WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

STAT371-L5
Midterm 1
Fall 2017

A. You are allowed one piece of paper for notes (both sides), and a hand calculator. Laptops, tablets, and smartphones are not allowed.

B. To receive full credit, you must show your work. Partial credit will be awarded when appropriate.

C. If you can't find the exact value you need in the Z table, state this and use the closest value you can find.

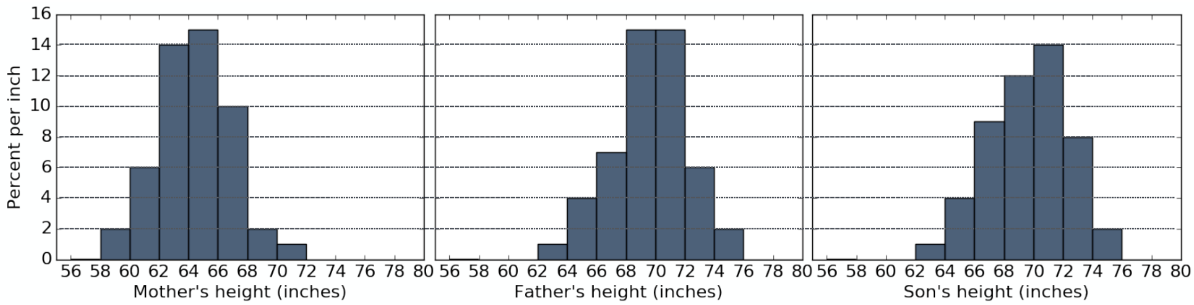
D. Do all your work in the space provided. If you need more space, you may ask for extra paper.

Name: ____________________ Discussion Section Number: __________________

For instructor's use:

1	20	
2	20	
3	15	
4	10	
5	15	
Total	80	

1. Steven measured the heights of **100 families** that each included 1 mother, 1 father, and some varying number of adult sons. The three histogram of heights below depict the distributions for all mothers, fathers and adult sons. **All bars are 2 inches wide. All bars' heights are integers.** The heights of all people in the data set are included in the histograms.



   (a) Calculate each quantity described below or write **unknown** if there is not enough information above to express the quantity as a single number(not a range). Show your work!

      i. The **percentage** of mothers that are at least 66 inches but less than 70 inches tall. (**4 points**)
      $10\% \times 2 + 2\% \times 2 = 24\%$

      ii. The **percentage** of fathers that are at least 66 inches but less than 71 inches tall. (**4 points**)
      Unknown

iii. The **number** of fathers that are at least 70 inches tall. (**4 points**)

$0.15 \times 2 + 0.06 \times 2 + 0.02 \times 2 = 0.46$

$0.46 \times 100 = 46$

iv. The **number** of sons that are at least 70 inches tall. (**4 points**)

Unknown

(b) If the sons' histogram were redrawn, replacing the two bins from 72-to-74 and from 74-to-76 with one bin from 72-to-76, what would be the height of its bar? If it's impossible to tell, write **unknown**. (**4 points**)

$0.08 \times 2 + 0.02 \times 2 = 0.2$

$0.2/4 = 0.05 = 5\%$

2. Three fans will be randomly selected at the football game in which Madison is playing Purdue this October. Since this is a home game, we assume 90% of the fans attending the game are Madison fans, while 10% are Purdue fans. Let X be the number of Madison fans selected, answer following questions:

   (a) List a table of **pmf** of X. (**5 points**)

X	pmf
0	0.001
1	0.027
2	0.243
3	0.729

   (b) Calculate the **expectation** and **variance** of X. (**10 points**)

   E(X)=3×0.9 = 2.7

   $VAR(X) = 3 \times 0.9 \times 0.1 = 0.27$

   (c) If we further assume two persons are friends only if they are same team's fans, then what's the probability that these selected three people are friends with each other? (**5 points**)

   P(X=0 or X=3)=P(X=0)+P(X=3)=0.73

3. A factory manager has been working in the factory for many years. Sometimes the old machine will break down in the middle of production which causes some problems for timely delivery. Assume that the time of an old machine stays in operation before a maintenance is needed is normally distributed with **mean 500 hours and standard deviation of 100 hours**.

   (a) What is the probability that an old machine could operate at least 600 hours before the next maintenance is arranged? (**5 points**)

   P(X$\geq$ 600) $= P(Z \geq \frac{600-500}{100}) = P(Z \geq 1) = 1 - 0.84 = 0.16$

   (b) If 10 machines are randomly selected, what is the probability that at least 2 would be able to operate at least 600 hours? Assume that the machines work independently. (**5 points**)

   Let B=the number of machines which are able to operate at least 600 hours, then $B \sim Bin(10, 0.16)$, then $P(B \geq 2) = 1 - P(B = 0) - P(B = 1) = 0.49$

(c) If 2 machines are randomly selected, what is the probability that one would be able to operate for at least 600 hours while the other could only operate at most 600 hours? (**5 points**)

$2 \times 0.16 \times 0.84 = 0.2688$

4. We have a fair four-sided die with sides numbered 1-4. If we roll it twice and got at least one 4, what is the probability of rolling a 3 for the first time and a 4 for the second time? (**10 points**)

   There are 7 outcomes that include at least one 4. So the probability of rolling a 3 for the first time and a 4 for the second time is 1/7.

5. Let θ be a patient's true blood pressure (in mm Hg). Suppose the patient's doctor has two devices for measuring blood pressure that are subject to random error. Let V be the blood pressure measurement taken by the first device, and let W be the blood pressure measurement taken by second device. Assume $E(V) = E(W) = \theta$ and that $Var(V) = 0.36$ while $Var(W) = 0.64$. Assume V and W are independent.

   (a) Suppose the doctor combines both measurements in the following way: she measures blood pressure using the first device and records V, then she measures it using the second device and records W, and records the final blood pressure measurement as $X = \frac{1}{2}(V + W)$. Calculate the **expectation** and **standard deviation** of X. (**6 points**)

   $$E(X) = E\left(\frac{V+W}{2}\right) = \frac{1}{2}E(V+W) = \frac{1}{2}(E(V) + E(W)) = \frac{1}{2}(\theta + \theta) = \theta$$

   (so X is *unbiased* for θ).

   $$Var(X) = Var\left(\frac{V+W}{2}\right) = \frac{1}{4}Var(V+W) = \frac{1}{4}(Var(V) + Var(W)) = \frac{1}{4}$$

   yielding standard deviation $SD(X) = 0.5$.

   (b) The doctor consults a statistician, who tells her she should combine the blood pressure measurements using the formula $Y = 0.64V + 0.36W$. Calculate the **expectation** and **standard deviation** of Y. (**6 points**)

   $$E(Y) = E(0.64V + 0.36W) = 0.64E(V) + 0.36E(W) = 0.64*\theta + 0.36*\theta = \theta.$$

   So Y is also *unbiased* for θ.

   $$Var(Y) = Var(0.64V + 0.36W) = (0.64^2)Var(V) + (0.36^2)Var(W)$$
   $$= (0.64^2) * 0.36 + (0.36^2) * 0.64 = 0.2304,$$

   yielding $SD(Y) = \sqrt{0.2304} = 0.48$.

(c) Which measurement of θ do you think is best: V, W, X, or Y? Briefly explain your answer. (**3 points**)

Choose Y because all the measurements are unbiased but Y has the smallest standard deviation. This is because Y adjusts the weight of V and W according to their variances, putting more weight on V because it's the more reliable measuring device.