Computational Structural Analysis 2D Direct Stiffness Code

Enis BAHADIR

10545700

June 24, 2017

1 Benchmarks

The accuracy and robustness of the code is tested by solving some sample problems, the exact solutions of which were computed apriori via the classical beam theory. The results are presented in this section using the following scheme:

- Demonstration of the problem
- Input file that addresses the problem
- Output file generated by the program after analysis
- Diagrams obtained by postprocessing
- Comparison of the results of numerical and analytical solutions

Each problem considered in the first part comprise a single or continuous beam subjected to a certain external loading and set of restraints. The second part deals with a one-bay one-storey frame under different loads and support conditions. Then some problems that involve more specialized cases of matrix structural analysis are investigated, such as beams with shear deformability, elastic restraints and links and thermal loadings. The fourth and final part is devoted to the analysis of prestressed beams.

The error in the desired quantities for each numerical solution is presented by using the true percent relative error according to the following definition:

$$\epsilon_{r,t} = \frac{|exact\ value - numerical\ value|}{|exact\ value|} \times 100$$

1.1 Single Beam Under Various Loading and Support Conditions

1.1.1 Problem 1: Simple Beam - Uniformly Distributed Load

```
2DSTIFF - INPUT FILE
[kN/m/C]

# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
```

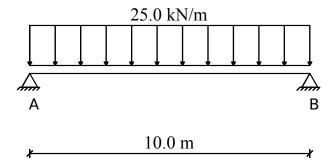


Figure 1: Problem 1: Loading, geometry and supports

```
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1 0.0 -25.0 -25.0 0.0 0.0
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
Node
             x-coord
                         y-coord
               0.00
                           0.00
 1
 2
              5.00
                           0.00
 3
              10.00
                           0.00
# SECTIONS
Sec.No
               .3200D+00
                                  .1707D-01
                                                       0.8000
                                                                          0.0000
 1
```

# MATERIA Mat.No	LS E		v		alpha	gamma
1	.3000D+0	18	0.20		.0000D+00	0.0000
# ELEMENT		P., 4	Nada	C+	Matania 1	
EI.NO 1	Start Node 1	End 2		Section 1	Material 1	
2	2	3		1	1	
# RESTRAI						
Restraint			Direc			
1		1	1			
2 3		1	2 1			
4		3	2			
•		Ü	-			
# LINKS						
Link No	Master	Node	Slave	Node	Direction	
	RESTRAINTS No Nod	10	Direc	tion	K	
E. Resti.	NO NOC	le	Direc	.01011	K	
# ELASTIC	LINKS					
E. Link.	No Node	: 1	Nod	le 2	Direction	K
# NODAL L						
Node	Direction		Magnitude			
# ELEMENT	I NADS					
		Py1	Py2	DTto	p DTbottom	
1		-25.00				
2	0.00	-25.00	-25.00	0.0	0.00	
# PRESTRE			•	_		
Element	e1	em	e2	P		
=======	=========	.=====			==========	.========
ANALYSIS	RESULTS					
MODAT	D T A DT A A DT A DT A DT					
>> NUDAL !	DISPLACEMENTS					
Node	DX		DY		PHI	
1	0.000000000D+	00	0.000000000	00D+00	-0.2034505208D-0)2
	0.000000000D+		-0.635782877		0.00000000D+0	
3	0.000000000D+	00	0.000000000	00D+00	0.2034505208D-0)2
>> MEMBER	END FORCES					
MEM N	1 T	' 1	M1	N2	T2	M2
					00 0.00000D+00	
					00 0.12500D+03	
=======						

Toggle Diagrams 1: Original configuration 2: Deformed shape 3: Axial force 4: Shear force 5: Bending moment f: Fill diagram v: Show values r: Reset camera 125 Displacement Scale

Figure 2: Problem 1, Deformed Shape

Shear force diagram

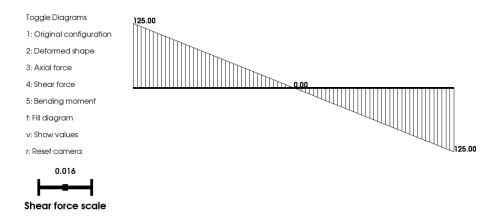


Figure 3: Problem 1, Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$rac{\omega l}{2}$	0.12500E+03	0.12500E+03	0.0%
M_{max}	$\frac{\omega l^2}{8}$	0.31250E+03	0.31250E+03	0.0%
δ_{max}	$\frac{5\omega l^4}{384EI}$	-0.63578E-02	-0.63578E-02	0.0%

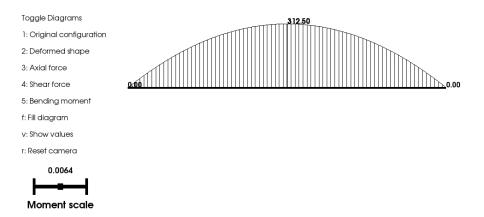


Figure 4: Problem 1, Bending Moment Diagram

1.1.2 Problem 5: Simple Beam - Load Increasing Uniformly To One End

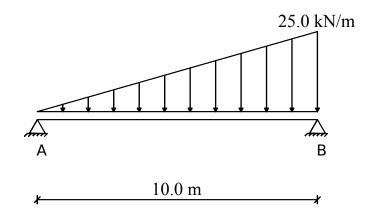


Figure 5: Problem 5: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
2 1
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1 0.0 0.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
```

NODE COORD			========		=========	=======
Nobe Cookb Node	x-coord	y-coord				
1	0.00	0.00				
2	10.00	0.00				
* SECTIONS						
Sec.No	A		I		h	Х
1	.3200D+0	0	.1707D-01		0.8000	0.0000
# MATERIALS						
Mat.No	Ε	•	V		alpha	gamma
1	.3000D+0	3	0.20		0000D+00	0.0000
# ELEMENTS					.	
El.No S 1	tart Node 1	End No 2	de Sect 1		Material 1	
1	1	2	1		1	
RESTRAINTS		,	D			
Restraint No			Direction	L		
1 2		1 1	1 2			
3		2	1			
4		2	2			
LINKS						
Link No	Master	Node	Slave Nod	le	Direction	
# ELASTIC RE	STRAINTS					
E. Restr. No	Node	е	Direction	L	K	
ELASTIC LI	NKS					
E. Link. No	Node	1	Node 2		Direction	K
NODAL LOAD	S					
lode	Direction	Ma	gnitude			
ELEMENT LO	ADS					
Element	Px	Py1	Py2	\mathtt{DTtop}	${\tt DTbottom}$	
1	0.00	0.00	-25.00	0.00	0.00	
PRESTRESSI	NG					
Element	e1	em	e2	P		
			=======	======		=======
ANALYSIS RES	ULTS =======			=======	=======================================	
>> NODAL DIS	PLACEMENTS					
lode	DX		DY		PHI	
1 0.0	00000000D+	0 0	000000000D+0	0 -0	.9494357639D-03	3
	00000000D+				.1085069444D-02	
> MEMBER EN	D FORCES					

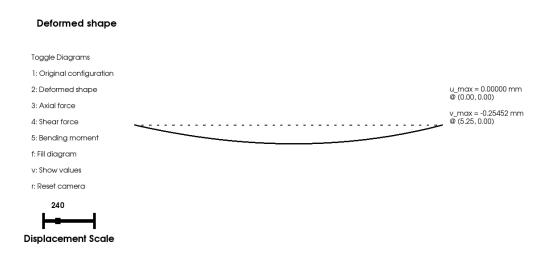


Figure 6: Problem 5, Deformed Shape

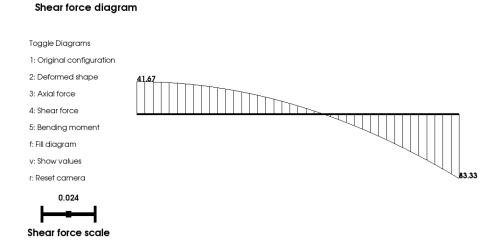


Figure 7: Problem 5, Shear Force Diagram

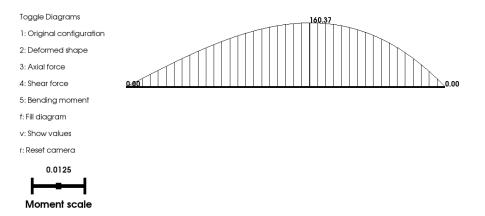


Figure 8: Problem 5, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{\omega l}{6}$	0.41667E+02	0.41667E+02	0.0%
V_B	$\frac{\omega l}{3}$	0.83333E+02	0.83333E+02	0.0%
M_{max}	$\frac{\omega l^2}{9\sqrt{3}}$	0.16037E+03	0.16037E + 03	0.0%

1.1.3 Problem 6: Simple Beam - Load Increasing Uniformly To Center

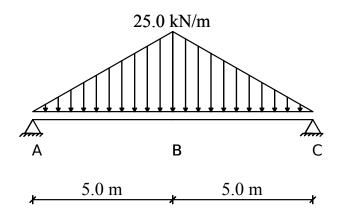


Figure 9: Problem 6: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
```

```
1 0.0 0.0 -25.0 0.0 0.0
2 0.0 -25.0 0.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
         x-coord
                    y-coord
Node
          0.00
                     0.00
 1
 2
           5.00
                      0.00
           10.00
                      0.00
 3
# SECTIONS
Sec.No
              Α
                                              h
                                                              X
                               Т
                         .1707D-01
            .3200D+00
 1
                                             0.8000
                                                             0.0000
# MATERIALS
             E
                                            alpha
                                                             gamma
            .3000D+08
                                                            0.0000
 1
                             0.20
                                           .0000D+00
# ELEMENTS
El.No
        Start Node
                      End Node
                                  Section
                                              Material
 1
            1
                       2
                                    1
                                              1
                       3
 2
                                                1
# RESTRAINTS
Restraint No
                 Node
                              Direction
 1
                  1
                               1
 2
                   1
                                 2
 3
                   3
                                 1
 4
                   3
                                 2
# LINKS
Link No
             Master Node
                            Slave Node
                                            Direction
# ELASTIC RESTRAINTS
E. Restr. No
              Node
                            Direction
# ELASTIC LINKS
E. Link. No
                              Node 2
               Node 1
                                           Direction
                                                          K
# NODAL LOADS
Node Direction Magnitude
# ELEMENT LOADS
                                       DTtop
                                                DTbottom
                   Py1
                            Py2
Element
          Px
          0.00
                                                  0.00
 1
                   0.00
                            -25.00
                                        0.00
          0.00
                   -25.00
                             0.00
                                        0.00
                                                  0.00
# PRESTRESSING
                               e2
                                         Ρ
Element e1
                     em
ANALYSIS RESULTS
______
>> NODAL DISPLACEMENTS
```

PHI

DY

Node

DX

```
0.000000000D+00
                                 0.00000000D+00
  1
                                                          -0.1271565755D-02
                                                       0.00000000D+00
  2
          0.000000000D+00
                                  -0.4069010417D-02
          0.000000000D+00
                                 0.00000000D+00
                                                           0.1271565755D-02
  3
>> MEMBER END FORCES
MEM
         N1
                        T1
                                       M1
                                                      N2
                                                                     T2
                                                                                    M2
 1 0.00000D+00 0.62500D+02 -0.39080D-13 0.00000D+00 0.00000D+00 0.20833D+03
 2 0.00000D+00 0.00000D+00 -0.20833D+03 0.00000D+00 0.62500D+02 0.39080D-13
  Deformed shape
 Toggle Diagrams
  1: Original configuration
                                                                                     u_max = 0.00000 mm
@ (0.00, 0.00)
 2: Deformed shape
 3: Axial force
                                                                                     v_max = -0.40690 mm
@ (5.00, 0.00)
 4: Shear force
 5: Bending moment
 f: Fill diagram
 v: Show values
 r: Reset camera
```

Figure 10: Problem 6, Deformed Shape

Displacement Scale

Shear force diagram

Toggle Diagrams 1: Original configuration 2: Deformed shape 3: Axial force 4: Shear force 5: Bending moment f: Fill diagram v: Show values r: Reset camera 0.032 Shear force scale

