Tugas 4
Probabilitas dan Proses Stokastika

Problem 21.2.
A gambler plays 120 hands of draw poker, 60 hands of black jack, and 20 hands of

stud poker per day. He wins a hand of draw poker with probability $1/6$, a hand of black jack with probability $1/2$, and a hand of stud poker with probability $1/5$.

(a) What is the expected number of hands the gambler wins in a day?

(b) What would the Markov bound be on the probability that the gambler will win at least 108 hands on a given day?

(c) Assume the outcomes of the card games are pairwise independent. What is the variance in the number of hands won per day?

(d) What would the Chebyshev bound be on the probability that the gambler will win at least 108 hands on a given day? You may answer with a numerical expression that is not completely evaluated.

Problem 21.3.
The hat-check staff has had a long day serving at a party, and at the end of the party they simply return people's hats at random. Assume that n people checked hats at the party.

(a) What is the expected number of people who get their own hat back?

Let $X_i = 1$ be the indicator variable for the ith person getting their own hat back. Let $S_n = \sum_{i=1}^{n} X_i$, so S_n is the total number of people who get their own hat back.

(b) Write a simple formula for $\mathrm{E}[X_i X_j]$ for $i \neq j$. *Hint:* What is $\Pr\{X_j = 1 \mid X_i = 1\}$?

(c) Explain why you cannot use the variance of sums formula to calculate $\mathrm{Var}[S_n]$.

(d) Show that $\mathrm{E}\left[S_n^2\right] = 2$. *Hint:* $X_i^2 = X_i$.

(e) What is the variance of S_n?

(f) Use the Chebyshev bound to show that the probability that 11 or more people get their own hat back is at most 0.01.