HOMEWORK 1

PROBLEMS RAISED IN LECTURES

Problem 1: For a free SDOF system with the EOM $\ddot{x} + 2\xi\omega_n\dot{x} + {\omega_n}^2 x = 0$, if the damping ratio $\xi < 1$, the homogeneous solution (i.e., the transient response) can be expressed as the following complex form $x_h = e^{-\xi\omega_n t}(\alpha e^{i\omega_d t} + \beta e^{-i\omega_d t})$. Based on this, please derive the homogeneous solution in real form. If the damping ratio $\xi = 1$, we can just obtain one particular solution as $e^{-\omega_d t}$, please derive another particular solution so that the homogeneous solution can be constructed. (10 points)

Problem 2: For a SDOF system subjected to harmonic excitations, the EOM is $m\ddot{x} + c\dot{x} + kx = A_0 \sin \omega_f t$, the particular solution (i.e., the steady-state response) can be expressed as the following real form

$$x_p = \frac{A_0/k}{\sqrt{\left(1-\left(\frac{\omega_f}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega_f}{\omega_n}\right)^2}} sin(\omega_f t - \phi_f), \text{ where } \phi_f = arctan\left(\frac{2\xi\frac{\omega_f}{\omega_n}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}\right).$$

Based on this, please derive the particular solution in complex form if the excitation force is $A_0 e^{i\omega_f t}$. (7 points)

DERIVE EOM AND CALCULATE NATURAL FREQUENCY

Problem 3: A disk of mass m and radius R rolls without slip while restrained by a dashpot with coefficient of viscous damping c in parallel with a spring of stiffness k, as shown in figure. Derive the differential equation by the Newton's second law for the displacement $x(t)$ of the disk mass center C and determine the viscous damping factor ζ and the frequency ω_n of undamped oscillation. (8 points)

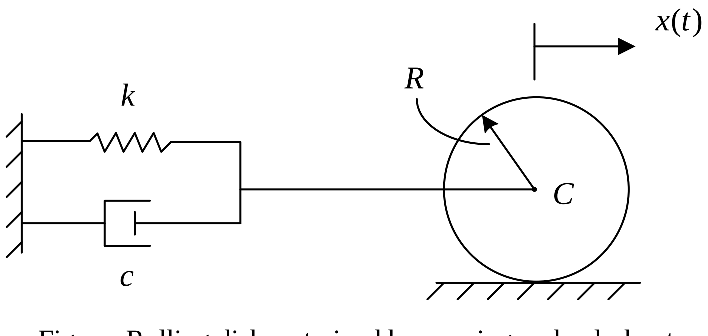


Figure: Rolling disk restrained by a spring and a dashpot

CALCULATE THE VALUE OF CRITICAL DAMPING

Problem 4: Derive the differential equation by the Newton's second law. Calculate the frequency of damped oscillation of the system shown in the following figure for the values $m = 1750\text{kg}$, $c = 3500 N \cdot \text{s/m}$, $k = 7 \times 10^5 \text{N/m}$ $a = 0.9m$ and $b = 2m$. Determine the value of the critical damping. (10 points)

FAET820005 MECHANICAL VIBRATIONS HW1

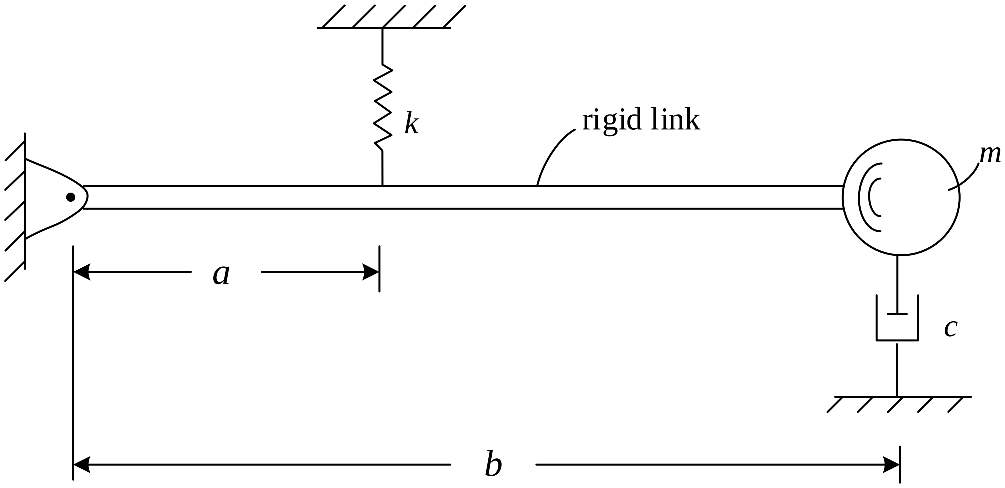
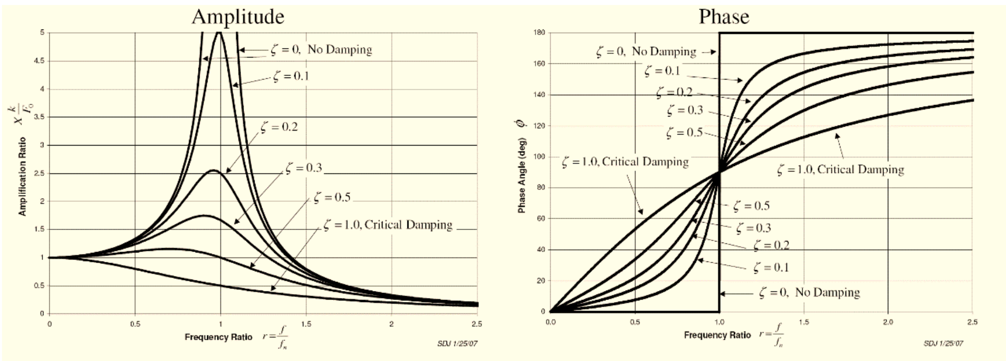


