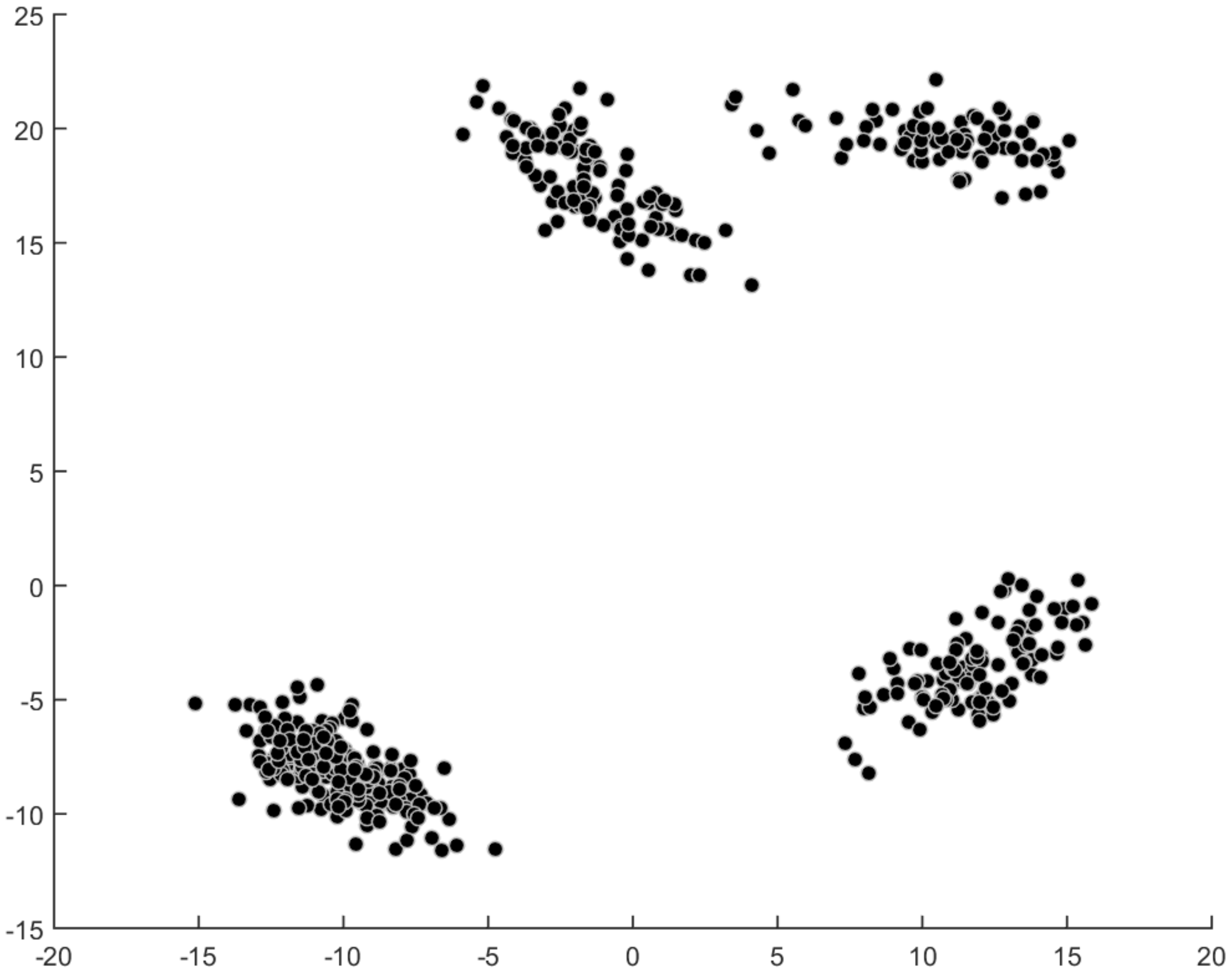
     $$p_i \propto d_i^2 \Rightarrow p_i = \frac{d_i^2}{\sum_{j=1}^{n} d_j^2}$$

     Can be done in $O(n)$.

     "probability that we choose $x_i$ as next mean"

  5. Keep returning to step 2 until we have k-means.
- Expected approximation ratio is O(log(k)).

# K-Means++



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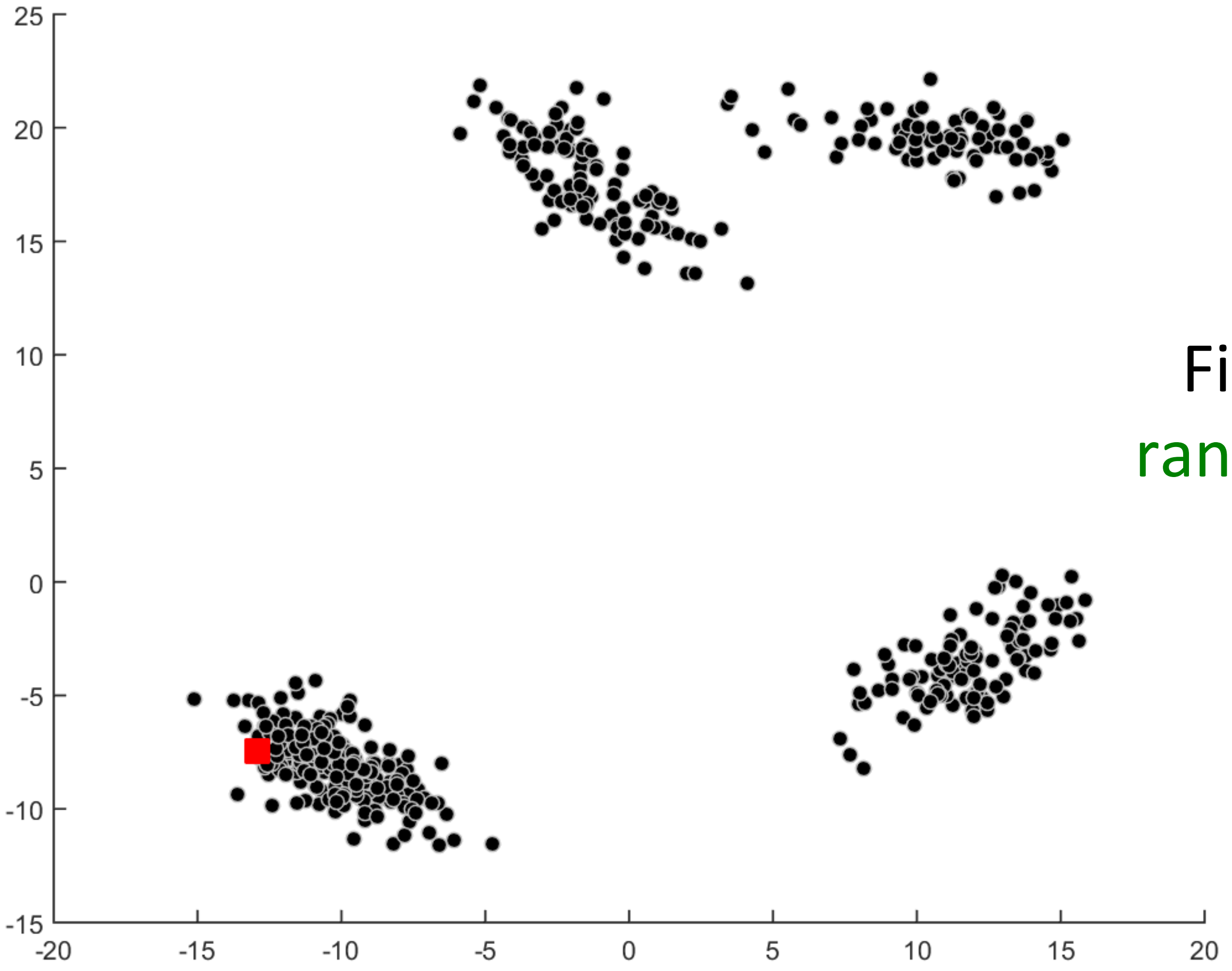
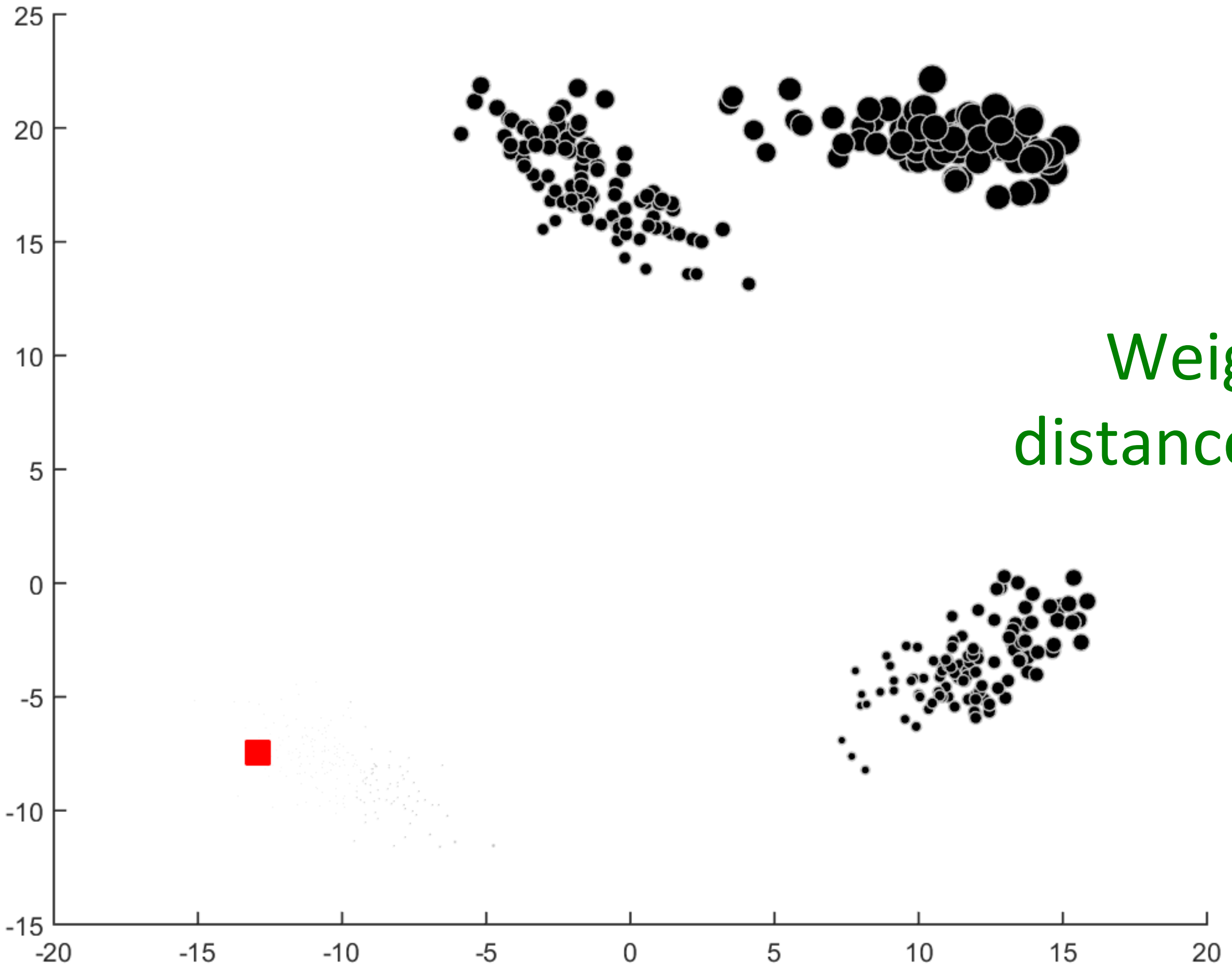


