Computational Structural Analysis
2D Direct Stiffness Code

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1 Benchmarks

The accuracy and robustness of the code is tested by solving some sample problems, the exact solutions of which were computed apriori via the classical beam theory. The results are presented in this section using the following scheme:

- Demonstration of the problem
- Input file that addresses the problem
- Output file generated by the program after analysis
- Diagrams obtaned by postprocessing
- Comparison of the results of numerical and analytical solutions

Each problem considered in the first part comprise a single or continous beam subjected to a certain external loading and support condition. The second part deals with a one-bay one-storey frame under different loads and restraints, and the third and final part is devoted to problems involving a prestressed beam, with each problem having different tendon layouts and/or support conditions.

The error in each numerical solution is presented by using the true percent relative error according to the following definition:

$$\epsilon_{r,t} = \frac{|exact\ value - numerical\ value|}{|exact\ value|} \times 100$$

1.1 Single Beam Under Various Loading and Support Conditions

1.1.1 Problem 1: Simple Beam - Uniformly Distributed Load

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
```

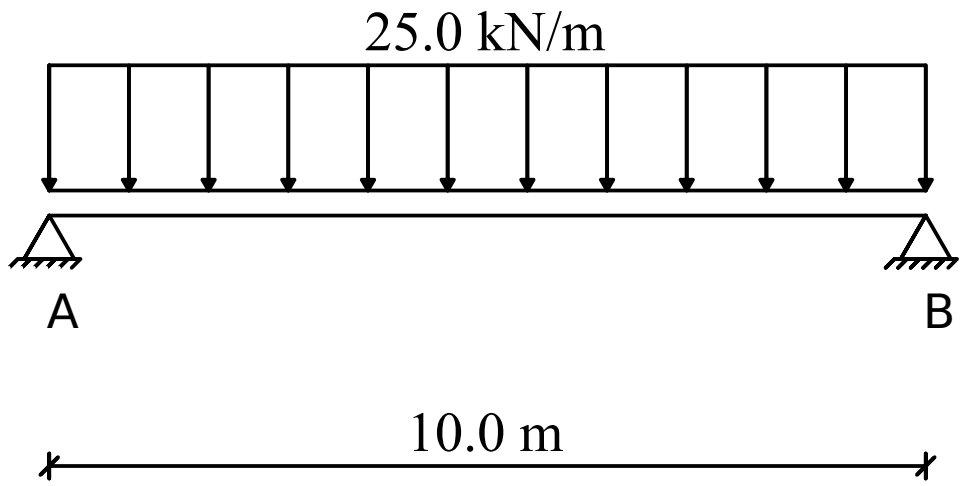

Figure 1: Problem 1: Loading, geometry and supports

```
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2
1 0.0 -25.0 -25.0 0.0 0.0
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
```

0
==========

==========
2DSTIFF - OUTPUT FILE
[kN/m/C]
==========

NODE COORDINATES

Node	x-coord	y-coord
1	0.00	0.00
2	5.00	0.00
3	10.00	0.00

SECTIONS

Sec.No	A	I	h	X
1	.3200D+00	.1707D-01	0.8000	0.0000

MATERIALS

Mat.No	E	v	alpha	gamma
1	.3000D+08	0.20	.0000D+00	0.0000

ELEMENTS

El.No	Start Node	End Node	Section	Material
1	1	2	1	1
2	2	3	1	1

RESTRAINTS

Restraint No	Node	Direction
1	1	1
2	1	2
3	3	1
4	3	2

LINKS

Link No	Master Node	Slave Node	Direction

ELASTIC RESTRAINTS

E. Restr. No	Node	Direction	K

ELASTIC LINKS

E. Link. No	Node 1	Node 2	Direction	K

NODAL LOADS

Node	Direction	Magnitude

ELEMENT LOADS

Element	Px	Py1	Py2	DTtop	DTbottom
1	0.00	-25.00	-25.00	0.00	0.00
2	0.00	-25.00	-25.00	0.00	0.00

PRESTRESSING