Figure: Mass supported by a spring and dashpot through a rigid bar

GENERATE FREQUENCY-RESPONSE DIAGRAM

Problem 5: Using Mathematica/MATLAB, please derive the steady-state response (i.e., the particular solution) of the following SDOF system $m\ddot{x} + c\dot{x} + kx = F_0\, sin(\,2\pi f_f t)$. Based on the response, please regenerate the following figures. In addition to the printed results, the Mathematica/Matlab code should be uploaded to the eLearning system. (10 points)



RESPONSE UNDER HARMONIC DISPLACEMENT EXCITATION

Problem 6: In practice, it's hard to apply a periodic force to a system. Consider the most fundamental of seismometers, which often measure the relative displacement directly using an optical transducer (e.g. a light beam deflected by a mirror on the mass). Now we will focus on another system that a vehicle traveling on a rough road. Let the vehicle velocity be uniform, $v = \text{const}$, and calculate the response $z(t)$, as well as the force transmitted to the vehicle. (10 points)

FAET820005 MECHANICAL VIBRATIONS HW1

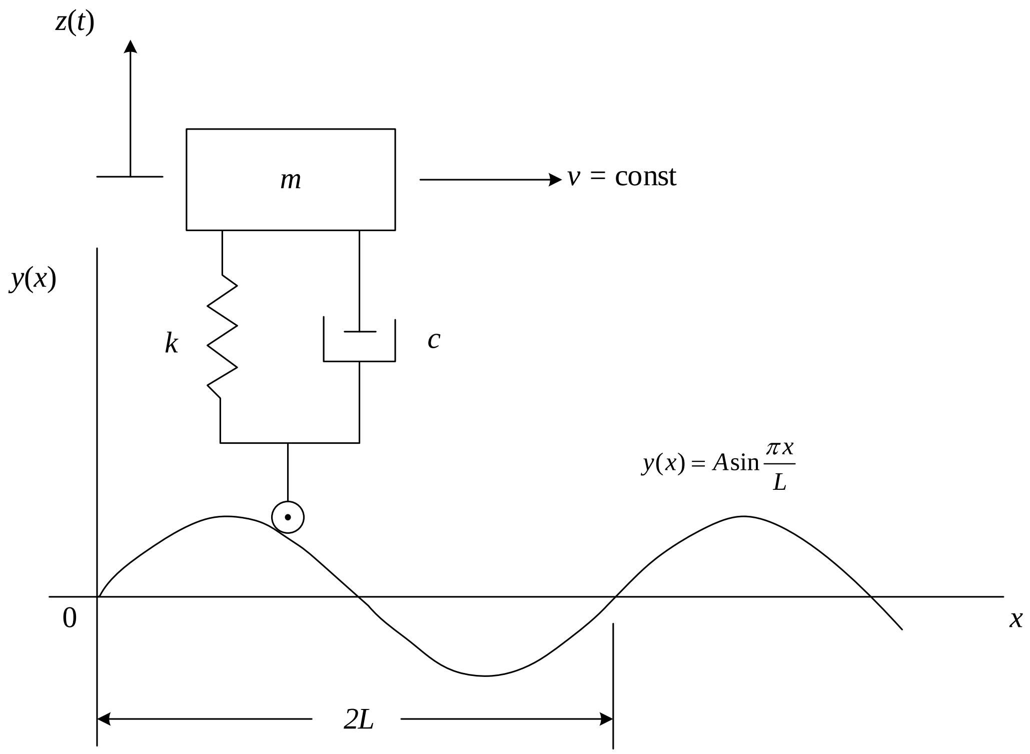


Figure: Vehicle traveling on a rough road

RESPONSE UNDER RANDOM PERIODIC EXCITATION VIA FOURIER ANALYSIS

Problem 7: Solve the differential equation

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

by means of a Fourier analysis, where $f(t)$ is the periodic function shown in the following figure, $m = 1$ kg, $c =$ 0.2 N·s/m, $k = 10$ N/m. Please use the function 'ode45' to generate and plot its response in MATLAB (15 points)