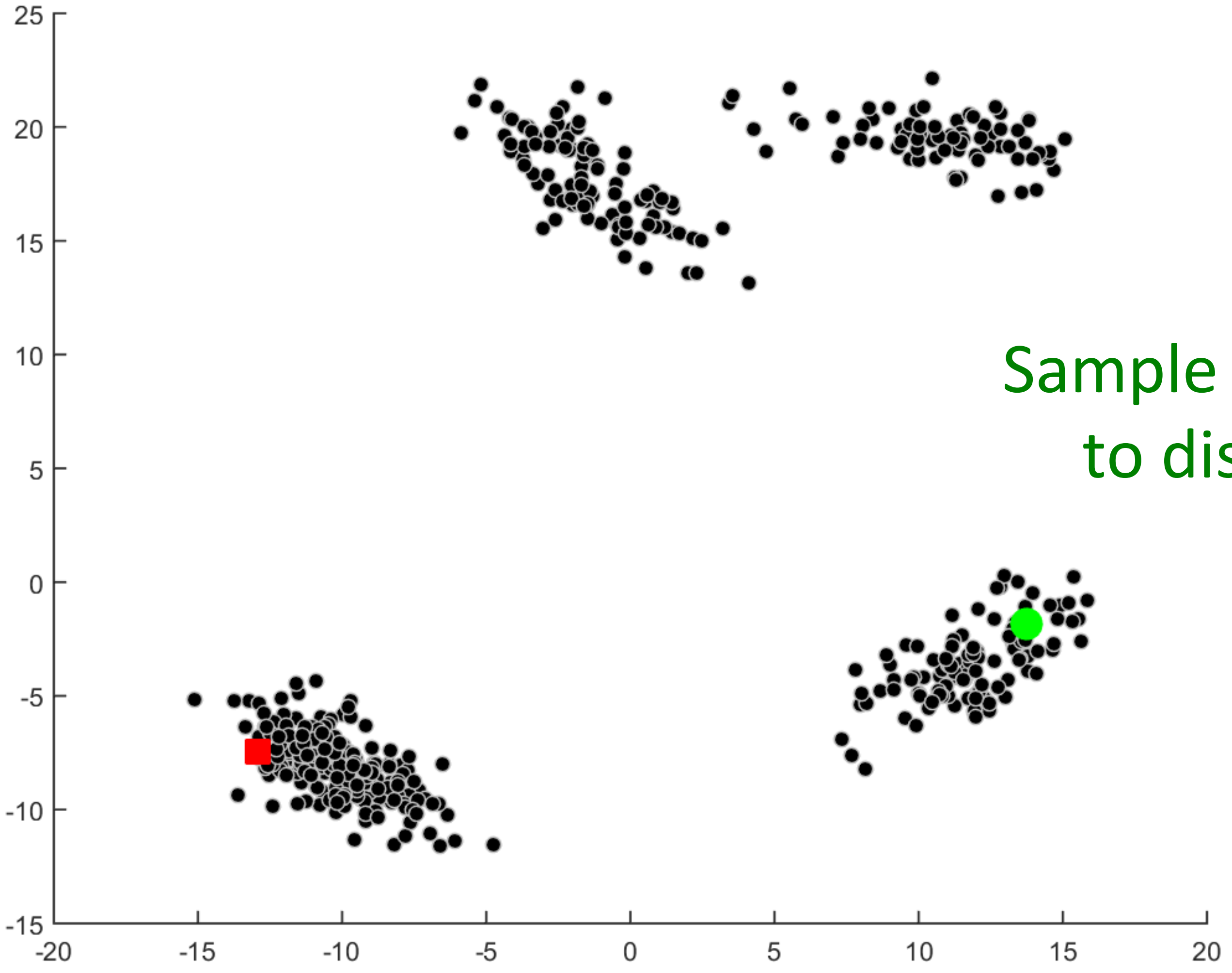

First mean is a random example.

# K-Means++



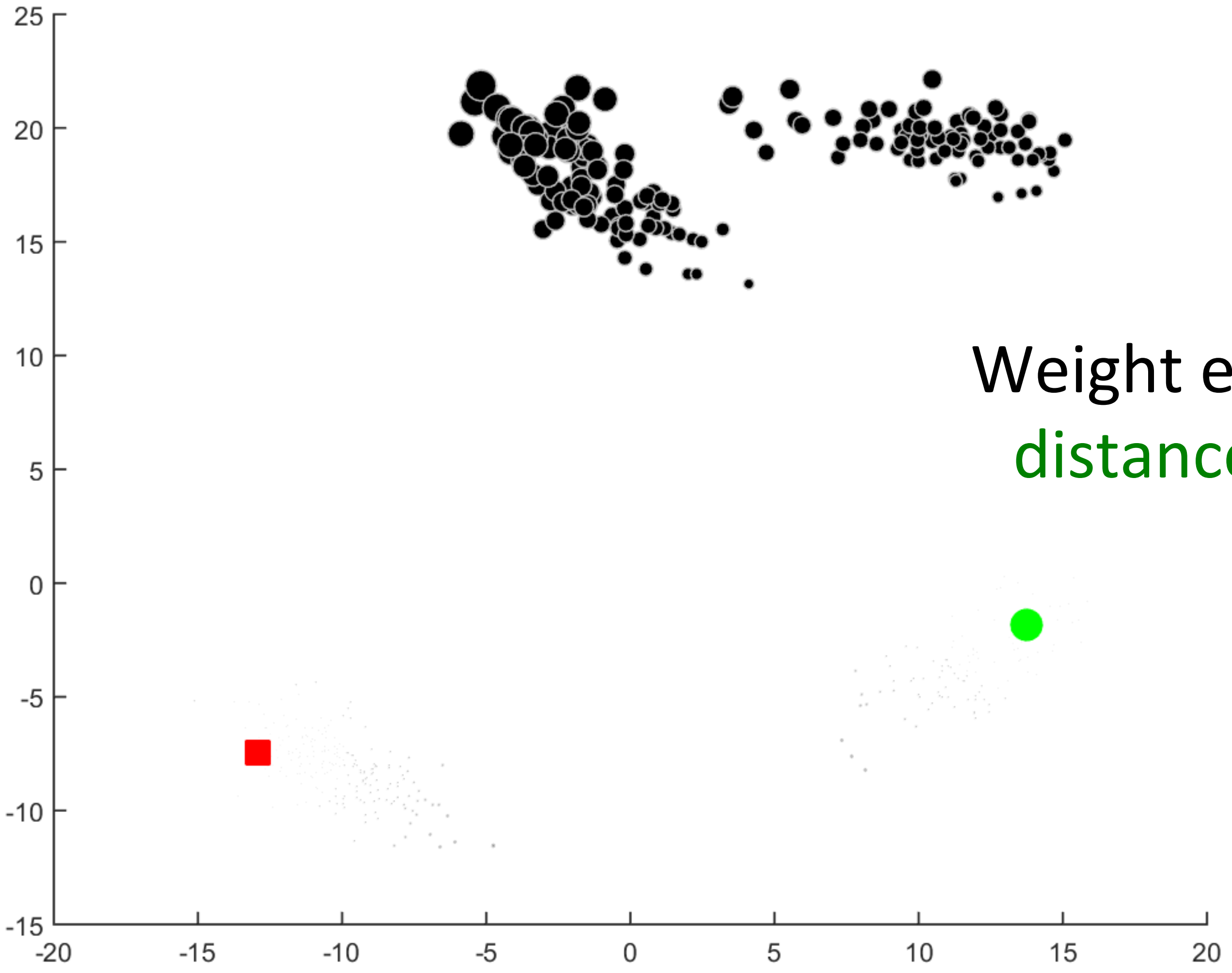

Weight examples by distance to mean squared.

# K-Means++



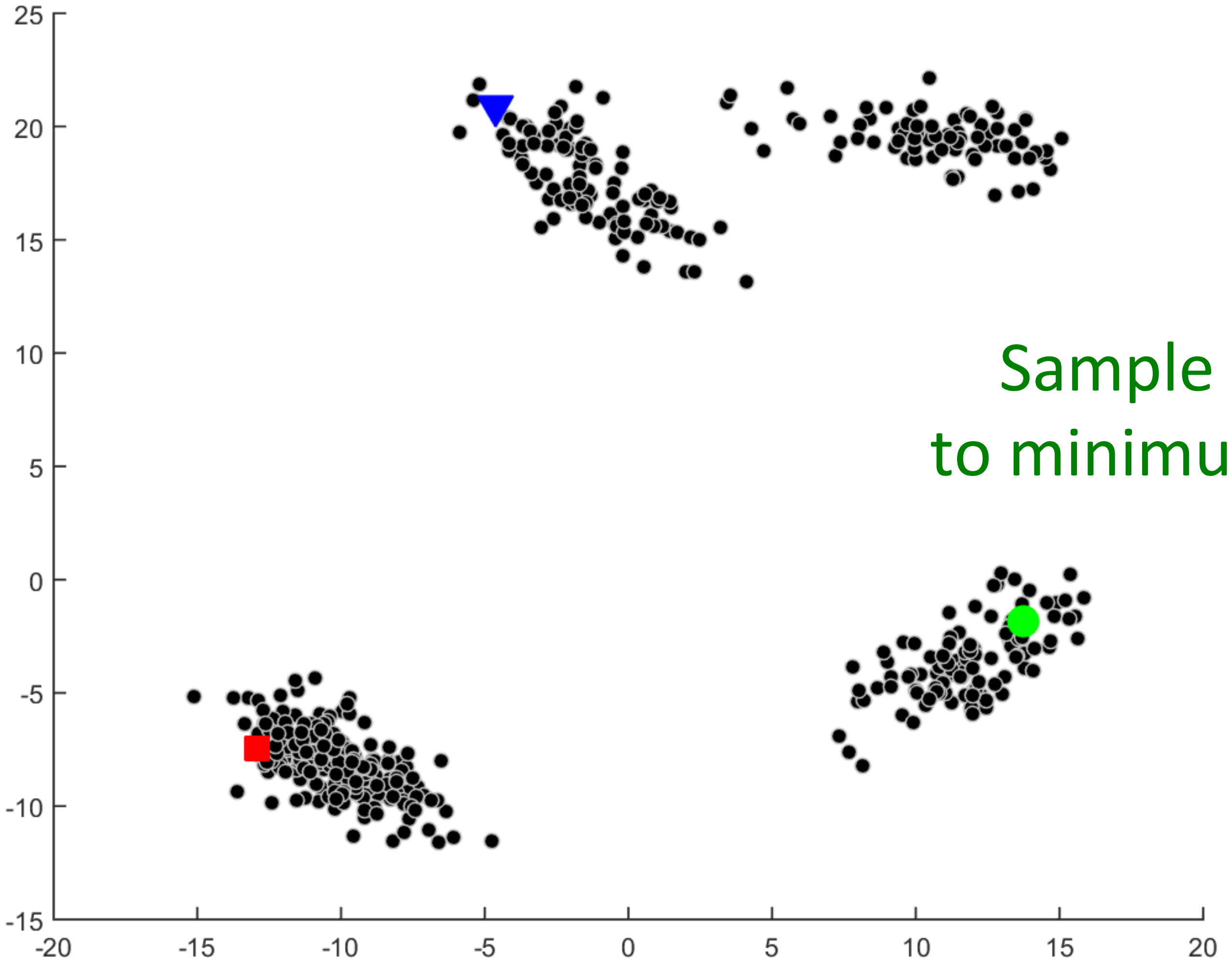

Sample mean proportional to distances squared.

