

CS 106X

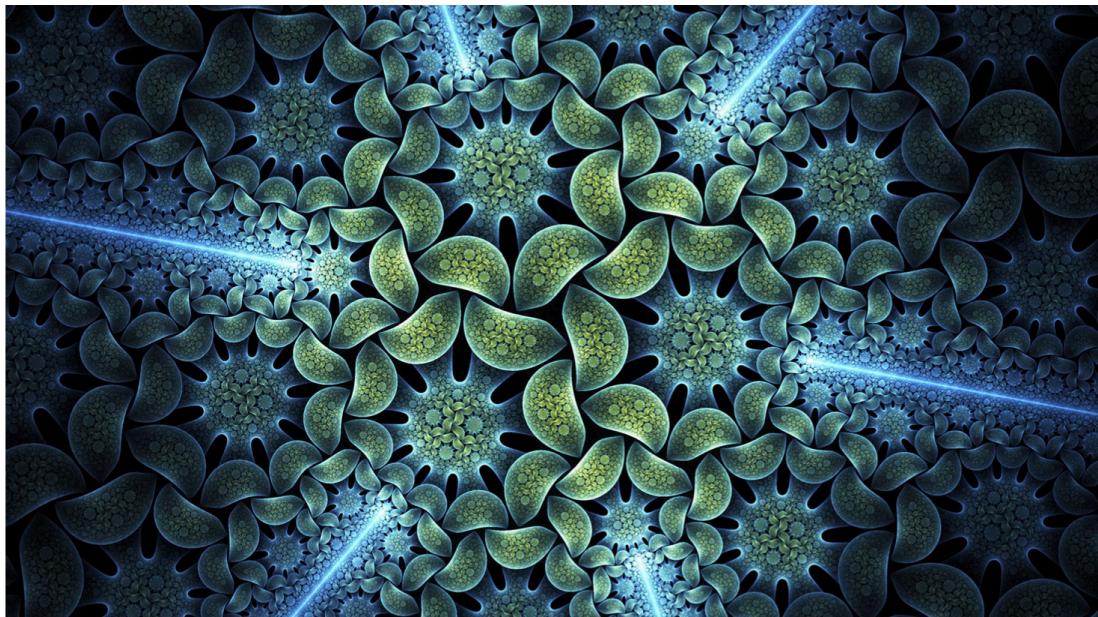
Lecture 8: Fractals

Friday, January 27, 2017

Programming Abstractions (Accelerated)
Winter 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 5.4-5.6



Today's Topics

- Logistics:
 - ADTs Due ~~Friday, January 27th~~, Saturday, January 28th, noon
 - Towers of Hanoi video featuring Keith Schwartz: <https://www.youtube.com/watch?v=2SUvWfNJSsM>
- Tiny Feedback
- Assignment 3: Recursion
 - Fractals
 - Grammar Solver
 - 20 Questions
- A more detailed recursion example
- Fractals



Tiny Feedback

- Give examples of when its more advantageous to use a loop over recursion, or vice versa. -- **Let's talk about this!**
 - ...sometimes on homework we're supposed to look up or use thing that we may not have gone over in class. So it would be nice to be a little more thorough with class instruction -- **Tough one. I can't tell you about every nuance in the assignments.**
- (1) **use Piazza!**
(2) **ask in office hours!**
(3) **look up online (but be a bit careful...)**



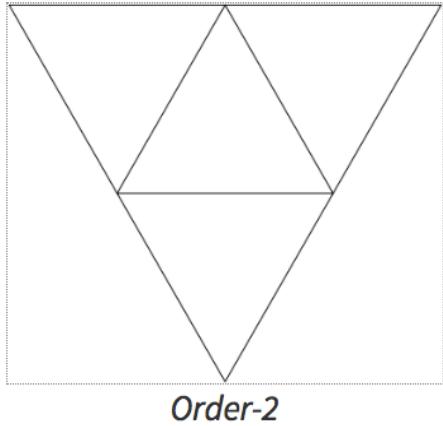
Assignment 3: Recursion

- (1) Fractals and Graphics
- (2) Grammar Solver
- (3) Twenty Questions

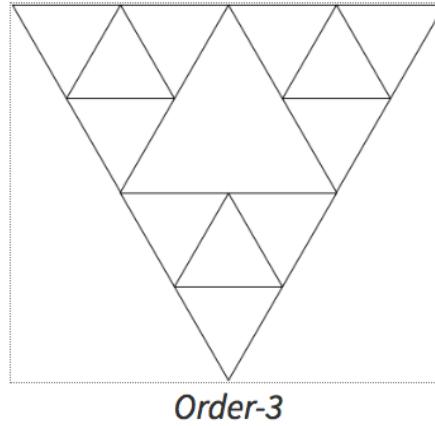


Assignment 3A: Fractals and Graphics

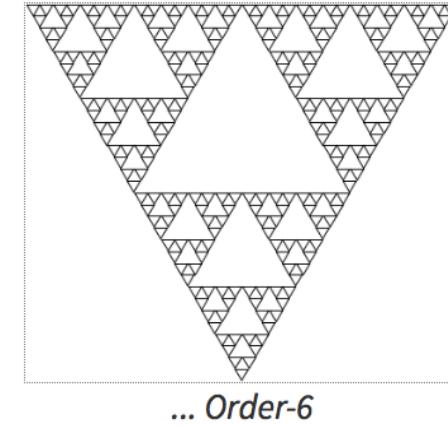
part 1
Sierpinski



Order-2



Order-3



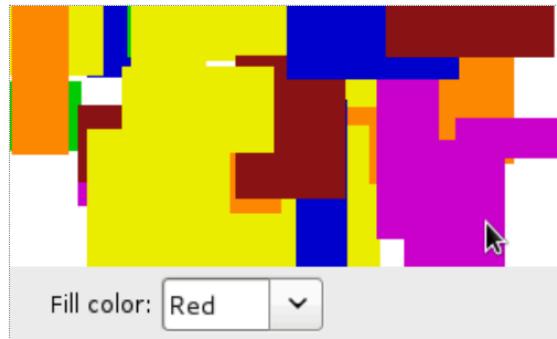
... Order-6

part 3
flood fill

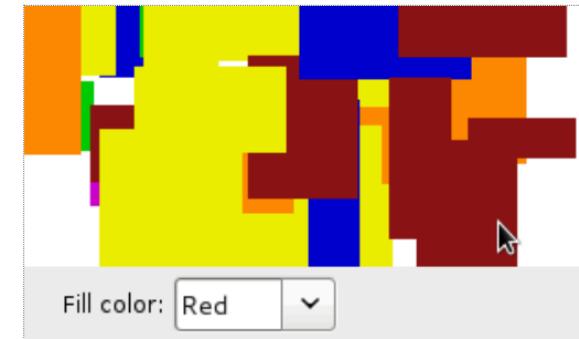
part 2
tree fractal



Order-5 tree fractal



Before flood fill



After flood fill



Assignment 3B: Grammar Solver

write a function for generating random sentences from a grammar.

example describing a small subset of the English language. Non-terminal names such as <s>, <np> and <tv> are short for linguistic elements such as sentences, noun phrases, and transitive verbs:

```
<s> ::= <np> <vp>
<np> ::= <dp> <adjp> <n> | <pn>
<dp> ::= the | a
<adjp> ::= <adj> | <adj> <adjp>
<adj> ::= big | fat | green | wonderful | faulty | subliminal | pretentious
<n> ::= dog | cat | man | university | father | mother | child | television
<pn> ::= John | Jane | Sally | Spot | Fred | Elmo
<vp> ::= <tv> <np> | <iv>
<tv> ::= hit | honored | kissed | helped
<iv> ::= died | collapsed | laughed | wept
```



