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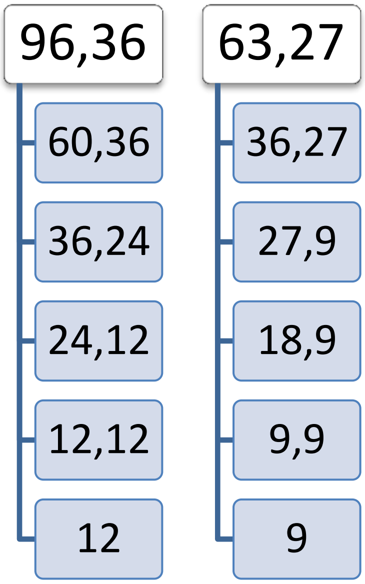
1. Euclid's algorithm computes the GCD of two numbers. It is based on the principle that the GCD of two numbers does not change when the smaller number is subtracted from the larger number. For example, the GCD of 35 and 14 is also the GCD of 21 and 14. Explain briefly why this is true.

Answer: Suppose that d is the GCD of the two input numbers. Then the larger number A can be written as “a × d” for some positive integer a; the smaller number B can be written as “b × d” for some positive integer b. While we use A minus B, we will get number C which can be described as “(a - b) × d”. Since a, b are both integers, the result of “a - b” should also be an integer. Clearly, d is still the divisor of the number C.

2. The property described in problem 1 can be applied repeatedly until the two numbers are equal. This number is the GCD of the original two numbers. An execution tree of this algorithm for 35 and 14 is shown below. Construct similar trees to find the GCD of 96 and 36 and the GCD of 63 and 27.

Answer:



3. Write the pseudo code for the algorithm described in problem 2.

Answer:

```
1)  Read 2 numbers a, b given as input.
2)  If “a>b”, use “a-b” as “a”.
3)  If “b>a”, use “b-a” as “b”.
4)  If a=b, stop. Otherwise go to line 2.
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5. Test your program by turning your birthdate into a decimal number. For example, if you were born on June 9th, 1995 you would use the number 19950609. To answer this question, write down your birthdate and the GCD of it and the numbers 75, 144, and 390.

Answer: My birthdate is 19920718.

1) The GCD of 19920718 and 75 is 1
2) The GCD of 19920718 and 144 is 2
3) The GCD of 19920718 and 390 is 2

6. The pseudo code for a more efficient implementation of Euclid's algorithm is given below:

```
Enter the larger number (the dividend)
Enter the smaller number (the divisor)
Calculate the remainder of dividing the dividend by the divisor
While the remainder is greater than zero
    Set the dividend to the value of the divisor
    Set the divisor to the value of the remainder
    Calculate the remainder of dividing the new dividend by the new divisor
Output the divisor as the GCD
```

Describe briefly the principle (similar to the one described in problem 2) on which this algorithm is based.

Answer: Suppose that d is the GCD of the two input numbers. Then the larger number A can be written as "a × d" for some positive integer a; the smaller number B can be written as "b × d" for some positive integer b. While we use A divided by B, the remainder R can be described as "a × d - k × b × d" or "(a - k × b) × d". Since a, b, k are all integers, the result of "a - k × b" should also be an integer. Clearly, d is still the divisor of the remainder R.

8. Use your program to compute the GCD of February 8, 2012 and the numbers 75, 144, and 390.

Answer:

1) The GCD of 20120208 and 75 is 3
2) The GCD of 20120208 and 144 is 48
3) The GCD of 20120208 and 390 is 6