# K-Means++



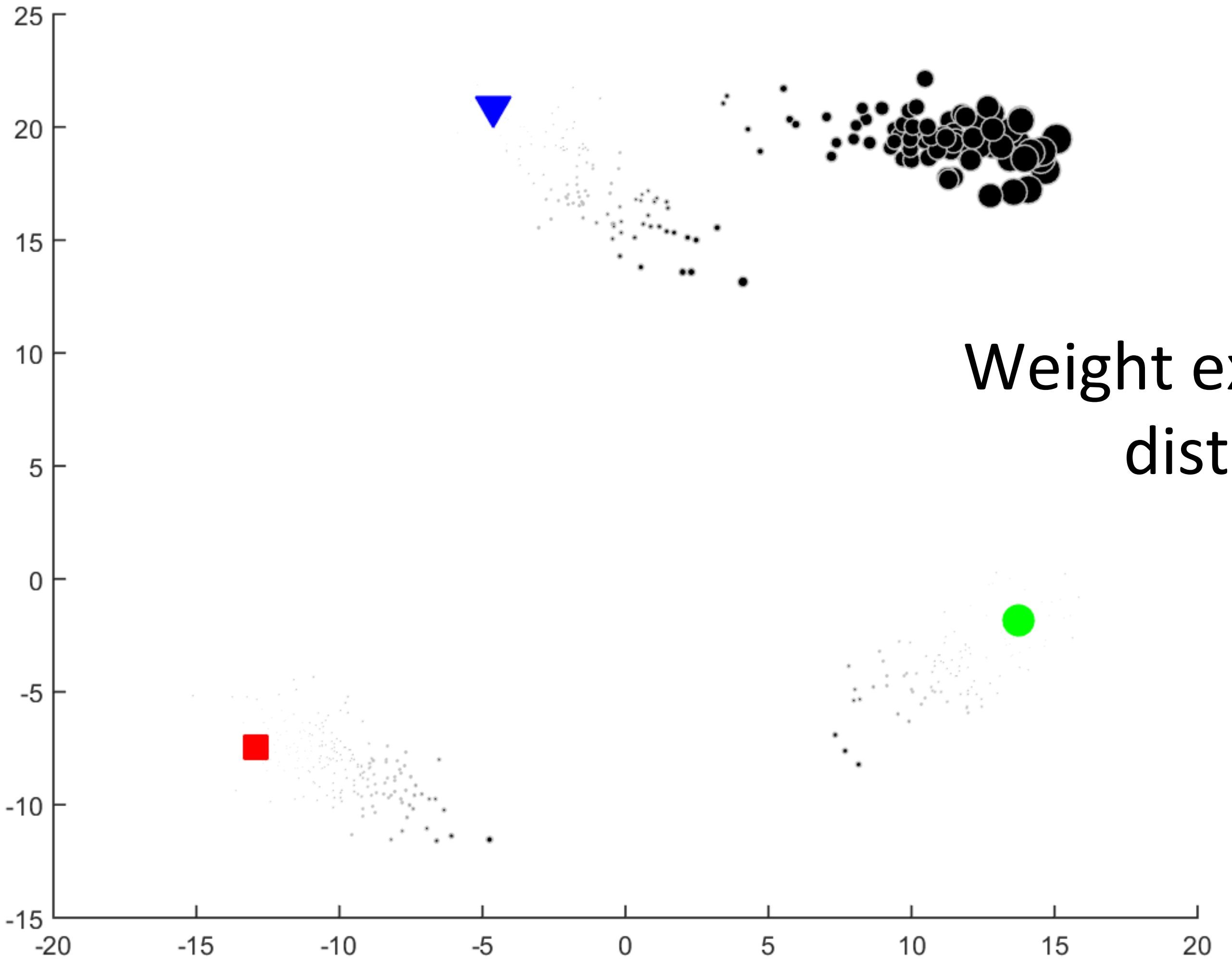

Weight examples by squared distance to nearest mean.

# K-Means++



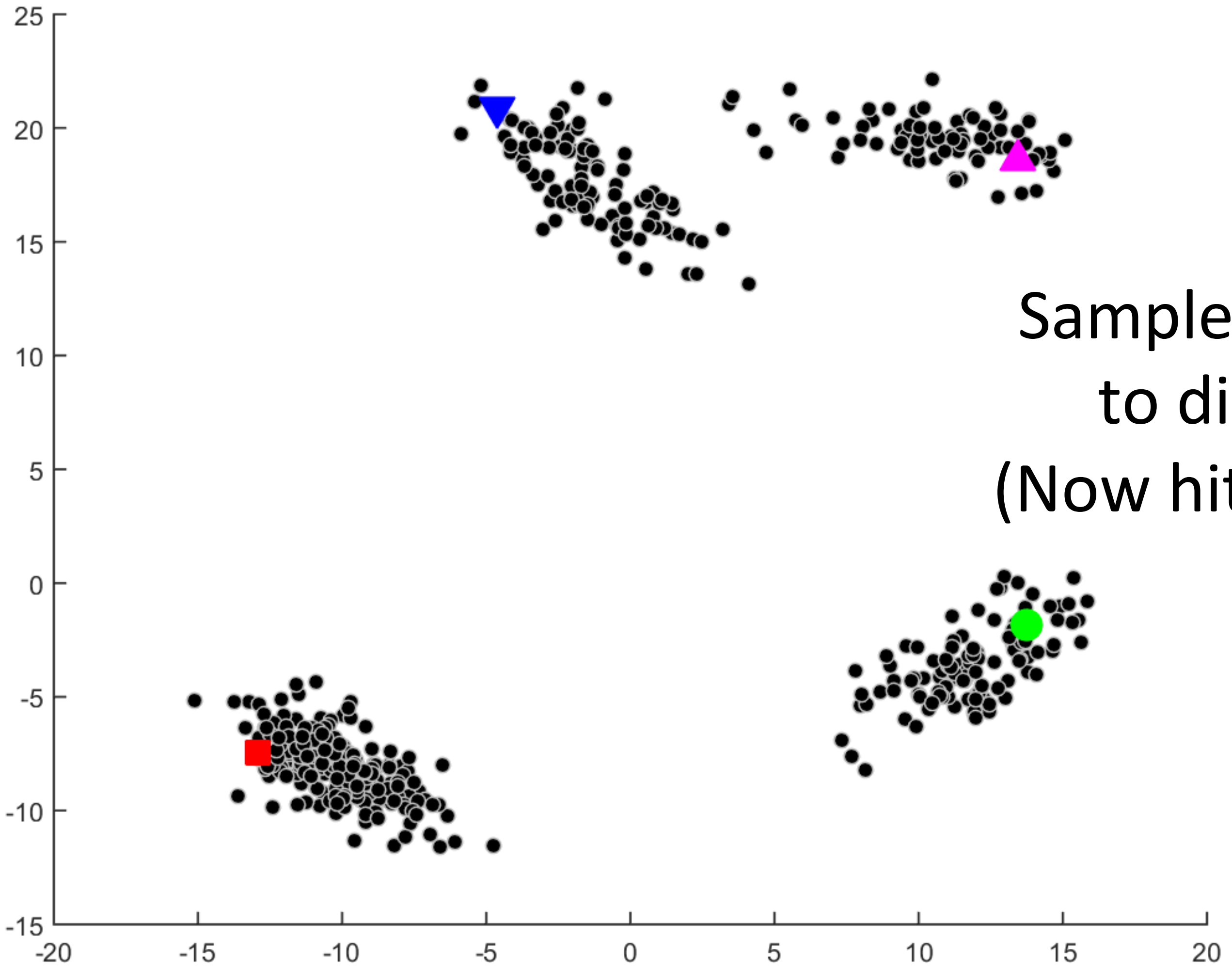

Sample mean proportional
to minimum distances squared

# K-Means++



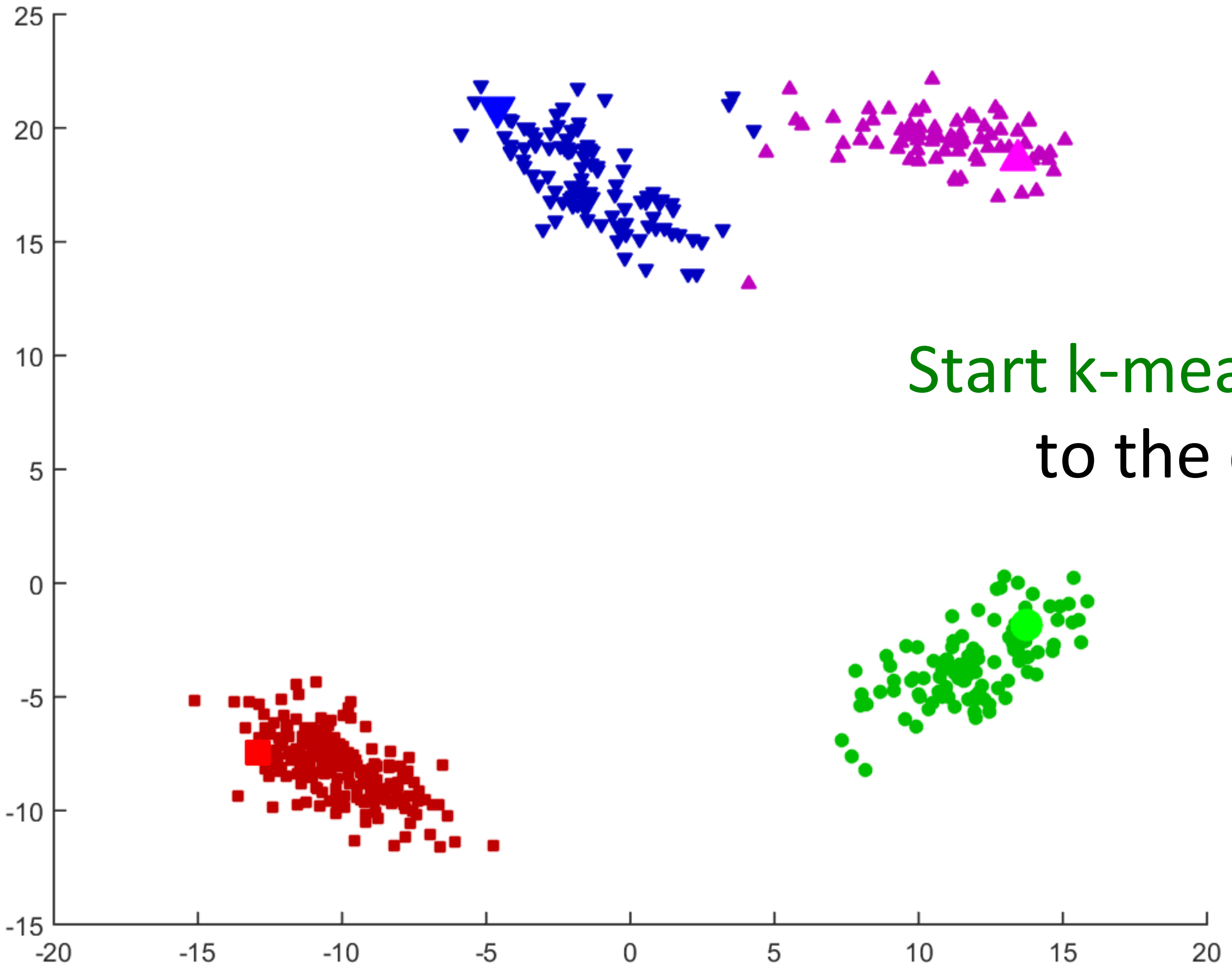

Weight examples by squared distance to mean.