Assignment 3C: Twenty Questions

Implement a yes/no guessing game called "20 Questions." Each round of the game begins by you (the human player) thinking of an object. The computer will try to guess your object by asking you a series of yes-or-no questions. Eventually the computer will have asked enough questions that it thinks it knows what object you are thinking of and makes a guess.

```
Q: Is it an animal?  
Q: Can it fly?  
A:bird  
Q: Does it have a tail?  
Q: Does it squeak?  
A:mouse  
A:lion  
A:spider  
Q: Does it have wheels?  
Q: Does it have an engine?  
A:car  
A:bicycle  
Q: Is it nice?  
A:section leader  
A:teacher
```

*input file **questions.txt***

```
Q: Is it an animal?  
y-- Q: Can it fly?  
|  
| y-- A:bird  
n-- Q: Does it have a tail?  
|  
| y-- Q: Does it squeak?  
|  
| y-- A:mouse  
|  
| n-- A:lion  
|  
| n-- A:spider  
  
n-- Q: Does it have wheels?  
|  
| y-- Q: Does it have an engine?  
|  
| y-- A:car  
|  
| n-- A:bicycle  
n-- Q: Is it nice?  
|  
| y-- A:section leader  
|  
| n-- A:teacher
```

*indented version of **questions.txt**
(actual input files will not be indented!)*



Three Musts of Recursion

1. Your code must have a case for all valid inputs
2. You must have a base case that makes no recursive calls
3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.



Recursion Example

Google

$((1+3)*(2*(4+1)))$



Google Search

I'm Feeling Lucky



Recursion Example

A screenshot of a web browser window. The address bar shows the URL [https://www.google.com/search?q=\(\(1*17\)+\(2*\(3+\(4*9\)\)\)\)&oq=\(\(1*17\)%2B\(2*\(3%2B4*9\)\)\)](https://www.google.com/search?q=((1*17)+(2*(3+(4*9))))&oq=((1*17)%2B(2*(3%2B4*9)))). The search query in the search bar is $((1*17)+(2*(3+(4*9))))$. Below the search bar, the word "Google" is displayed in its signature colors. Underneath the search bar, there are links for "All", "Maps", "News", "Shopping", "Images", "More", and "Search tools". A message indicates "About 43,200,000 results (0.64 seconds)". Below this, a snippet from a search result shows a digital calculator interface. The calculator has a numeric keypad and various function keys like Rad, Inv, sin, ln, etc. The display screen of the calculator shows the expression $(1 * 17) + (2 * (3 + (4 * 9))) =$ followed by the result **95**.

$$((1*17)+(2*(3+(4*9)))) \rightarrow 95$$



Challenge

Implement a function which evaluates an expression string:

"((1+3)*(2*(4+1)))"

"(7+6)"

"(((4*(1+2))+6)*7)"

(only needs to implement * or +)



Anatomy of an Expression

An expression is always one of these three things

number

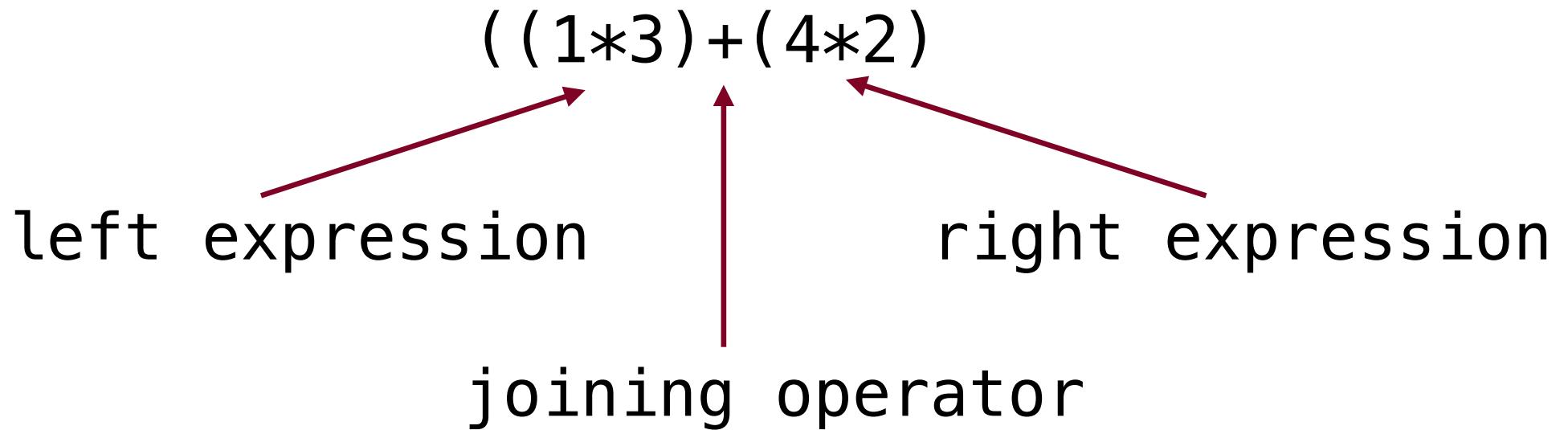
expression

(expression + expression)

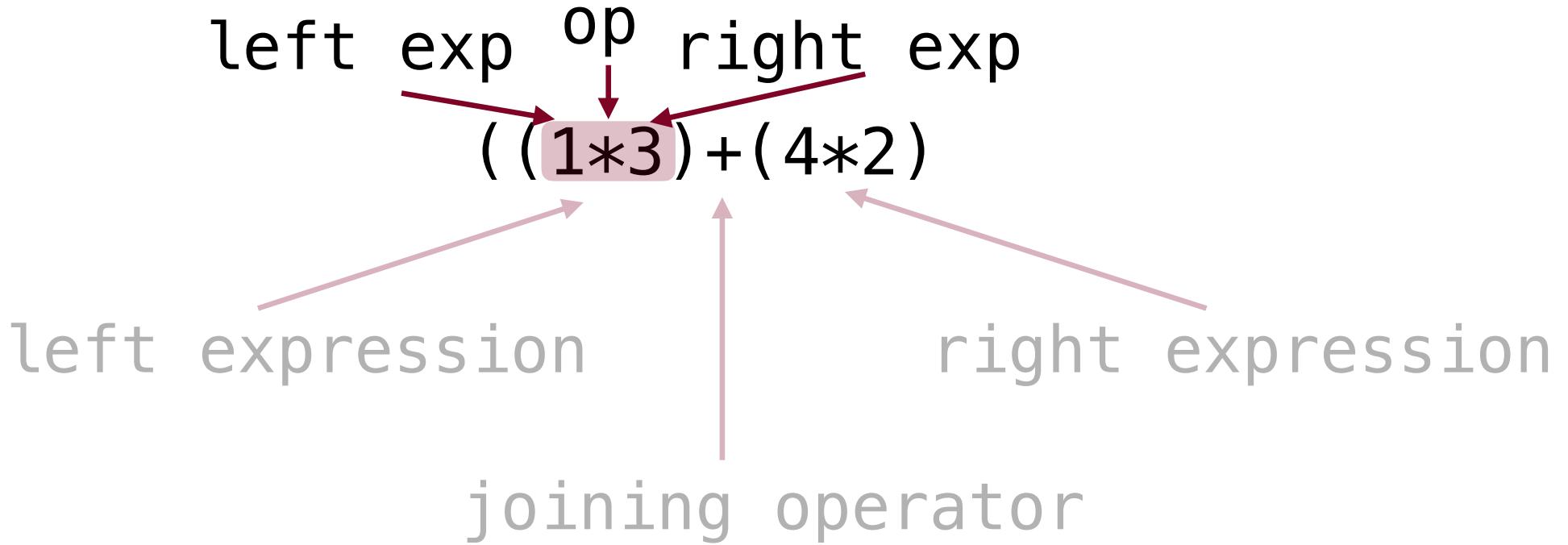
(expression * expression)



