Computational Structural Analysis
2D Direct Stiffness Code

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1 Benchmarks

The accuracy and robustness of the code is tested by solving some sample problems, the exact solutions of which were computed apriori via the classical beam theory. The results are presented in this section using the following scheme:

- Demonstration of the problem
- Input file that addresses the problem
- Output file generated by the program after analysis
- Diagrams obtained by postprocessing
- Comparison of the results of numerical and analytical solutions

Each problem considered in the first part comprise a single or continous beam subjected to a certain external loading and set of restraints. The second part deals with a one-bay one-storey frame under different loads and support conditions. Then some problems that involve more specialized cases of matrix structural analysis are investigated, such as beams with shear deformability, elastic restraints and links and thermal loadings. The fourth and final part is devoted to the analysis of prestressed beams.

The error in the desired quantities for each numerical solution is presented by using the true percent relative error according to the following definition:

$$\epsilon_{r,t} = \frac{|exact\ value - numerical\ value|}{|exact\ value|} \times 100$$

1.1 Single Beam Under Various Loading and Support Conditions

1.1.1 Problem 1: Simple Beam - Uniformly Distributed Load

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
```

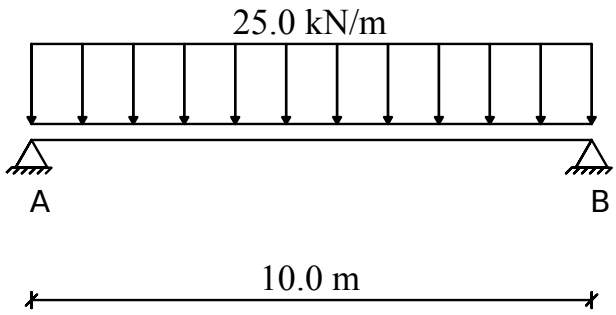

Figure 1: Problem 1: Loading, geometry and supports

```
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2
1 0.0 -25.0 -25.0 0.0 0.0
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================


================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            5.00        0.00
  3           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000
```

```
# MATERIALS
Mat.No              E                   v               alpha               gamma
  1             .3000D+08             0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No          Node            Direction
  1                     1                1
  2                     1                2
  3                     3                1
  4                     3                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom
  1          0.00       -25.00      -25.00        0.00        0.00
  2          0.00       -25.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node             DX                    DY                    PHI

  1      0.0000000000D+00      0.0000000000D+00     -0.2034505208D-02
  2      0.0000000000D+00     -0.6357828776D-02      0.0000000000D+00
  3      0.0000000000D+00      0.0000000000D+00      0.2034505208D-02


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.12500D+03  0.78160D-13  0.00000D+00  0.00000D+00  0.31250D+03
 2  0.00000D+00  0.00000D+00 -0.31250D+03  0.00000D+00  0.12500D+03 -0.78160D-13
================================================================================
```

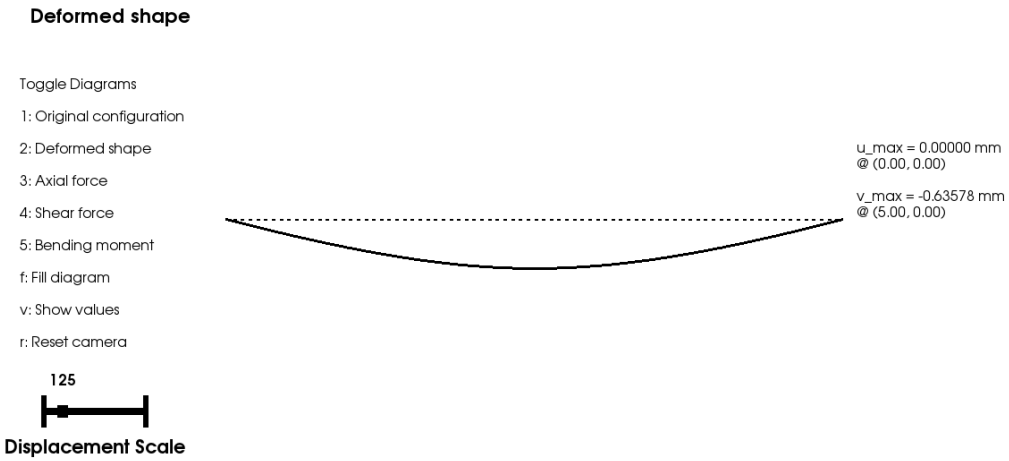

Figure 2: Problem 1, Deformed Shape

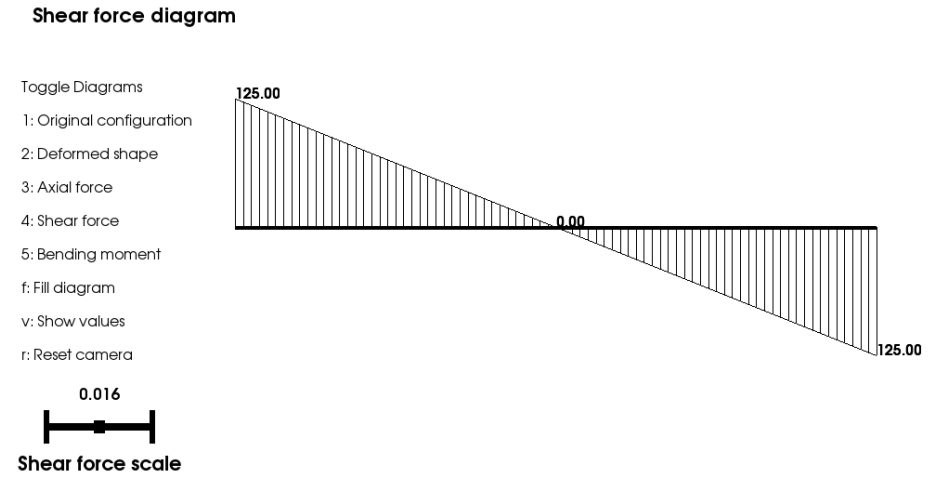

Figure 3: Problem 1, Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{\omega l}{2}$	0.12500E+03	0.12500E+03	0.0%
M_{max}	$\frac{\omega l^2}{8}$	0.31250E+03	0.31250E+03	0.0%
δ_{max}	$\frac{5\omega l^4}{384EI}$	-0.63578E-02	-0.63578E-02	0.0%

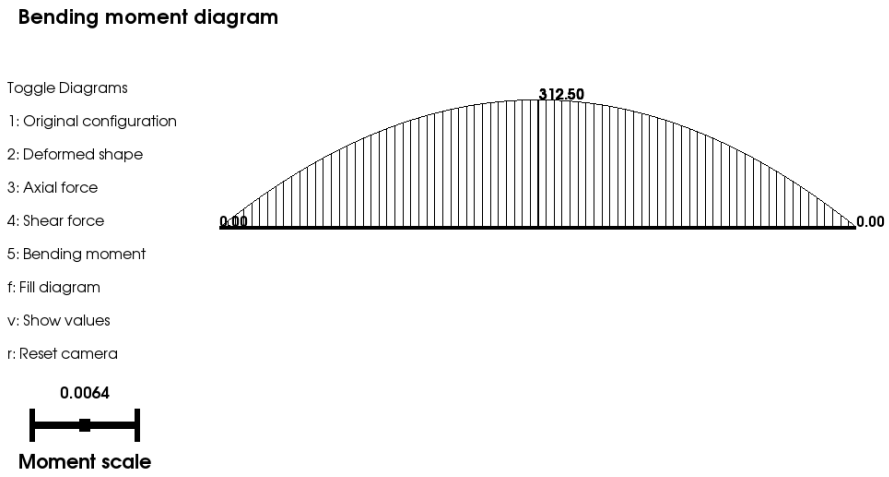

Figure 4: Problem 1, Bending Moment Diagram

1.1.2 Problem 5: Simple Beam - Load Increasing Uniformly To One End

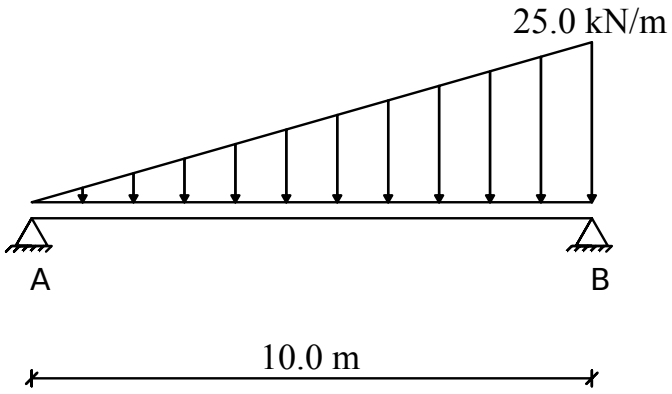

Figure 5: Problem 5: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
2
0.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
1
1 2
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
2 1
2 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1
1 0.0 0.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
```

```
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2           10.00        0.00

# SECTIONS
Sec.No             A                  I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                  v               alpha               gamma
  1            .3000D+08            0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    2                1
  4                    2                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom
  1          0.00        0.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1           em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                    DY                    PHI

  1      0.0000000000D+00      0.0000000000D+00     -0.9494357639D-03
  2      0.0000000000D+00      0.0000000000D+00      0.1085069444D-02


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.41667D+02 -0.14211D-13  0.00000D+00  0.83333D+02  0.00000D+00
```

==========

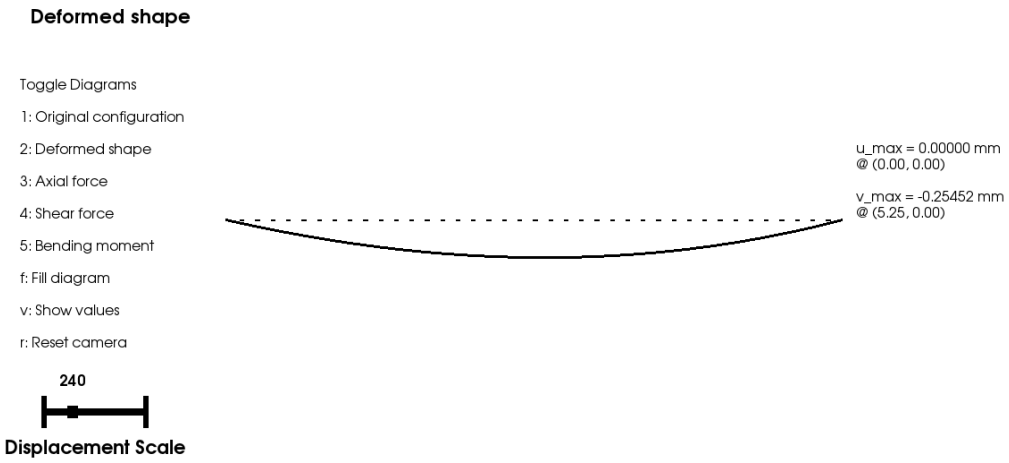

Figure 6: Problem 5, Deformed Shape

Figure 7: Problem 5, Shear Force Diagram

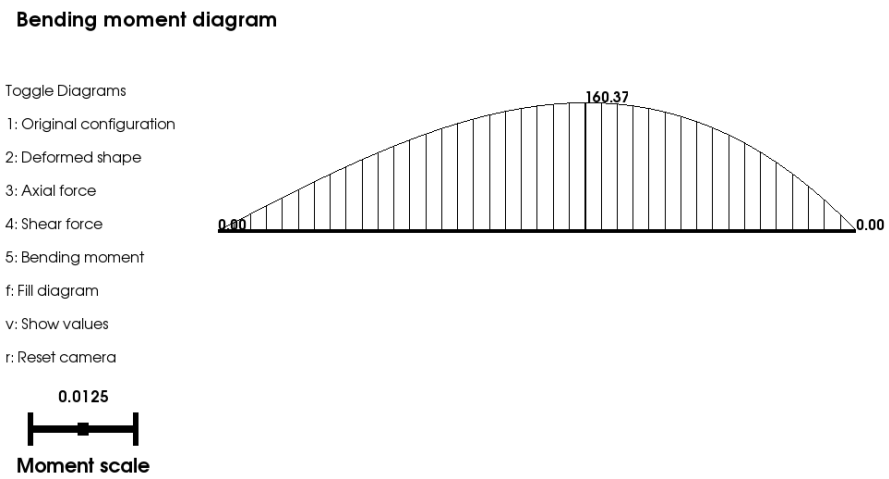

Figure 8: Problem 5, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{\omega l}{6}$	0.41667E+02	0.41667E+02	0.0%
V_B	$\frac{\omega l}{3}$	0.83333E+02	0.83333E+02	0.0%
M_{max}	$\frac{\omega l^2}{9\sqrt{3}}$	0.16037E+03	0.16037E+03	0.0%

1.1.3 Problem 6: Simple Beam - Load Increasing Uniformly To Center

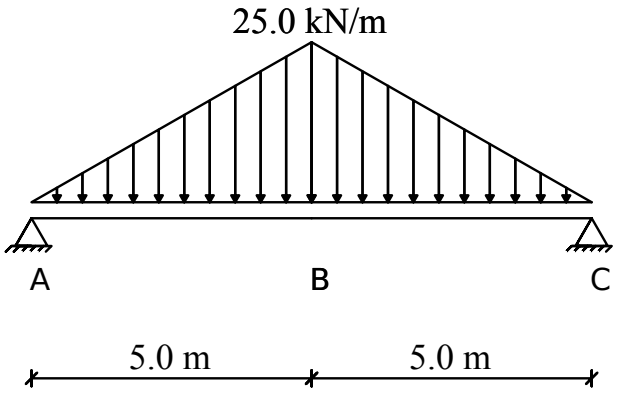

Figure 9: Problem 6: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
3 1
3 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
```

```
2
1 0.0 0.0 -25.0 0.0 0.0
2 0.0 -25.0 0.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            5.00        0.00
  3           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                   v               alpha               gamma
  1            .3000D+08            0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1               1
  2                    1               2
  3                    3               1
  4                    3               2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom
  1          0.00         0.00      -25.00        0.00         0.00
  2          0.00       -25.00        0.00        0.00         0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                  DY                  PHI
```