# K-Means++



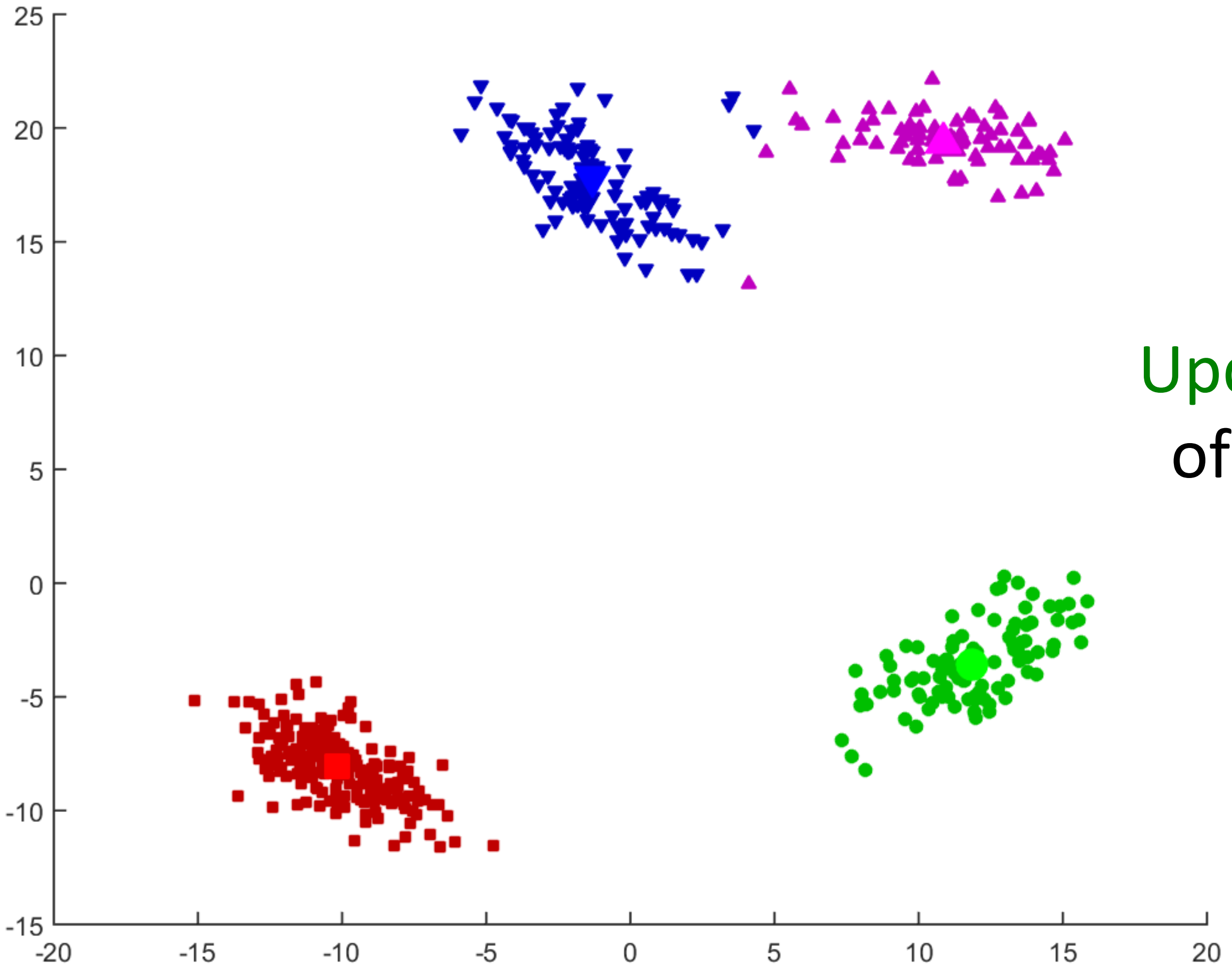

Sample mean proportional
to distances squared.
(Now hit chosen target k=4.)

# K-Means++



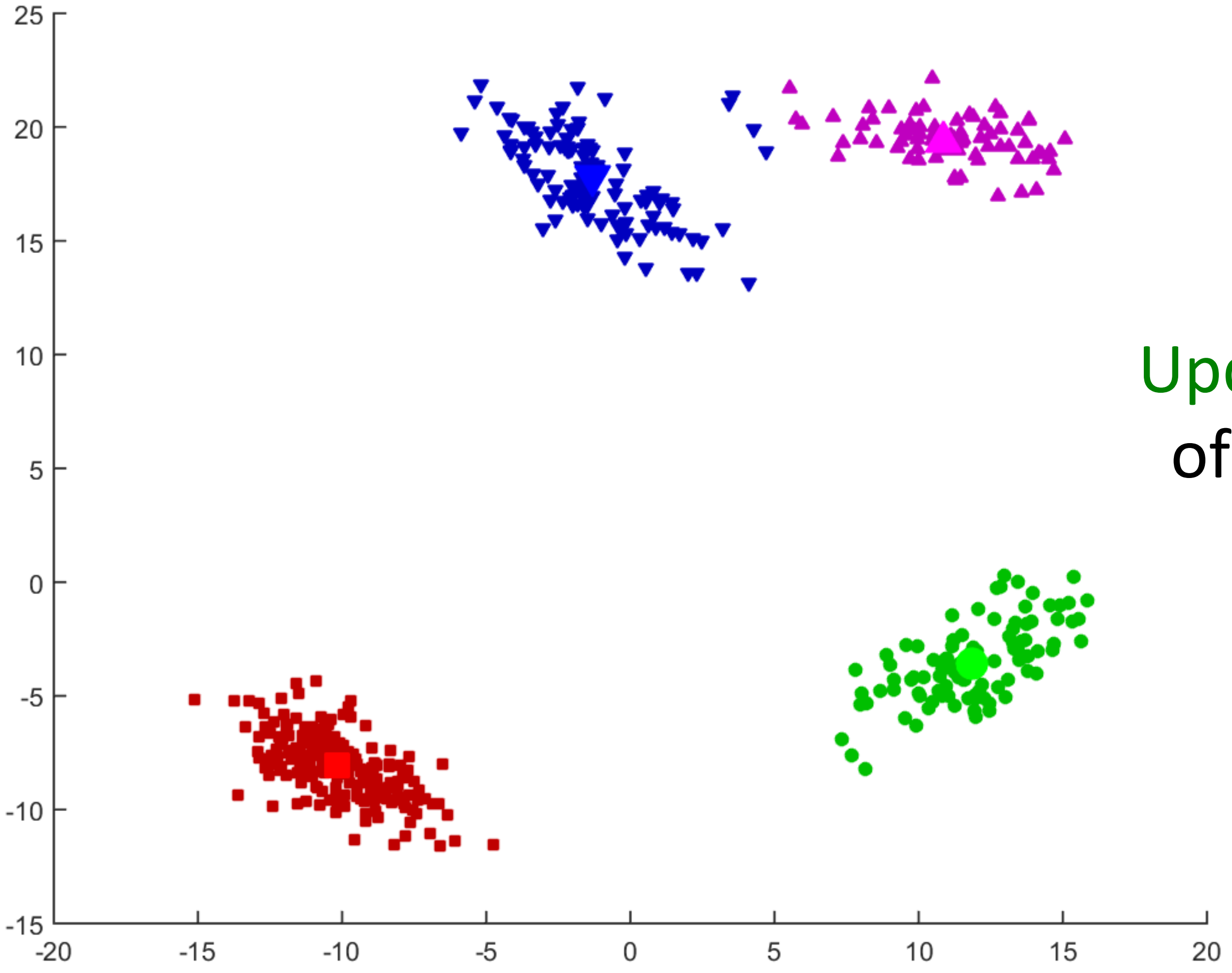

Start k-means: assign objects to the closest mean.

# K-Means++




Update the mean of each cluster.

# K-Means++



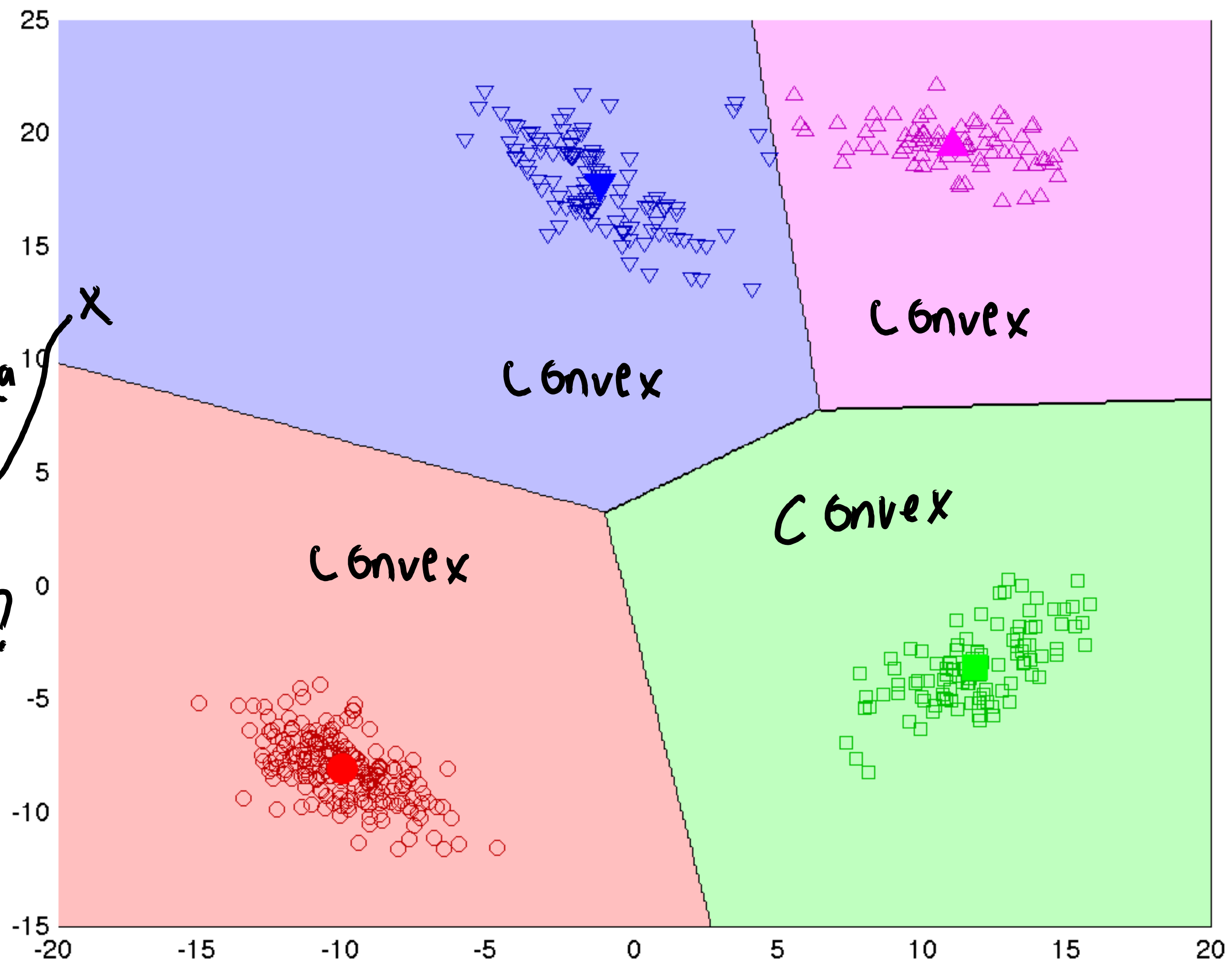

Update the mean
of each cluster.

In this case: just 2 iterations!