Anatomy of an Expression

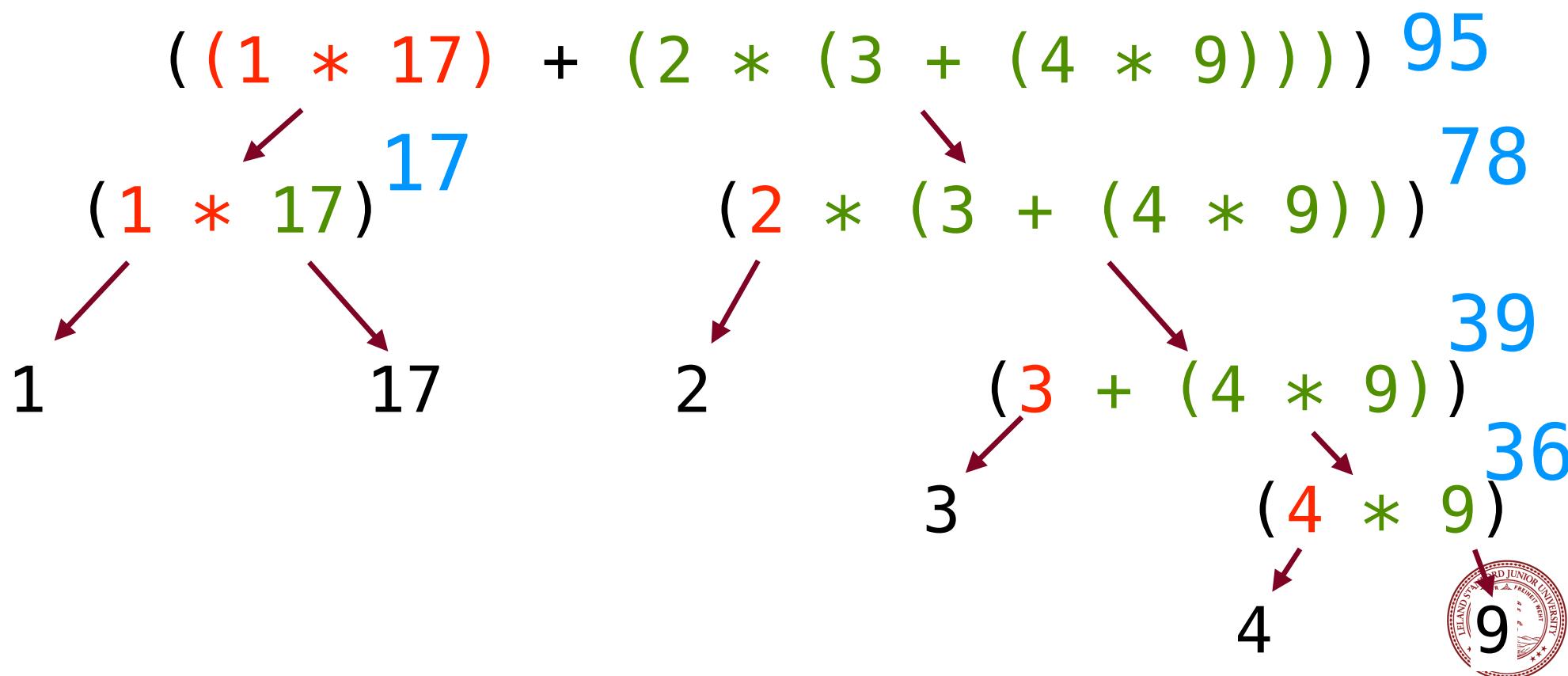


Anatomy of an Expression



Anatomy of an Expression

How do we evaluate $((1*17)+(2*(3+(4*9))))$?



Is it Recursive? Yes!

$$((1*3)+(4+2))$$

The big instance of this problem is:

$$((1*3)+(4+2))$$

The smaller instances are:

$$(1*3) \text{ and } (4+2)$$



Task

Write this function: `int evaluate(string exp);`

`"((1*3)+(4+2))" // returns 9`

Using these library functions:
`stringIsInteger(exp)`
`stringToInteger(exp)`

And these exp helper functions:

```
//returns '+'
char op = getOperator(exp);
//returns "(1*3)"
string left = getLeftExp(exp);
//returns "(4+2)"
string right = getRightExp(exp);
```



Solution (Pseudocode)

"((1*3)+(4+2))"

int evaluate(expression):

- if *expression* is a number, return *expression*
- Otherwise, break up *expression* by its operator:
 - *leftResult* = evaluate(leftExpression)
 - *rightResult* = evaluate(rightExpression)
 - return *leftResult* operator *rightResult*



Solution

```
int evaluate(string exp) {  
    if (stringIsInteger(exp)) {  
        return stringToInteger(exp);  
    } else {  
        char op = getOperator(exp);  
        string left = getLeftExp(exp);  
        string right = getRightExp(exp);  
        int leftResult = evaluate(left);  
        int rightResult = evaluate(right);  
        if (op == '+') {  
            return leftResult + rightResult;  
        } else if (op == '*') {  
            return leftResult * rightResult;  
        }  
    }  
}
```

exp = "((1*3)+(4*5)+2)"

op = '+'

left = "(1*3)"

right = "((4*5)+2)"

leftResult = 3

rightResult = 22



Helper Methods

Here is the key function behind the helper methods:

```
int getOppIndex(string exp){  
    int parens = 0;  
    // ignore first left paren  
    for (int i = 1; i < exp.length(); i++) {  
        char c = exp[i];  
        if (c == '(') {  
            parens++;  
        } else if (c == ')') {  
            parens--;  
        }  
        if (parens == 0 && (c == '+' || c == '*')) {  
            return i;  
        }  
    }  
}
```



By the way...

We could also have solved this with a stack!



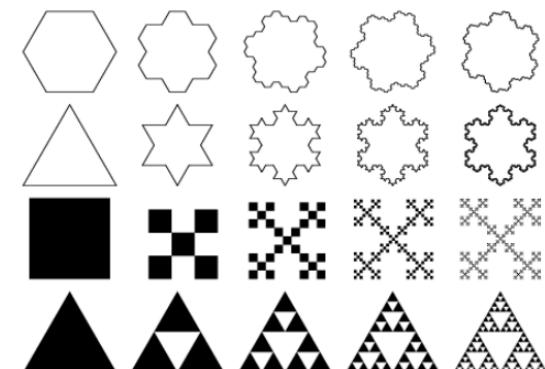
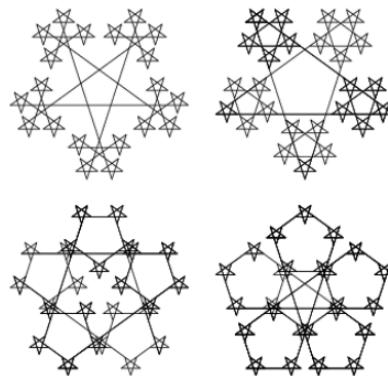
Today

Recursion you can see



Fractal

fractal: A recurring graphical pattern. Smaller instances of the same shape or pattern occur within the pattern itself.