Figure 11: Problem 6, Shear Force Diagram

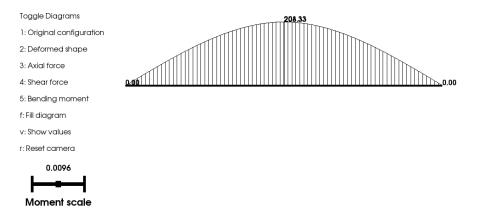


Figure 12: Problem 6, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{\omega l}{4}$	0.62500E+02	0.62500E+02	0.0%
M_{max}	$\frac{\omega l^2}{12}$	0.20833E+03	0.20833E+03	0.0%
δ_{max}	$\frac{\omega l^4}{120EI}$	-0.40690E-02	-0.40690E-02	0.0%

1.1.4 Problem 9: Simple Beam - Two Equal Concentrated Loads Symmetrically Placed

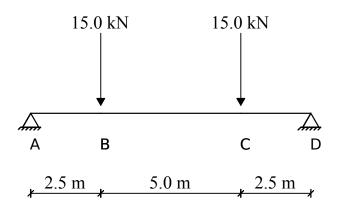


Figure 13: Problem 9: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
2.5 0.0
5.0 0.0
7.5 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.017066666666666 0.8 0.0
# SECTION INCIDENCES
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
5 1
5 2
# LINKS <number of links> <master node, slave node, direction>
```

```
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2 2 -15.0
4 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
______
# NODE COORDINATES
Node
          x-coord
                     y-coord
 1
            0.00
                       0.00
                       0.00
 2
            2.50
 3
            5.00
                       0.00
 4
            7.50
                       0.00
            10.00
 5
                       0.00
# SECTIONS
              Α
Sec.No
                                 I
                                                 h
                                                                  Х
                             .1707D-01
 1
             .3200D+00
                                                0.8000
                                                                0.0000
# MATERIALS
Mat.No
                Ε
                                                alpha
                                                                 gamma
             .3000D+08
                                0.20
                                              .0000D+00
                                                                0.0000
 1
# ELEMENTS
El.No
         Start Node
                       End Node
                                    Section
                                                Material
 1
             1
                         2
                                     1
                                                 1
 2
             2
                         3
                                       1
                                                   1
 3
             3
                         4
 4
             4
                         5
                                       1
                                                   1
# RESTRAINTS
Restraint No
                  Node
                                Direction
 1
                   1
                                   1
 2
                                   2
                    1
 3
                    5
                                   1
 4
                    5
                                    2
# I.TNKS
Link No
             Master Node
                              Slave Node
                                                Direction
# ELASTIC RESTRAINTS
E. Restr. No
                                Direction
            Node
                                                    K
# ELASTIC LINKS
E. Link. No
                Node 1
                               Node 2
                                              Direction K
# NODAL LOADS
Node
     Direction
                          Magnitude
                           -15.00
 2
              2
                           -15.00
# ELEMENT LOADS
Element
                    Py1
                          Py2
                                       DTtop
          Px
                                                   DTbottom
```

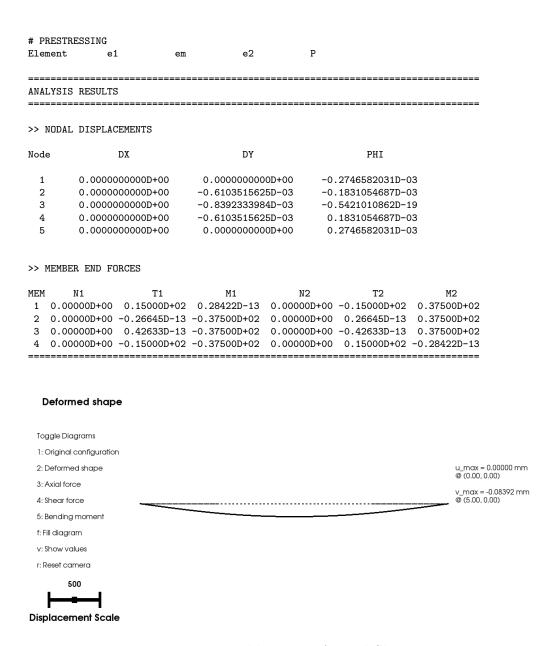


Figure 14: Problem 9, Deformed Shape

	Exact Expression	Exact Value	Computed Value	% RE
V	Р	0.15000E+02	0.15000E+02	0.0%
M_{max}	Pa	0.37500E+02	0.37500E+02	0.0%
δ_{max}	$\frac{Pa}{24EI}(3l^2 - 4a^2)$	-0.83923E-03	-0.83923E-03	0.0%

Shear force diagram

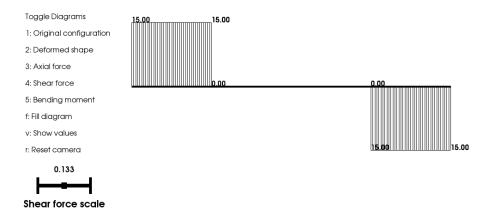


Figure 15: Problem 9, Shear Force Diagram

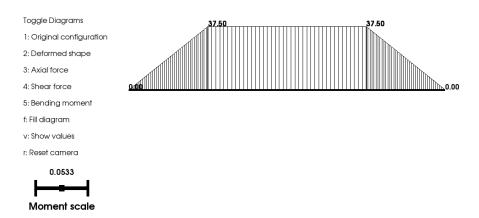


Figure 16: Problem 9, Bending Moment Diagram

1.1.5 Problem 14: Cantilever Beam - Concentrated Load At Any Point

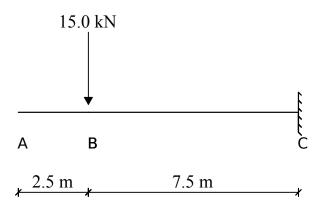


Figure 17: Problem 14: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
2.5 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
```

0		======				
[kN/m/C]					==========	
	COORDINATES					
Wode	x-coord	у-соо:	rd			
1	0.00	0.0	0			
2	2.50					
3	10.00	0.0	Ü			
# SECTIO	INS					
Sec.No	A		I		h	X
1	.3200D+	00	.1707D-01		0.8000	0.0000
# MATERI	ALS					
Mat.No	E		v		alpha	gamma
1	.3000D+	80	0.20		.0000D+00	0.0000
# ELEMEN	ita					
		End	Node Sec	tion	Material	
1	1	2		1	1	
2	2	3		1	1	
# RESTRA		ode	Directio	on.		
1		3	1			
2		3	2			
3		3	3			
# LINKS Link No	Maste	r Node	Slave No	ode	Direction	
# FI ASTI	C RESTRAINTS					
	. No No	de	Directio	on	К	
# ELASTI E. Link.	C LINKS No Nod	e 1	Node 2	2	Direction	K
# NODAL Node 2	LOADS Direction 2		Magnitude -15.00			
# ELEMEN Element		Py1	Py2	DTtop	DTbottom	
# PRESTF	RESSING e1	em	e2	P		
ANALYSIS	RESULTS					
>> NODAL	. DISPLACEMENTS					
Node	DX		DY		PHI	
1 2	0.0000000000 0.0000000000		-0.6179809570D-		0.8239746094D-03 0.8239746094D-03	

0.00000000D+00

0.000000000D+00

3

0.00000000D+00

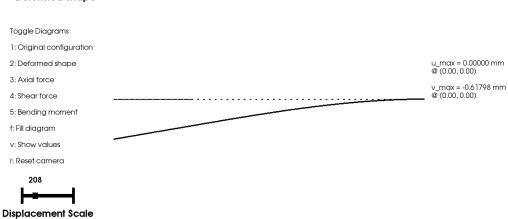


Figure 18: Problem 14, Deformed Shape

Toggle Diagrams 1: Original configuration 2: Deformed shape 3: Axial force 4: Shear force 5: Bending moment f: Fill diagram v: Show values r: Reset camera 0.133 Shear force scale

Shear force diagram

Figure 19: Problem 14, Shear Force Diagram

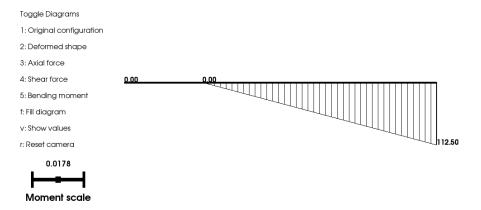


Figure 20: Problem 14, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	Р	0.15000E+02	0.15000E+02	0.0%
M_{max}	Pb	0.11250E+03	0.11250E + 03	0.0%
δ_{max}	$\frac{Pb^2}{6EI}(3l-b)$ Pb^3	-0.61798E-02	-0.61798E-02	0.0%
δ_B	$\frac{Pb^3}{3EI}$	-0.41199E-02	-0.41199E-02	0.0%

1.1.6 Problem 15: Beam Fixed at One End, Supported at Other- Uniformly Distributed Load

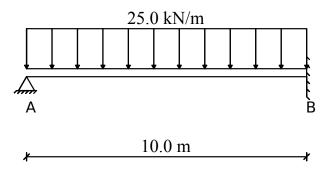


Figure 21: Problem 15: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
4.215 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1 0.0 -25.0 -25.0 0.0 0.0
```

```
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
2DSTIFF - OUTPUT FILE
[kN/m/C]
______
# NODE COORDINATES
                     y-coord
Node
          x-coord
 1
            0.00
                      0.00
            4.21
                       0.00
 2
                       0.00
 3
           10.00
# SECTIONS
Sec.No
              Α
                                 Ι
                                                h
                                                                 Х
            .3200D+00
                             .1707D-01
                                               0.8000
                                                               0.0000
# MATERIALS
Mat.No
               Ε
                                               alpha
                                                                gamma
             .3000D+08
                               0.20
                                             .0000D+00
                                                               0.0000
 1
# ELEMENTS
         Start Node
                       End Node
                                    Section
                                                Material
El.No
                       2
 1
            1
                                     1
                                                1
 2
            2
                         3
                                      1
                                                   1
# RESTRAINTS
                  Node
                               Direction
Restraint No
 1
                   1
 2
                   1
                                   2
 3
                   3
                                   1
 4
                   3
                                   2
 5
                   3
                                   3
# LINKS
Link No
             Master Node
                             Slave Node
                                              Direction
# ELASTIC RESTRAINTS
E. Restr. No
                               Direction
                                                   K
                Node
# ELASTIC LINKS
E. Link. No
                               Node 2
                                                             K
                Node 1
                                              Direction
# NODAL LOADS
                    Magnitude
Node
          Direction
# ELEMENT LOADS
Element
           Px
                    Py1
                               Py2
                                         DTtop
                                                  DTbottom
                   -25.00
                             -25.00
                                                    0.00
           0.00
                                         0.00
 1
                   -25.00
                             -25.00
                                                    0.00
 2
           0.00
                                          0.00
# PRESTRESSING
Element
                                e2
                                           P
ANALYSIS RESULTS
>> NODAL DISPLACEMENTS
Node
             DX
                                DY
                                                   PHI
```

-0.1017252604D-02

0.000000000D+00

0.000000000D+00

1

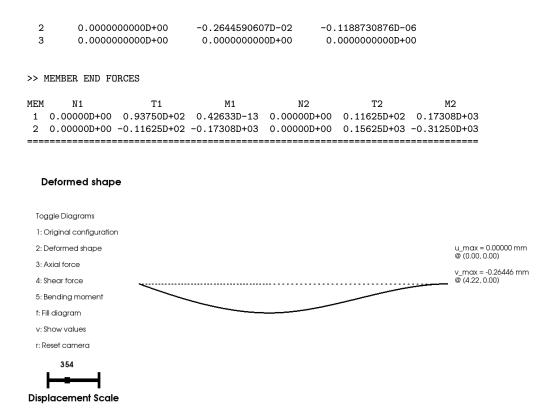


Figure 22: Problem 15, Deformed Shape

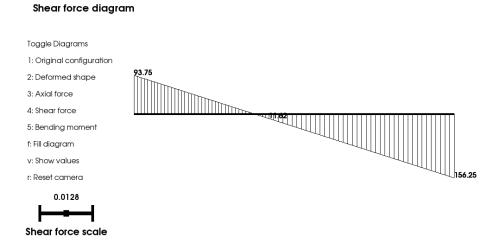


Figure 23: Problem 15, Shear Force Diagram

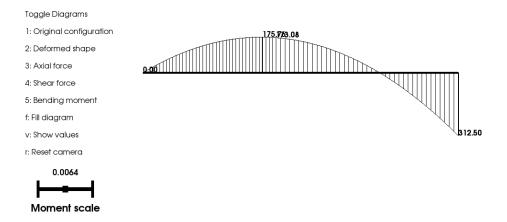


Figure 24: Problem 15, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{3\omega l}{8}$	0.93750E+02	0.93750E+02	0.0%
V_B	$\frac{5\omega l}{8}$	0.15625E+03	0.15625E+03	0.0%
M_{max}^-	$\frac{\omega l^2}{8}$	0.31250E+03	0.31250E+03	0.0%
M_{max}^+	$\frac{9\omega l^2}{128}$	0.17578E+03	0.17576E+03	0.014%
δ_{max}	$\frac{\omega l^4}{185EI}$	-0.26394E-02	-0.26446E-02	0.197%

1.1.7 Problem 19: Beam Overhanging One Support - Uniformly Distributed Load on Overhang

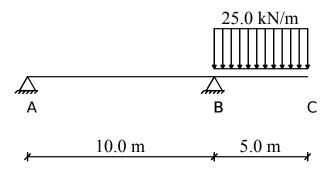
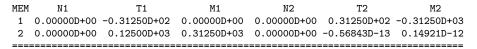


Figure 25: Problem 19: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
10.0 0.0
15.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
2 1
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
```