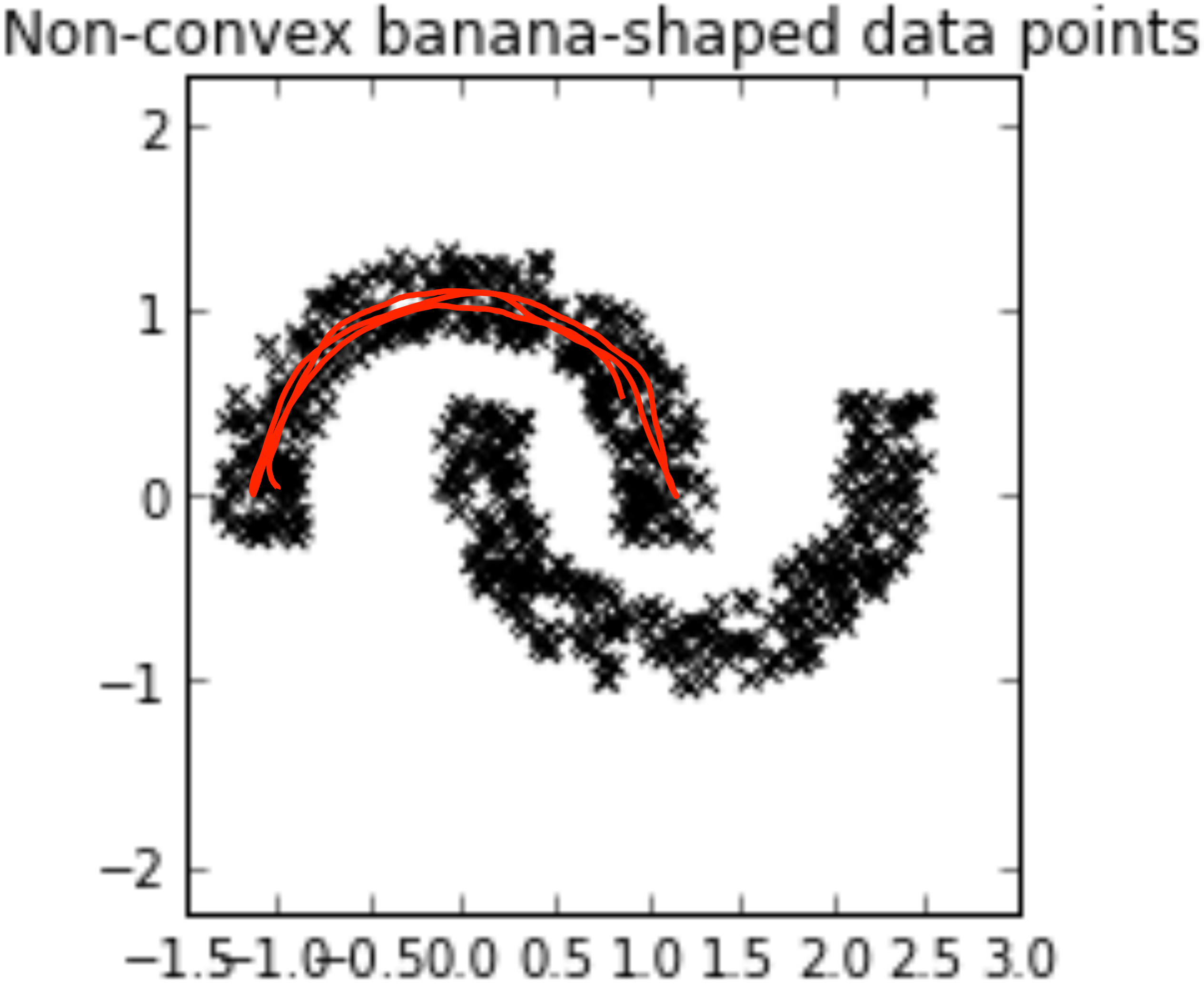
# Shape of K-Means Clusters

New issues:

1. Are clusters in data always convex?
2. Is this really part of the blue cluster?



# K-Means with Non-Convex Clusters






https://corelifesciences.com/human-long-non-coding-rna-expression-microarray-service.html

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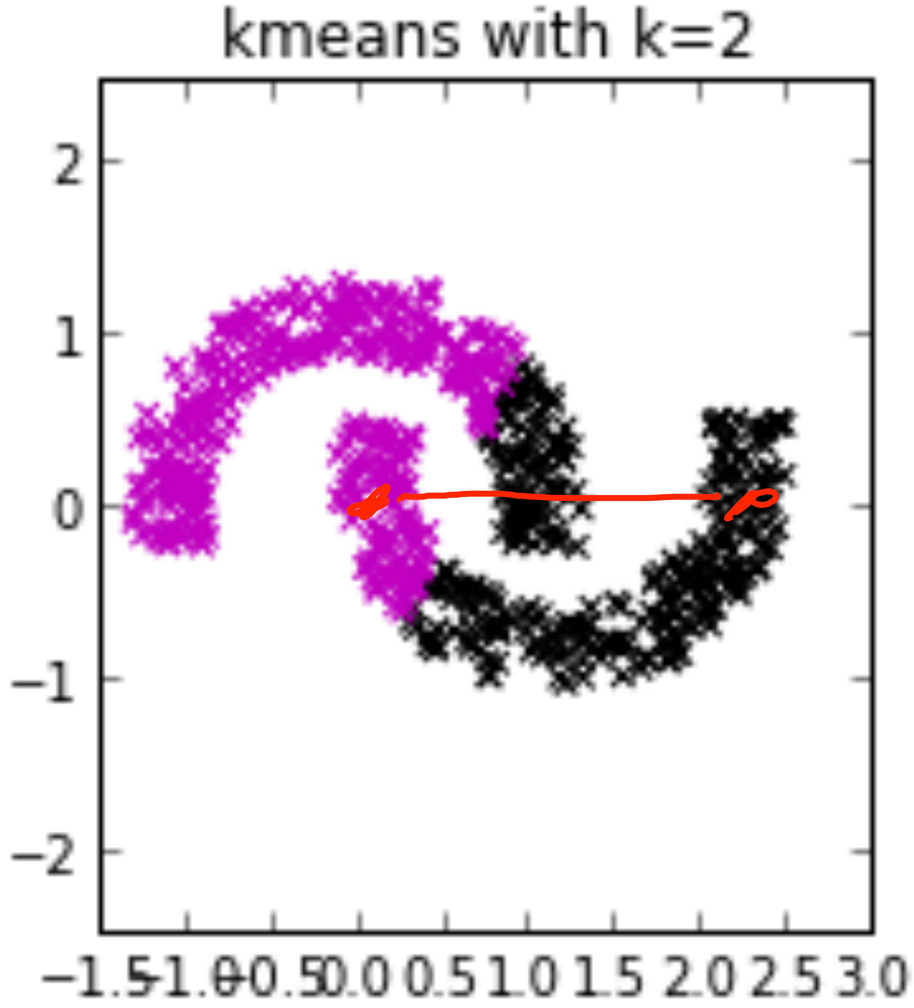


K-means cannot separate non-convex clusters



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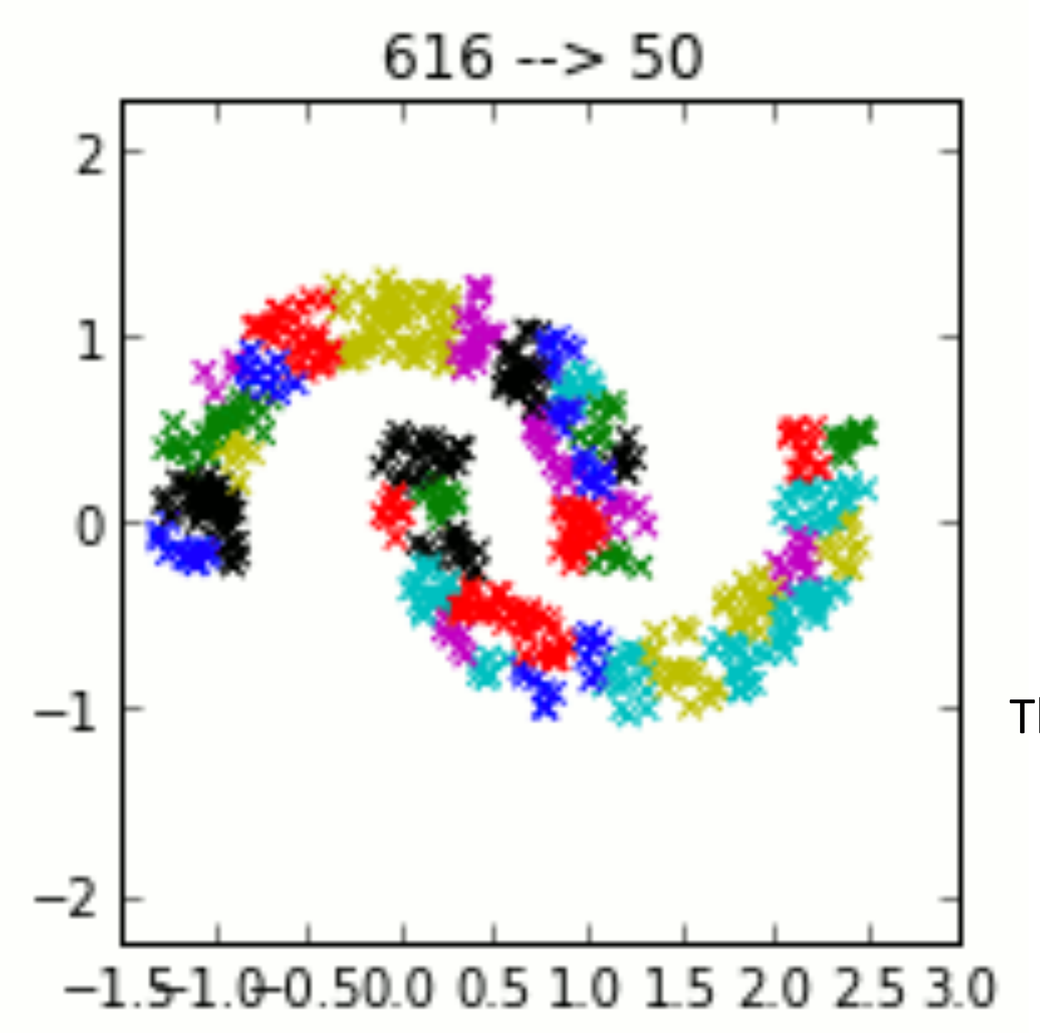
# K-Means with Non-Convex Clusters




K-means cannot separate non-convex clusters

Though over-clustering can help (next class)

https://corelifesciences.com/human-long-non-coding-rna-expression-microarray-service.html

# Application: Elephant Range Map

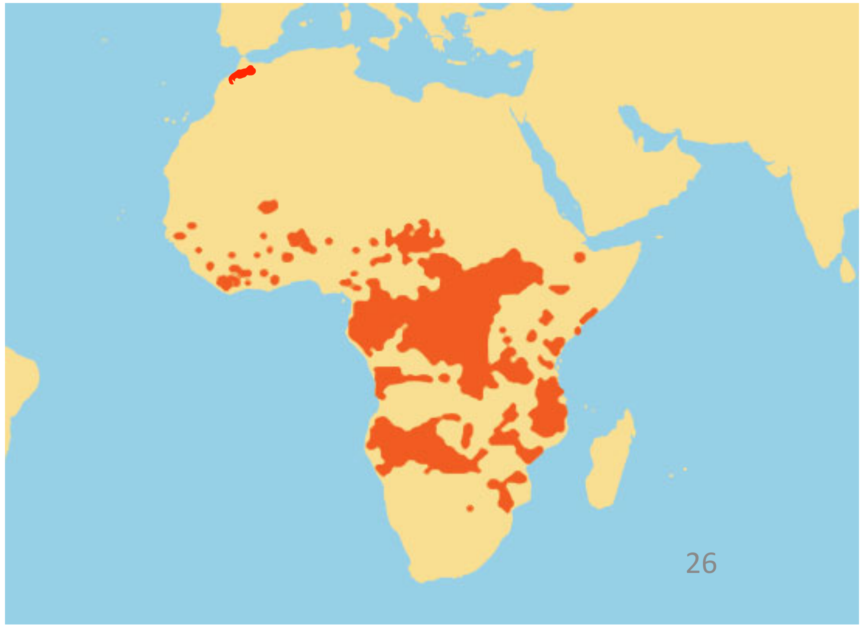
- Find habitat area of African elephants.
  - Useful for assessing/protecting population.
- Build clusters from observations of locations.
- Clusters are non-convex:
  - affected by vegetation, mountains, rivers, water access, etc.
- We don't want to "partition" data:
  - Some points have no cluster.



http://www.defenders.org/elephant/basic-facts

# Motivation for Density-Based Clustering

- Density-based clustering:
  - Clusters are defined by all the objects in “dense” regions.
  - Objects in non-dense regions don’t get clustered.
- It’s a non-parametric clustering method:
  - Clusters can become more complicated the more data we have.
  - No fixed number of clusters ‘k’.