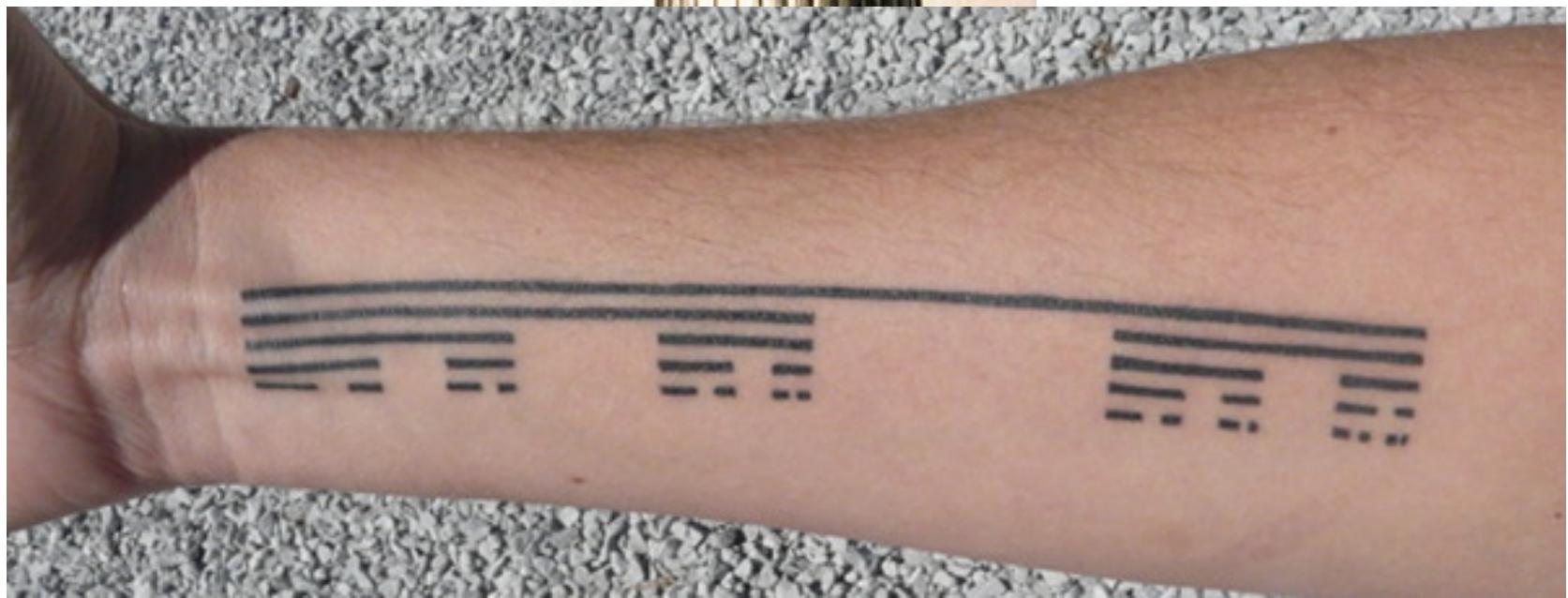
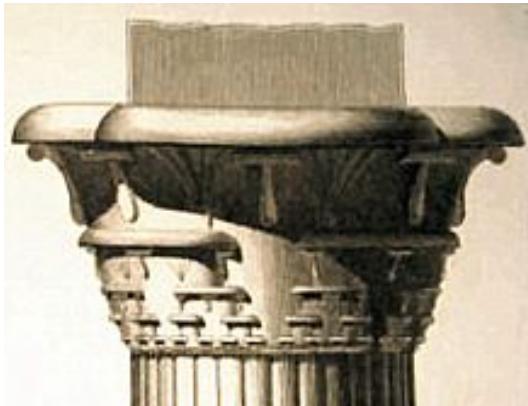
Fractal

Many natural phenomena generate fractal patterns:

1. earthquake fault lines
2. animal color patterns
3. clouds
4. mountain ranges
5. snowflakes
6. crystals
7. DNA
8. ...



The Cantor Fractal



Cantor Fractal

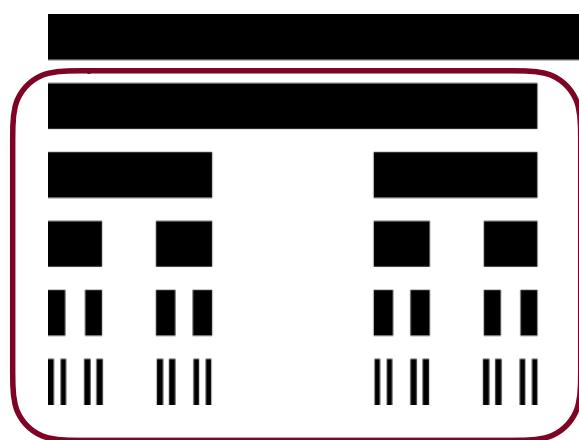


Parts of a cantor set image ... are Cantor set images



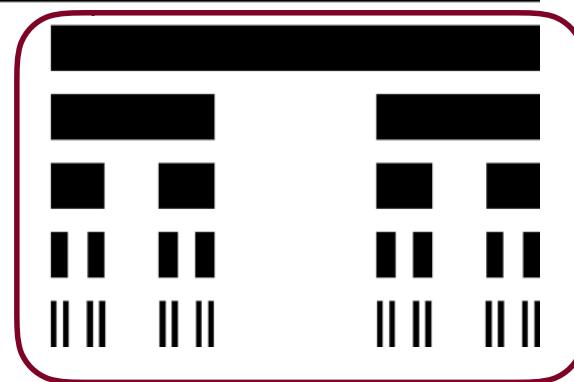
Cantor Fractal

Start



Another cantor set

End



Also a cantor set



Levels of Cantor



6 levels



Levels of Cantor



5 levels



Levels of Cantor



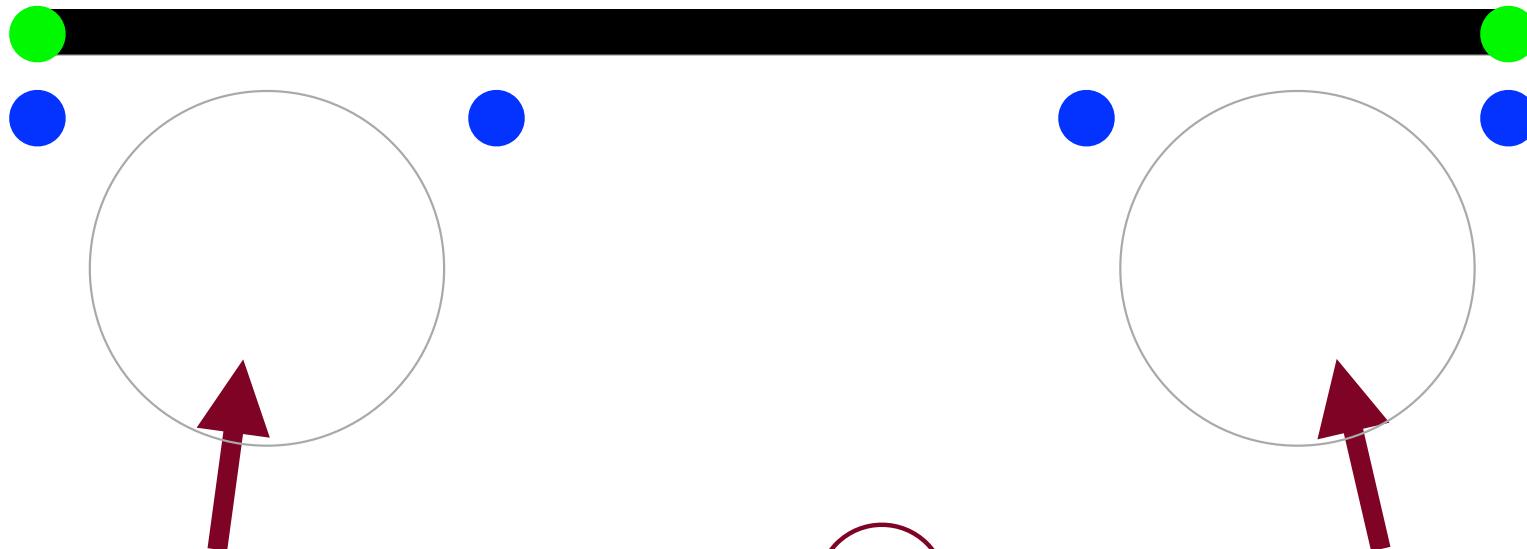
How to Draw a Level 1 Cantor



How to Draw a Level n Cantor

1

Draw a line from start to finish.