0						
	- OUTPUT FILE					
======				======	=======================================	=======
	COORDINATES					
Node 1	x-coord 0.00	y-coord 0.00	i			
2	10.00	0.00				
3	15.00	0.00				
# SECTIO	ONS					
Sec.No	A		I		h	Х
1	.3200D+0	00	.1707D-01		0.8000	0.0000
# MATERI	IALS					
Mat.No	E		v		alpha	gamma
1	.3000D+0	8	0.20		.0000D+00	0.0000
# ELEMEN	NTS					
	Start Node		Node Se			
1	1	2		1	1	
2	2	3		1	1	
# RESTRA	AINTS					
Restrair	nt No No		Directi	on		
1 2		1	1 2			
3		2	1			
4		2	2			
# LINKS						
Link No	Master	Node	Slave N	ode	Direction	
# ELASTI E. Restr	IC RESTRAINTS . No Noc	le	Directi	on	K	
L. ROBUI	. 10 100		DITCCUI	OII	II.	
# ELASTI				_		
E. Link.	. No Node	1	Node	2	Direction	K
# NODAL	LOADS					
Node	Direction	l	Magnitude			
# ELEMEN	T LOADS					
Element	Px	Py1	Py2	DTtop	DTbottom	
2	0.00	-25.00	-25.00	0.00	0.00	
# PRESTF	RESSING					
Element	e1	em	e2	P		
	======================================	:======		======		=======
>> NODAL	L DISPLACEMENTS					
Node	DX		DY		PHI	
1	0.000000000D+	-00 (0.00000000D	+00	0.1017252604D-02	
2	0.00000000DH	-00 (0.000000000D		0.2034505208D-02	
3	0.00000000D+	-00 -0	0.1398722331D	-01 -	0.3051757812D-02	

>> MEMBER END FORCES



Deformed shape

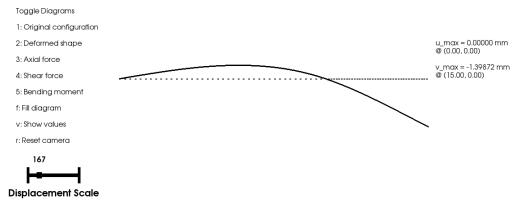


Figure 26: Problem 19, Deformed Shape

Shear force diagram

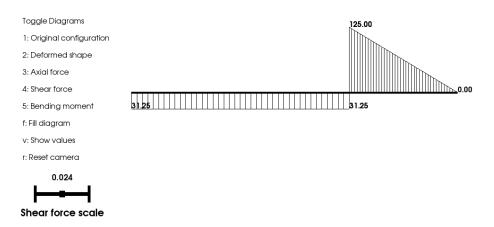


Figure 27: Problem 19, Shear Force Diagram

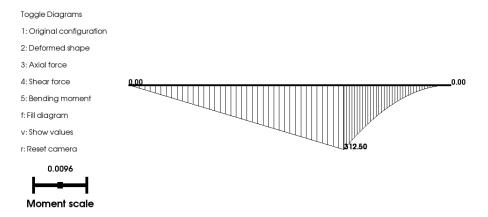


Figure 28: Problem 19, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{\omega a^2}{2l}$	0.31250E+02	0.31250E+02	0.0%
V_B	ωa	0.12500E+03	0.12500E + 03	0.0%
M_{max}	$\frac{\omega a^2}{2}$	0.31250E+03	0.31250E+03	0.0%
δ_{max}	$\frac{\omega a^3}{24EI}(4l+3a)$	-0.13987E-01	-0.13987E-01	0.0%

1.1.8 Problem 24: Beam Fixed at Both Ends - Concentrated Load at Center

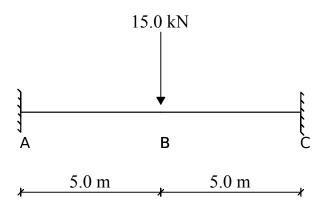


Figure 29: Problem 24: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
1 3
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
```

```
2 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
# PRESTRESSING <number> <Element, e1, em, e2, P>
_____
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
        x-coord
                  y-coord
Node
         0.00
5.00
                    0.00
 1
 2
                    0.00
         10.00
                    0.00
 3
# SECTIONS
Sec.No
            Α
                                                         X
                            Т
                                          h
           .3200D+00 .1707D-01
                                                        0.0000
 1
                                         0.8000
# MATERIALS
Mat.No
            E
                                        alpha
                                                        gamma
           .3000D+08
                                                       0.0000
1
                           0.20
                                        .0000D+00
# ELEMENTS
El.No
        Start Node
                    End Node
                               Section
                                          Material
 1
           1
                    2
                                 1
                                          1
                     3
 2
# RESTRAINTS
Restraint No
               Node
                            Direction
 1
                 1
                            1
 2
                 1
                               2
 3
                               3
                 1
 4
                 3
                              1
 5
                 3
                               2
 6
                 3
                               3
# LINKS
                          Slave Node
                                        Direction
Link No
           Master Node
# ELASTIC RESTRAINTS
E. Restr. No
                          Direction
                                           K
           Node
# ELASTIC LINKS
                           Node 2
E. Link. No
              Node 1
                                       Direction
# NODAL LOADS
Node Direction
                     Magnitude
                       -15.00
 2
# ELEMENT LOADS
                  Py1
                                  DTtop
Element Px
                          Py2
                                           DTbottom
# PRESTRESSING
       e1
                            e2
                                     Р
Element
                   em
ANALYSIS RESULTS
______
>> NODAL DISPLACEMENTS
```

PHI

DY

Node

DΧ

```
0.000000000D+00
                                   0.000000000D+00
                                                             0.000000000D+00
  1
          0.00000000D+00
                                  -0.1525878906D-03
                                                             0.000000000D+00
  2
  3
          0.00000000D+00
                                   0.00000000D+00
                                                             0.000000000D+00
>> MEMBER END FORCES
MEM
         N1
                         T1
                                        M1
                                                       N2
                                                                      T2
                                                                                     M2
 1 0.00000D+00 0.75000D+01 0.18750D+02 0.00000D+00 -0.75000D+01 0.18750D+02
 2 0.00000D+00 -0.75000D+01 -0.18750D+02 0.00000D+00 0.75000D+01 -0.18750D+02
  Deformed shape
 Toggle Diagrams
  1: Original configuration
                                                                                        u_max = 0.00000 mm
@ (0.00, 0.00)
 2: Deformed shape
 3: Axial force
                                                                                        v_max = -0.01526 mm
@ (5.00, 0.00)
 4: Shear force
 5: Bending moment
 f: Fill diagram
 v: Show values
 r: Reset camera
           1e+03
Displacement Scale
```

Figure 30: Problem 24, Deformed Shape

Shear force diagram

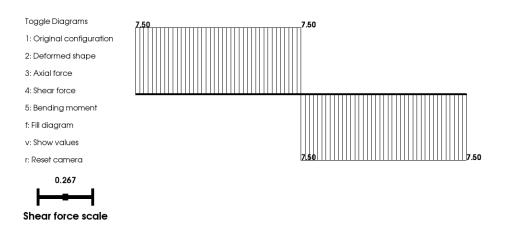


Figure 31: Problem 24, Shear Force Diagram

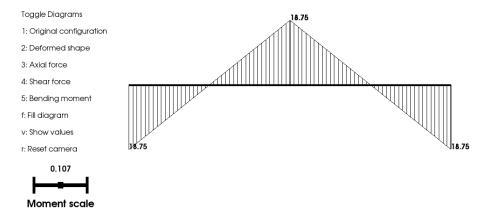


Figure 32: Problem 24, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{P}{2}$	0.75000E+01	0.75000E+01	0.0%
M_{max}	$\frac{Pl}{8}$	0.18750E+02	0.18750E+02	0.0%
δ_{max}	$\frac{Pl^3}{192EI}$	-0.15259E-03	-0.15259E-03	0.0%

1.1.9 Problem 27: Continuous Beam - Two Equal Spans - Concentrated Load At Center of One Span

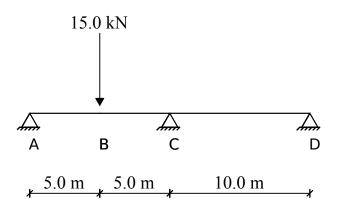


Figure 33: Problem 27: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
5.0 0.0
10.0 0.0
20.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
\mbox{\# SECTIONS} <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
3 1
3 2
4 1
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
```

```
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
______
# NODE COORDINATES
Node
         x-coord
                   y-coord
           0.00
                      0.00
 1
           5.00
 2
                      0.00
 3
           10.00
                       0.00
 4
           20.00
                      0.00
# SECTIONS
Sec.No
                                                                 Х
               Α
                                 Ι
                                                h
            .3200D+00
                             .1707D-01
                                              0.8000
                                                               0.0000
 1
# MATERIALS
              E
Mat.No
                                              alpha
                                                                gamma
            .3000D+08
                               0.20
                                             .0000D+00
                                                               0.0000
 1
# ELEMENTS
El.No
      Start Node
                       End Node
                                   Section
                                               Material
 1
            1
                       2
                                      1
                                                1
 2
            2
                        3
                                      1
                                                  1
 3
# RESTRAINTS
Restraint No
                  Node
                               Direction
 1
                   1
 2
                                   2
                   1
 3
                   3
                                   1
 4
 5
                   4
                                   1
 6
                                   2
# LINKS
Link No
             Master Node
                             Slave Node
                                              Direction
# ELASTIC RESTRAINTS
E. Restr. No
                               Direction
             Node
# ELASTIC LINKS
E. Link. No
               Node 1
                                Node 2
                                              Direction
                                                            K
# NODAL LOADS
Node Direction
                         Magnitude
 2
                          -15.00
             2
# ELEMENT LOADS
                     Py1
                               Py2
                                       DTtop
Element Px
                                                  DTbottom
# PRESTRESSING
Element e1
                                e2
                                          Ρ
```

ANALYSIS RESULTS >> NODAL DISPLACEMENTS Node DX DY PHI -0.1373291016D-03 0.00000000D+00 0.00000000D+00 2 0.00000000D+00 -0.4386901855D-03 0.1144409180D-04 3 0.000000000D+00 0.00000000D+00 0.9155273437D-04 0.000000000D+00 0.00000000D+00 -0.4577636719D-04 4 >> MEMBER END FORCES MEM T1 N1 M1 N2 T2 M2 1 0.00000D+00 0.60938D+01 0.57732D-14 0.00000D+00 -0.60938D+01 0.30469D+02 2 0.00000D+00 -0.89062D+01 -0.30469D+02 0.00000D+00 0.89062D+01 -0.14062D+02 3 0.00000D+00 0.14062D+01 0.14062D+02 0.00000D+00 -0.14062D+01 0.00000D+00 Deformed shape Toggle Diagrams 1: Original configuration 2: Deformed shape u_max = 0.00000 mm @ (0.00, 0.00) 3: Axial force v_max = -0.04397 mm @ (4.75, 0.00) 4: Shear force 5: Bending moment f: Fill diagram v: Show values r: Reset camera 1e+03