```
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     0.0000000000D+00     -0.2034505208D-02
  2      0.0000000000D+00    -0.6357828776D-02      0.0000000000D+00
  3      0.0000000000D+00     0.0000000000D+00      0.2034505208D-02


>> MEMBER END FORCES

Member     N1          T1          M1          N2          T2          M2
  1        0.00      125.00        0.00        0.00        0.00      312.50
  2        0.00        0.00     -312.50        0.00      125.00       -0.00
================================================================================
```

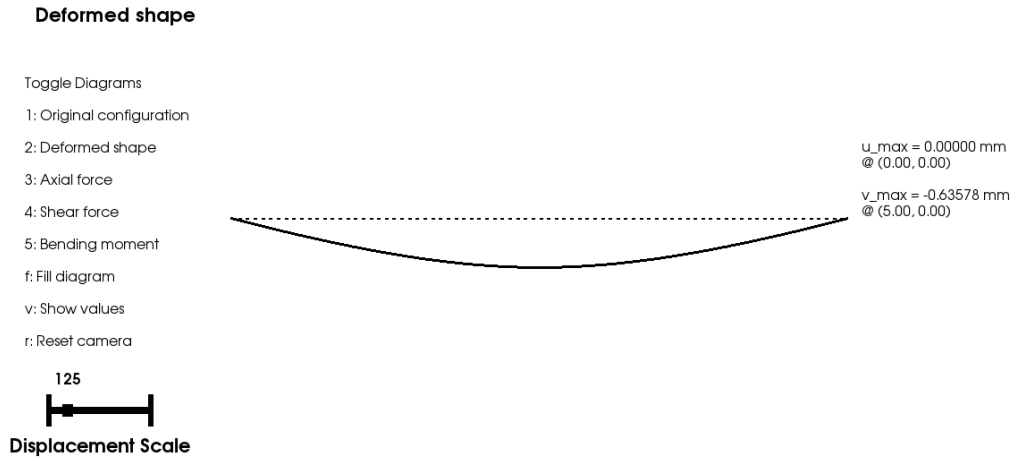

Figure 2: Problem 1, Deformed Shape

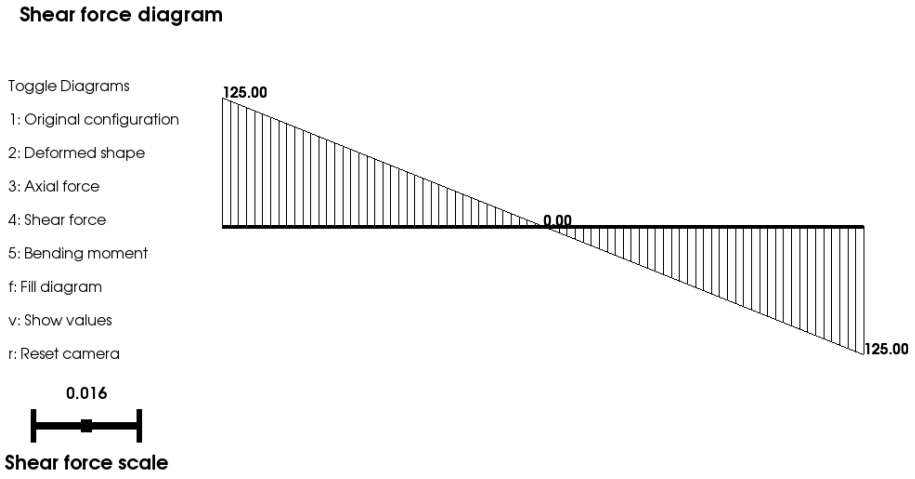

Figure 3: Problem 1, Shear Force Diagram

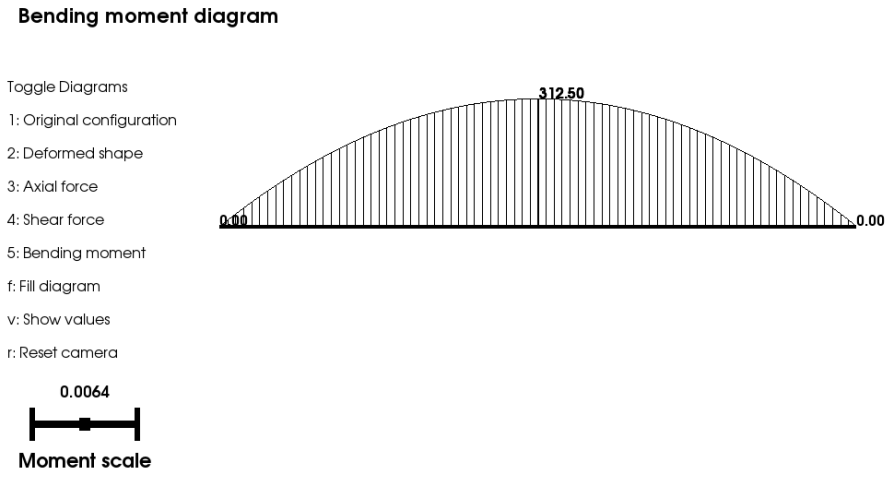

Figure 4: Problem 1, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{\omega l}{2}$	125.00	125.00	0.0%
M_{max}	$\frac{\omega l^2}{8}$	312.50	312.50	0.0%
δ_{max}	$\frac{5\omega l^4}{384EI}$	0.6357828776E-02	0.6357828776E-02	0.0%

1.1.2 Problem 5: Simple Beam - Load Increasing Uniformly To One End

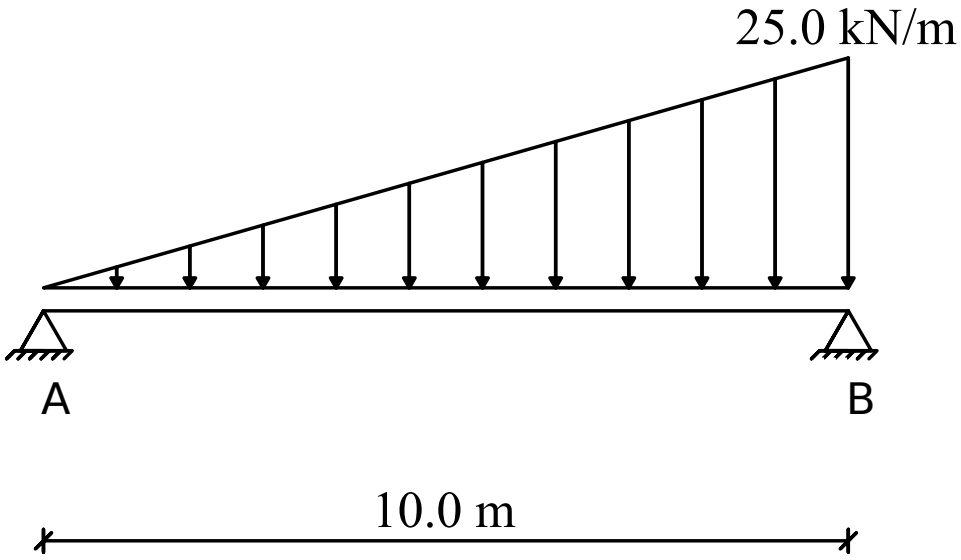

Figure 5: Problem 5: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
2
0.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1
1 2
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
2 1
```

```
2 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1
1 0.0 0.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node          x-coord      y-coord
  1             0.00         0.00
  2            10.00         0.00

# SECTIONS
Sec.No              A                    I                 h                  X
  1            .3200D+00           .1707D-01            0.8000             0.0000

# MATERIALS
Mat.No              E                    v               alpha               gamma
  1            .3000D+08              0.20            .0000D+00             0.0000

# ELEMENTS
El.No       Start Node      End Node       Section       Material
  1             1              2               1              1

# RESTRAINTS
Restraint No          Node            Direction
  1                     1                1
  2                     1                2
  3                     2                1
  4                     2                2

# LINKS
Link No           Master Node          Slave Node          Direction

# ELASTIC RESTRAINTS
E. Restr. No         Node              Direction              K

# ELASTIC LINKS
```

```
E. Link. No        Node 1              Node 2          Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom
  1          0.00        0.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     0.0000000000D+00     -0.9494357639D-03
  2      0.0000000000D+00     0.0000000000D+00      0.1085069444D-02


>> MEMBER END FORCES

Member     N1          T1          M1          N2          T2          M2
  1        0.00       41.67       -0.00        0.00       83.33        0.00
================================================================================
```