2

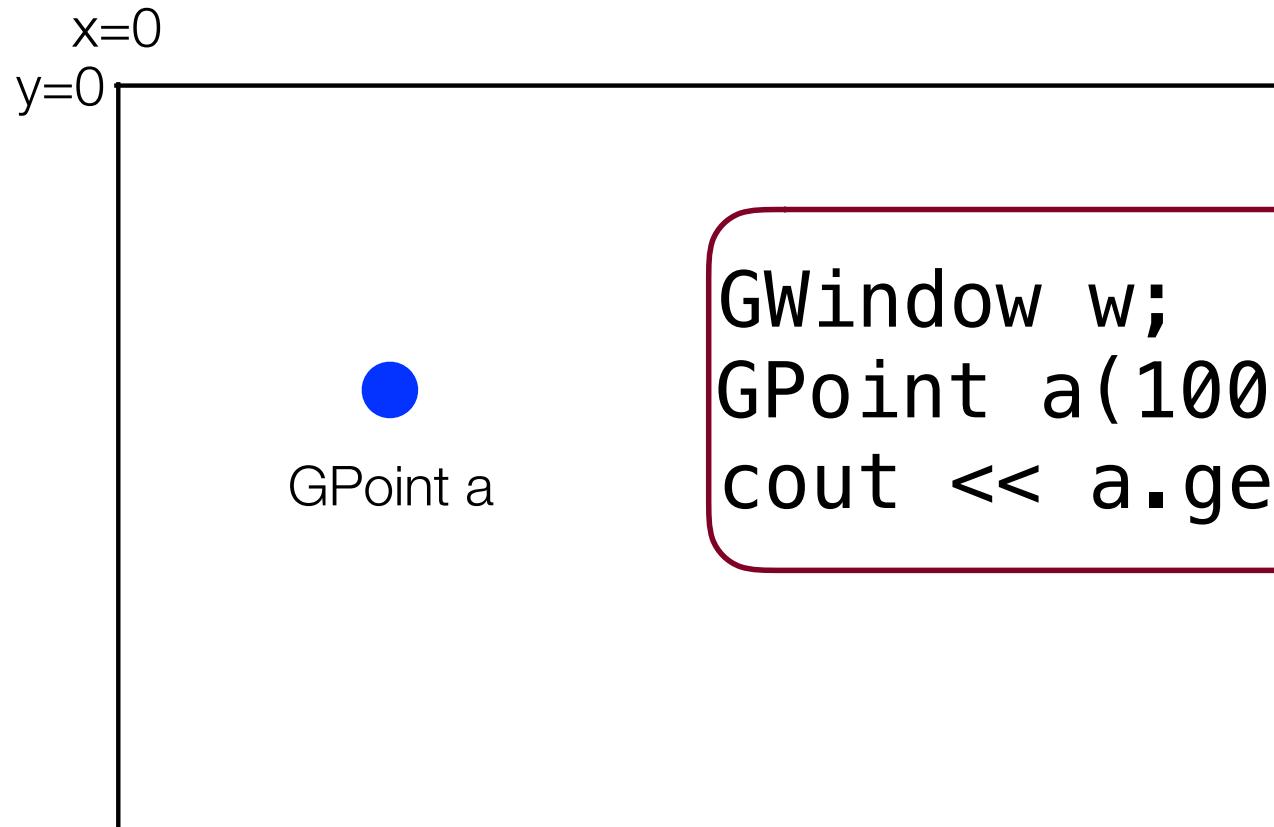
Draw a Cantor of size $n-1$

2

Draw a Cantor of size $n-1$



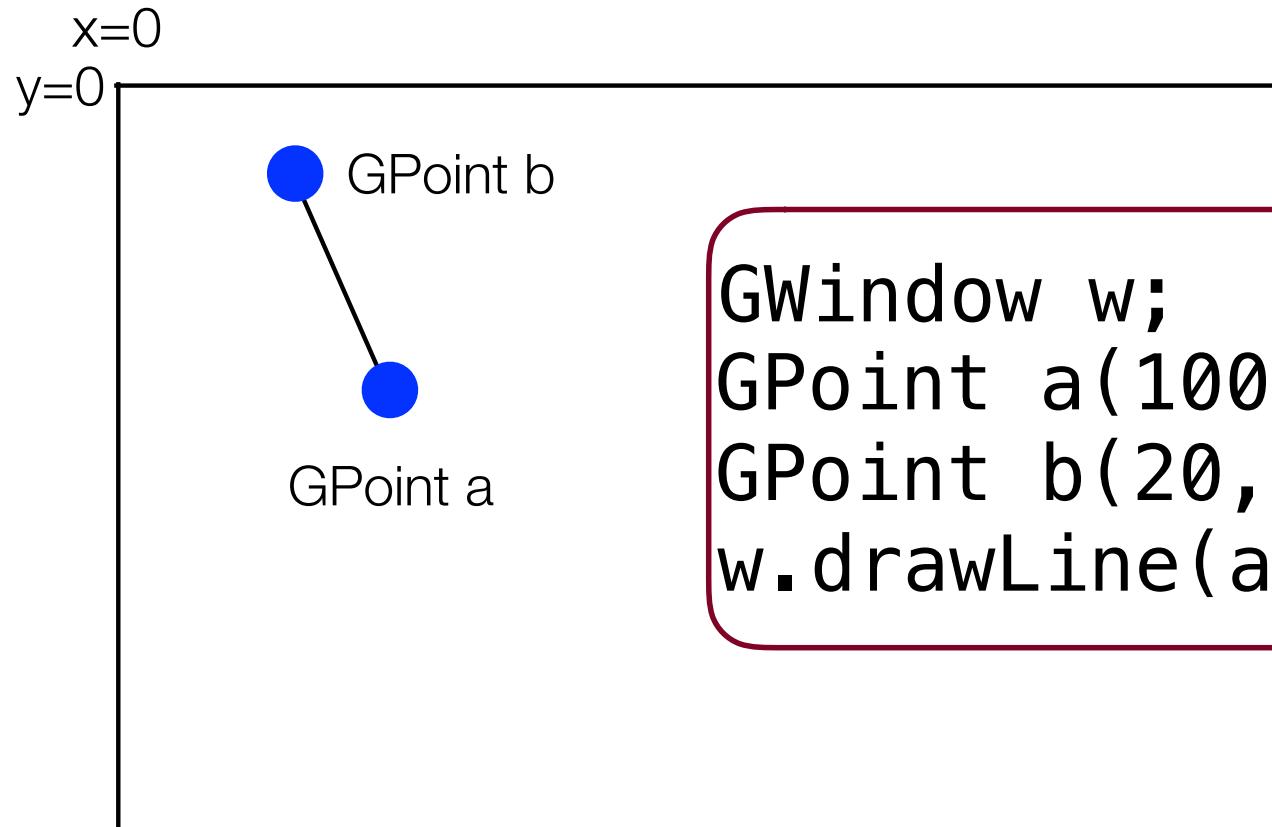
Graphics in C++ with the Stanford Libs: GPoint



```
GWindow w;  
GPoint a(100, 100);  
cout << a.getX() << endl;
```