```
  1       0.0000000000D+00       0.0000000000D+00      -0.1271565755D-02
  2       0.0000000000D+00      -0.4069010417D-02       0.0000000000D+00
  3       0.0000000000D+00       0.0000000000D+00       0.1271565755D-02

>> MEMBER END FORCES

MEM     N1            T1            M1            N2            T2            M2
 1  0.00000D+00  0.62500D+02 -0.39080D-13  0.00000D+00  0.00000D+00  0.20833D+03
 2  0.00000D+00  0.00000D+00 -0.20833D+03  0.00000D+00  0.62500D+02  0.39080D-13
================================================================================
```

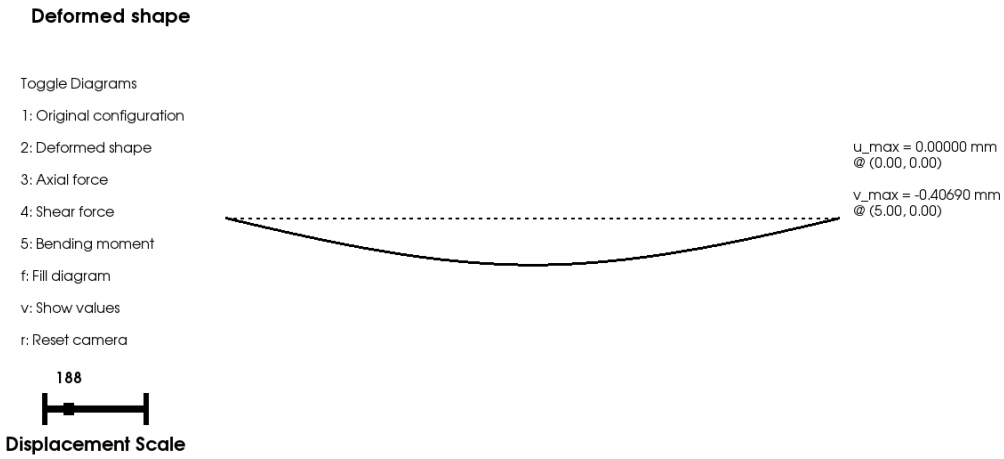

Figure 10: Problem 6, Deformed Shape

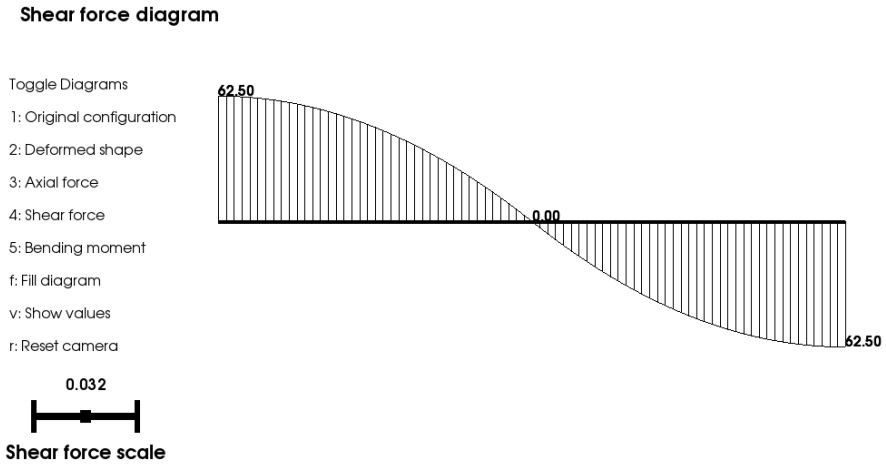

Figure 11: Problem 6, Shear Force Diagram

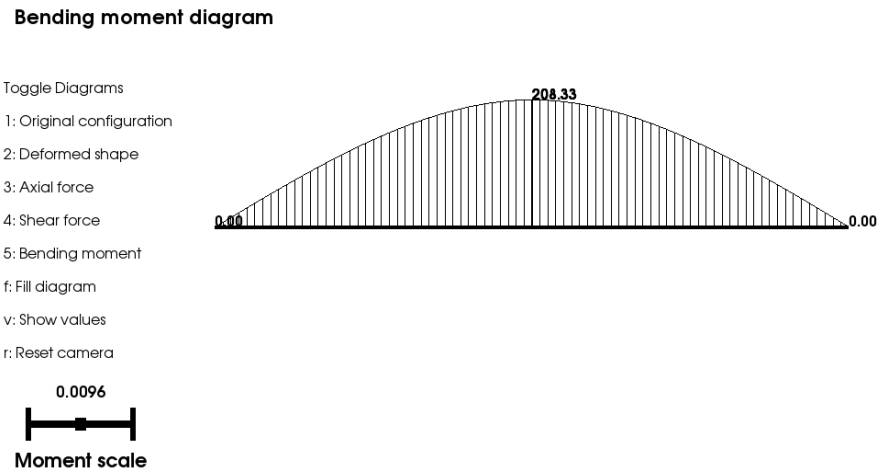

Figure 12: Problem 6, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{\omega l}{4}$	0.62500E+02	0.62500E+02	0.0%
M_{max}	$\frac{\omega l^2}{12}$	0.20833E+03	0.20833E+03	0.0%
δ_{max}	$\frac{\omega l^4}{120EI}$	-0.40690E-02	-0.40690E-02	0.0%

1.1.4 Problem 9: Simple Beam - Two Equal Concentrated Loads Symmetrically Placed

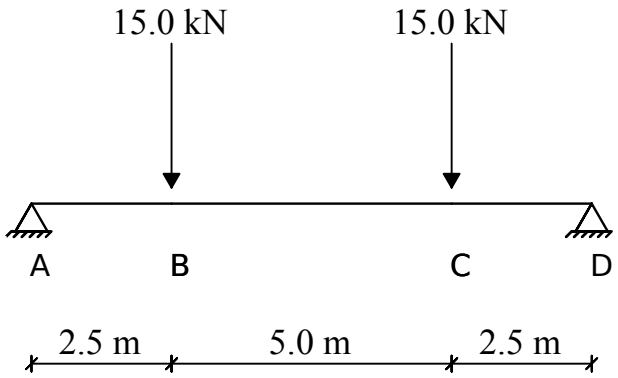

Figure 13: Problem 9: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
2.5 0.0
5.0 0.0
7.5 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
4
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
5 1
5 2
# LINKS <number of links> <master node, slave node, direction>
0
```

```
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
2
2 2 -15.0
4 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            2.50        0.00
  3            5.00        0.00
  4            7.50        0.00
  5           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                   v               alpha               gamma
  1            .3000D+08             0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1
  3            3             4              1             1
  4            4             5              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    5                1
  4                    5                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude
  2              2             -15.00
  4              2             -15.00

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom
```

```
# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                  DY                  PHI

  1      0.0000000000D+00      0.0000000000D+00     -0.2746582031D-03
  2      0.0000000000D+00     -0.6103515625D-03     -0.1831054687D-03
  3      0.0000000000D+00     -0.8392333984D-03     -0.5421010862D-19
  4      0.0000000000D+00     -0.6103515625D-03      0.1831054687D-03
  5      0.0000000000D+00      0.0000000000D+00      0.2746582031D-03


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.15000D+02  0.28422D-13  0.00000D+00 -0.15000D+02  0.37500D+02
 2  0.00000D+00 -0.26645D-13 -0.37500D+02  0.00000D+00  0.26645D-13  0.37500D+02
 3  0.00000D+00  0.42633D-13 -0.37500D+02  0.00000D+00 -0.42633D-13  0.37500D+02
 4  0.00000D+00 -0.15000D+02 -0.37500D+02  0.00000D+00  0.15000D+02 -0.28422D-13
================================================================================
```

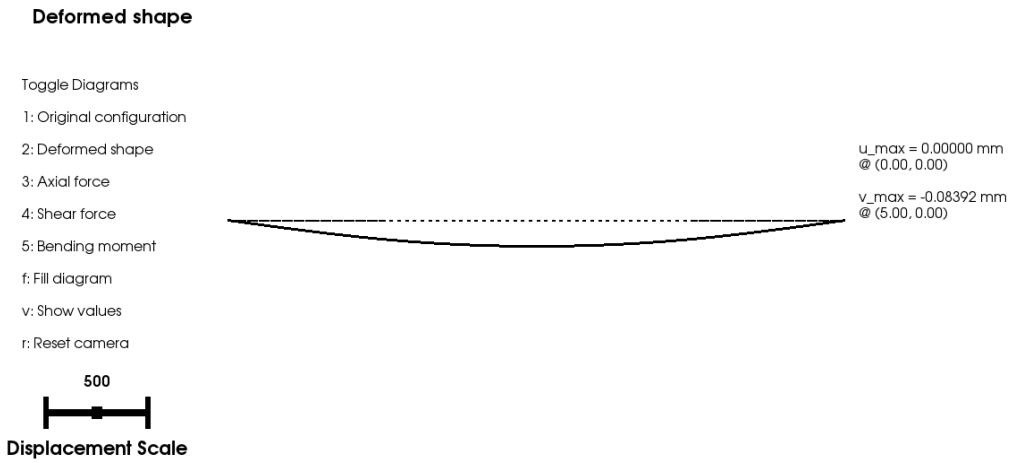

Figure 14: Problem 9, Deformed Shape

	Exact Expression	Exact Value	Computed Value	% RE
V	P	0.15000E+02	0.15000E+02	0.0%
M_{max}	Pa	0.37500E+02	0.37500E+02	0.0%
δ_{max}	$\frac{Pa}{24EI}(3l^2 - 4a^2)$	-0.83923E-03	-0.83923E-03	0.0%

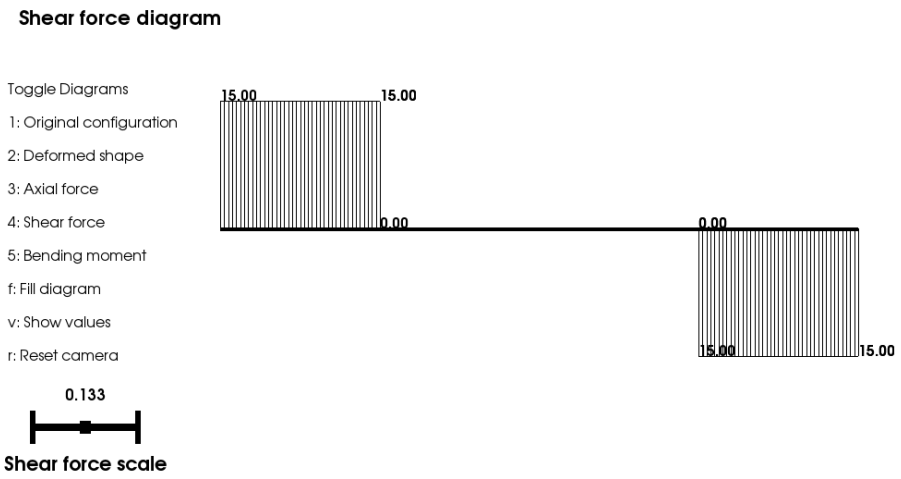

Figure 15: Problem 9, Shear Force Diagram

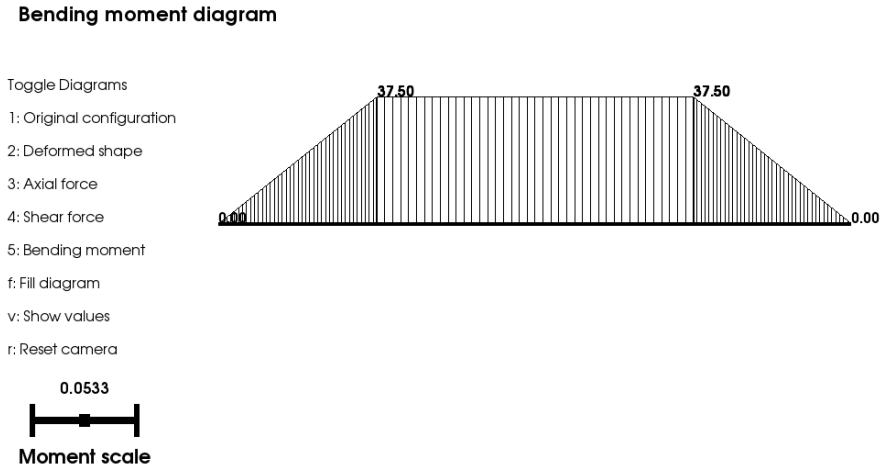

Figure 16: Problem 9, Bending Moment Diagram

1.1.5 Problem 14: Cantilever Beam - Concentrated Load At Any Point

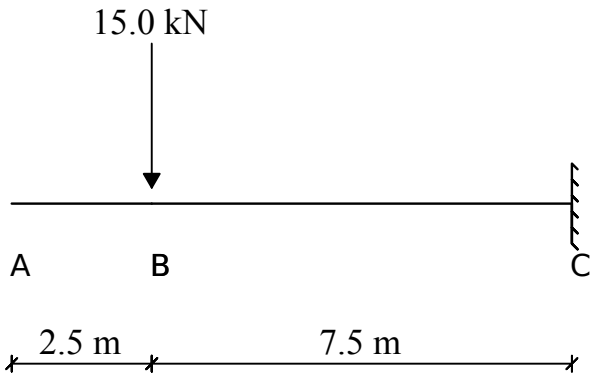

Figure 17: Problem 14: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
2.5 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
3
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
```

```
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            2.50        0.00
  3           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                   v               alpha               gamma
  1            .3000D+08             0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2               1             1
  2            2             3               1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    3                1
  2                    3                2
  3                    3                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude
  2              2             -15.00

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     -0.6179809570D-02      0.8239746094D-03
  2      0.0000000000D+00     -0.4119873047D-02      0.8239746094D-03
  3      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00
```