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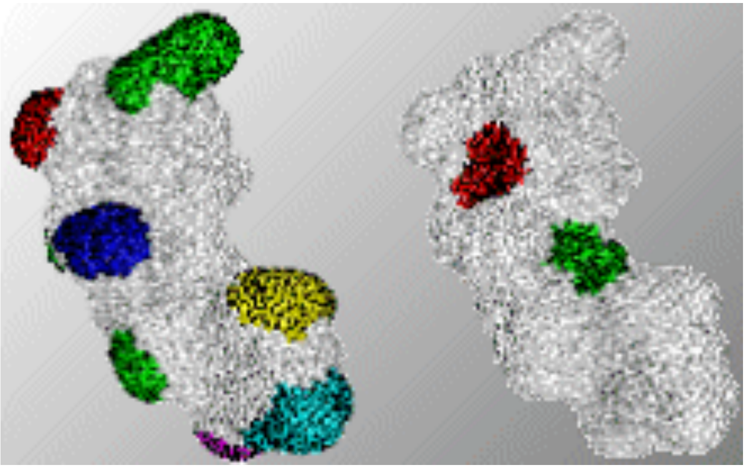
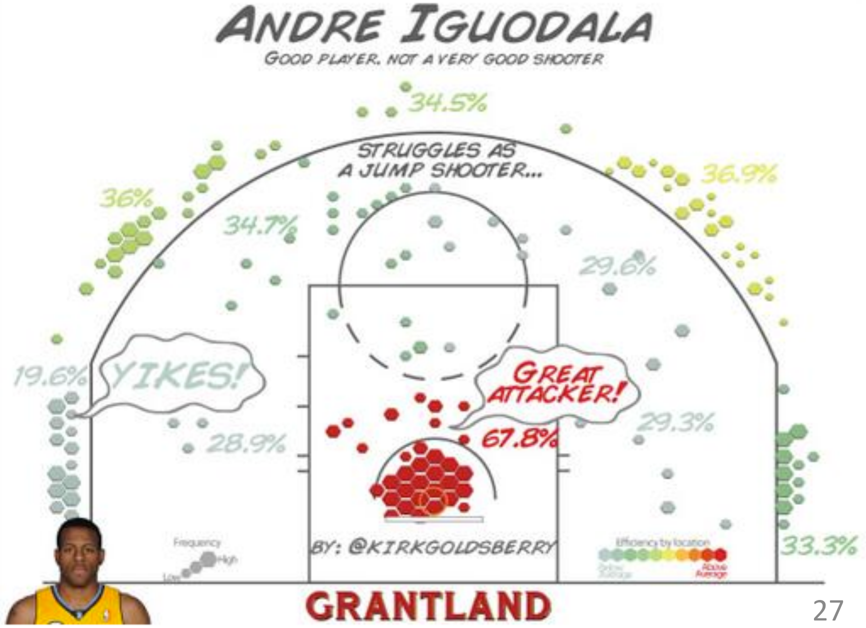
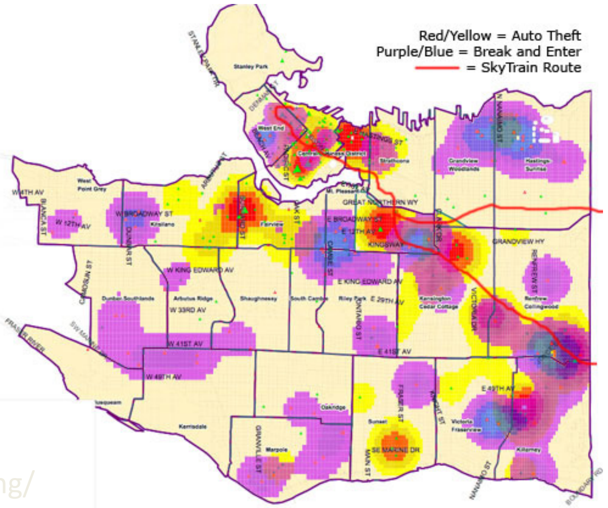
# Other Potential Applications

- Where are high crime regions of a city?
- Where should taxis patrol?
- Which products are similar to this one?
- Which pictures are in the same place?
- Where can protein ‘dock’?



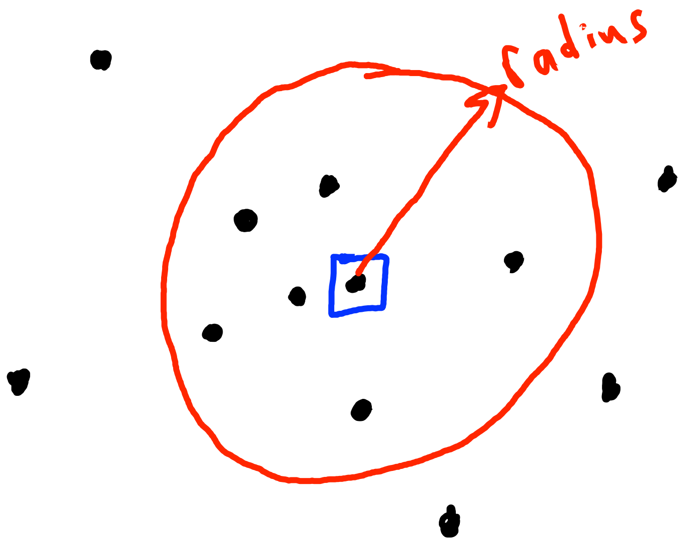



https://en.wikipedia.org/wiki/Cluster_analysis
https://www.flickr.com/photos/dbarefoot/420194128/
http://letsgowarriors.com/replacing-jarrett-jack/2013/10/04/
http://www.dbs.informatik.uni-muenchen.de/Forschung/KDD/Clustering/

# Density-Based Clustering

- Density-based clustering algorithm (DBSCAN) has two parameters:
  - Radius: minimum distance between points to be considered ‘close’.
    - Objects within this radius are called “reachable”.



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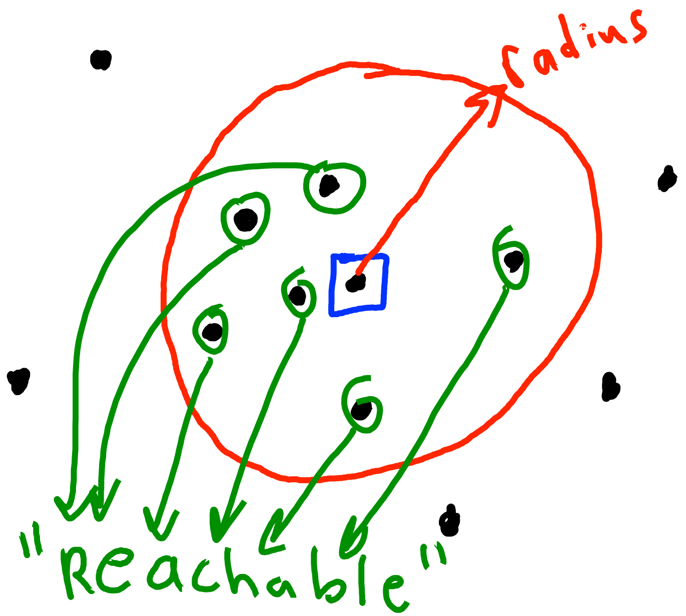
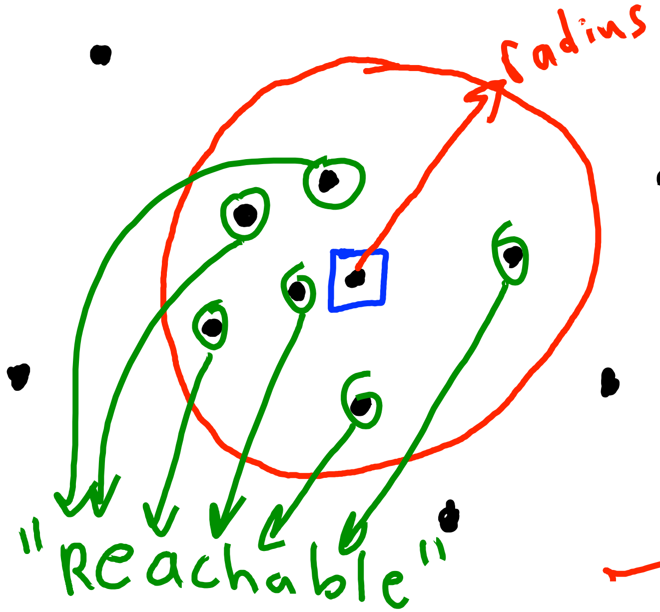


# Density-Based Clustering

- Density-based clustering algorithm (DBSCAN) has two parameters:
  - Radius: maximum distance between points to be considered ‘close’.
    - Objects within this radius are called “reachable”.
  - MinPoints: number of reachable points needed to define a cluster.
    - If you have minPoints “reachable points”, you are called a “core” point.




E.g., if minPoints = 3 then this is a "core" point since 6 points are "reachable"

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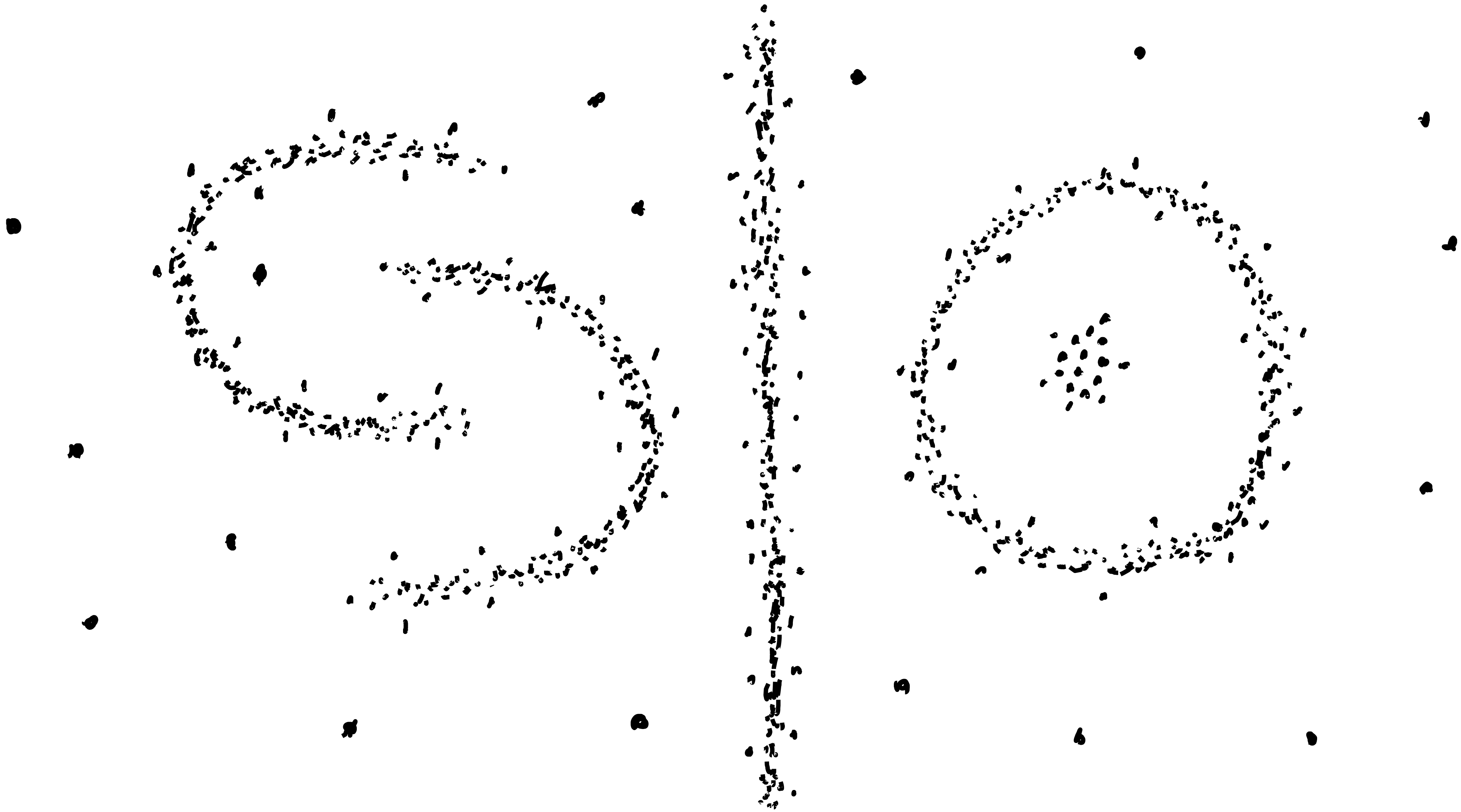