Figure 34: Problem 27, Deformed Shape

Displacement Scale

Shear force diagram

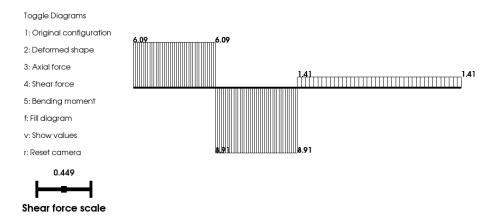


Figure 35: Problem 27, Shear Force Diagram

Bending moment diagram

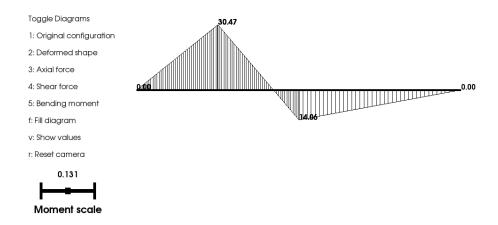


Figure 36: Problem 27, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{13}{32}P$	0.60938E+01	0.60938E+01	0.0%
V_C	$\frac{19}{32}P$	0.89062E+01	0.89062E+01	0.0%
V_D	$\frac{3}{32}P$	0.14062E+01	0.14062E+01	0.0%
M_C	$\frac{3}{32}Pl$	0.14062E+02	0.14062E+02	0.0%
M_{max}	$\frac{13}{64}Pl$	0.30469E+02	0.30469E+02	0.0%

1.1.10 Problem 29: Continuous Beam - Two Equal Spans - Uniformly Distributed Load

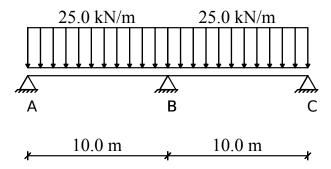


Figure 37: Problem 29: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
3.75 0.0
10.0 0.0
16.25 0.0
20.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
3 1
3 2
5 1
5 2
# LINKS <number of links> <master node, slave node, direction>
```

```
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1 0.0 -25.0 -25.0 0.0 0.0
2 0.0 -25.0 -25.0 0.0 0.0
3 0.0 -25.0 -25.0 0.0 0.0
4 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
         x-coord
                      y-coord
Node
 1
            0.00
                       0.00
 2
            3.75
                        0.00
 3
            10.00
                        0.00
 4
            16.25
                         0.00
 5
            20.00
                        0.00
# SECTIONS
Sec.No
                Α
                                    Т
                                                    h
                                                                      X
 1
              .3200D+00
                               .1707D-01
                                                   0.8000
                                                                     0.0000
# MATERIALS
               E
Mat.No
                                                   alpha
                                                                     gamma
              .3000D+08
                                  0.20
                                                 .0000D+00
                                                                     0.0000
 1
# ELEMENTS
El.No
         Start Node
                        End Node
                                      Section
                                                   Material
 1
             1
                         2
                                        1
              2
                          3
 2
                                         1
                                                      1
 3
              3
                           4
                                         1
                                                      1
 4
# RESTRAINTS
Restraint No
                   Node
                                  Direction
                    1
 1
                                     1
 2
                     1
                                      2
 3
                     3
                                      1
 4
                     3
                                      2
 5
                     5
                                      1
 6
# LINKS
Link No
              Master Node
                                Slave Node
                                                  Direction
# ELASTIC RESTRAINTS
E. Restr. No
                                Direction
                                                      K
               Node
# ELASTIC LINKS
                                  Node 2
E. Link. No
                 Node 1
                                                   Direction
                                                                  K
# NODAL LOADS
Node Direction Magnitude
# ELEMENT LOADS
```

Element		,	Py2	DTtop	DTbottom	
1 2	0.0			0.00	0.00 0.00	
3	0.0			0.00	0.00	
4	0.0			0.00	0.00	
	RESSING		. 0	D		
Element	e1	. em	e2	Р		
						=====
ANALYSI	S RESULTS	5				
						=====
>> NODA	L DISPLAC	CEMENTS				
Node		DX	DY		PHI	
1	0.00000	00000D+00	0.00000000	OD+00 -0.	1017252604D-02	
2	0.00000	00000D+00	-0.260770320	9D-02 -0.	1589457194D-03	
3		00000D+00	0.000000000		6776263578D-19	
4		00000D+00	-0.260770320		1589457194D-03	
5	0.00000	00000D+00	0.00000000	ор+оо о.	1017252604D-02	
>> MEMB	ER END FO	RCES				
MEM	N1	T1	M1	N2	T2	M2
1 0.0	0000D+00	0.93750D+02	0.16342D-12	0.0000D+00	-0.12790D-12 0.175	78D+03
	0000D+00		-0.17578D+03	0.0000D+00	0.15625D+03 -0.312	
	0000D+00	0.15625D+03	0.31250D+03	0.00000D+00		578D+03
4 0.0		-0.56843D-13	-0.17578D+03	0.0000D+00	0.93750D+02 0.000	000D+00
Defor	med shap	e				
Toggle D	iagrams					
1: Origina	al configuration	n				
2: Deforn	ned shape					u_max = 0.00000 mm @ (0.00, 0.00)
3: Axial fo	orce					v max = -0.26336 mm
4: Shear	force					@ (4.06, 0.00)
5: Bendir	ng moment				<u> </u>	
f: Fill diag	gram					
v: Show	values					
r: Reset c	camera					
	500					
	-					
Disblace	ement Scale	9				

Figure 38: Problem 29, Deformed Shape

Shear force diagram

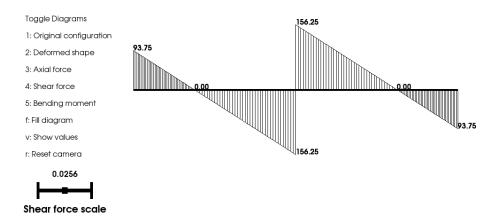


Figure 39: Problem 29, Shear Force Diagram

Bending moment diagram

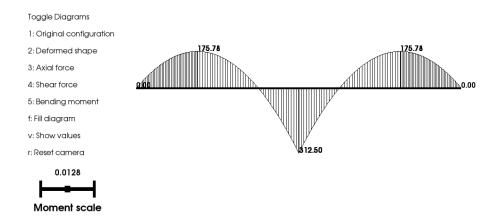


Figure 40: Problem 29, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{3\omega l}{8}$	0.93750E+02	0.93750E+02	0.0%
V_B	$\frac{5\omega l}{8}$	0.15625E+03	0.15625E+03	0.0%
M_{max}^+	$\frac{9\omega l^2}{128}$	0.17578E+03	0.17578E+03	0.0%
M_{max}^-	$\frac{\omega l^2}{8}$	0.31250E+03	0.31250E+03	0.0%
δ_{max}	$\frac{\omega l^4}{185EI}$	-0.26394E-02	-0.26336E-03	0.220%

1.2 One-Bay, One-Storey Frame with Pin and Roller Supports and a Tie Member

The first frame to be analyzed under three different load cases is demonstrated in the figure below. It will be referred to as FR1 in the following parts.

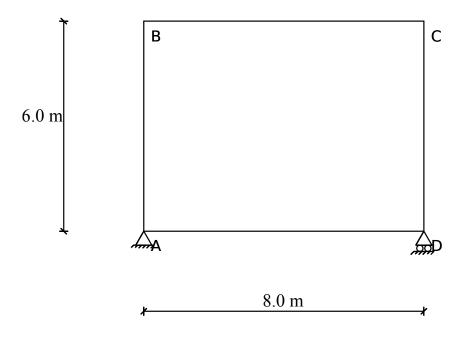


Figure 41: Frame FR1

Three different cross sections are assigned to the four members that the frame is composed of, and the areas are amplified by a factor of $10^7 I_i/l_i^2$ to mimic axial rigidity by ensuring that the axial stiffness is orders of magnitude greater than the bending stiffness for each member. Cross sectional properties of each member are summarized via the following table:

Member	b(m)	h(m)	$\mathrm{I}(m^4)$	l(m)	$\mathrm{A}(m^2)$
AB	0.3	0.6	0.54000E- 02	6.0	0.15000000E + 04
BC	0.3	0.5	0.31250 E-02	8.0	0.48828125E + 03
CD	0.3	0.6	0.54000E- 02	6.0	0.15000000E+04
AD	0.3	0.3	0.67500 E-03	8.0	0.10546875E + 03

To ease the process of parametric analysis, some factors are used which take the different cross sectional properties into account. These factors are used in the parts to come to compute the analytical values of the desired quantities, and therefore presented below for the sake of completeness.

Factor	Expression	Value
	7	
k_1	$rac{I_{BC}}{I_{AD}}$	4.62963
k_2	$\frac{I_{BC}}{I_{AB}}\frac{h}{l}$	0.43403
R_1	$2k_2 + 3$	3.86806
R_2	$3k_1 + 2k_2$	14.75694
F_1	$R_1 R_2 - k_2^2$	56.89230
F_2	$1 + k_1 + 6k_2$	8.23380

1.2.1 Load Case 1

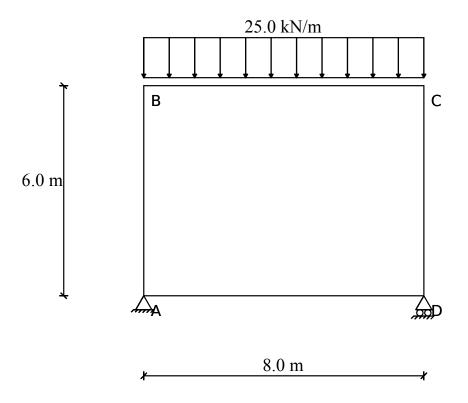


Figure 42: Problem 1: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]

# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
0.0 6.0
4.0 6.0
8.0 6.0
8.0 0.0
```

```
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
4 5
1 5
# SECTIONS <number of sections> <A, I, h, X>
0.10546875D+03 0.67500D-03 0.3 0
0.15000000D+04 0.54000D-02 0.6 0
0.48828125D+03 0.31250D-02 0.5 0
# SECTION INCIDENCES
3
3
2
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
3
1 1
1 2
5 2
# LINKS <number of links> <master node, slave node, direction>
\mbox{\tt\#} ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2 0.0 -25.0 -25.0 0.0 0.0
3 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
           x-coord
                         y-coord
Node
 1
              0.00
                          0.00
 2
              0.00
                           6.00
 3
              4.00
                           6.00
 4
              8.00
                           6.00
 5
              8.00
                           0.00
# SECTIONS
Sec.No
                  Α
                                       Т
                                                        h
                                                                            X
 1
               .1055D+03
                                  .6750D-03
                                                       0.3000
                                                                          0.0000
               .1500D+04
                                  .5400D-02
                                                       0.6000
                                                                          0.0000
 2
                                                       0.5000
 3
               .4883D+03
                                  .3125D-02
                                                                          0.0000
```