```
>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.22737D-12  0.68212D-12  0.00000D+00 -0.22737D-12  0.11369D-11
 2  0.00000D+00 -0.15000D+02 -0.56843D-12  0.00000D+00  0.15000D+02 -0.11250D+03
================================================================================
```

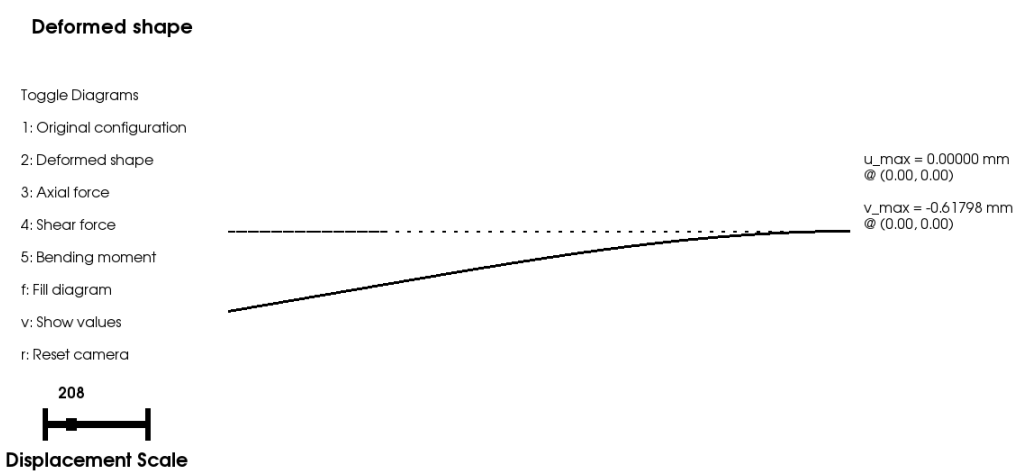

Figure 18: Problem 14, Deformed Shape

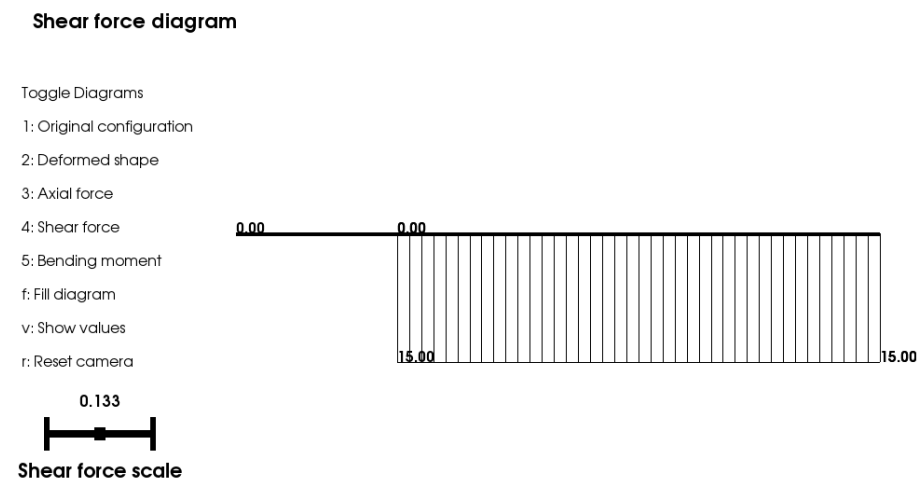

Figure 19: Problem 14, Shear Force Diagram

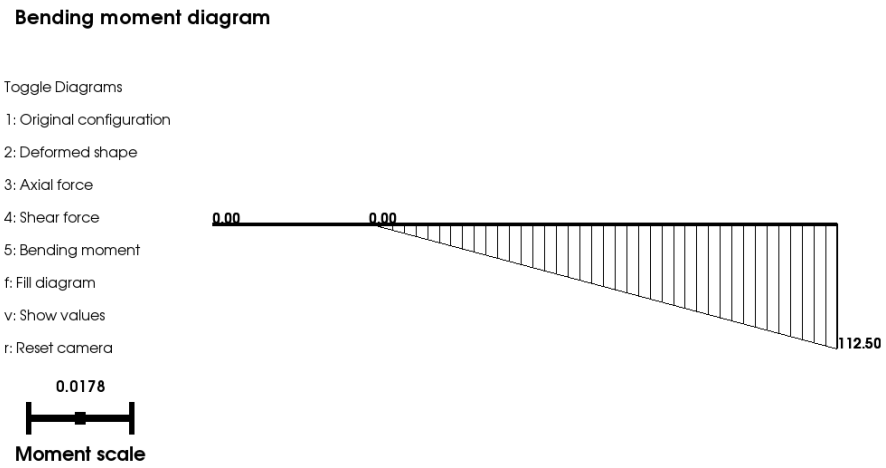

Figure 20: Problem 14, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	P	0.15000E+02	0.15000E+02	0.0%
M_{max}	Pb	0.11250E+03	0.11250E+03	0.0%
δ_{max}	$\frac{Pb^2}{6EI}(3l-b)$	-0.61798E-02	-0.61798E-02	0.0%
δ_B	$\frac{Pb^3}{3EI}$	-0.41199E-02	-0.41199E-02	0.0%

1.1.6 Problem 15: Beam Fixed at One End, Supported at Other - Uniformly Distributed Load

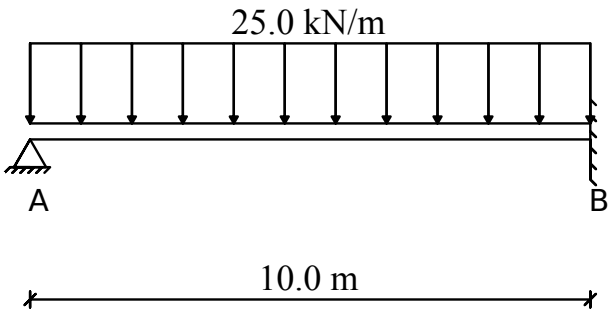

Figure 21: Problem 15: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
4.215 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
5
1 1
1 2
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2
1 0.0 -25.0 -25.0 0.0 0.0
```

```
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            4.21        0.00
  3           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                   v               alpha               gamma
  1            .3000D+08             0.20            .0000D+00             0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2               1             1
  2            2             3               1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    3                1
  4                    3                2
  5                    3                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom
  1          0.00       -25.00      -25.00        0.00        0.00
  2          0.00       -25.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node             DX                    DY                    PHI

  1      0.0000000000D+00      0.0000000000D+00     -0.1017252604D-02
```

```
  2      0.0000000000D+00      -0.2644590607D-02      -0.1188730876D-06
  3      0.0000000000D+00       0.0000000000D+00       0.0000000000D+00

>> MEMBER END FORCES

MEM     N1             T1             M1             N2             T2             M2
 1  0.00000D+00  0.93750D+02  0.42633D-13  0.00000D+00  0.11625D+02  0.17308D+03
 2  0.00000D+00 -0.11625D+02 -0.17308D+03  0.00000D+00  0.15625D+03 -0.31250D+03
================================================================================
```

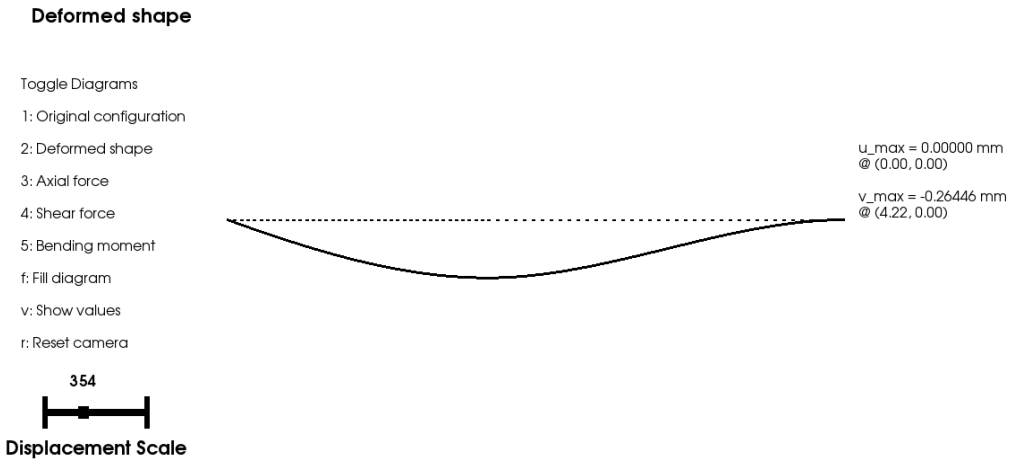

Figure 22: Problem 15, Deformed Shape

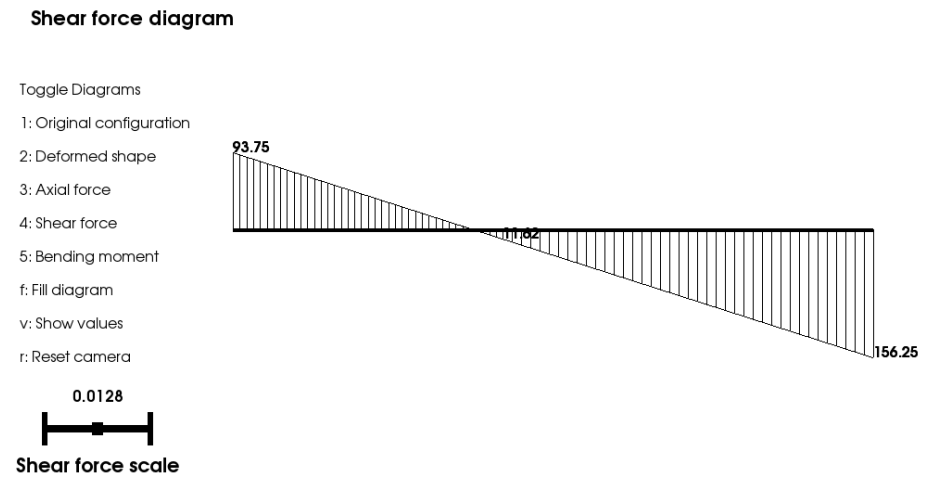

Figure 23: Problem 15, Shear Force Diagram

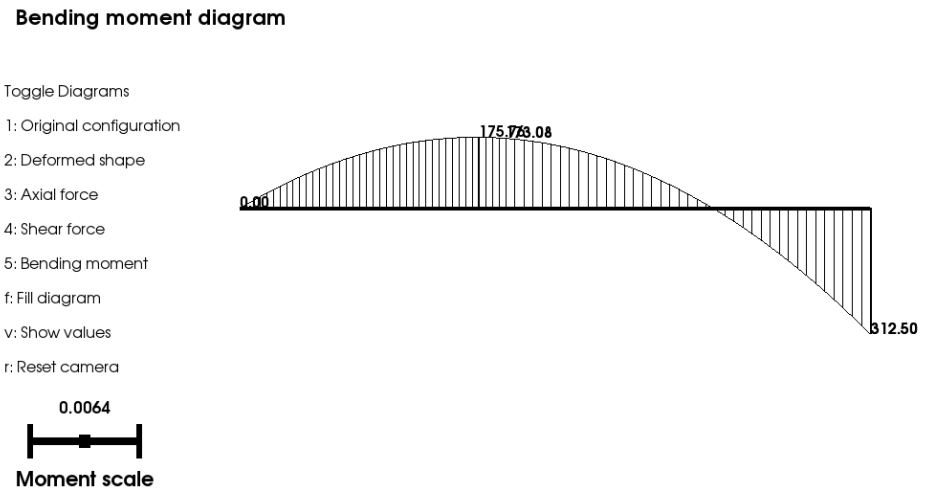

Figure 24: Problem 15, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{3\omega l}{8}$	0.93750E+02	0.93750E+02	0.0%
V_B	$\frac{5\omega l}{8}$	0.15625E+03	0.15625E+03	0.0%
M^-_{max}	$\frac{\omega l^2}{8}$	0.31250E+03	0.31250E+03	0.0%
M^+_{max}	$\frac{9\omega l^2}{128}$	0.17578E+03	0.17576E+03	0.014%
δ_{max}	$\frac{\omega l^4}{185EI}$	-0.26394E-02	-0.26446E-02	0.197%

1.1.7 Problem 19: Beam Overhanging One Support - Uniformly Distributed Load on Overhang

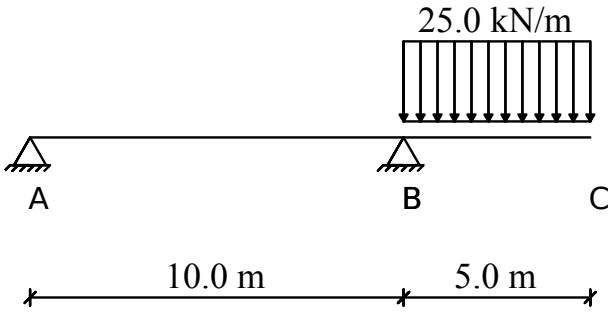

Figure 25: Problem 19: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
10.0 0.0
15.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
4
1 1
1 2
2 1
2 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1
2 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
```

```
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2           10.00        0.00
  3           15.00        0.00

# SECTIONS
Sec.No             A                  I                 h                 X
  1           .3200D+00          .1707D-01          0.8000            0.0000

# MATERIALS
Mat.No             E                  v               alpha              gamma
  1           .3000D+08            0.20           .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    2                1
  4                    2                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom
  2          0.00      -25.00      -25.00         0.00         0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     0.0000000000D+00     0.1017252604D-02
  2      0.0000000000D+00     0.0000000000D+00    -0.2034505208D-02
  3      0.0000000000D+00    -0.1398722331D-01    -0.3051757812D-02
```