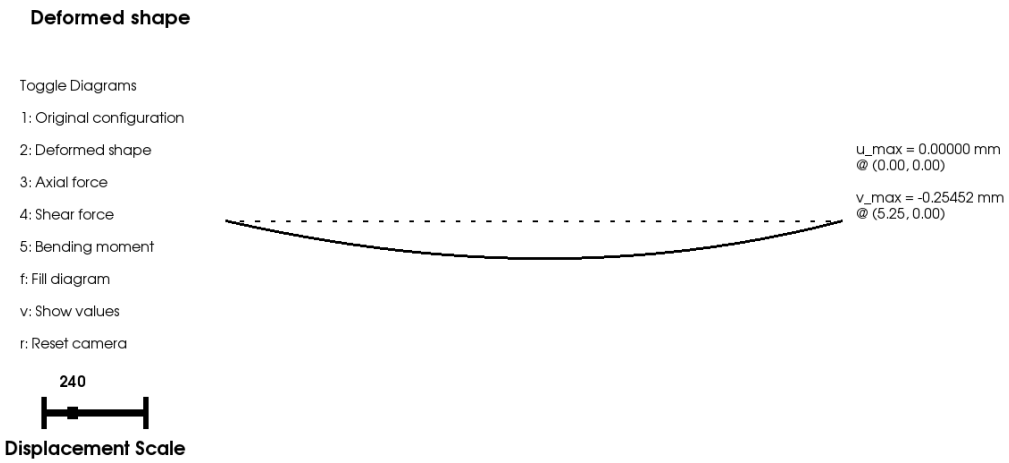

Figure 6: Problem 5, Deformed Shape

Figure 7: Problem 5, Shear Force Diagram

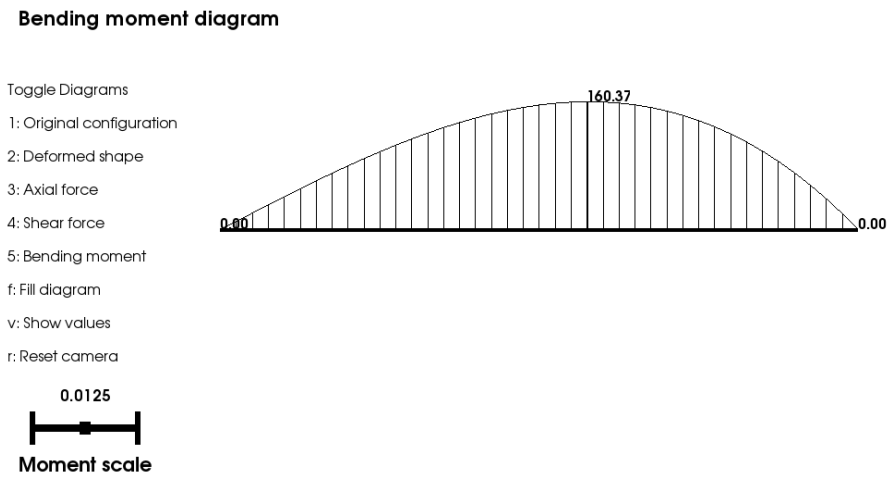

Figure 8: Problem 5, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{\omega l}{6}$	41.67	41.67	0.0%
V_B	$\frac{\omega l}{3}$	83.33	41.67	0.0%
M_{max}	$\frac{\omega l^2}{9\sqrt{3}}$	160.37	160.37	0.0%