# MATERIALS	_					
Mat.No 1	E .3000D+0	10	v 0.20		alpha 0000D+00	gamma
1	.30000+0	0	0.20	•	00000+00	0.0000
# ELEMENTS						
El.No S	tart Node	End No	ode Sec	tion	Material	
1	1	2		2	1	
2	2	3		3	1	
3	3	4		3	1	
4	4	5		2	1	
5	1	5		1	1	
# RESTRAINTS						
Restraint No			Directio	n		
1		1	1			
2		1	2			
3		5	2			
# LINKS						
Link No	Master	Node	Slave No	aha	Direction	
LIIK NO	Haster	Node	Diave No	ae	Direction	
# ELASTIC RE	STRAINTS					
E. Restr. No		le	Directio	n	K	
# ELASTIC LI	NKS					
E. Link. No	Node	: 1	Node 2	!	Direction	K
# NODAL LOAD	S					
Node	Direction	Ma	agnitude			
_						
# ELEMENT LO	ADS					
Element	Px	Py1	Py2	-	DTbottom	
			-25.00	0.00		
3	0.00	-25.00	-25.00	0.00	0.00	
# DDEGEDEGGI	NO					
# PRESTRESSI Element		em	e2	P		
Flement	eı	em	62	Г		
========						
ANALYSIS RES	ULTS					
>> NODAL DIS	PLACEMENTS					
N 1 .	DV		DV		DIIT	
Node	DX		DY		PHI	
1 0.0	00000000D+	.00	.000000000D+	.00 0	.6027754160D-0	9
	736458578D-		. 13333333333D-		.1262074710D-0	
	250377120D-		. 133333333355 - 5368607197D		.1876177978D-18	
	764295662D-		. 13333333333D-		.1262074710D-0	
	500754241D-		. 0000000000D+		.6027754160D-0	
5 0.4	500754241D-	.07	. 000000000000	-00 -0	.602//54160D-0	5
>> MEMBER EN	D FORCES					
MEM N1	т	'1	M1	N2	Т2	M2
					0.17801D+02	
2 0.17801D					-0.71054D-14	
					0.10000D+03	
					-0.17801D+02	
					-0.28866D-14	
					=========	

Toggle Diagrams 1: Original configuration 2: Deformed shape 3: Axial force 4: Shear force 5: Bending moment f: Fill diagram v: Show values r: Reset camera 195 Displacement Scale

Figure 43: FR1, Load Case 1: Deformed Shape

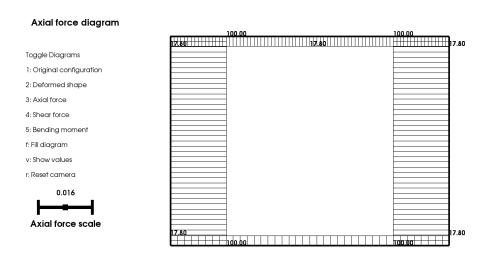


Figure 44: FR1, Load Case 1: Axial Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_A = M_D$	$\frac{\omega l^2}{4} \frac{k_2}{F_1}$	0.30516E+01	0.30516E+01	0.0%
$M_B = M_C$	$\frac{-\omega l^2}{4} \frac{R_2}{F_1}$	-0.10375E+02	-0.10375E+02	0.0%
M_{max}	$\frac{\omega l^2}{8} + M_B$	0.96246E+02	0.96247E+02	0.001%
$V_A = V_D$	$\frac{\omega l}{2}$	0.10000E+03	0.10000E+03	0.0%
	I			

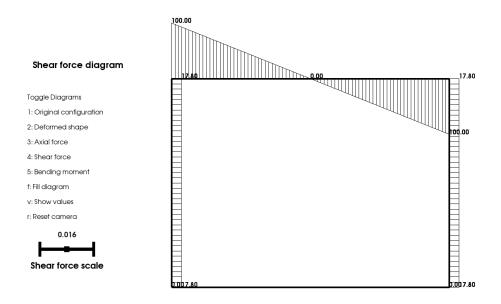


Figure 45: FR1, Load Case 1: Shear Force Diagram

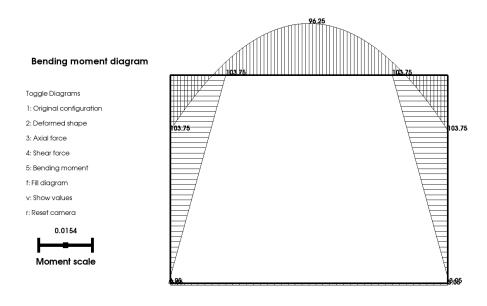


Figure 46: FR1, Load Case 1: Bending Moment Diagram

1.2.2 Load Case 2

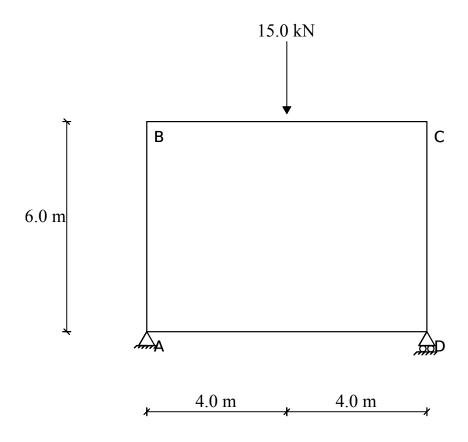


Figure 47: Problem 1: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
0.0 6.0
4.0 6.0
8.0 6.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
0.10546875D+03 0.67500D-03 0.3 0
0.1500000D+04 0.54000D-02 0.6 0
0.48828125D+03 0.31250D-02 0.5 0
# SECTION INCIDENCES
```

```
3
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
5 2
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
3 2 -50.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
                     y-coord
Node
          x-coord
 1
            0.00
                         0.00
 2
             0.00
                         6.00
                         6.00
 3
             4.00
 4
             8.00
                         6.00
 5
             8.00
                         0.00
# SECTIONS
               Α
                                                                        Х
Sec.No
                                    Т
                                                     h
 1
              .1055D+03
                                .6750D-03
                                                    0.3000
                                                                      0.0000
              .1500D+04
                                .5400D-02
                                                    0.6000
                                                                      0.0000
 2
              .4883D+03
                                                    0.5000
                                .3125D-02
 3
                                                                      0.0000
# MATERIALS
               E
Mat.No
                                   v
                                                    alpha
                                                                       gamma
              .3000D+08
                                   0.20
                                                  .0000D+00
                                                                      0.0000
 1
# ELEMENTS
El.No
          Start Node
                         End Node
                                        Section
                                                     Material
                                                     1
 1
              1
                          2
                                          2
 2
              2
                           3
                                           3
                                                        1
 3
              3
                           4
                                           3
                                                        1
 4
              4
                          5
                                           2
                                                       1
# RESTRAINTS
Restraint No
                    Node
                                   Direction
```

1

1

1

2 # LINKS Master Node Link No Slave Node Direction # ELASTIC RESTRAINTS E. Restr. No Node Direction K # ELASTIC LINKS Node 1 Node 2 Direction K E. Link. No # NODAL LOADS Node Direction Magnitude 3 2 -50.00 # ELEMENT LOADS Element Px Py1 Py2 DTtop DTbottom # PRESTRESSING e2 P Element e1 em ANALYSIS RESULTS _____ >> NODAL DISPLACEMENTS

Node	DX	DY	PHI
1	0.000000000D+00	0.000000000D+00	0.2260407810D-03
2	0.1026171967D-07	-0.333333333D-08	-0.4732780161D-03
3	0.8438914201D-08	-0.2368781588D-02	-0.1588186776D-19
4	0.6616108733D-08	-0.333333333D-08	0.4732780161D-03
5	0.1687782840D-07	0.00000000D+00	-0.2260407810D-03

>> MEMBER END FORCES

MEM	N1	T1	M1	N2	T2	M2
1	0.25000D+02	-0.66753D+01	-0.11443D+01	-0.25000D+02	0.66753D+01	-0.38908D+02
2	0.66753D+01	0.25000D+02	0.38908D+02	-0.66753D+01	-0.25000D+02	0.61092D+02
3	0.66753D+01	-0.25000D+02	-0.61092D+02	-0.66753D+01	0.25000D+02	-0.38908D+02
4	0.25000D+02	0.66753D+01	0.38908D+02	-0.25000D+02	-0.66753D+01	0.11443D+01
5 -	-0.66753D+01	0.88818D-15	0.11443D+01	0.66753D+01	-0.88818D-15	-0.11443D+01
====						

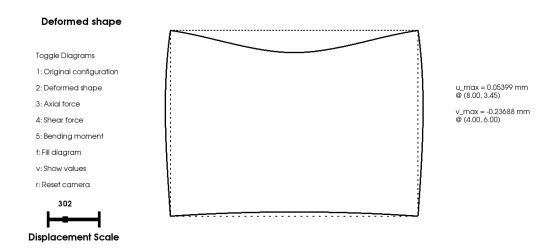


Figure 48: FR1, Load Case 2: Deformed Shape

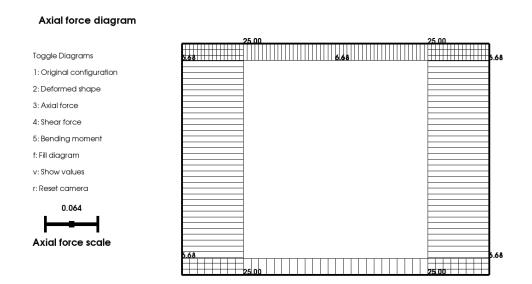


Figure 49: FR1, Load Case 2: Axial Force Diagram

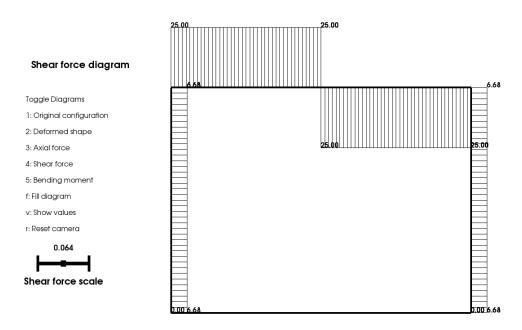


Figure 50: FR1, Load Case 2: Shear Force Diagram

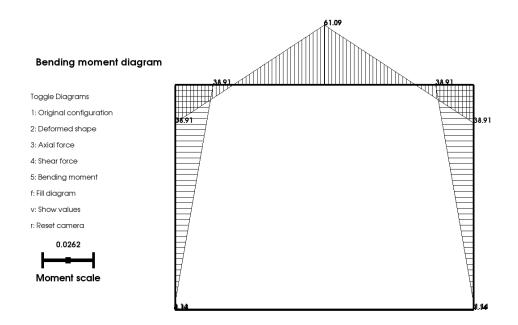


Figure 51: FR1, Load Case 2: Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_A = M_D$	$\frac{3}{8}Pl\frac{k_2}{F_1}$	0.11443E+01	0.11443E+01	0.0%
$M_B = M_C$	$\frac{-\omega l^2}{4} \frac{R_2}{F_1}$	-0.38908E+02	-0.38908E+02	0.0%
M_{max}	$\frac{pl}{4} + M_B$	0.61092E+02	0.61092E+02	0.0%
$V_A = V_D$	$\frac{P}{2}$	0.25000E+02	0.25000E+02	0.0%

1.2.3 Load Case 3

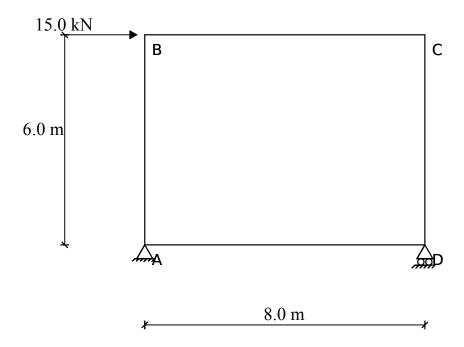


Figure 52: Problem 1: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
______
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
0.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
1 4
# SECTIONS <number of sections> <A, I, h, X>
0.10546875D+03 0.67500D-03 0.3 0
0.1500000D+04 0.54000D-02 0.6 0
0.48828125D+03 0.31250D-02 0.5 0
# SECTION INCIDENCES
3
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
```

```
1
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
4 2
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2 1 50.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
Node
           x-coord
                        y-coord
              0.00
                          0.00
 1
                          6.00
 2
              0.00
 3
              8.00
                          6.00
 4
              8.00
                          0.00
# SECTIONS
                                                                           Х
Sec.No
                 Α
                                      Т
                                                       h
 1
               .1055D+03
                                 .6750D-03
                                                      0.3000
                                                                         0.0000
              .1500D+04
                                 .5400D-02
                                                      0.6000
                                                                         0.0000
 2
 3
              .4883D+03
                                 .3125D-02
                                                      0.5000
                                                                         0.0000
# MATERIALS
                  Ε
                                                      alpha
Mat.No
                                     V
                                                                          gamma
               .3000D+08
                                    0.20
                                                    .0000D+00
                                                                         0.0000
 1
# ELEMENTS
El.No
          Start Node
                          End Node
                                         Section
                                                       Material
                            2
                                            2
 1
              1
                                                          1
  2
              2
                            3
                                            3
                                                          1
                                            2
 3
              3
                            4
                                                          1
  4
                                            1
                                                          1
# RESTRAINTS
Restraint No
                    Node
                                    Direction
 1
                      1
                                        1
 2
                                        2
                      1
 3
                      4
                                        2
# LINKS
Link No
                Master Node
                                    Slave Node
                                                       Direction
# ELASTIC RESTRAINTS
E. Restr. No
                   Node
                                    Direction
                                                           K
# ELASTIC LINKS
E. Link. No
                                     Node 2
                                                      Direction
                  Node 1
                                                                        K
```

```
# NODAL LOADS
Node
             Direction
                               Magnitude
                                50.00
 2
# ELEMENT LOADS
                         Py1
                                     Py2
                                                DTtop
                                                           {\tt DTbottom}
Element
# PRESTRESSING
Element
                                      e2
ANALYSIS RESULTS
>> NODAL DISPLACEMENTS
Node
                \mathtt{DX}
                                      DY
                                                             PHI
  1
        0.000000000D+00
                               0.00000000D+00
                                                    -0.2761385424D-02
 2
         0.1567258161D-01
                               0.3602052035D-08
                                                    -0.1536881566D-02
 3
        0.1567256795D-01
                              -0.3602052035D-08
                                                    -0.1536871305D-02
         0.6320981152D-07
                               0.000000000D+00
                                                    -0.2761371969D-02
>> MEMBER END FORCES
MEM
                                   M1
                                                N2
                                                             T2
 1 -0.27015D+02 0.25000D+02 0.41938D+02 0.27015D+02 -0.25000D+02 0.10806D+03
 2 0.25000D+02 -0.27015D+02 -0.10806D+03 -0.25000D+02 0.27015D+02 -0.10806D+03
 3 0.27015D+02 0.25000D+02 0.10806D+03 -0.27015D+02 -0.25000D+02 0.41938D+02
 4 -0.25000D+02 -0.10485D+02 -0.41938D+02 0.25000D+02 0.10485D+02 -0.41938D+02
```