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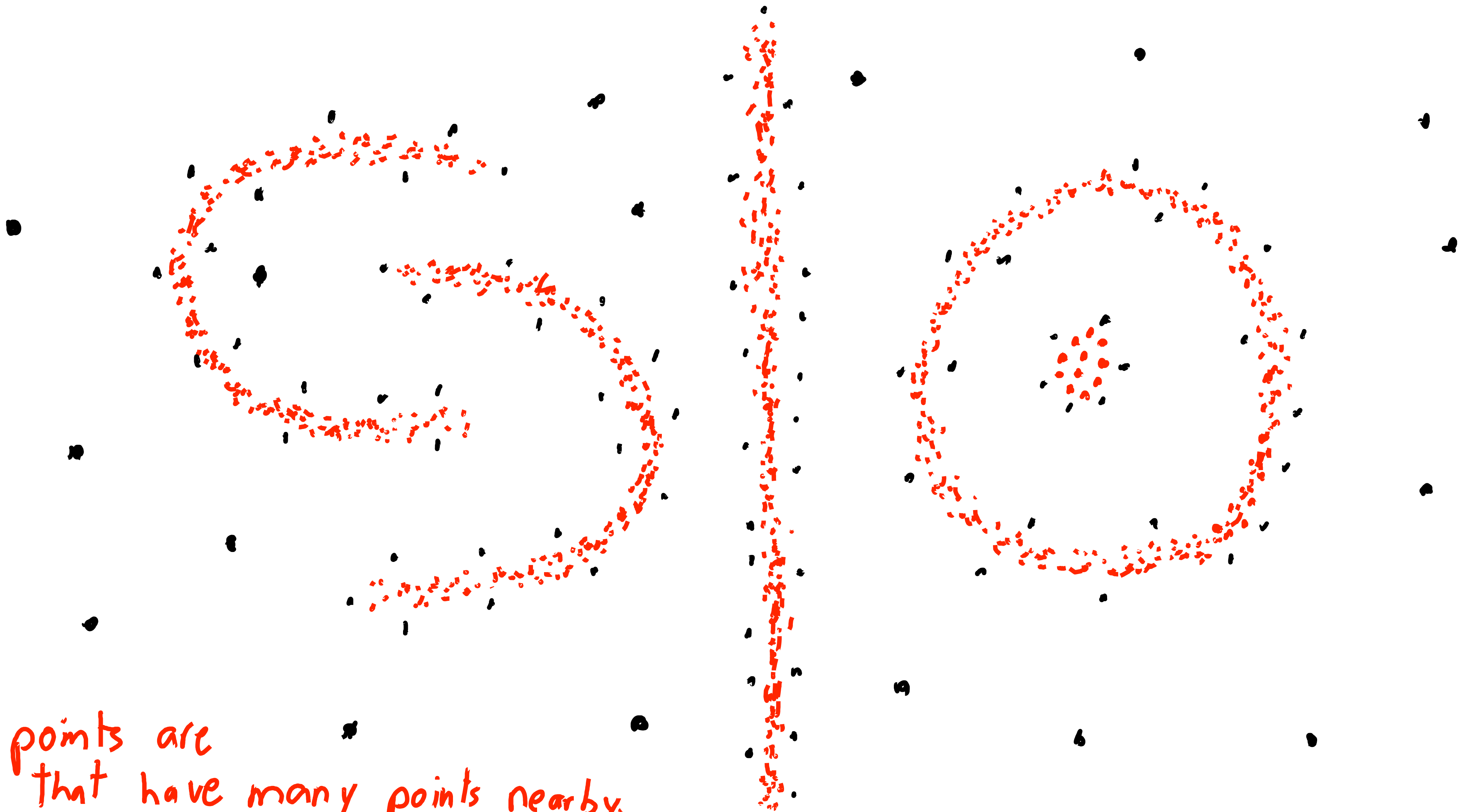

"Core" points are points that have many points nearby.

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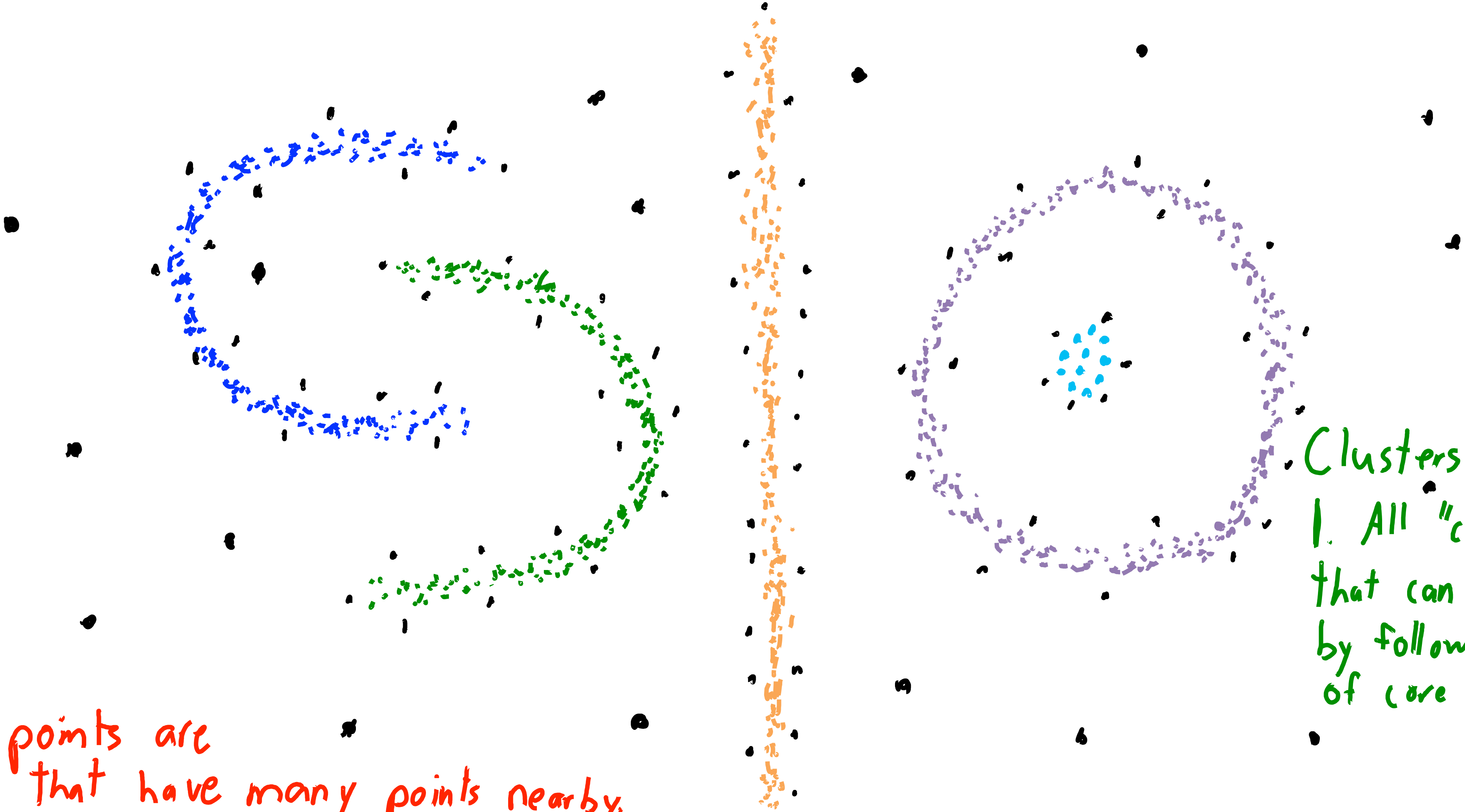

Clusters contain:
1. All "core" points that can be reached by following a sequence of core points.

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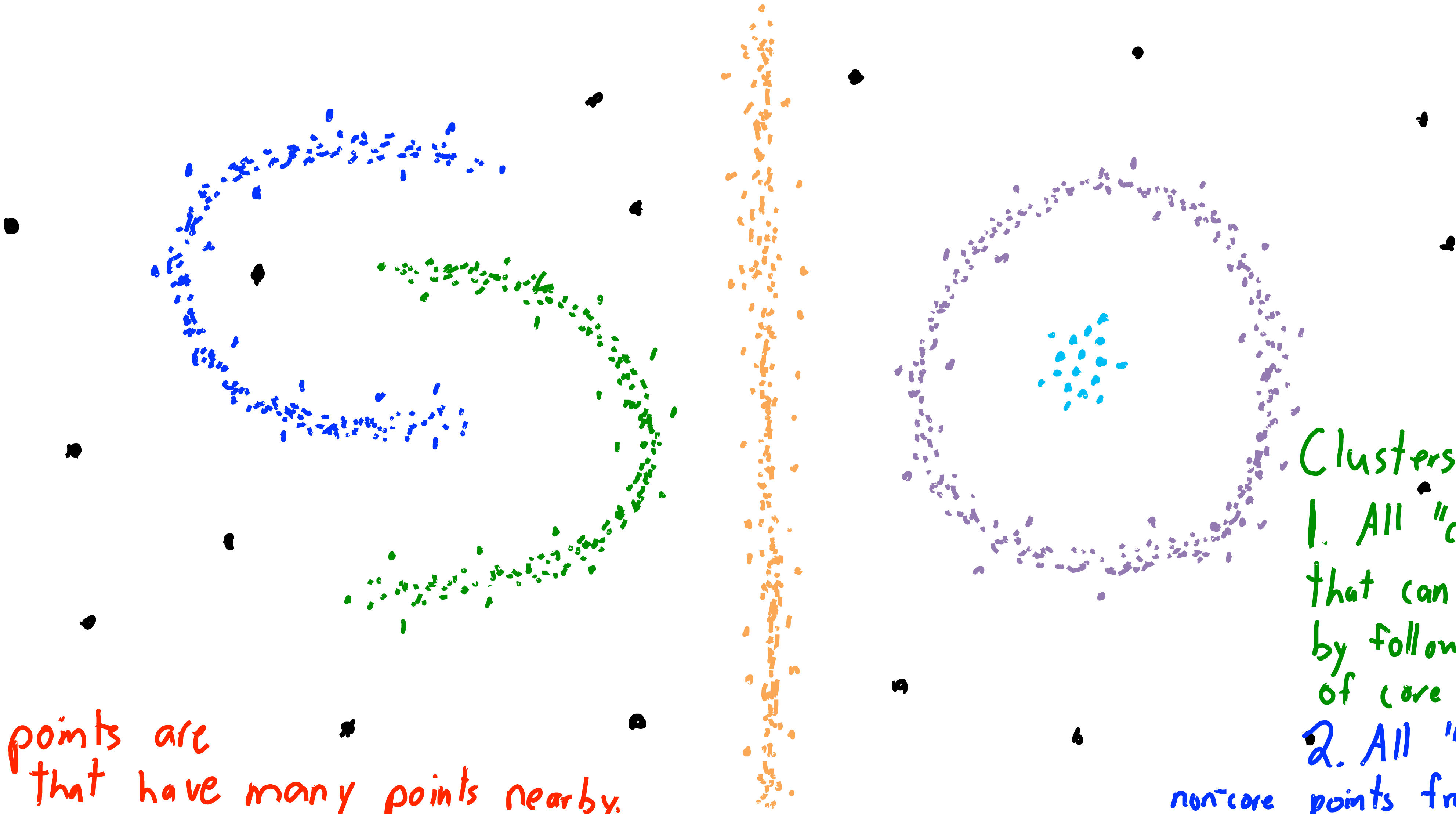


"Core" points are points that have many points nearby.