1.1.3 Problem 6: Simple Beam - Load Increasing Uniformly To Center

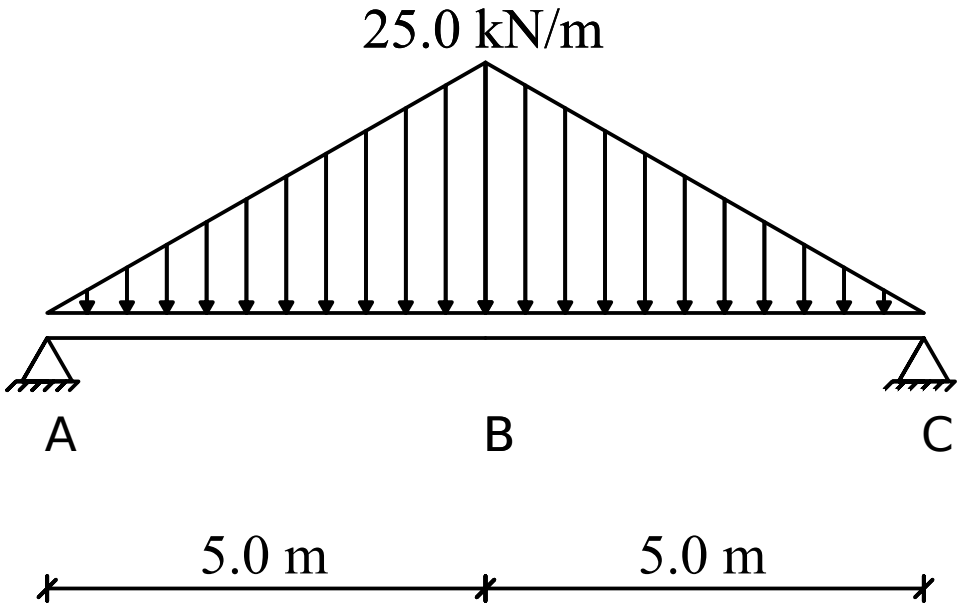

Figure 9: Problem 6: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
```

```
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2
1 0.0 0.0 -25.0 0.0 0.0
2 0.0 -25.0 0.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            5.00        0.00
  3           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                   v               alpha               gamma
  1            .3000D+08             0.20            .0000D+00           0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    3                1
```

```
  4                   3                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction             K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom
  1          0.00        0.00      -25.00        0.00        0.00
  2          0.00      -25.00        0.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     0.0000000000D+00    -0.1271565755D-02
  2      0.0000000000D+00    -0.4069010417D-02     0.0000000000D+00
  3      0.0000000000D+00     0.0000000000D+00     0.1271565755D-02