```
>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00 -0.31250D+02  0.00000D+00  0.00000D+00  0.31250D+02 -0.31250D+03
 2  0.00000D+00  0.12500D+03  0.31250D+03  0.00000D+00 -0.56843D-13  0.14921D-12
================================================================================
```

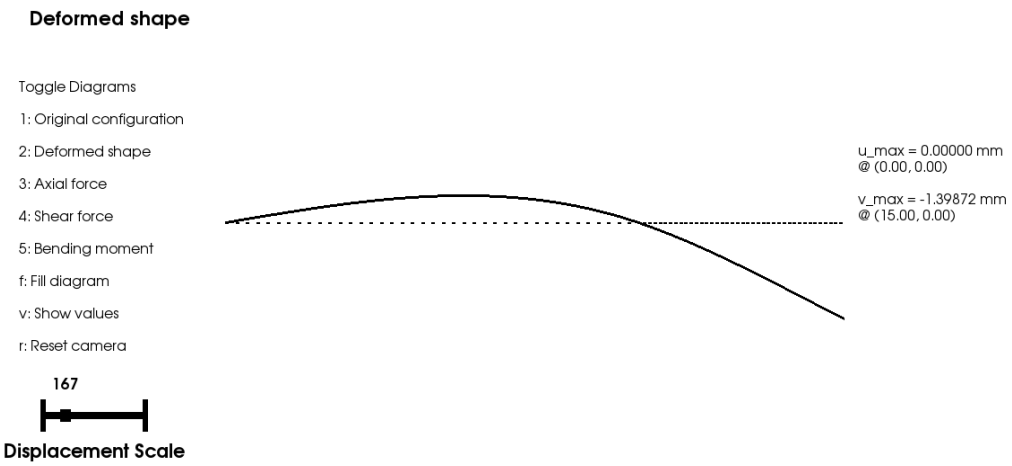

Figure 26: Problem 19, Deformed Shape

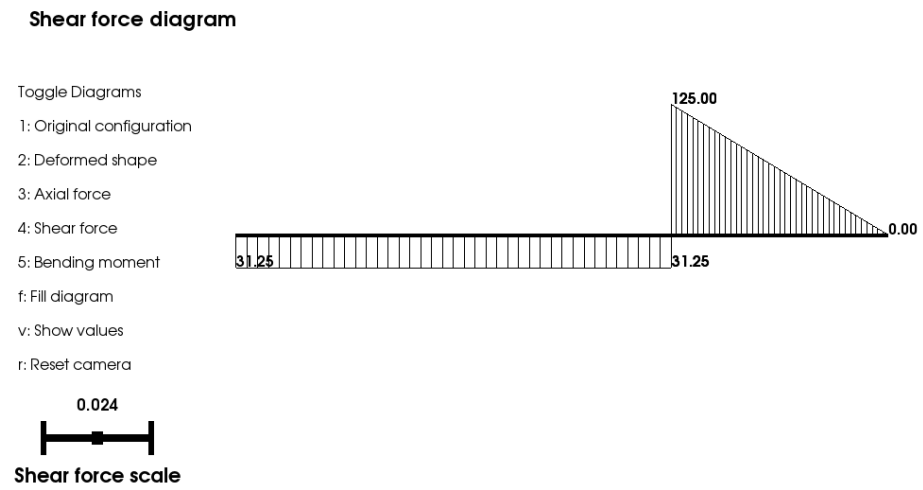

Figure 27: Problem 19, Shear Force Diagram

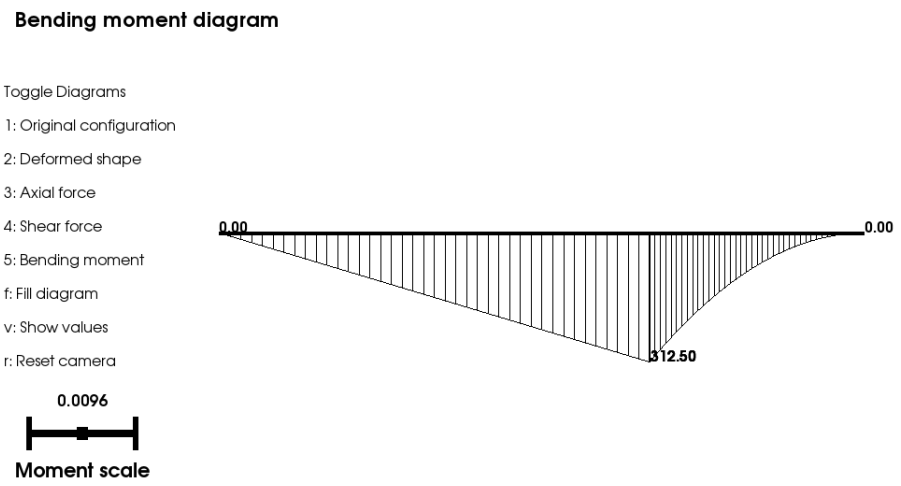

Figure 28: Problem 19, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{\omega a^2}{2l}$	0.31250E+02	0.31250E+02	0.0%
V_B	ωa	0.12500E+03	0.12500E+03	0.0%
M_{max}	$\frac{\omega a^2}{2}$	0.31250E+03	0.31250E+03	0.0%
δ_{max}	$\frac{\omega a^3}{24EI}(4l+3a)$	-0.13987E-01	-0.13987E-01	0.0%

1.1.8 Problem 24: Beam Fixed at Both Ends - Concentrated Load at Center

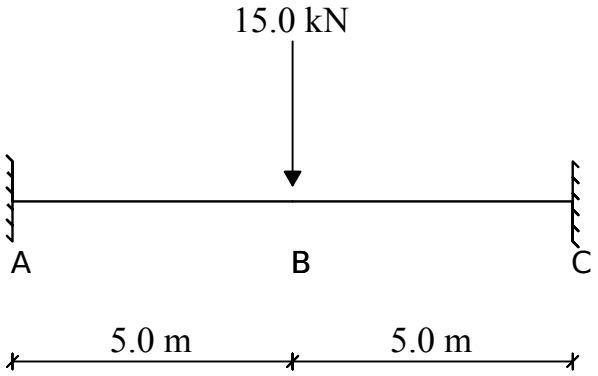

Figure 29: Problem 24: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
5.0 0.0
10.0 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
1 3
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
```

```
2 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            5.00        0.00
  3           10.00        0.00

# SECTIONS
Sec.No             A                   I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                   v               alpha               gamma
  1            .3000D+08            0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    1                3
  4                    3                1
  5                    3                2
  6                    3                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude
  2              2             -15.00

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI
```

```
  1       0.0000000000D+00       0.0000000000D+00       0.0000000000D+00
  2       0.0000000000D+00      -0.1525878906D-03       0.0000000000D+00
  3       0.0000000000D+00       0.0000000000D+00       0.0000000000D+00

>> MEMBER END FORCES

MEM     N1            T1            M1            N2            T2            M2
 1  0.00000D+00  0.75000D+01  0.18750D+02  0.00000D+00 -0.75000D+01  0.18750D+02
 2  0.00000D+00 -0.75000D+01 -0.18750D+02  0.00000D+00  0.75000D+01 -0.18750D+02
================================================================================
```

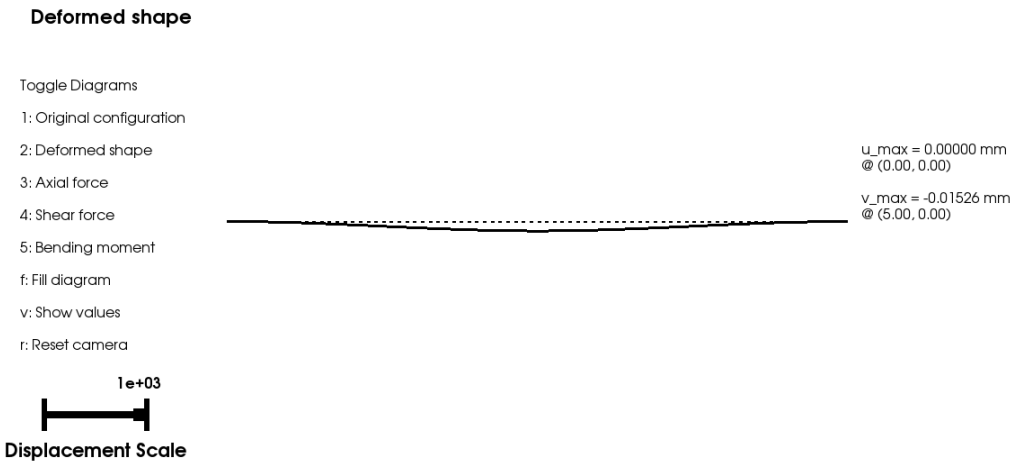

Figure 30: Problem 24, Deformed Shape

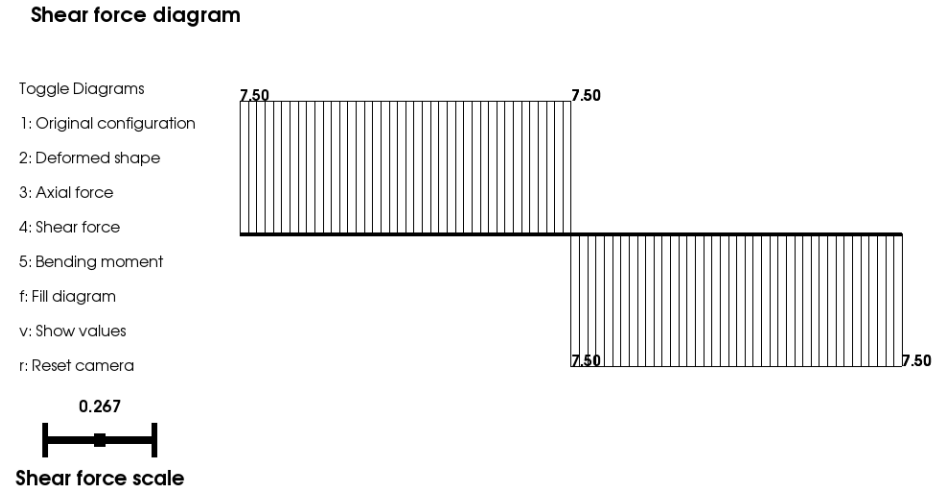

Figure 31: Problem 24, Shear Force Diagram

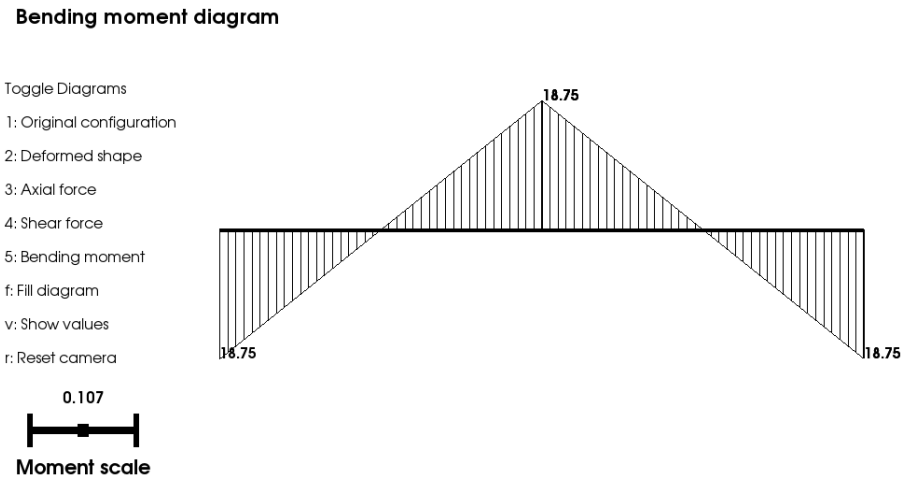

Figure 32: Problem 24, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V	$\frac{P}{2}$	0.75000E+01	0.75000E+01	0.0%
M_{max}	$\frac{Pl}{8}$	0.18750E+02	0.18750E+02	0.0%
δ_{max}	$\frac{Pl^3}{192EI}$	-0.15259E-03	-0.15259E-03	0.0%

1.1.9 Problem 27: Continuous Beam - Two Equal Spans - Concentrated Load At Center of One Span

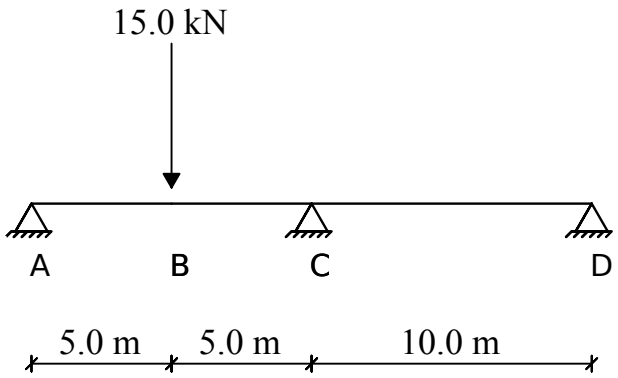

Figure 33: Problem 27: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
4
0.0 0.0
5.0 0.0
10.0 0.0
20.0 0.0
# ELEMENTS <number of elements> <start node, end node>
3
1 2
2 3
3 4
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
3 1
3 2
4 1
4 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
```

```
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 2 -15.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            5.00        0.00
  3           10.00        0.00
  4           20.00        0.00

# SECTIONS
Sec.No             A                  I                 h                  X
  1            .3200D+00          .1707D-01          0.8000             0.0000

# MATERIALS
Mat.No             E                  v               alpha               gamma
  1            .3000D+08            0.20           .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1
  3            3             4              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    3                1
  4                    3                2
  5                    4                1
  6                    4                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude
  2              2             -15.00

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1           em          e2          P

================================================================================
```

```
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                  DY                  PHI

  1      0.0000000000D+00    0.0000000000D+00    -0.1373291016D-03
  2      0.0000000000D+00   -0.4386901855D-03     0.1144409180D-04
  3      0.0000000000D+00    0.0000000000D+00     0.9155273437D-04
  4      0.0000000000D+00    0.0000000000D+00    -0.4577636719D-04