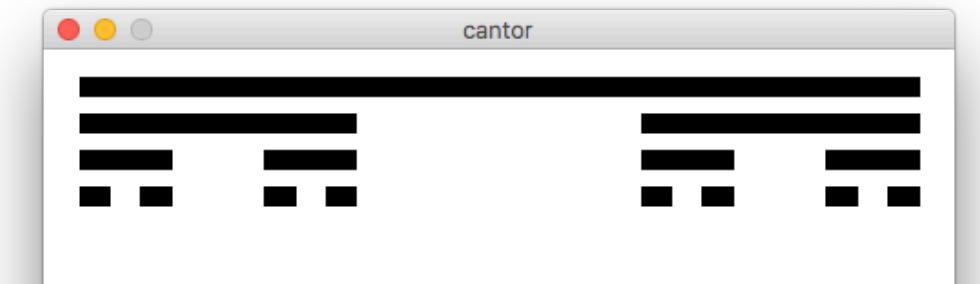
Graphics in C++ with the Stanford Libs: GPoint



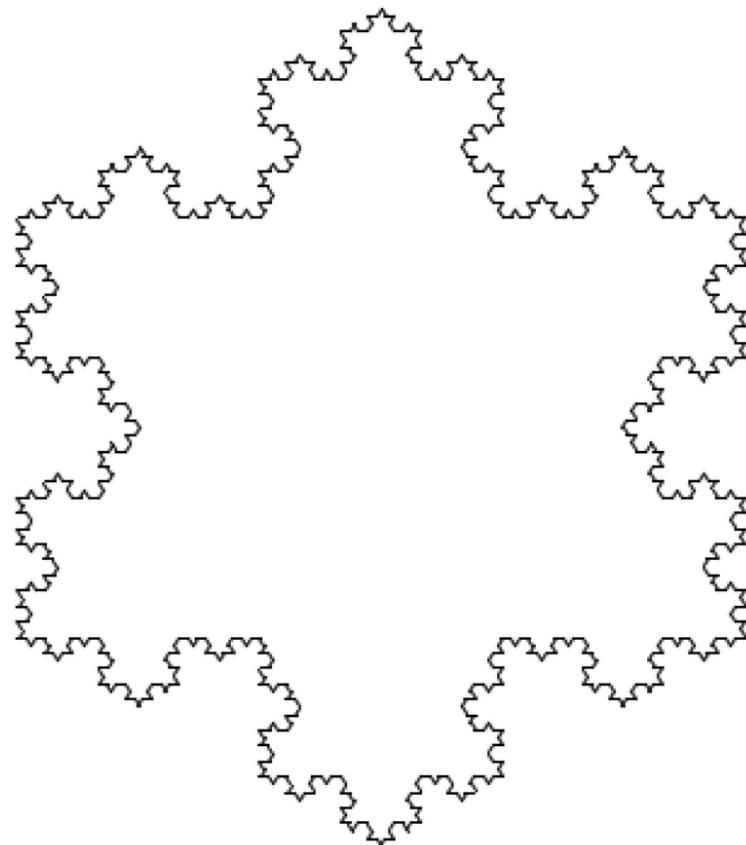
```
GWindow w;  
GPoint a(100, 100);  
GPoint b(20, 20);  
w.drawLine(a, b);
```