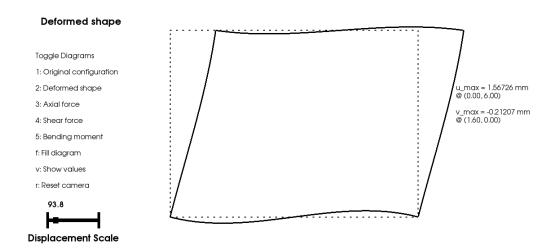


Figure 53: FR1, Load Case 3: Deformed Shape

Axial force diagram

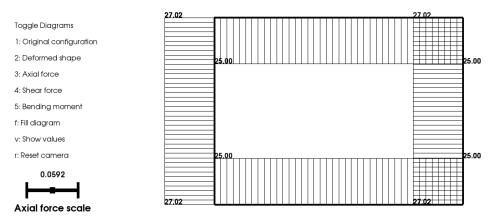


Figure 54: FR1, Load Case 3: Axial Force Diagram

Shear force diagram

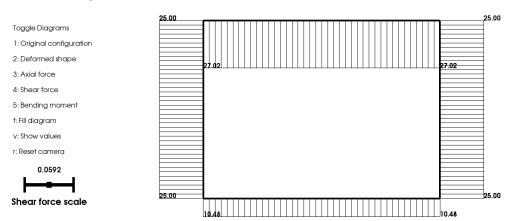


Figure 55: FR1, Load Case 3: Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_B = -M_C$	$\frac{Ph}{2}\frac{k_1+3k_2}{F_2}$	0.10806E+03	0.10806E+03	0.0%
$M_A = -M_D$	$\frac{Ph}{2}\frac{3k_2+1}{F_2}$	0.41938E+02	0.41938E+02	0.0%
$V_D = -V_A$	$-\frac{P}{2} = N_{DC} + T_{DA}$	0.37500E+02	0.37500E+02	0.0%
$H_A = H_B$	$-\frac{P}{2}$	-0.25000E+02	-0.25000E+02	0.0%

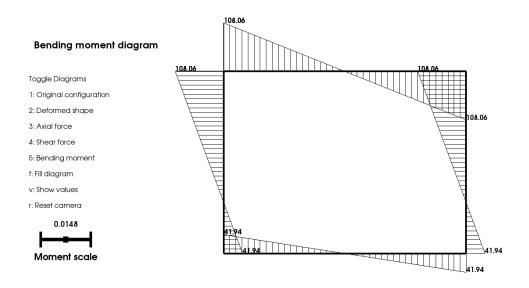


Figure 56: FR1, Load Case 3: Bending Moment Diagram

1.3 One-Bay, One-Storey Frame with Clamped Ends

1.3.1 Load Case 1

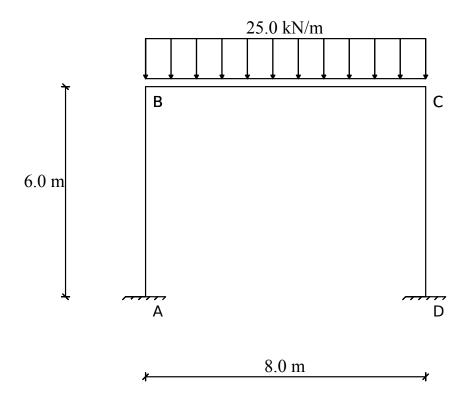


Figure 57: Problem 1: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
0.0 6.0
4.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
0.48828125D+03 0.31250D-02 0.5 0
0.1500000D+04 0.54000D-02 0.6 0
# SECTION INCIDENCES
1
1
```

```
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
1 3
5 1
5 2
5 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2 0.0 -25.0 -25.0 0.0 0.0
3 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
______
# NODE COORDINATES
Node
          x-coord
                       y-coord
            0.00
                        0.00
 1
 2
             0.00
                        6.00
 3
             4.00
                        6.00
             8.00
                        6.00
 4
 5
             8.00
                        0.00
# SECTIONS
               A
Sec.No
                                   Ι
                                                   h
                                                                      Х
             .4883D+03
                               .3125D-02
                                                  0.5000
                                                                    0.0000
 1
 2
             .1500D+04
                               .5400D-02
                                                  0.6000
                                                                    0.0000
# MATERIALS
               E
Mat.No
                                   v
                                                  alpha
                                                                     gamma
             .3000D+08
                                  0.20
                                                 .0000D+00
                                                                    0.0000
 1
# ELEMENTS
El.No
          Start Node
                        End Node
                                       Section
                                                   Material
 1
             1
                          2
                                         2
                                                      1
 2
             2
                          3
                                         1
                                                      1
 3
             3
                          4
                                         1
                                                      1
 4
             4
                          5
                                         2
                                                      1
# RESTRAINTS
Restraint No
                   Node
                                  {\tt Direction}
 1
                    1
                                     1
 2
                                      2
                     1
```

3	1	3				
4	5	1				
5	5	2				
6	5	3				
# LINKS Link No	Master Node	Slave Node		Direction		
# ELASTIC REE		Direction		К		
# ELASTIC LI E. Link. No	NKS Node 1	Node 2		Direction	K	
# NODAL LOAD Node	S Direction	Magnitude				
# ELEMENT LO		P0	DT4	DTb a t t a m		
Element 2	Px Py1 0.00 -25.00	Py2 -25.00	DTtop 0.00	DTbottom 0.00		
3	0.00 -25.00		0.00	0.00		
· ·	20.00	20100	0.00	0.00		
# PRESTRESSI	NG					
Element	e1 em	e2	P			
========						
ANALYSIS RES	ULTS					
========			======			
>> NODAL DIS	PLACEMENTS					
Node	DX	DY		PHI		
1 0.00	00000000D+00	0.000000000D+00	0	000000000D+00		
	479137028D-08	-0.1333333333D-07		1014425375D-02		
	098203032D-17	-0.4873308528D-02		1355252716D-18		
4 -0.7	479137025D-08	-0.133333333D-07	0.	1014425375D-02		
5 0.00	00000000D+00	0.00000000D+00	0.	000000000D+00		
>> MEMBER END FORCES						
MEM N1	T1	M1	N2	T2	M2	
		-0.54779D+02 -0.10		0.27389D+02 -0.		
	+02 0.10000D+03				90442D+02	
3 0.27389D		-0.90442D+02 -0.27		0.10000D+03 -0	.10956D+03	
4 0.10000D	+03 0.27389D+02	0.10956D+03 -0.10	000D+03	-0.27389D+02 0.	.54779D+02	

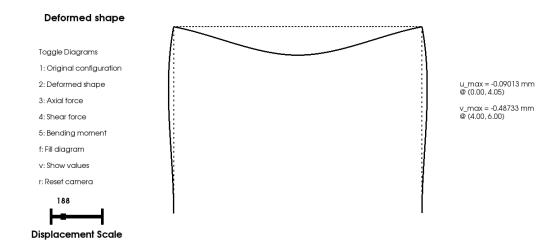


Figure 58: FR2, Load Case 1: Deformed Shape

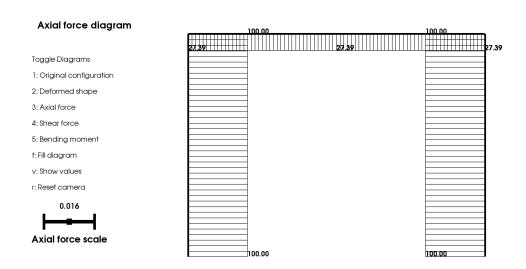


Figure 59: FR2, Load Case 1: Axial Force Diagram

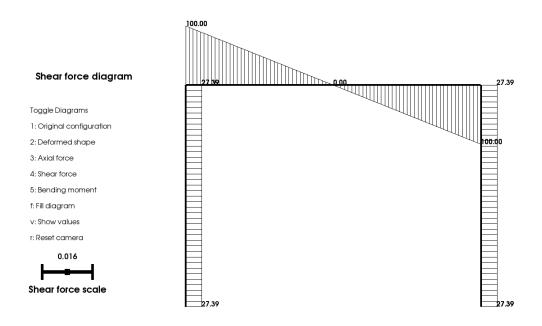


Figure 60: FR2, Load Case 1: Shear Force Diagram

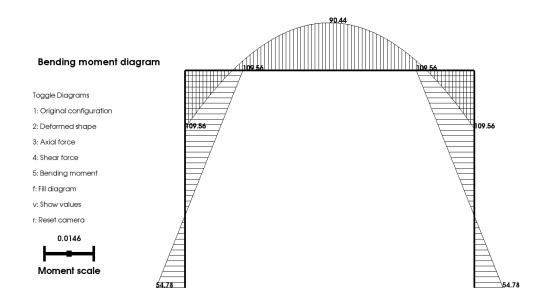


Figure 61: FR2, Load Case 1: Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_A = M_D$	$\frac{\omega l^2}{12(k+2)}$	0.54779E+02	0.54779E+02	0.0%
$M_B = M_C$	$-2M_A$	-0.10956E+03	-0.10956E+03	0.0%
M_{max}	$\frac{\omega l^2}{8} + M_B$	0.90442E+02	0.90442E+02	0.0%
$V_A = V_D$	$rac{\omega l}{2}$	0.10000E+03	0.10000E+03	0.0%
$H_A = H_D$	$rac{3M_A}{h}$	0.27389E+02	0.27389E+03	0.0%

1.3.2 Load Case 2

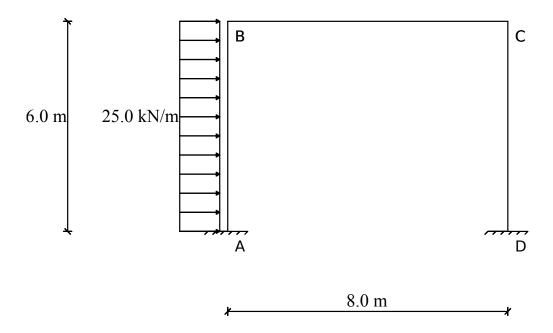


Figure 62: Problem 1: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
_____
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
0.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
3
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X> \,
0.48828125D+03 0.31250D-02 0.5 0
0.1500000D+04 0.54000D-02 0.6 0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
```

```
1 2
1 3
4 1
4 2
4 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
                        y-coord
Node
          x-coord
              0.00
                          0.00
 1
 2
              0.00
                          6.00
                          6.00
 3
             8.00
                          0.00
              8.00
 4
# SECTIONS
Sec.No
                 Α
                                     Т
                                                      h
                                                                          Х
 1
              .4883D+03
                                 .3125D-02
                                                      0.5000
                                                                         0.0000
              .1500D+04
                                 .5400D-02
                                                      0.6000
                                                                         0.0000
 2
# MATERIALS
Mat.No
                Ε
                                     v
                                                      alpha
                                                                          \operatorname{\mathtt{gamma}}
 1
               .3000D+08
                                    0.20
                                                    .0000D+00
                                                                         0.0000
# ELEMENTS
El.No
          Start Node
                          End Node
                                         Section
                                                       Material
                            2
                                            2
 1
              1
                                                       1
 2
              2
                            3
                                            1
                                                          1
 3
                                                          1
# RESTRAINTS
Restraint No
                    Node
                                    Direction
 1
                      1
                                        1
 2
                      1
                                        2
 3
                                        3
                      1
 4
                      4
                                        1
 5
                      4
                                        2
 6
                                        3
# LINKS
               Master Node
                                 Slave Node
                                                     Direction
Link No
# ELASTIC RESTRAINTS
E. Restr. No
                   Node
                                    Direction
                                                           K
# ELASTIC LINKS
E. Link. No
                  Node 1
                                      Node 2
                                                    Direction
                                                                  K
# NODAL LOADS
```