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.60938D+01  0.57732D-14  0.00000D+00 -0.60938D+01  0.30469D+02
 2  0.00000D+00 -0.89062D+01 -0.30469D+02  0.00000D+00  0.89062D+01 -0.14062D+02
 3  0.00000D+00  0.14062D+01  0.14062D+02  0.00000D+00 -0.14062D+01  0.00000D+00
================================================================================
```

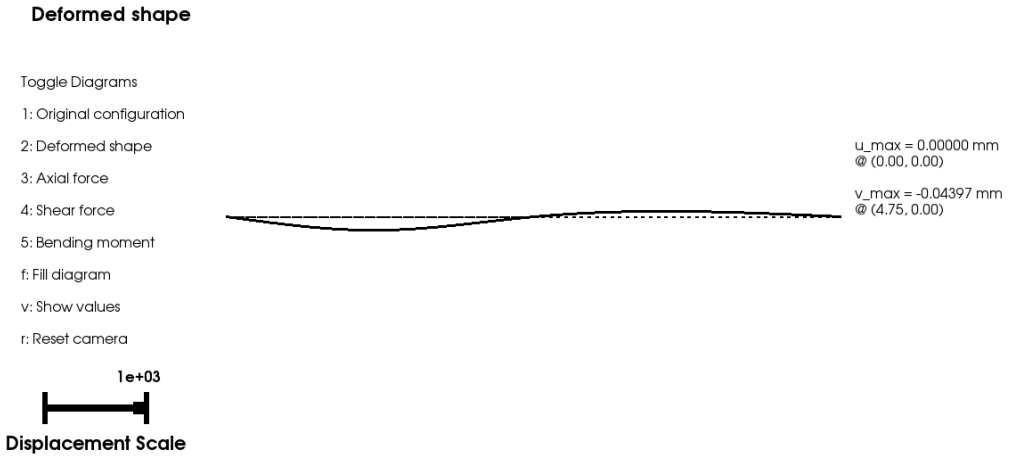

Figure 34: Problem 27, Deformed Shape

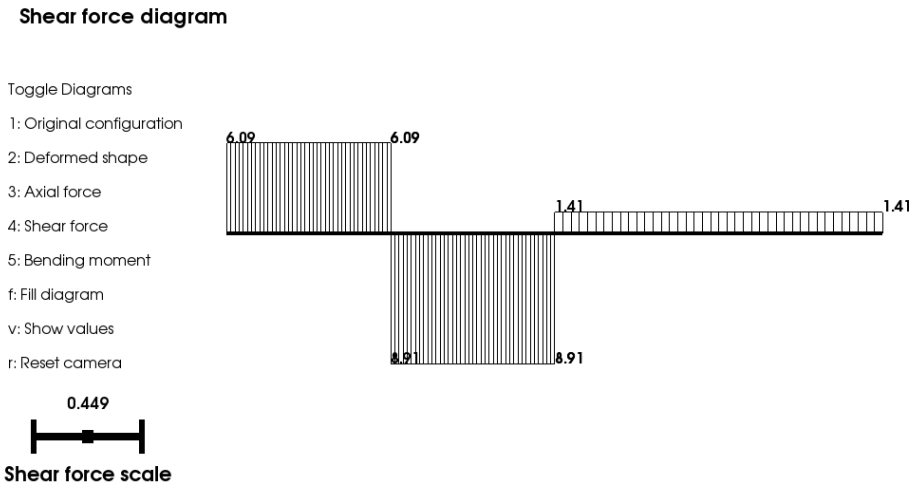

Figure 35: Problem 27, Shear Force Diagram

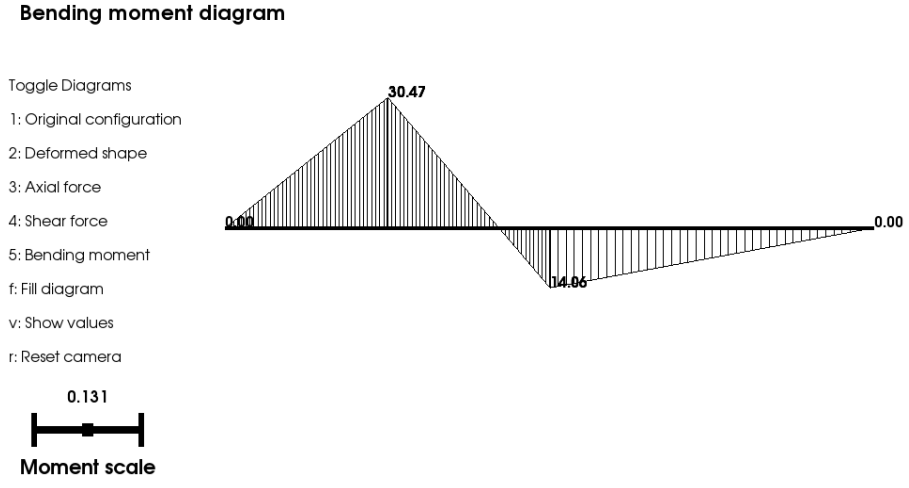

Figure 36: Problem 27, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{13}{32}P$	0.60938E+01	0.60938E+01	0.0%
V_C	$\frac{19}{32}P$	0.89062E+01	0.89062E+01	0.0%
V_D	$\frac{3}{32}P$	0.14062E+01	0.14062E+01	0.0%
M_C	$\frac{3}{32}Pl$	0.14062E+02	0.14062E+02	0.0%
M_{max}	$\frac{13}{64}Pl$	0.30469E+02	0.30469E+02	0.0%

1.1.10 Problem 29: Continuous Beam - Two Equal Spans - Uniformly Distributed Load

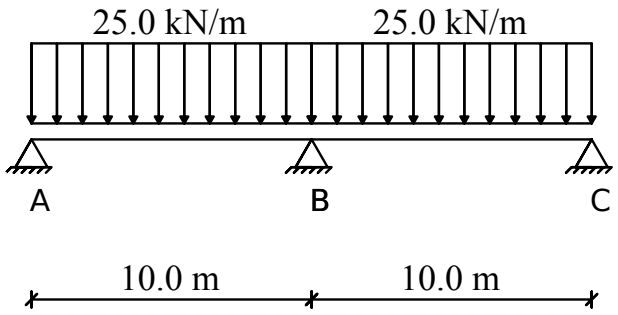

Figure 37: Problem 29: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
3.75 0.0
10.0 0.0
16.25 0.0
20.0 0.0
# ELEMENTS <number of elements> <start node, end node>
4
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
1
0.32 0.0170666666666667 0.8 0.0
# SECTION INCIDENCES
1
1
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
3 1
3 2
5 1
5 2
# LINKS <number of links> <master node, slave node, direction>
0
```

```
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
4
1 0.0 -25.0 -25.0 0.0 0.0
2 0.0 -25.0 -25.0 0.0 0.0
3 0.0 -25.0 -25.0 0.0 0.0
4 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            3.75        0.00
  3           10.00        0.00
  4           16.25        0.00
  5           20.00        0.00

# SECTIONS
Sec.No             A                  I                 h                  X
  1            .3200D+00          .1707D-01           0.8000             0.0000

# MATERIALS
Mat.No             E                  v               alpha               gamma
  1            .3000D+08            0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1
  3            3             4              1             1
  4            4             5              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    3                1
  4                    3                2
  5                    5                1
  6                    5                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
```

```
Element       Px         Py1         Py2        DTtop      DTbottom
  1         0.00      -25.00      -25.00         0.00         0.00
  2         0.00      -25.00      -25.00         0.00         0.00
  3         0.00      -25.00      -25.00         0.00         0.00
  4         0.00      -25.00      -25.00         0.00         0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                  DY                  PHI

  1      0.0000000000D+00    0.0000000000D+00    -0.1017252604D-02
  2      0.0000000000D+00   -0.2607703209D-02    -0.1589457194D-03
  3      0.0000000000D+00    0.0000000000D+00     0.6776263578D-19
  4      0.0000000000D+00   -0.2607703209D-02     0.1589457194D-03
  5      0.0000000000D+00    0.0000000000D+00     0.1017252604D-02


>> MEMBER END FORCES

MEM     N1           T1           M1           N2           T2           M2
 1  0.00000D+00  0.93750D+02  0.16342D-12  0.00000D+00 -0.12790D-12  0.17578D+03
 2  0.00000D+00  0.14211D-13 -0.17578D+03  0.00000D+00  0.15625D+03 -0.31250D+03
 3  0.00000D+00  0.15625D+03  0.31250D+03  0.00000D+00  0.28422D-13  0.17578D+03
 4  0.00000D+00 -0.56843D-13 -0.17578D+03  0.00000D+00  0.93750D+02  0.00000D+00
================================================================================
```

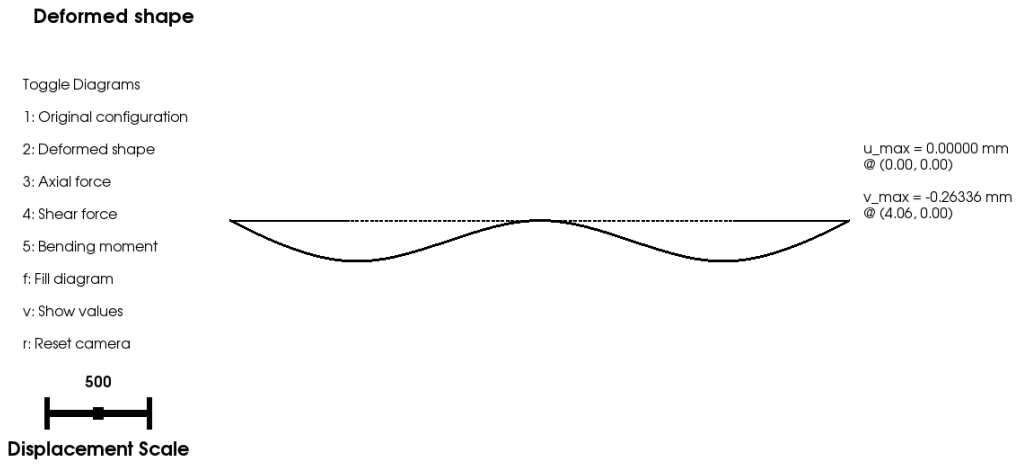

Figure 38: Problem 29, Deformed Shape

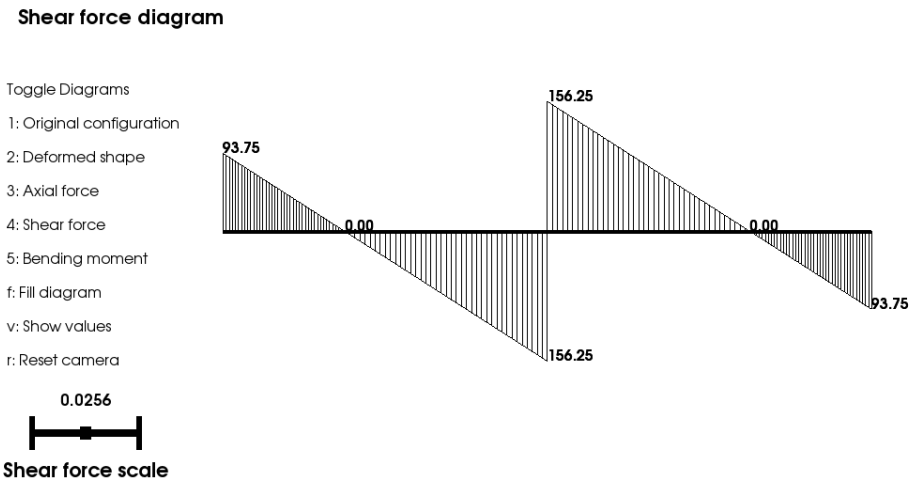

Figure 39: Problem 29, Shear Force Diagram

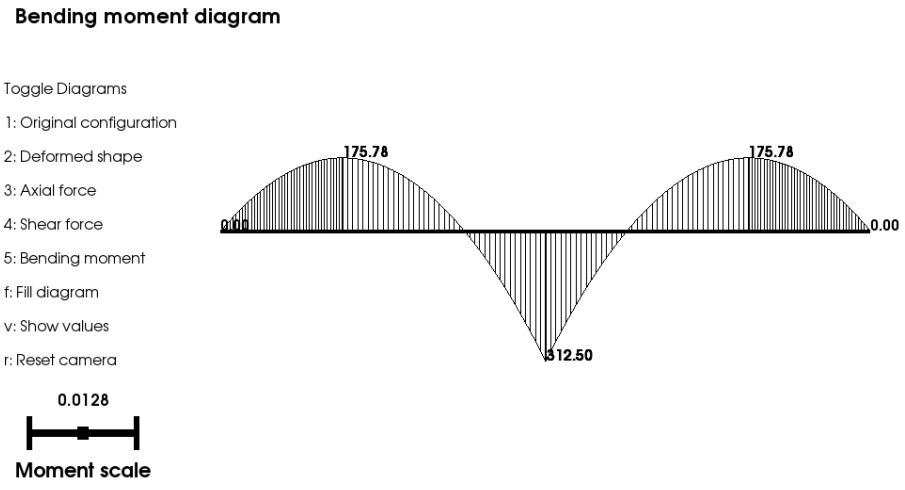

Figure 40: Problem 29, Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
V_A	$\frac{3\omega l}{8}$	0.93750E+02	0.93750E+02	0.0%
V_B	$\frac{5\omega l}{8}$	0.15625E+03	0.15625E+03	0.0%
M^+_{max}	$\frac{9\omega l^2}{128}$	0.17578E+03	0.17578E+03	0.0%
M^-_{max}	$\frac{\omega l^2}{8}$	0.31250E+03	0.31250E+03	0.0%
δ_{max}	$\frac{\omega l^4}{185EI}$	-0.26394E-02	-0.26336E-03	0.220%

1.2 One-Bay, One-Storey Frame with Pin and Roller Supports and a Tie Member

The first frame to be analyzed under three different load cases is demonstrated in the figure below. It will be referred to as FR1 in the following parts.