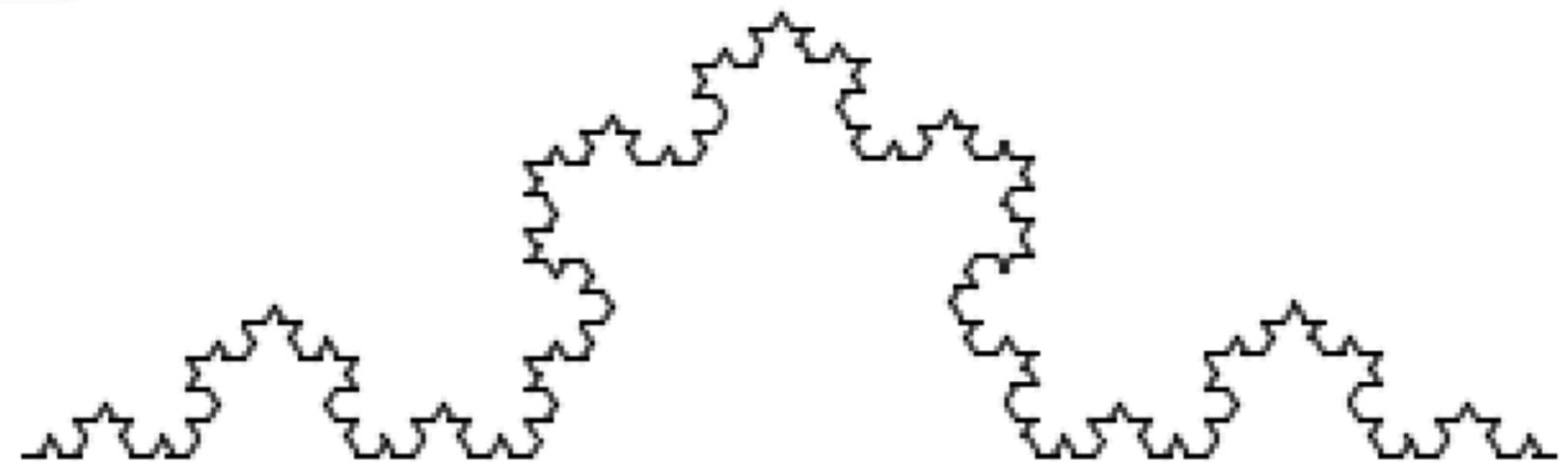
Cantor Fractal



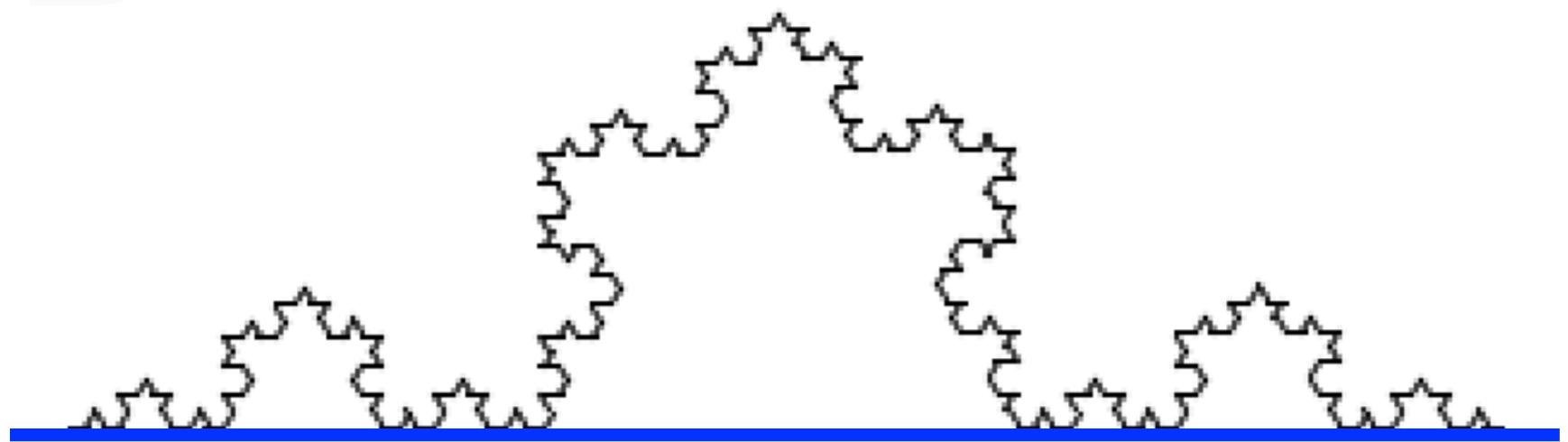
Snoflake Fractal



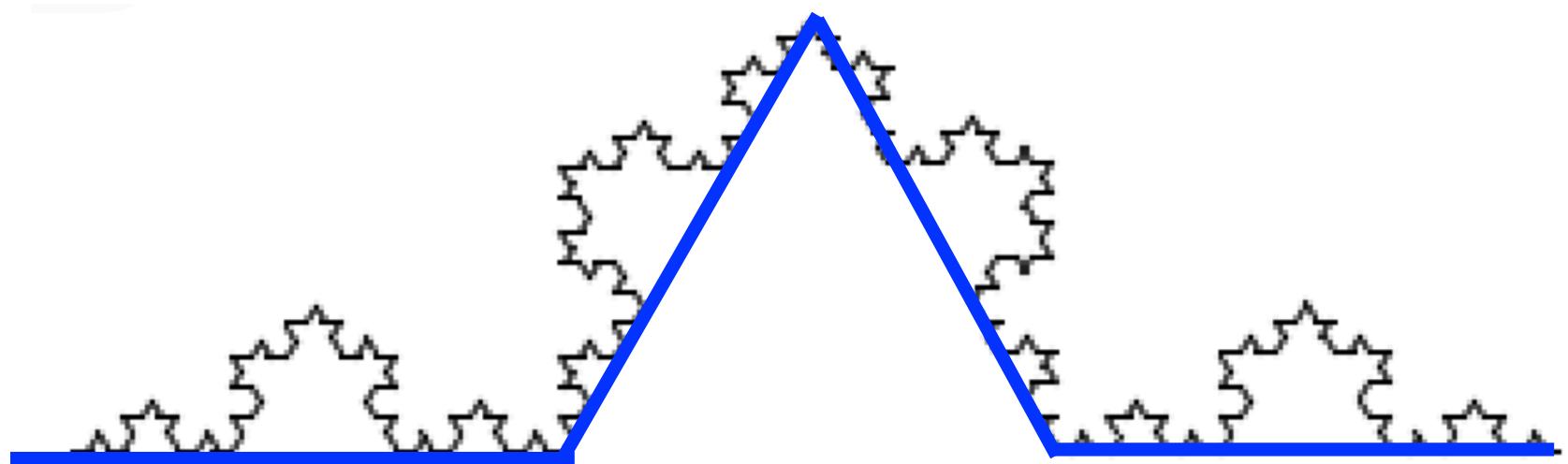
Snowflake Fractal



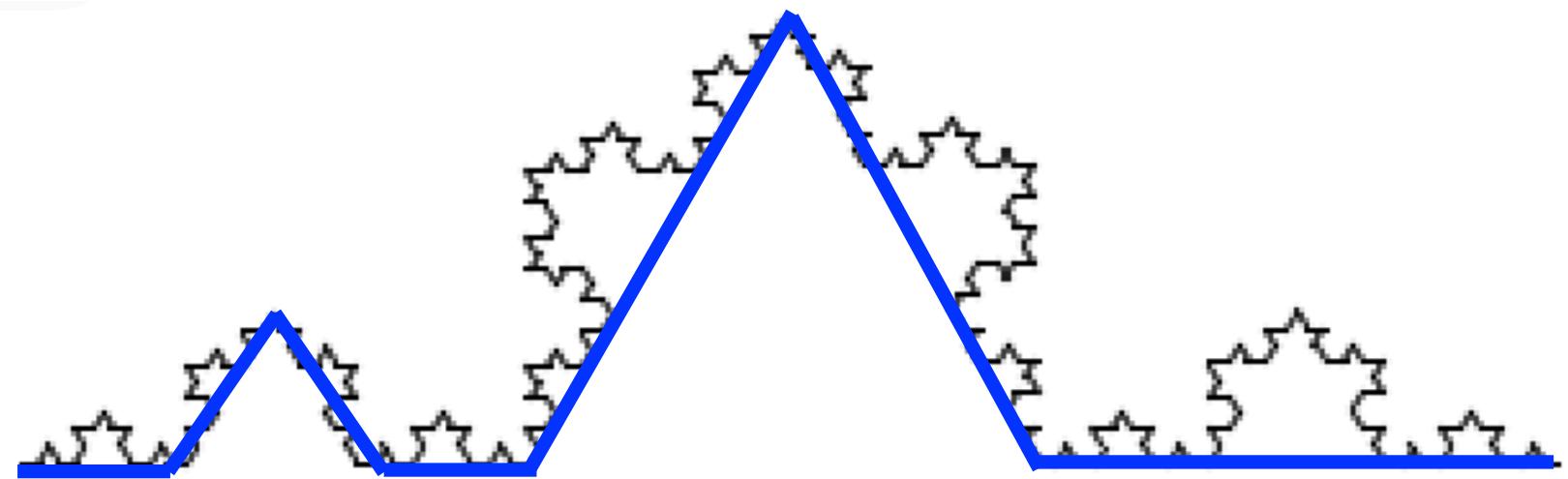
Depth 1 Snowflake Line



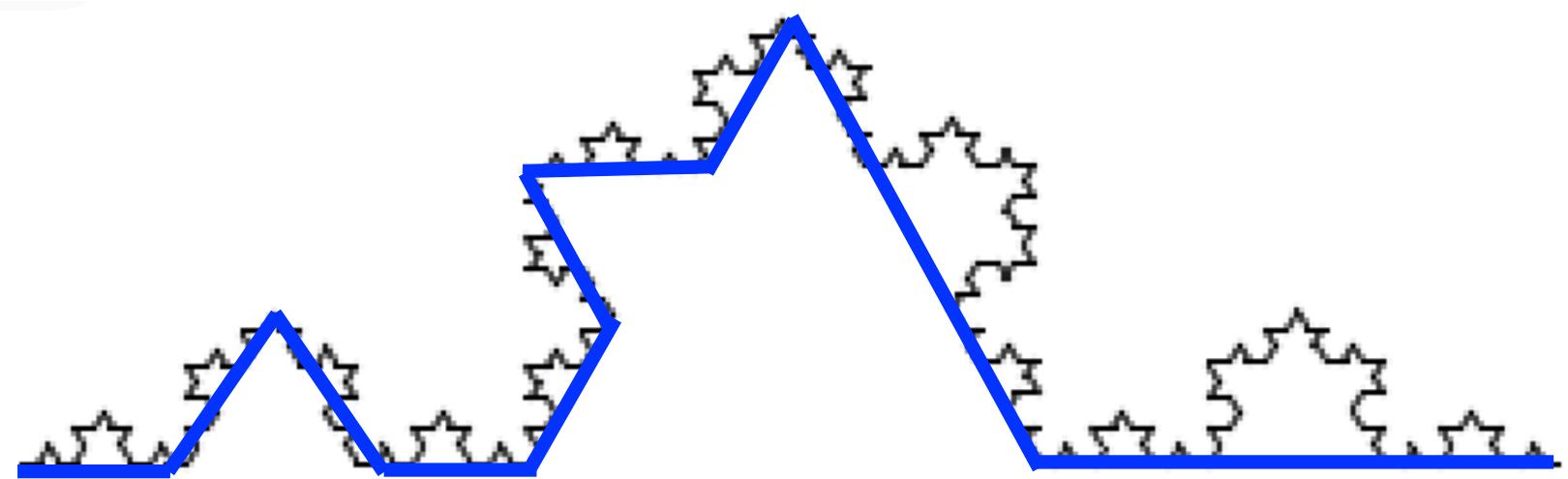
Depth 2 Snowflake Line



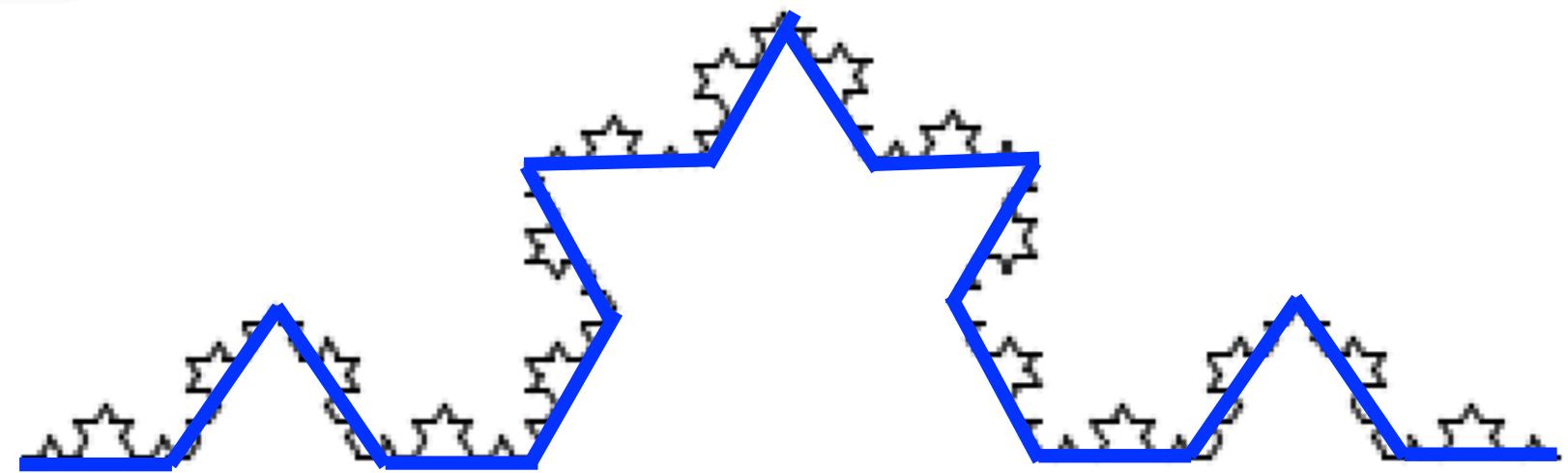
Depth 3 Snowflake Line (in progress)



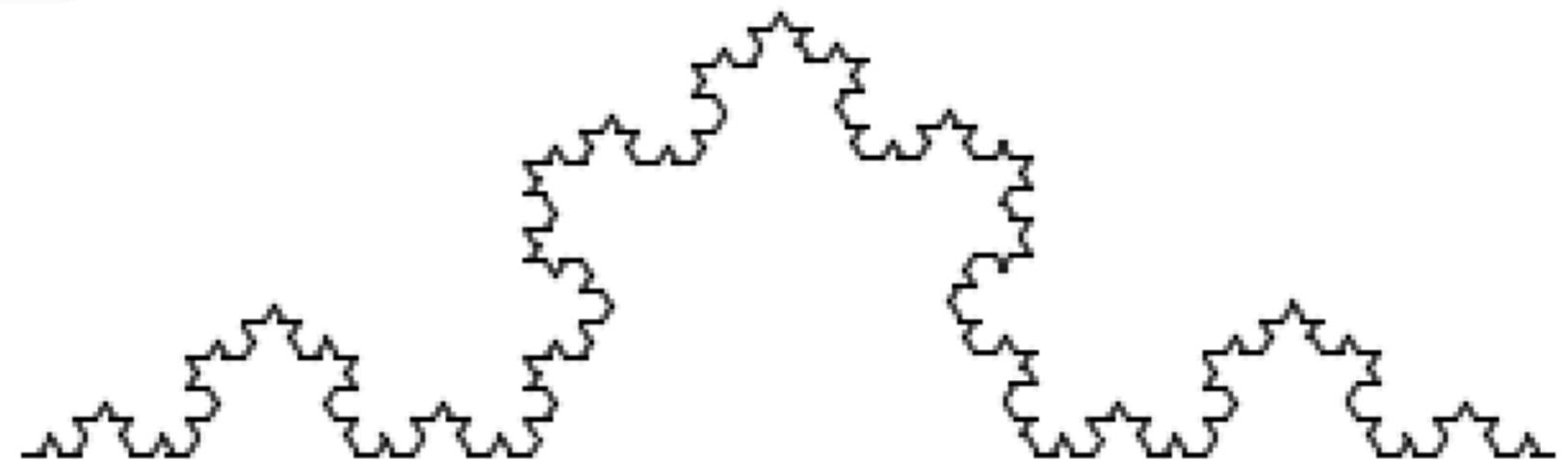
Depth 3 Snowflake Line (in progress)



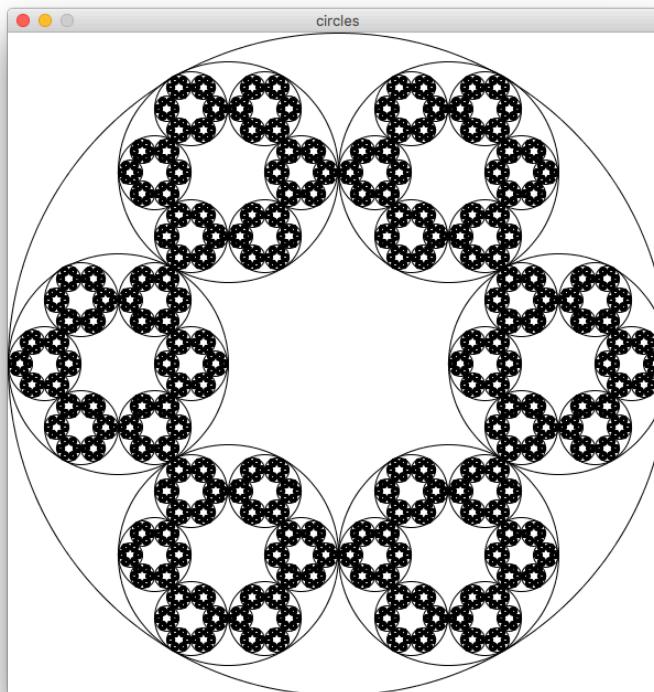
Depth 3 Snowflake Line (in progress)



Depth 3 Snowflake Line (in progress)



Another Example On the Website



Recap

•Fractals

- Fractals are self-referential, and that makes for nice recursion problems!
- Break the problem into a smaller, self-similar part, and don't forget your base case!



References and Advanced Reading

- **References:**

- <http://www.cs.utah.edu/~germain/PPS/Topics/recursion.html>
- Why is iteration generally better than recursion? <http://stackoverflow.com/a/3093/561677>

- **Advanced Reading:**

- Tail recursion: <http://stackoverflow.com/questions/33923/what-is-tail-recursion>
- Interesting story on the history of recursion in programming languages: <http://goo.gl/P6Einb>



Extra Slides