>> MEMBER END FORCES

Member     N1          T1          M1          N2          T2          M2
  1        0.00       62.50       -0.00        0.00        0.00      208.33
  2        0.00        0.00     -208.33        0.00       62.50        0.00
================================================================================
```

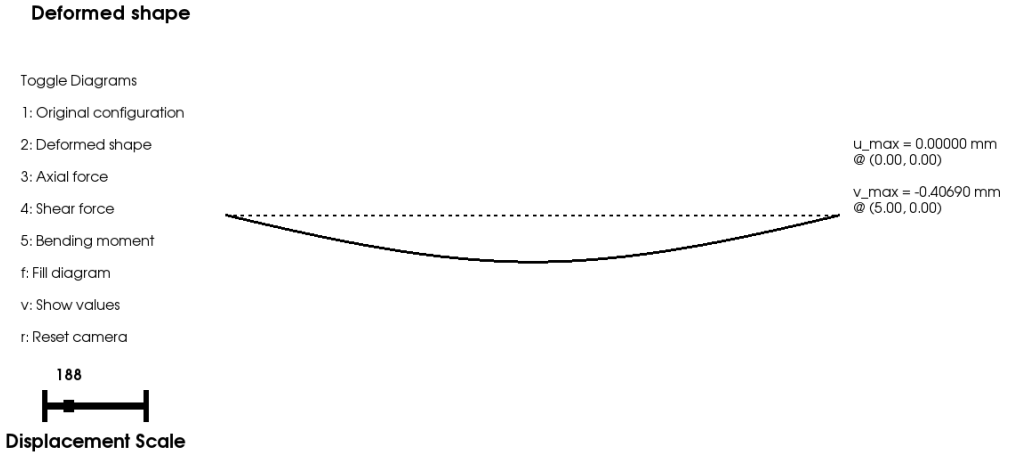

Figure 10: Problem 6, Deformed Shape

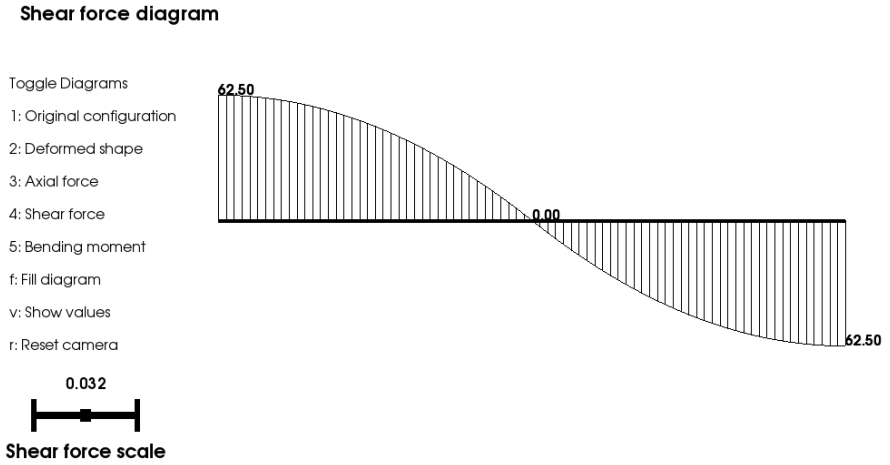

Figure 11: Problem 6, Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{\omega l}{4}$	62.50	62.50	0.0%
M_{max}	$\frac{\omega l^2}{12}$	208.33	208.33	0.0%
δ_{max}	$\frac{\omega l^4}{120EI}$	0.4069010417E-02	0.4069010417E-02	0.0%

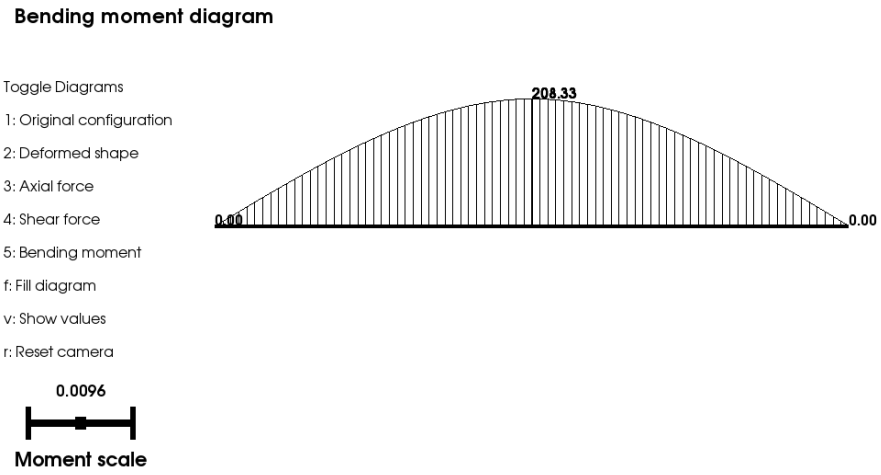

Figure 12: Problem 6, Bending Moment Diagram

1.1.4 Problem 9: Simple Beam - Two Equal Concentrated Loads Symmetrically Placed

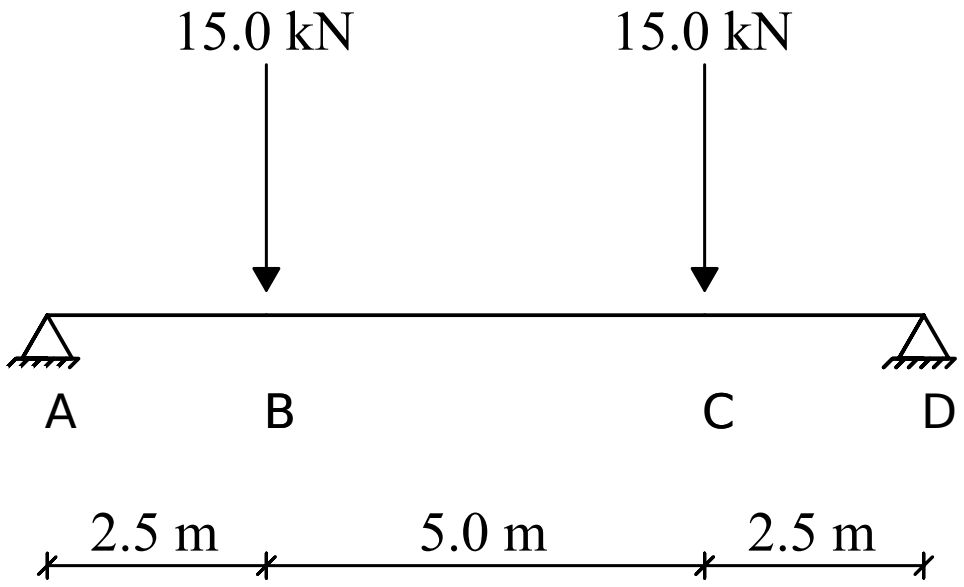

Figure 13: Problem 9: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
2.5 0.0
5.0 0.0
7.5 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
4
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
1
1
```

```
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
5 1
5 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2
2 2 -15.0
4 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            2.50        0.00
  3            5.00        0.00
  4            7.50        0.00
  5           10.00        0.00

# SECTIONS
Sec.No             A                  I                h                  X
  1            .3200D+00          .1707D-01          0.8000             0.0000

# MATERIALS
Mat.No             E                  v              alpha              gamma
  1            .3000D+08            0.20           .0000D+00          0.0000

# ELEMENTS
```

```
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1
  3            3             4              1             1
  4            4             5              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    5                1
  4                    5                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction             K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude
  2              2             -15.00
  4              2             -15.00

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00      0.0000000000D+00     -0.2746582031D-03
  2      0.0000000000D+00     -0.6103515625D-03     -0.1831054687D-03
  3      0.0000000000D+00     -0.8392333984D-03     -0.5421010862D-19
  4      0.0000000000D+00     -0.6103515625D-03      0.1831054687D-03
  5      0.0000000000D+00      0.0000000000D+00      0.2746582031D-03


>> MEMBER END FORCES

Member     N1          T1          M1          N2          T2          M2
  1        0.00       15.00        0.00        0.00      -15.00       37.50
```