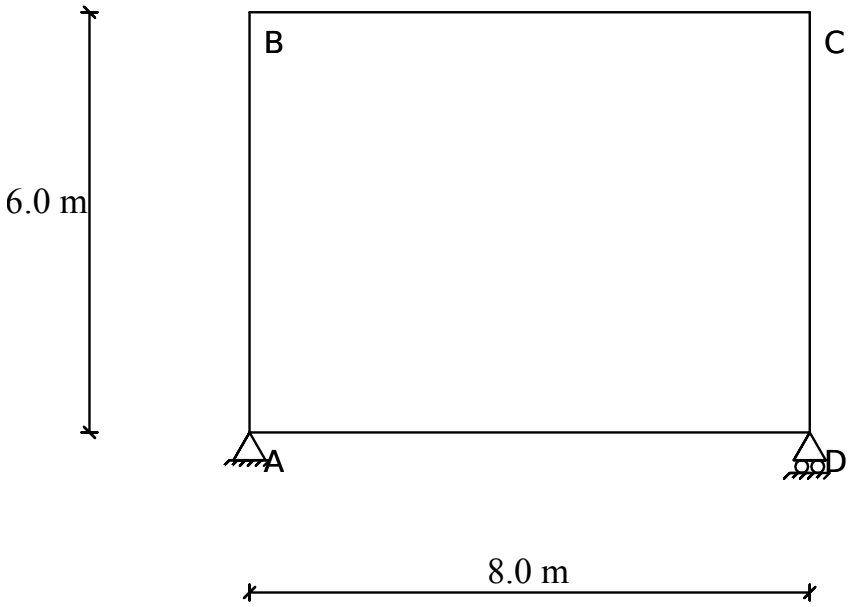

Figure 41: Frame FR1

Three different cross sections are assigned to the four members that the frame is composed of, and the areas are amplified by a factor of $10^7 I_i/l_i^2$ to mimic axial rigidity by ensuring that the axial stiffness is orders of magnitude greater than the bending stiffness for each member. Cross sectional properties of each member are summarized via the following table:

Member	b(m)	h(m)	I(m^4)	l(m)	A(m^2)
AB	0.3	0.6	0.54000E-02	6.0	0.15000000E+04
BC	0.3	0.5	0.31250E-02	8.0	0.48828125E+03
CD	0.3	0.6	0.54000E-02	6.0	0.15000000E+04
AD	0.3	0.3	0.67500E-03	8.0	0.10546875E+03

To ease the process of parametric analysis, some factors are used which take the different cross sectional properties into account. These factors are used in the parts to come to compute the analytical values of the desired quantities, and therefore presented below for the sake of completeness.

Factor	Expression	Value
k_1	$\frac{I_{BC}}{I_{AD}}$	4.62963
k_2	$\frac{I_{BC}}{I_{AB}}\frac{h}{l}$	0.43403
R_1	$2k_2 + 3$	3.86806
R_2	$3k_1 + 2k_2$	14.75694
F_1	$R_1 R_2 - k_2^2$	56.89230
F_2	$1 + k_1 + 6k_2$	8.23380

1.2.1 Load Case 1

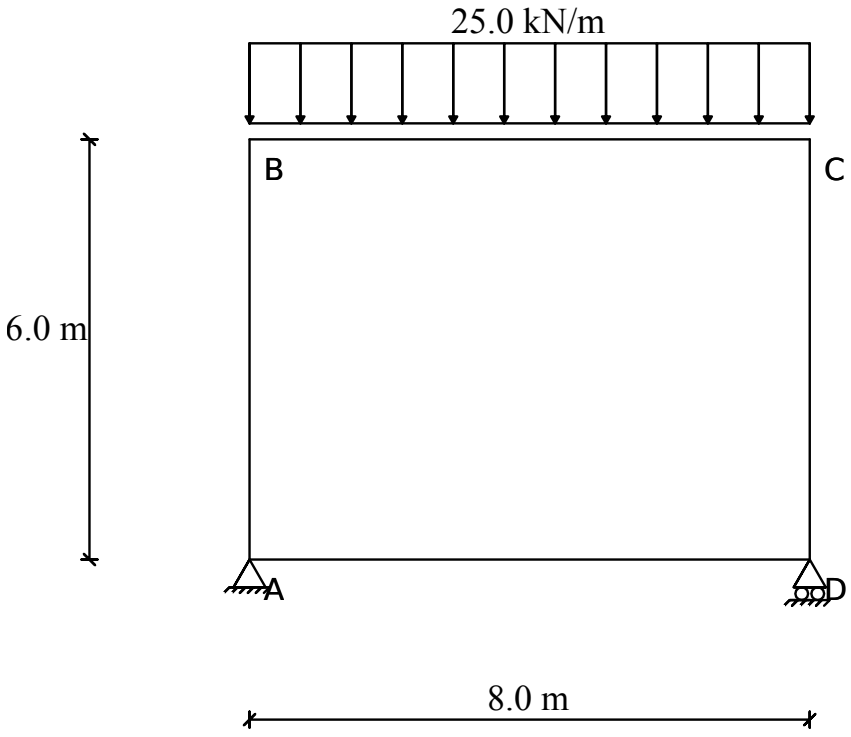

Figure 42: Problem 1: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
0.0 6.0
4.0 6.0
8.0 6.0
8.0 0.0
```

```
# ELEMENTS <number of elements> <start node, end node>
5
1 2
2 3
3 4
4 5
1 5
# SECTIONS <number of sections> <A, I, h, X>
3
0.10546875D+03 0.67500D-03 0.3 0
0.15000000D+04 0.54000D-02 0.6 0
0.48828125D+03 0.31250D-02 0.5 0
# SECTION INCIDENCES
2
3
3
2
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
3
1 1
1 2
5 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2
2 0.0 -25.0 -25.0 0.0 0.0
3 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            0.00        6.00
  3            4.00        6.00
  4            8.00        6.00
  5            8.00        0.00

# SECTIONS
Sec.No             A                  I                 h                  X
  1            .1055D+03          .6750D-03          0.3000             0.0000
  2            .1500D+04          .5400D-02          0.6000             0.0000
  3            .4883D+03          .3125D-02          0.5000             0.0000
```

```
# MATERIALS
Mat.No              E                   v               alpha               gamma
  1            .3000D+08             0.20            .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              2             1
  2            2             3              3             1
  3            3             4              3             1
  4            4             5              2             1
  5            1             5              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    5                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction       K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom
  2          0.00       -25.00      -25.00        0.00        0.00
  3          0.00       -25.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00      0.0000000000D+00      0.6027754160D-03
  2      0.2736458578D-07     -0.1333333333D-07     -0.1262074710D-02
  3      0.2250377120D-07     -0.5368607197D-02     -0.1876177978D-18
  4      0.1764295662D-07     -0.1333333333D-07      0.1262074710D-02
  5      0.4500754241D-07      0.0000000000D+00     -0.6027754160D-03


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.10000D+03 -0.17801D+02 -0.30516D+01 -0.10000D+03  0.17801D+02 -0.10375D+03
 2  0.17801D+02  0.10000D+03  0.10375D+03 -0.17801D+02 -0.71054D-14  0.96247D+02
 3  0.17801D+02  0.28422D-13 -0.96247D+02 -0.17801D+02  0.10000D+03 -0.10375D+03
 4  0.10000D+03  0.17801D+02  0.10375D+03 -0.10000D+03 -0.17801D+02  0.30516D+01
 5 -0.17801D+02  0.28866D-14  0.30516D+01  0.17801D+02 -0.28866D-14 -0.30516D+01
================================================================================
```

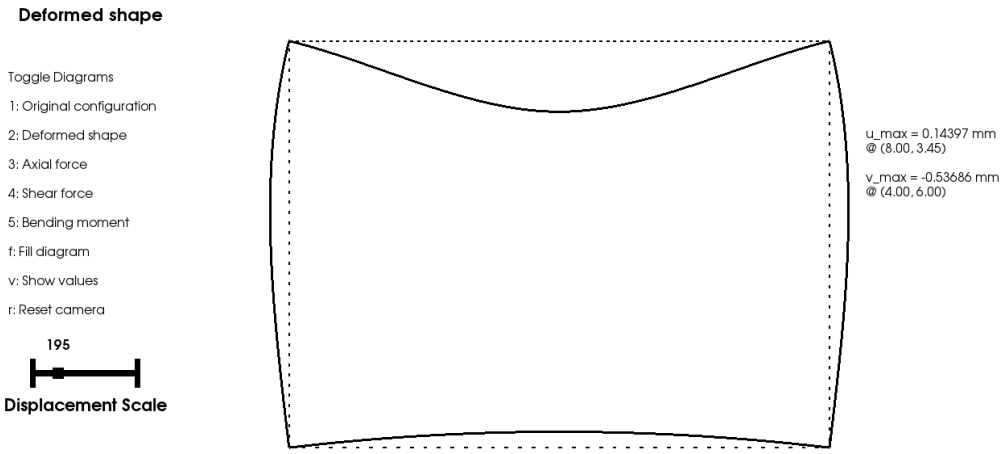

Figure 43: FR1, Load Case 1: Deformed Shape

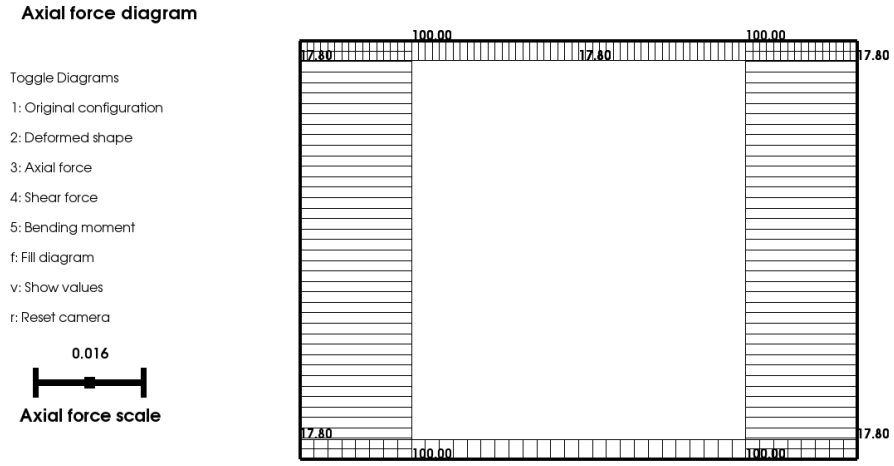

Figure 44: FR1, Load Case 1: Axial Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_A = M_D$	$\frac{\omega l^2}{4}\frac{k_2}{F_1}$	0.30516E+01	0.30516E+01	0.0%
$M_B = M_C$	$\frac{-\omega l^2}{4}\frac{R_2}{F_1}$	-0.10375E+02	-0.10375E+02	0.0%
M_{max}	$\frac{\omega l^2}{8} + M_B$	0.96246E+02	0.96247E+02	0.001%
$V_A = V_D$	$\frac{\omega l}{2}$	0.10000E+03	0.10000E+03	0.0%

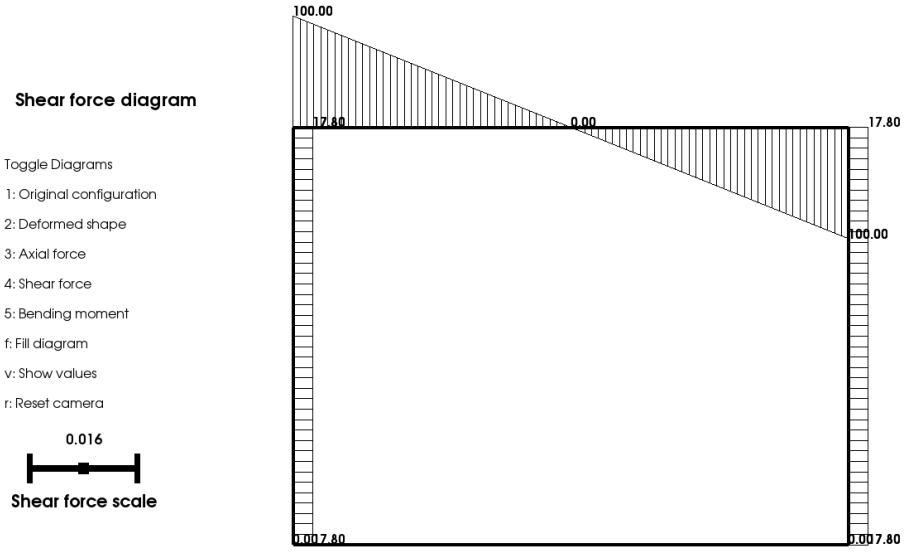

Figure 45: FR1, Load Case 1: Shear Force Diagram

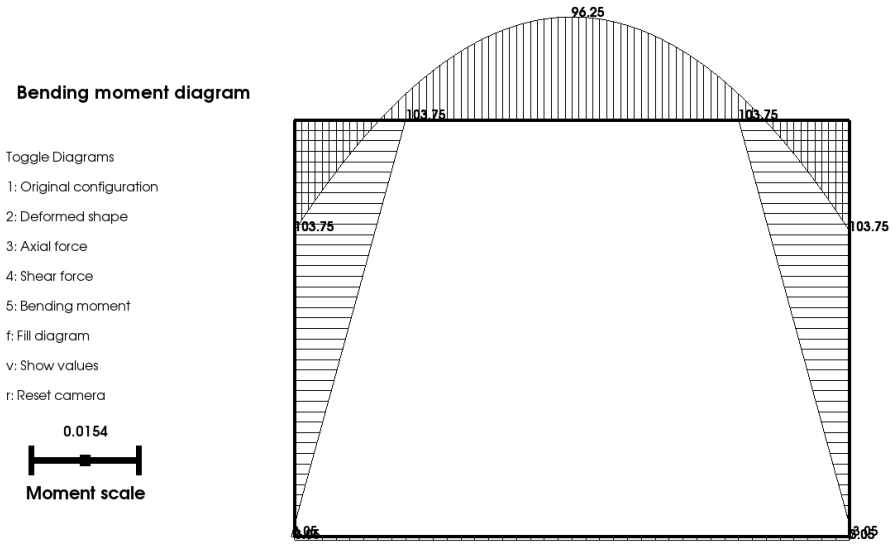

Figure 46: FR1, Load Case 1: Bending Moment Diagram

1.2.2 Load Case 2

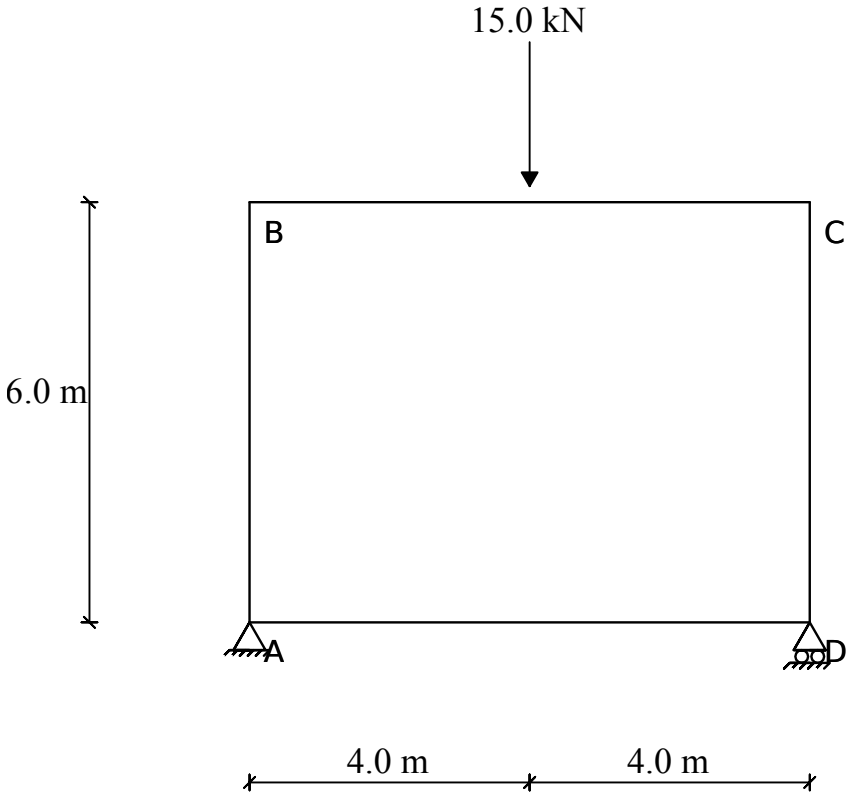

Figure 47: Problem 1: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
0.0 6.0
4.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
5
1 2
2 3
3 4
4 5
1 5
# SECTIONS <number of sections> <A, I, h, X>
3
0.10546875D+03 0.67500D-03 0.3 0
0.15000000D+04 0.54000D-02 0.6 0
0.48828125D+03 0.31250D-02 0.5 0
# SECTION INCIDENCES
2
3
```

```
3
2
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
3
1 1
1 2
5 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
3 2 -50.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            0.00        6.00
  3            4.00        6.00
  4            8.00        6.00
  5            8.00        0.00

# SECTIONS
Sec.No             A                 I                 h                 X
  1           .1055D+03         .6750D-03          0.3000            0.0000
  2           .1500D+04         .5400D-02          0.6000            0.0000
  3           .4883D+03         .3125D-02          0.5000            0.0000

# MATERIALS
Mat.No             E                 v              alpha              gamma
  1           .3000D+08            0.20           .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              2             1
  2            2             3              3             1
  3            3             4              3             1
  4            4             5              2             1
  5            1             5              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
```

```
  2                   1                2
  3                   5                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction             K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction       K

# NODAL LOADS
Node         Direction         Magnitude
  3              2             -50.00

# ELEMENT LOADS
Element       Px          Py1          Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1           em           e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00      0.0000000000D+00      0.2260407810D-03
  2      0.1026171967D-07     -0.3333333333D-08     -0.4732780161D-03
  3      0.8438914201D-08     -0.2368781588D-02     -0.1588186776D-19
  4      0.6616108733D-08     -0.3333333333D-08      0.4732780161D-03
  5      0.1687782840D-07      0.0000000000D+00     -0.2260407810D-03


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.25000D+02 -0.66753D+01 -0.11443D+01 -0.25000D+02  0.66753D+01 -0.38908D+02
 2  0.66753D+01  0.25000D+02  0.38908D+02 -0.66753D+01 -0.25000D+02  0.61092D+02
 3  0.66753D+01 -0.25000D+02 -0.61092D+02 -0.66753D+01  0.25000D+02 -0.38908D+02
 4  0.25000D+02  0.66753D+01  0.38908D+02 -0.25000D+02 -0.66753D+01  0.11443D+01
 5 -0.66753D+01  0.88818D-15  0.11443D+01  0.66753D+01 -0.88818D-15 -0.11443D+01
================================================================================
```