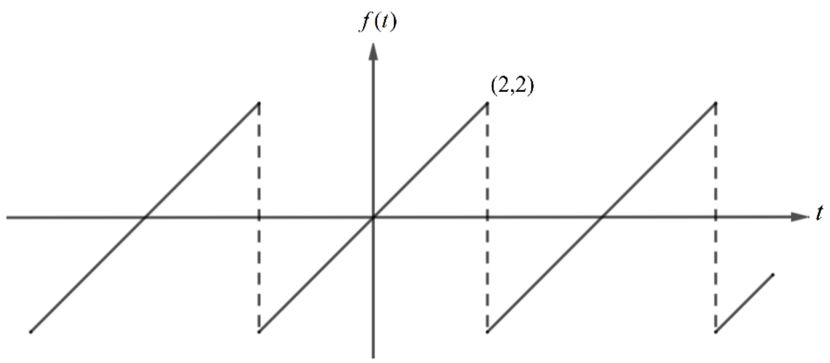


Figure: Periodic excitation

RESPONSE UNDER RANDOM NON-PERIODIC EXCITATION VIA SUPERPOSITION PRINCIPLE

Problem 8: Use the superposition principle to derive the response of the single-degree-of-freedom system in Problem 10 but at the triangular pulse shown in. (15 points)

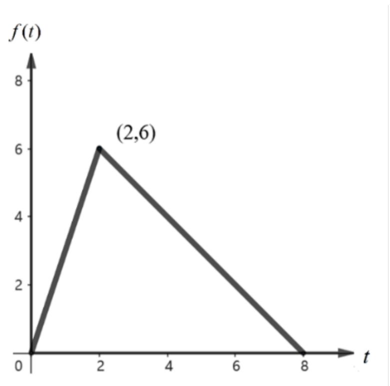


Figure: Fore in the form of a triangular pulse

RESPONSE UNDER RANDOM NON-PERIODIC EXCITATION VIA CONVOLUTION INTEGRAL

Problem 9: Derive the response of a viscously damped single-degree-of-freedom system to the force $F(t) = F_0(1 - e^{-\alpha t})u(t)$ by means of the convolution integral. The system parameters are: $m = 12\text{kg}$, $c = 24N \cdot \text{s/m}, k = 4800\text{N/m}$. Use convolution integral to plot the response to a step function of magnitude $F_0 = 100N$, $\alpha = 1$. Then, compare the result with the simulation using ode45 method. (15 points)

Hint: the impulse response of a viscously damped single-degree-of-freedom system is

$$g(t) = \frac{1}{m\omega_d} e^{-\xi\omega_n t} \sin(\omega_d t)$$

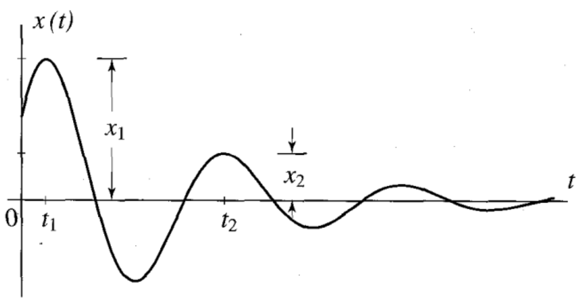
ADDITIONAL PROBLEMS (INVERSE PROBLEM)

Additional Problem: Measurements of the response peak amplitudes of a vibrating single-degree-of-freedom system has yielded the values $x_1 = 14.88$, $x_2 = 12.99$, $x_3 = 11.49$, $x_4 = 10.56$, $x_5 = 9.16$, $x_6 = 7.85$, $x_7 = 7.04$ and $x_8 = 5.96$. Develop a least squares approach to determine the "best" viscous damping factor ζ. (10 points)

Hint: You can use some basic transformations like $z_i = ln(x_i)$ to minimize the sum of the squares of loss.

$$\frac{x_1}{x_2} = \frac{e^{-\zeta\omega_n t_1}\cos(\omega_d t_1 - \phi)}{e^{-\zeta\omega_n (t_1+T)}\cos[\omega_d(t_1+T) - \phi]} = \frac{e^{\zeta\omega_n T}\cos(\omega_d t_1 - \phi)}{\cos(\omega_d t_1 - \phi + 2\pi)}$$

$$= e^{\zeta\omega_n T} = e^{2\pi\zeta\omega_n/\omega_d} = e^{2\pi\zeta/\sqrt{1-\zeta^2}}$$



ADDITIONAL PROBLEMS (INVERSE PROBLEM)

Additional Problem: Please read the "Reading Material 1", and solve the problems listed at the end (30 points)