2	0.00	-0.00	-37.50	0.00	0.00	37.50
3	0.00	0.00	-37.50	0.00	-0.00	37.50
4	0.00	-15.00	-37.50	0.00	15.00	-0.00

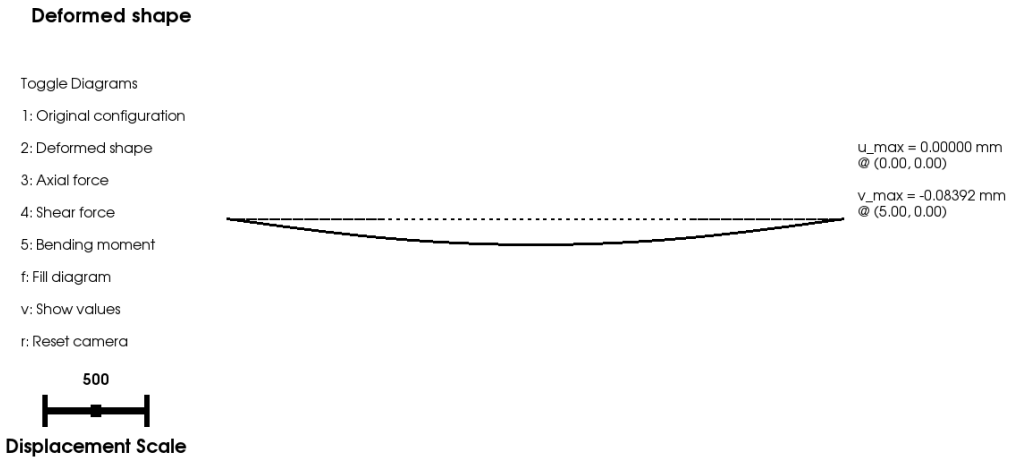

Figure 14: Problem 9, Deformed Shape

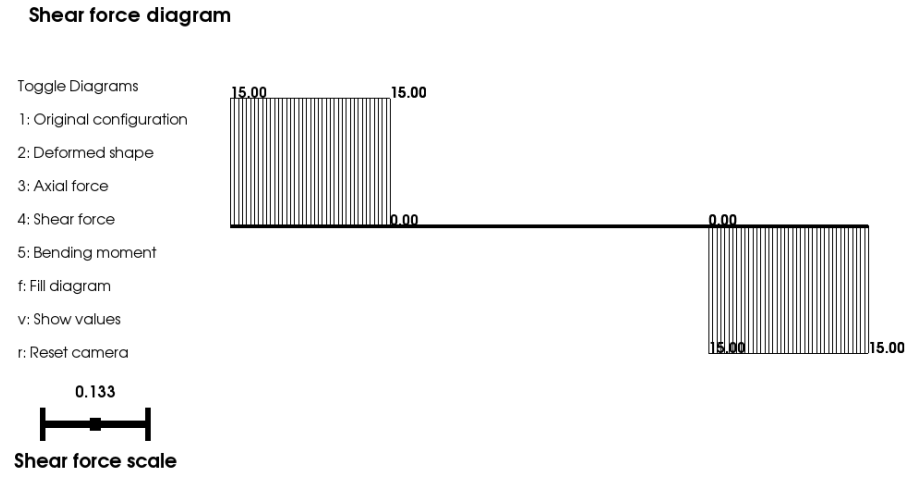

Figure 15: Problem 9, Shear Force Diagram

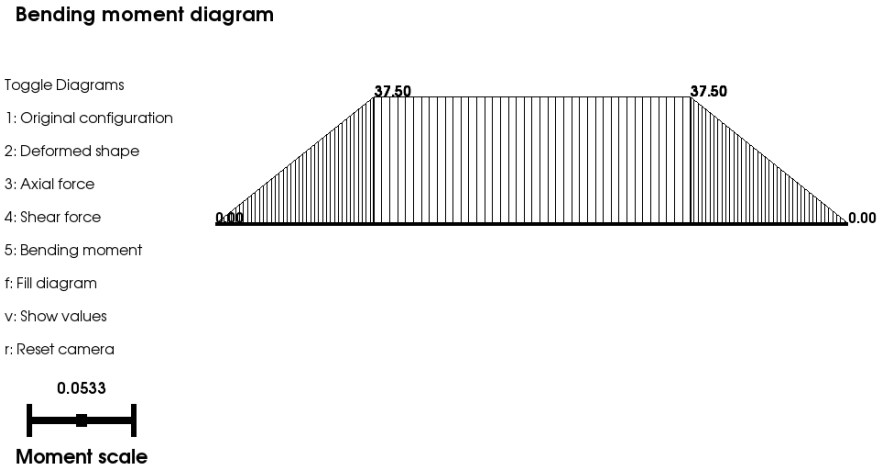

Figure 16: Problem 9, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	P	15.00	15.00	0.0%
M_{max}	Pa	37.50	37.50	0.0%
δ_{max}	$\frac{Pa}{24EI}(3l^2 - 4a^2)$	0.8392333984E-03	0.8392333984E-02	0.0%