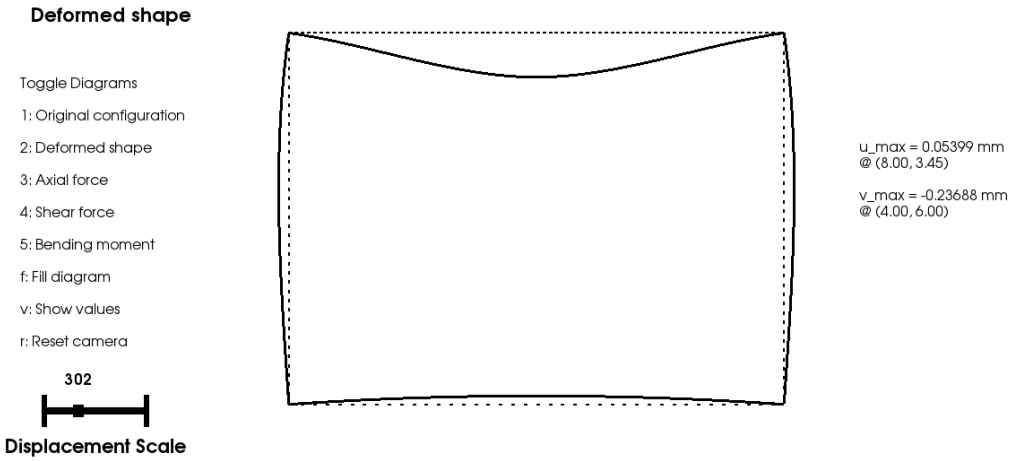

Figure 48: FR1, Load Case 2: Deformed Shape

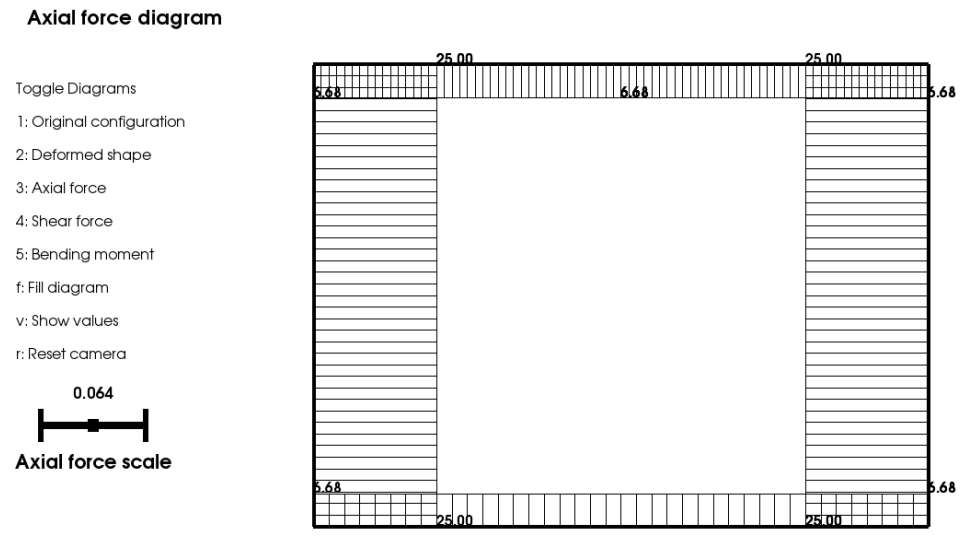

Figure 49: FR1, Load Case 2: Axial Force Diagram

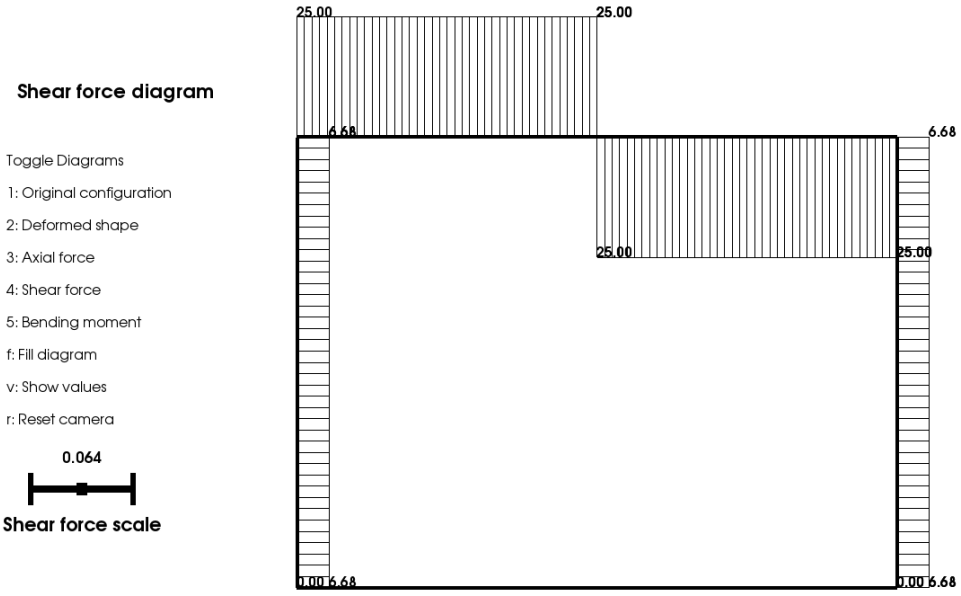

Figure 50: FR1, Load Case 2: Shear Force Diagram

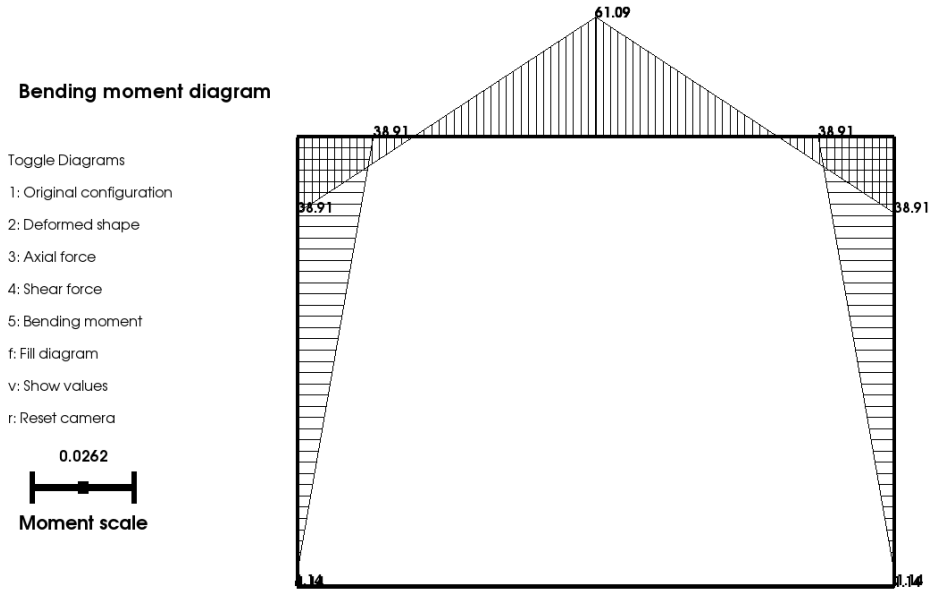

Figure 51: FR1, Load Case 2: Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_A = M_D$	$\frac{3}{8}Pl\frac{k_2}{F_1}$	0.11443E+01	0.11443E+01	0.0%
$M_B = M_C$	$\frac{-\omega l^2}{4}\frac{R_2}{F_1}$	-0.38908E+02	-0.38908E+02	0.0%
M_{max}	$\frac{pl}{4} + M_B$	0.61092E+02	0.61092E+02	0.0%
$V_A = V_D$	$\frac{P}{2}$	0.25000E+02	0.25000E+02	0.0%

1.2.3 Load Case 3

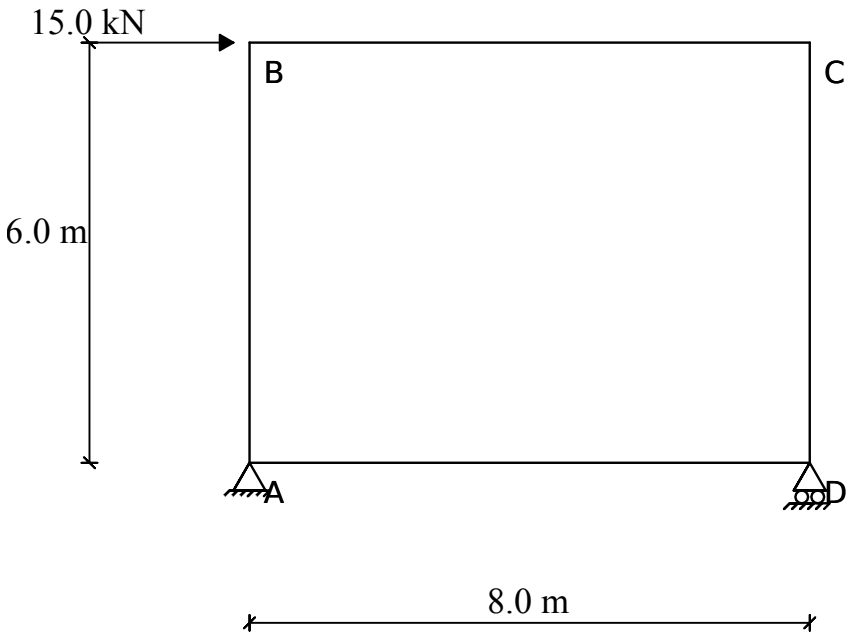

Figure 52: Problem 1: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
4
0.0 0.0
0.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
4
1 2
2 3
3 4
1 4
# SECTIONS <number of sections> <A, I, h, X>
3
0.10546875D+03 0.67500D-03 0.3 0
0.15000000D+04 0.54000D-02 0.6 0
0.48828125D+03 0.31250D-02 0.5 0
# SECTION INCIDENCES
2
3
2
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
```