Directio	on	Magnitude						
# ELEMENT LOADS								
Px	Pv1	Pv2	DTtop	DTbottom				
0.00	-25.00	-25.00	0.00	0.00				
# PRESTRESSING								
e1	em	e2	P					
ULTS								
PLACEMENT	rs							
DX		DY		PHI				
000000000	D+00	0.000000000	00D+00 0	.000000000D+0	00			
478814510	D-02	0.180635808	38D-08 -0	.4854082024D-0)3			
478798237	7D-02	-0.180635808	38D-08 -0	.1056018269D-0)2			
000000000	D+00	0.000000000	00D+00 0	.000000000D+0	00			
>> MEMBER END FORCES								
	T1	M1	N2	T2	M2			
+02 0.12	2020D+03	0.22372D+03	0.13548D+02	0.29797D+02	0.47504D+02			
+02 -0.13	3548D+02 -	0.47504D+02	-0.29797D+02	0.13548D+02	-0.60878D+02			
02 0120					0.11790D+03			
	DADS Px 0.00 ING e1 SULTS SPLACEMENT DX 00000000000 6478814510 6478798237 000000000000000000000000000000000000	Px Py1 0.00 -25.00 ING e1 em SULTS SPLACEMENTS DX 00000000000000+00 4478814510D-02 4478798237D-02 0000000000D+00 ID FORCES T1 0+02 0.12020D+03 0+02 -0.13548D+02 -0+02 0.29797D+02	DADS Px Py1 Py2 0.00 -25.00 -25.00 ENG e1 em e2 SULTS SPLACEMENTS DX DY 0.0000000000000000000000000000000000	DADS Px Py1 Py2 DTtop 0.00 -25.00 -25.00 0.00 ENG e1 em e2 P SULTS SPLACEMENTS DX DY 0.0000000000000000000000000000000000	DADS Px Py1 Py2 DTtop DTbottom 0.00 -25.00 -25.00 0.00 0.00 ENG e1 em e2 P SULTS SPLACEMENTS DX DY PHI 00000000000D+00 0.00000000D+00 0.000000000D+0 6478814510D-02 0.1806358088D-08 -0.4854082024D-0 6478798237D-02 -0.1806358088D-08 -0.1056018269D-0 000000000D+00 0.00000000D+00 0.000000000D+00 DD FORCES T1 M1 N2 T2 0+02 0.12020D+03 0.22372D+03 0.13548D+02 0.29797D+02 0+02 -0.13548D+02 -0.47504D+02 -0.29797D+02 0.13548D+02			

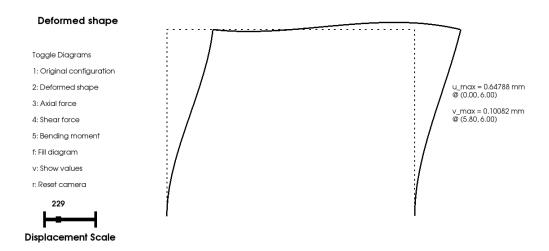


Figure 63: FR2, Load Case 2: Deformed Shape

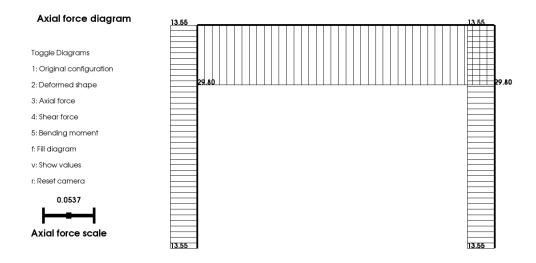


Figure 64: FR2, Load Case 2: Axial Force Diagram

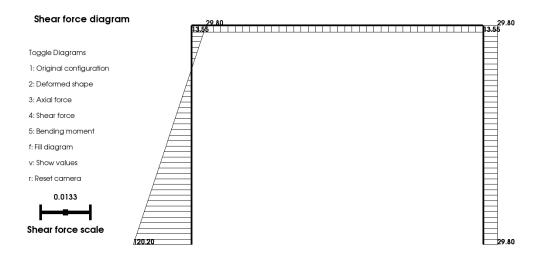


Figure 65: FR2, Load Case 2: Shear Force Diagram

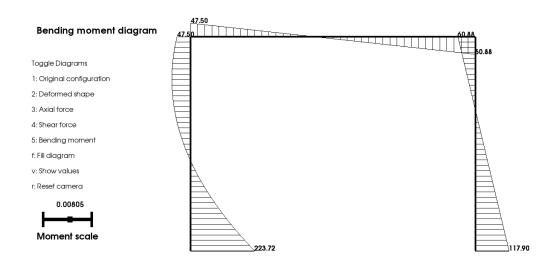


Figure 66: FR2, Load Case 2: Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
M_A	$\frac{\omega h^2}{4} \left[-\frac{k+3}{6(k+2)} - \frac{4k+1}{6k+1} \right]$	-0.22372E+03	-0.22372E+03	0.0%
M_D	$\frac{\omega h^2}{4} \left[-\frac{k+3}{6(k+2)} + \frac{4k+1}{6k+1} \right]$	0.11790E+03	0.11790E+03	0.0%
M_B	$\frac{\omega h^2}{4} \left[-\frac{k}{6(k+2)} + \frac{2k}{6k+1} \right]$	0.47504E+02	0.47504E+02	0.0%
M_C	$\frac{\omega h^2}{4} \left[-\frac{k}{6(k+2)} - \frac{2k}{6k+1} \right]$	-0.60878E+02	0.60878E+02	0.0%
H_D	$\omega h \frac{2k+3}{8(k+2)}$	-0.29797E+02	-0.29797E+02	0.0%
H_A	$-(\omega h - H_D)$	-0.12020E+03	-0.12020E+03	0.0%

1.3.3 Load Case 3

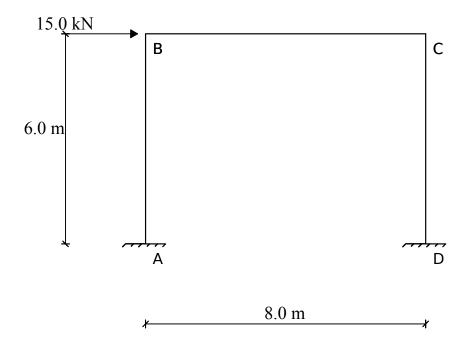


Figure 67: Problem 1: Loading, geometry and supports

```
2DSTIFF - INPUT FILE
[kN/m/C]
______
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
0.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
3 4
# SECTIONS <number of sections> <A, I, h, X>
0.48828125D+03 0.31250D-02 0.5 0
0.1500000D+04 0.54000D-02 0.6 0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
```

```
1 1
1 2
1 3
4 1
4 2
4 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2 1 50
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
# NODE COORDINATES
Node
            x-coord
                        y-coord
 1
              0.00
                          0.00
             0.00
                          6.00
 2
             8.00
                          6.00
 3
 4
              8.00
                          0.00
# SECTIONS
Sec.No
                 Α
                                      Ι
                                                       h
              .4883D+03
                                 .3125D-02
                                                     0.5000
                                                                        0.0000
 1
 2
              .1500D+04
                                 .5400D-02
                                                     0.6000
                                                                        0.0000
# MATERIALS
Mat.No
                 Ε
                                                     alpha
                                                                         gamma
                                                    .0000D+00
              .3000D+08
                                    0.20
                                                                        0.0000
 1
# ELEMENTS
El.No
          Start Node
                          End Node
                                         Section
                                                      Material
 1
              1
                            2
                                            2
                                                         1
                            3
 2
              2
                                            1
                                                         1
 3
                            4
                                            2
                                                          1
# RESTRAINTS
Restraint No
                    Node
                                    Direction
                      1
                                        1
 2
                                        2
                      1
 3
                      1
                                        3
 4
                      4
                                        1
 5
                      4
                                        2
  6
                      4
                                        3
# LINKS
Link No
                Master Node
                                    Slave Node
                                                      Direction
# ELASTIC RESTRAINTS
E. Restr. No
                  Node
                                  Direction
                                                          K
# ELASTIC LINKS
E. Link. No
                  Node 1
                                     Node 2
                                                     Direction
                                                                       K
```

```
# NODAL LOADS
Node
            Direction
                              Magnitude
                               50.00
 2
# ELEMENT LOADS
                        Py1
                                    Py2
                                               DTtop
                                                          {\tt DTbottom}
Element
# PRESTRESSING
Element
                                      e2
ANALYSIS RESULTS
>> NODAL DISPLACEMENTS
                                                           PHI
Node
                \mathtt{DX}
                                     DY
 1
        0.00000000D+00
                             0.00000000D+00
                                                   0.000000000D+00
 2
        0.5089924312D-02
                              0.1806358088D-08
                                                   -0.7707146381D-03
 3
        0.5089910658D-02
                             -0.1806358088D-08
                                                   -0.7707118334D-03
        0.000000000D+00
                             0.00000000D+00
                                                    0.00000000D+00
>> MEMBER END FORCES
MEM
                                  M1
                                               N2
                                                            T2
1 -0.13548D+02 0.25000D+02 0.95809D+02 0.13548D+02 -0.25000D+02 0.54191D+02
 2 0.25000D+02 -0.13548D+02 -0.54191D+02 -0.25000D+02 0.13548D+02 -0.54191D+02
3 0.13548D+02 0.25000D+02 0.54191D+02 -0.13548D+02 -0.25000D+02 0.95809D+02
```

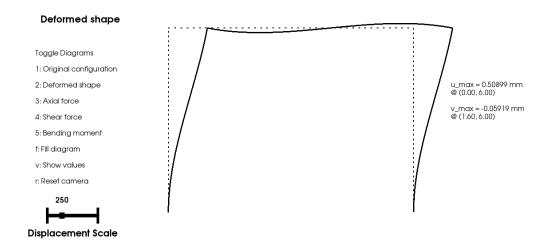


Figure 68: FR2, Load Case 3: Deformed Shape

Axial force diagram

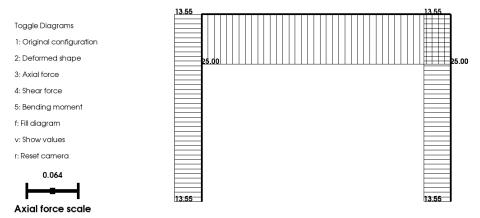


Figure 69: FR2, Load Case 3: Axial Force Diagram

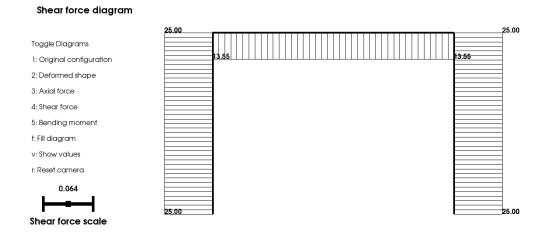


Figure 70: FR2, Load Case 3: Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_D = -M_A$	$\frac{Ph}{2} \left[-\frac{3k+1}{6k+1} \right] 0.95809E + 02$	0.95809E+02	0.0%	
$M_B = -M_C$	$\frac{Ph}{2} \left[\frac{3k}{6k+1} \right] 0.54191E + 02$	0.54191E+02	0.0%	
$V_D = -V_A$	$\frac{2M_B}{/}l$	0.13548E+02	0.13548E+02	0.0%
$H_D = H_A$	P/2	-0.25000E+02	-0.25000E+02	0.0%

