Chapter 9: Comparing two paired populations

(Ott & Longnecker Sections: 6.4)

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Part 1: the paired T-test

https://dzwang91.github.io/stat371/

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Example

- Musculoskeletal disorders of the neck and shoulders are common in office workers because of repetitive tasks. Long periods of upper-arm elevation above 30 degrees have been shown to be related to disorders. It was thought that varying working conditions over the course of the day could alleviate some of these problems.
- 8 office workers were randomly selected. They were observed for one work day under the standard conditions, and the percentage of time that their dominant upper-arm was below 30 degrees was recorded. The next day, these same individuals had their work diversified, and again were observed.
- The results are below:

Participant	1	2	3	4	5	6	7	8
Diverse	78	91	79	65	67	72	71	96
Standard	81	87	86	59	66	70	73	92

Example continued

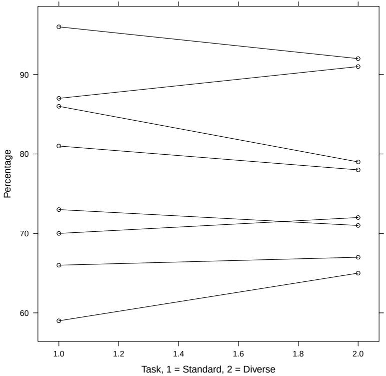
- We'd like to know whether the diversification changes the percentage of time where arm angle is below 30 degrees.
- So, we want to test:

$$H_0 : \mu_{diverse} - \mu_{standard} = 0 \text{ } vs. \text{ } H_A : \mu_{diverse} - \mu_{standard} \neq 0$$

- The data is clearly paired, since we are taking two measurements on each individual.

Example continued

- Dotplots connected for each individual:



It seems that some go up, some go down, and some stay the same. It's unlikely that we will see much difference.

Example continued

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  NO because of the dependence.
- How could we remove the dependence?
  Take the differences for each pair. The data is then the change from standard to diverse, rather than the raw percentage. This fixes the problem because the workers are independent.
- Now we have only one observation (the change) for each one. We can run one sample T-test on the differences.

Example continued

- diverse - standard: -3, 4, -7, 6, 1, 2, -2, 4.
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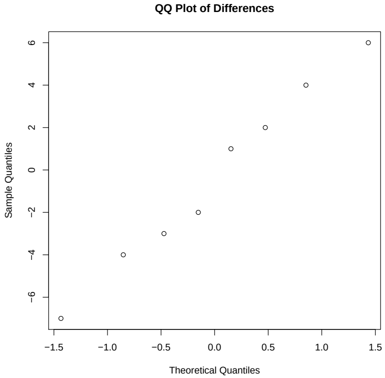

This looks quite straight, so we can assume normality.

Example continued

- If we let μ_d be the true mean difference, then the original hypotheses are equivalent to $H_0 : \mu_d = 0$ vs. $H_A : \mu_d \neq 0$.
- If we let d_i denote the observed differences, we also define $\bar{d}$ to be the sample mean of the differences and s_d to be the sample standard deviation of the differences. Then our test statistic is:

$$T = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} \sim T_{n-1},$$

where n is the number of *pairs*.

Example continued

- For this data we get $\bar{d} = 0.63$ and $s_d = 4.34$, so our statistic is:
$$t_{obs} = \frac{0.63}{\frac{4.34}{\sqrt{8}}} = 0.41.$$
- Compare to a T_7 distribution, p-value is about 0.702. Thus we would not reject. It seems that diversification of work has little effect on arm angle.

Recap of the paired T-test

- The data consists of paired observations. Let:
  - $X_{1,i}$ = i-th data point from population 1
  - μ_1 = true mean of population 1
  - $X_{2,i}$ = i-th data point from population 2
  - μ_2 = true mean of population 2
  - $D_i = X_{1,i} - X_{2,i}$ = the difference for pair i
  - n = number of pairs
- We wish to test: $H_0 : \mu_1 - \mu_2 = \delta$ vs. $H_A : \mu_1 - \mu_2 \neq \delta$.
- Good graphs for exploring the data include dotplots where the observations for each pair are connected by lines, and QQ plots of the differences to check normality.

Recap of the paired T-test continued

- If based on our prior knowledge and after exploring the data we are willing to assume:
  - All of the *pairs* are indepdendent
  - The differences follow a normal distribution

  Then the test statistic is:

  $$t = \frac{\bar{D} - \delta}{\frac{S_D}{\sqrt{n}}}$$

  Where $\bar{D}$ is the mean of the differences and S_D is the sample standard deviation of the differences.
- In the end, compare to a T distribution on $n-1$ degrees of freedom.

What's the next?

We'll discuss the sign test in two paired populations in the next lecture.