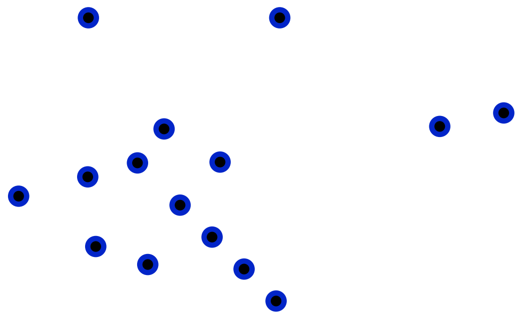
Clusters contain:

1. All "core" points that can be reached by following a sequence of core points.
2. All "reachable" non-core points from these "core" points ("boundary" points)

# Density-Based Clustering in Action

# Density-Based Clustering

- Each "core" point defines a cluster:
  - Consisting of "core" point and all its "reachable" points.
- Merge clusters if "core" points are "reachable" from each other.



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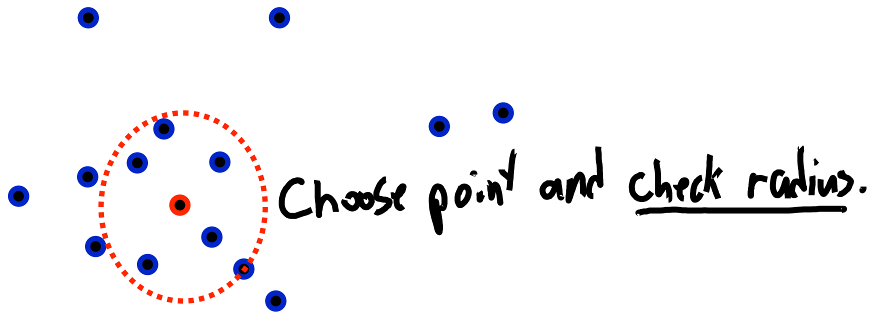

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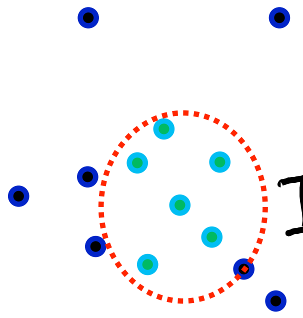

It's a "core" point, so make a new cluster with points within radius

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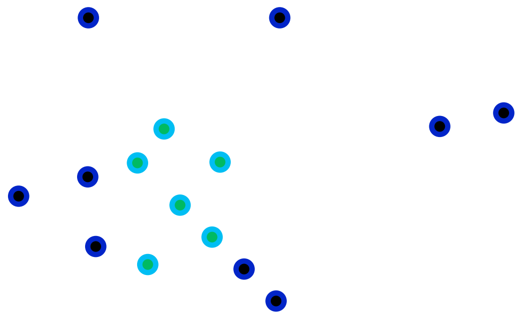

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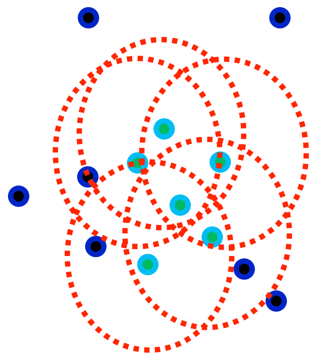

"Expand" the new cluster points that are "core" points

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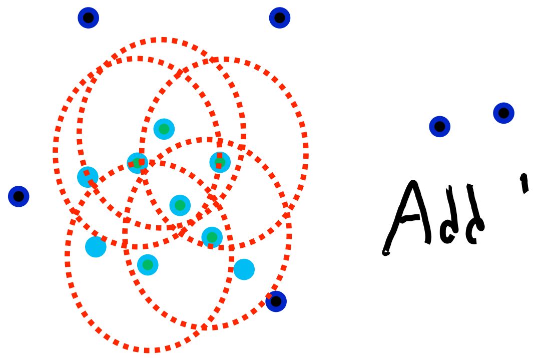

Add "reachable" points to cluster.

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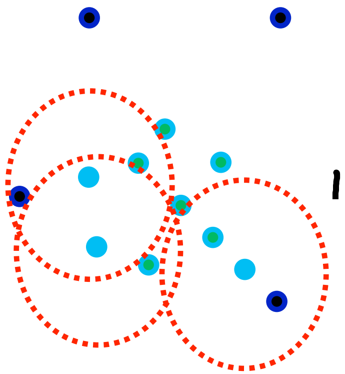

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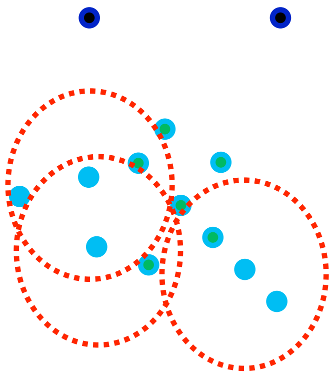

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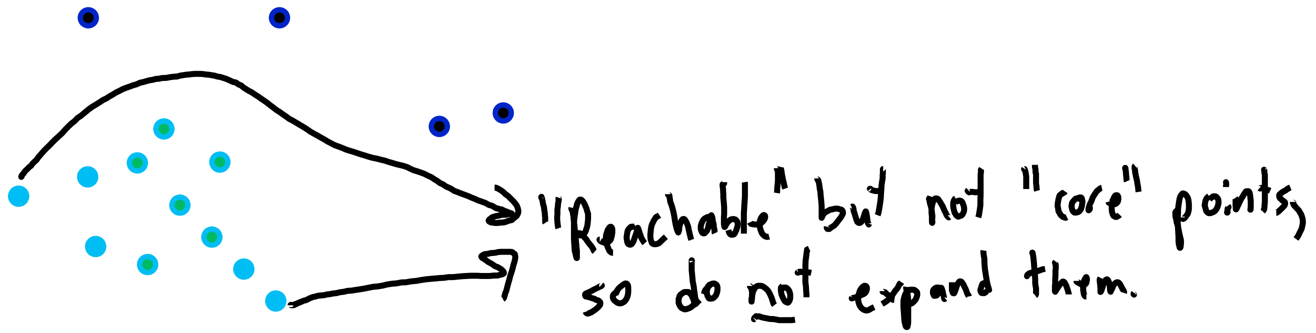

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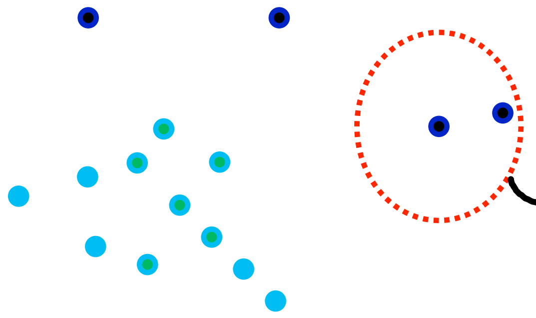

We're done "expanding", try a new point.

Not enough points in radius => not a "core" point.

Don't assign cluster

# Density-Based Clustering

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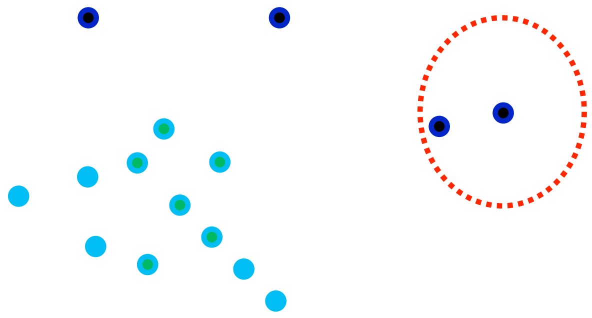
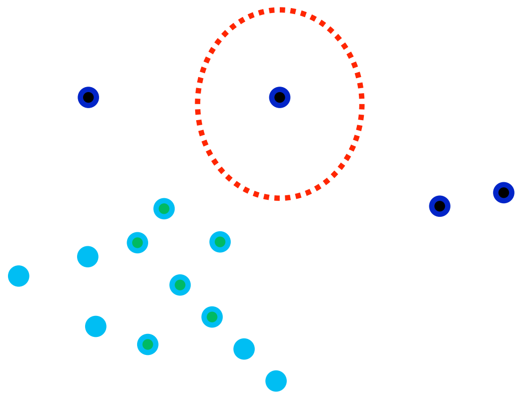
  - Consisting of "core" point and all its "reachable" points.
- Merge clusters if "core" points are "reachable" from each other.



# Density-Based Clustering

- Each "core" point defines a cluster:
  - Consisting of "core" point and all its "reachable" points.
- Merge clusters if "core" points are "reachable" from each other.



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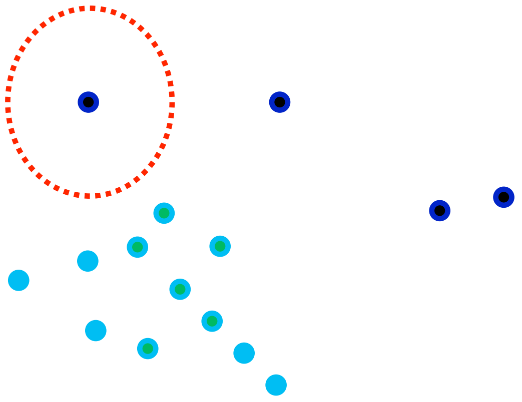
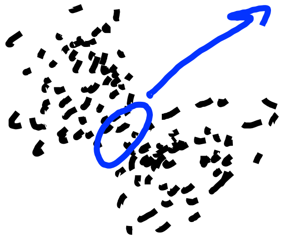


# Density-Based Clustering

- Pseudocode for DBSCAN:
  - For each example $x_i$:
    - If $x_i$ is already assigned to a cluster, do nothing.
    - Test whether $x_i$ is a 'core' point (less than minPoints neighbours with distances ≤ 'r').
      - If $x_i$ is not core point, do nothing.
      - If $x_i$ is a core point, "expand" cluster.
  - "Expand" cluster function:
    - Assign all $x_j$ within distance 'r' of core point $x_i$ to cluster.
    - For each newly-assigned neighbour $x_j$ that is a core point, "expand" cluster.

# Density-Based Clustering Issues

- Some points are not assigned to a cluster.
  - Good or bad, depending on the application.
- Ambiguity of "non-core" (boundary) points:



Points at "boundary" between clusters.

- Sensitive to the choice of radius and minPoints.
  - Otherwise, not sensitive to initialization (except for boundaries).

- If you get a new example, finding cluster is expensive.
  - Need to compute distances to training points.
- In high-dimensions, need a lot of points to 'fill' the space.

# Summary

- Norms:
  - Ways to measure "size" in higher dimensions.
- K-means++:
  - Randomized initialization of k-means with good expected performance.
- Shape of K-means clusters:
  - Intersection of half-spaces, which forms convex sets.
- Density-based clustering:
  - "Expand" and "merge" dense regions of points to find clusters.
  - Useful for finding non-convex connected clusters.

- Next time:
  - Discovering the tree of life.