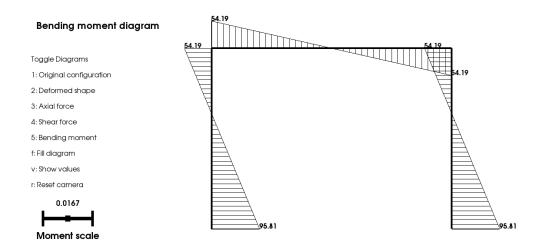


Figure 71: FR2, Load Case 3: Bending Moment Diagram

1.4 Clamped Beam with Nodal Load, Influence of Shear Deformability

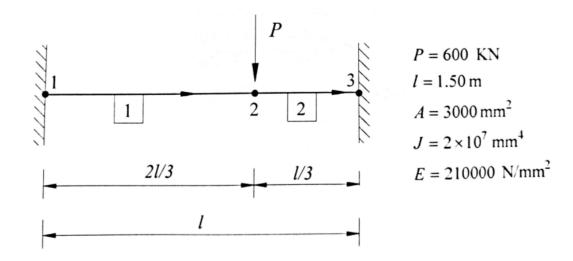


Figure 72: Specialized Problem 1: Loading, geometry and supports

1.4.1 Without Shear Deformability

```
2DSTIFF - INPUT FILE
[kN/m/C]
# NODES <number of nodes> <x coord, y coord>
0.0 0.0
1.0 0.0
1.5 0.0
# ELEMENTS <number of elements> <start node, end node>
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.30000D-02 0.20000D-04 0.5 0.0
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
0.21000D+09 0.0 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
1 3
3 1
```

```
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 2 -600.00
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
Node
         x-coord
                        y-coord
              0.00
                          0.00
 1
 2
              1.00
                          0.00
              1.50
                          0.00
# SECTIONS
Sec.No
                                      Ι
                                                                          Х
                 Α
                                                       h
              .3000D-02
                                                                        0.0000
                                 .2000D-04
                                                     0.5000
 1
# MATERIALS
                  E
                                                     alpha
Mat.No
                                      ٧
                                                                         gamma
 1
              .2100D+09
                                    0.00
                                                    .0000D+00
                                                                        0.0000
# ELEMENTS
El.No
          Start Node
                          End Node
                                         Section
                                                       Material
                            2
 1
              1
                                            1
                                                         1
 2
              2
                            3
# RESTRAINTS
Restraint No
                    Node
                                    Direction
 1
                      1
                                        1
 2
                      1
                                        2
 3
                                        3
                      1
 4
                      3
                                        1
 5
                      3
                                        2
 6
                                        3
                      3
# LINKS
Link No
                Master Node
                                   Slave Node
                                                     Direction
# ELASTIC RESTRAINTS
E. Restr. No
                  Node
                                   Direction
                                                          K
# ELASTIC LINKS
E. Link. No
                  Node 1
                                     Node 2
                                                     Direction
                                                                       K
# NODAL LOADS
                              Magnitude
Node
           Direction
 2
                               -600.00
# ELEMENT LOADS
                        Py1
                                 Py2
                                              DTtop
Element
             Px
                                                         DTbottom
```

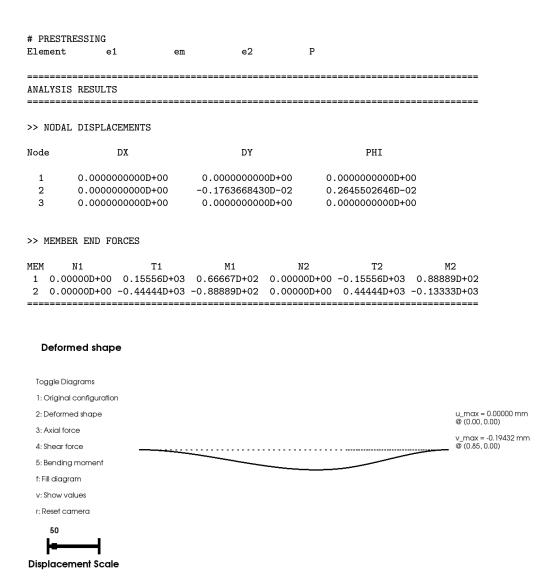


Figure 73: FR1, Load Case 1: Deformed Shape

	Exact Expression	Exact Value	Computed Value	% RE
v_B	$-\frac{P(\frac{2}{3}l)^3(\frac{1}{3}l)^3}{3EJl^3}$	-0.17637E-02	-0.17637E-02	0.0%
$arphi_B$	$-\frac{P(\frac{2}{3}l)^2(\frac{1}{3}l)^3}{2EJl^3}$	0.26455E-02	0.26455E-02	0.0%

Shear force diagram

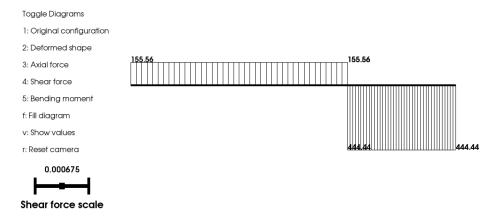


Figure 74: FR1, Load Case 1: Shear Force Diagram

Bending moment diagram

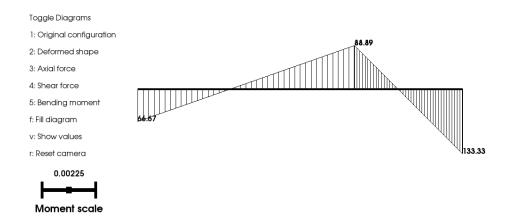


Figure 75: FR1, Load Case 1: Bending Moment Diagram

1.4.2 With Shear Deformability

Shear deformability is introduced by setting $\chi=2.5$ and $\nu=0.3$, while all other parameters were held constant. The results are presented below.

```
2DSTIFF - INPUT FILE
[kN/m/C]

# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
1.0 0.0
1.5 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
```

```
2 3
# SECTIONS <number of sections> <A, I, h, X>
0.30000D-02 0.20000D-04 0.5 2.5
# SECTION INCIDENCES
# MATERIALS <number of materials> <E, v, alpha, gamma>
0.21000D+09 0.3 0.0 0.0
# MATERIAL INCIDENCES
# RESTRAINTS <number of restraints> <node number, direction>
1 1
1 2
1 3
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2 2 -600.00
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
# PRESTRESSING <number> <Element, e1, em, e2, P>
2DSTIFF - OUTPUT FILE
[kN/m/C]
# NODE COORDINATES
                         y-coord
Node
        x-coord
              0.00
                          0.00
 1
 2
              1.00
                          0.00
 3
              1.50
                          0.00
# SECTIONS
                                                                           Х
Sec.No
                  Α
                                      Ι
                                                        h
 1
               .3000D-02
                                  . 2000D-04
                                                       0.5000
                                                                          2.5000
# MATERIALS
Mat.No
                  Ε
                                                       alpha
                                                                           gamma
               .2100D+09
                                     0.30
                                                     .0000D+00
                                                                          0.0000
 1
# ELEMENTS
El.No
           Start Node
                          End Node
                                          Section
                                                        Material
 1
              1
                             2
                                             1
                                                           1
  2
# RESTRAINTS
Restraint No
                     Node
                                     Direction
 1
                      1
                                        1
 2
                       1
                                         2
 3
                                         3
                      1
 4
                                         1
```

```
5
  6
# LINKS
                 Master Node
                                      Slave Node
                                                         Direction
Link No
# ELASTIC RESTRAINTS
E. Restr. No
                                      Direction
                                                             K
                    Node
# ELASTIC LINKS
E. Link. No
                   Node 1
                                        Node 2
                                                         Direction
                                                                          K
# NODAL LOADS
             {\tt Direction}
                               Magnitude
Node
  2
                 2
                                -600.00
# ELEMENT LOADS
                                                 DTtop
Element
                         Py1
                                      Py2
                                                            DTbottom
# PRESTRESSING
                                       e2
                                                   Ρ
Element
                          em
ANALYSIS RESULTS
______
>> NODAL DISPLACEMENTS
                DX
                                      DY
                                                             PHI
Node
         0.000000000D+00
                               0.00000000D+00
                                                      0.000000000D+00
         0.000000000D+00
                              -0.3868546215D-02
                                                      0.2148873990D-02
  2
  3
         0.00000000D+00
                               0.00000000D+00
                                                      0.000000000D+00
>> MEMBER END FORCES
MEM
        N1
                      T1
                                   M1
                                                 N2
                                                              T2
 1 0.00000D+00 0.16390D+03 0.72924D+02 0.00000D+00 -0.16390D+03 0.90975D+02
 2 0.00000D+00 -0.43610D+03 -0.90975D+02 0.00000D+00 0.43610D+03 -0.12708D+03
  Deformed shape
 Toggle Diagrams
  1: Original configuration
                                                                              u_max = 0.00000 mm
@ (0.00, 0.00)
 2: Deformed shape
 3: Axial force
                                                                              v_max = -0.39444 mm
@ (0.93, 0.00)
 4: Shear force
 5: Bending moment
 f: Fill diagram
 v: Show values
 r: Reset camera
    50
```

Figure 76: FR1, Load Case 1: Deformed Shape

Displacement Scale

Shear force diagram

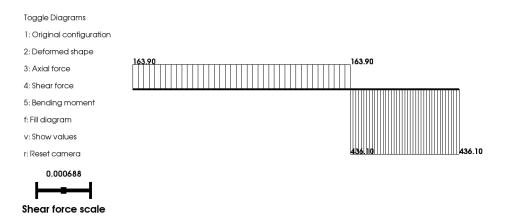


Figure 77: FR1, Load Case 1: Shear Force Diagram

Bending moment diagram

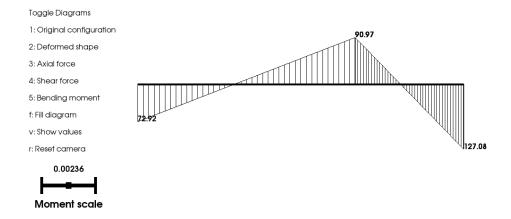


Figure 78: FR1, Load Case 1: Bending Moment Diagram

	Exact Value	Computed Value	% RE
v_B	-0.3869E-02	-0.3869E-02	0.0%
ϕ_B	0.2149E-02	0.2149E-02	0.0%

As can be observed from the presnted values, introducing shear deformability results in a significant variation in nodal displacement and rotation:

$$\frac{\delta v}{v_t} = \frac{v_t - v_m}{v_t} = \frac{|-0.3869E - 02 - (-0.1764E - 02)|}{|-0.3869E - 02|} = 54.4\% \frac{\delta \varphi}{\varphi_t} = \frac{\varphi_t - \varphi_m}{\varphi_t} = \frac{|0.2149E - 02|}{|0.2149E - 02|} = \frac{|-0.3869E - 02|}{|-0.3869E - 02|} = \frac{|-0.3869E - 02$$

However, the variation in the internal force resultants are rather low when compared to that of the displacements, for instance the variation in the moments at A and C:

$$\frac{\delta M_A}{M_{A,t}} = \frac{M_{A,t} - M_{A,m}}{M_{A,t}} = \frac{|-0.72924E + 02 - (-0.66667E + 02)|}{|-0.72924E + 02|} = 8.6\% \frac{\delta M_C}{M_{C,t}} = \frac{M_{C,t} - M_{C,m}}{M_{C,t}} = \frac{M_{C,t} - M_{C,m}}{M_{C,t}} = \frac{M_{C,t} - M_{C,t}}{M_{C,t}} = \frac{M_{C,t}}{M_{C,t}} = \frac{M_{C$$