```
1
1
# RESTRAINTS <number of restraints> <node number, direction>
3
1 1
1 2
4 2
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 1 50.0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            0.00        6.00
  3            8.00        6.00
  4            8.00        0.00

# SECTIONS
Sec.No             A                 I                 h                 X
  1            .1055D+03         .6750D-03          0.3000            0.0000
  2            .1500D+04         .5400D-02          0.6000            0.0000
  3            .4883D+03         .3125D-02          0.5000            0.0000

# MATERIALS
Mat.No             E                 v               alpha              gamma
  1            .3000D+08            0.20           .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              2             1
  2            2             3              3             1
  3            3             4              2             1
  4            1             4              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    4                2

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction             K

# ELASTIC LINKS
E. Link. No        Node 1             Node 2           Direction       K
```

```
# NODAL LOADS
Node         Direction         Magnitude
  2              1              50.00

# ELEMENT LOADS
Element       Px          Py1          Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1           em           e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                    DY                    PHI

  1      0.0000000000D+00      0.0000000000D+00     -0.2761385424D-02
  2      0.1567258161D-01      0.3602052035D-08     -0.1536881566D-02
  3      0.1567256795D-01     -0.3602052035D-08     -0.1536871305D-02
  4      0.6320981152D-07      0.0000000000D+00     -0.2761371969D-02


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1 -0.27015D+02  0.25000D+02  0.41938D+02  0.27015D+02 -0.25000D+02  0.10806D+03
 2  0.25000D+02 -0.27015D+02 -0.10806D+03 -0.25000D+02  0.27015D+02 -0.10806D+03
 3  0.27015D+02  0.25000D+02  0.10806D+03 -0.27015D+02 -0.25000D+02  0.41938D+02
 4 -0.25000D+02 -0.10485D+02 -0.41938D+02  0.25000D+02  0.10485D+02 -0.41938D+02
================================================================================
```

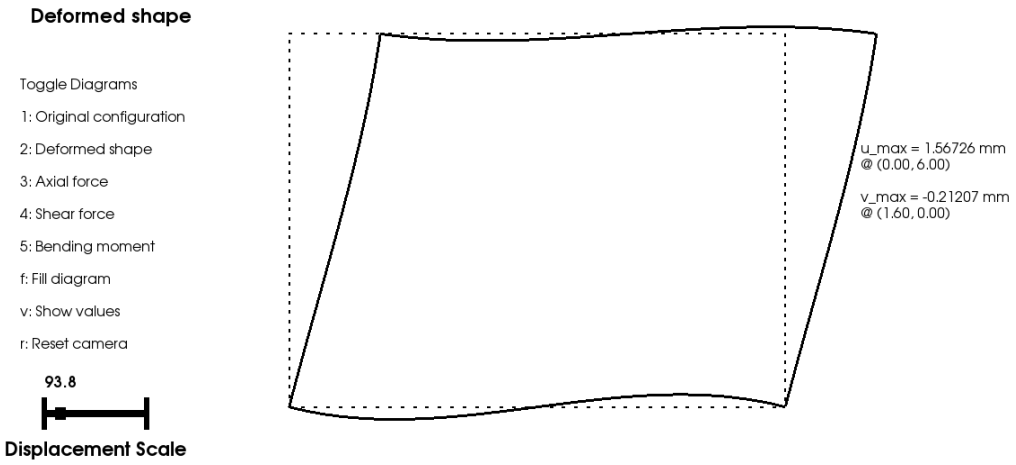

Figure 53: FR1, Load Case 3: Deformed Shape

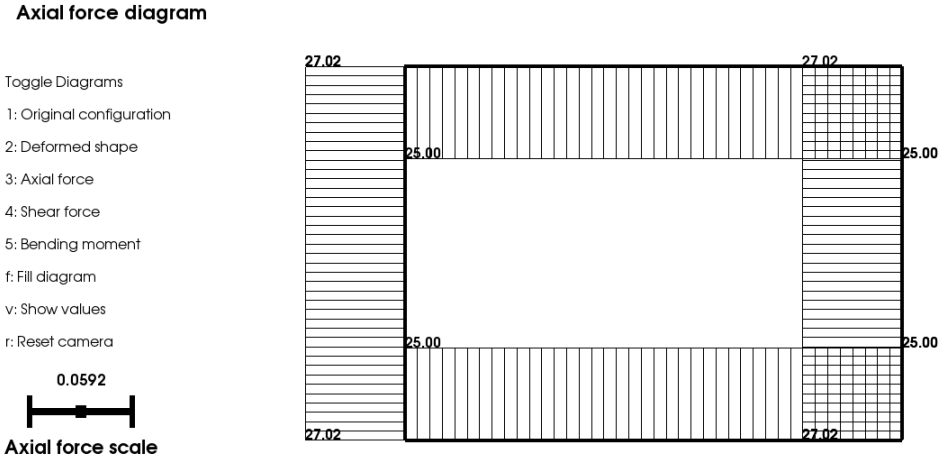

Figure 54: FR1, Load Case 3: Axial Force Diagram

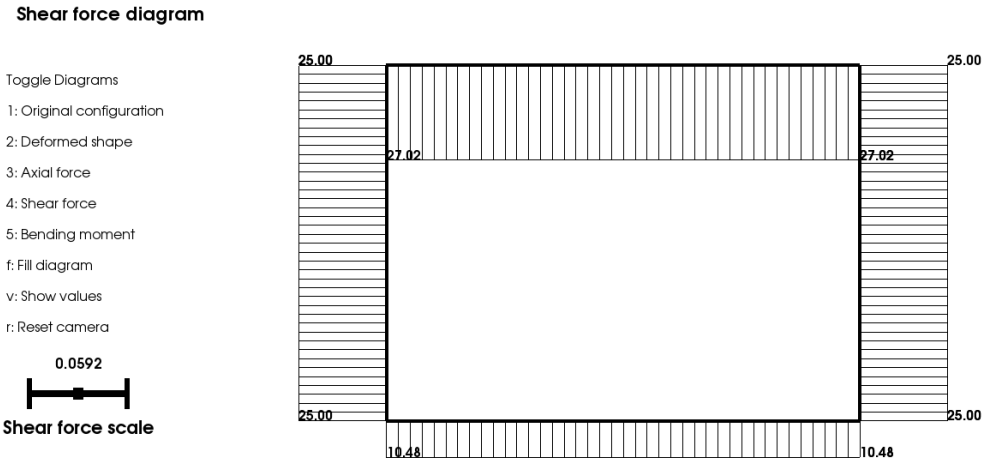

Figure 55: FR1, Load Case 3: Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_B = -M_C$	$\frac{Ph}{2}\frac{k_1+3k_2}{F_2}$	0.10806E+03	0.10806E+03	0.0%
$M_A = -M_D$	$\frac{Ph}{2}\frac{3k_2+1}{F_2}$	0.41938E+02	0.41938E+02	0.0%
$V_D = -V_A$	$-\frac{P}{2} = N_{DC} + T_{DA}$	0.37500E+02	0.37500E+02	0.0%
$H_A = H_B$	$-\frac{P}{2}$	-0.25000E+02	-0.25000E+02	0.0%

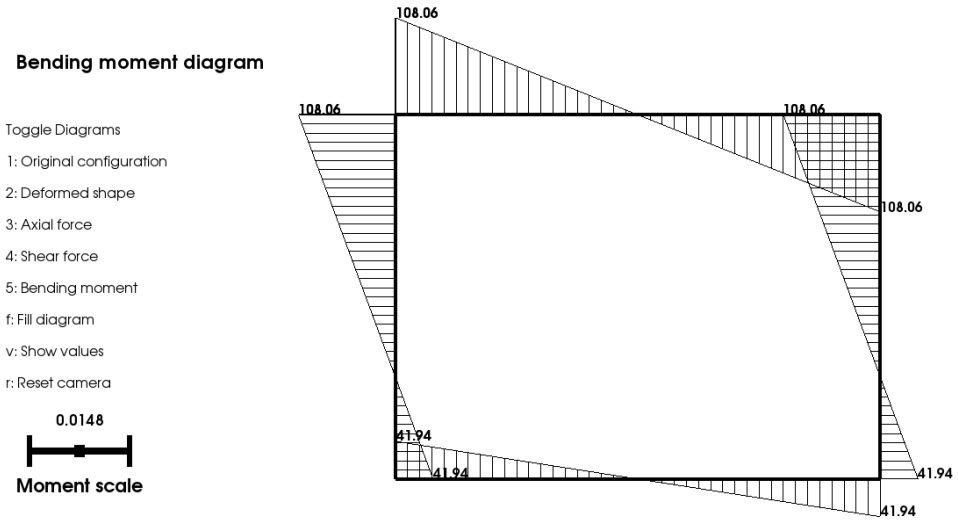

Figure 56: FR1, Load Case 3: Bending Moment Diagram

1.3 One-Bay, One-Storey Frame with Clamped Ends

1.3.1 Load Case 1

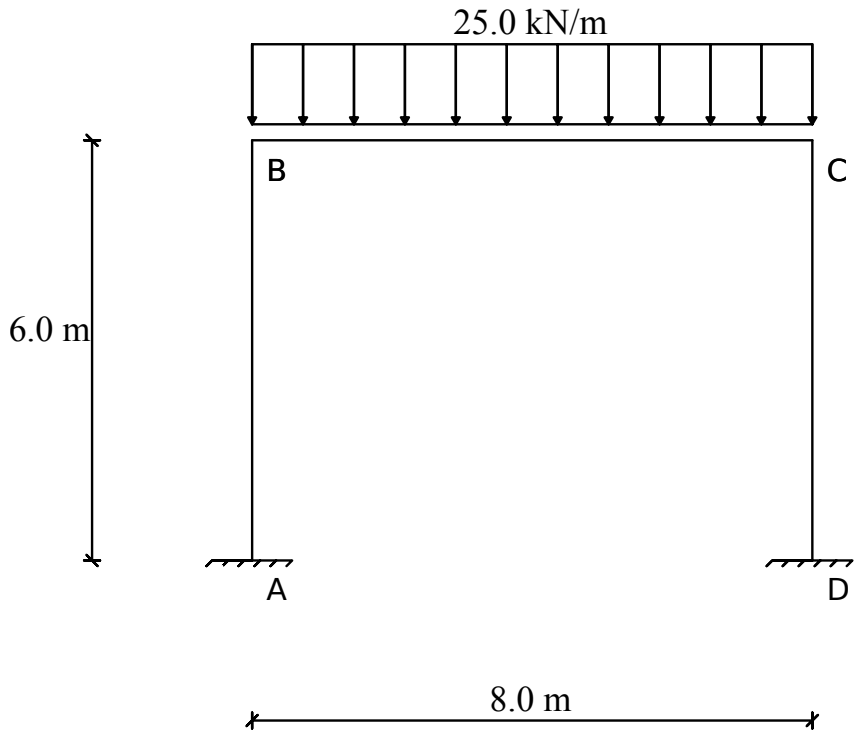

Figure 57: Problem 1: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
5
0.0 0.0
0.0 6.0
4.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
4
1 2
2 3
3 4
4 5
# SECTIONS <number of sections> <A, I, h, X>
2
0.48828125D+03 0.31250D-02 0.5 0
0.15000000D+04 0.54000D-02 0.6 0
# SECTION INCIDENCES
2
1
1
```

```
2
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
1 3
5 1
5 2
5 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
2
2 0.0 -25.0 -25.0 0.0 0.0
3 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            0.00        6.00
  3            4.00        6.00
  4            8.00        6.00
  5            8.00        0.00

# SECTIONS
Sec.No             A                  I                h                  X
  1            .4883D+03          .3125D-02          0.5000             0.0000
  2            .1500D+04          .5400D-02          0.6000             0.0000

# MATERIALS
Mat.No             E                  v               alpha               gamma
  1            .3000D+08            0.20           .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              2             1
  2            2             3              1             1
  3            3             4              1             1
  4            4             5              2             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
```

```
  3                   1                3
  4                   5                1
  5                   5                2
  6                   5                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px          Py1         Py2        DTtop      DTbottom
  2          0.00       -25.00      -25.00        0.00        0.00
  3          0.00       -25.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     0.0000000000D+00     0.0000000000D+00
  2      0.7479137028D-08    -0.1333333333D-07    -0.1014425375D-02
  3      0.1098203032D-17    -0.4873308528D-02     0.1355252716D-18
  4     -0.7479137025D-08    -0.1333333333D-07     0.1014425375D-02
  5      0.0000000000D+00     0.0000000000D+00     0.0000000000D+00


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.10000D+03 -0.27389D+02 -0.54779D+02 -0.10000D+03  0.27389D+02 -0.10956D+03
 2  0.27389D+02  0.10000D+03  0.10956D+03 -0.27389D+02  0.71054D-14  0.90442D+02
 3  0.27389D+02  0.71054D-14 -0.90442D+02 -0.27389D+02  0.10000D+03 -0.10956D+03
 4  0.10000D+03  0.27389D+02  0.10956D+03 -0.10000D+03 -0.27389D+02  0.54779D+02
================================================================================
```

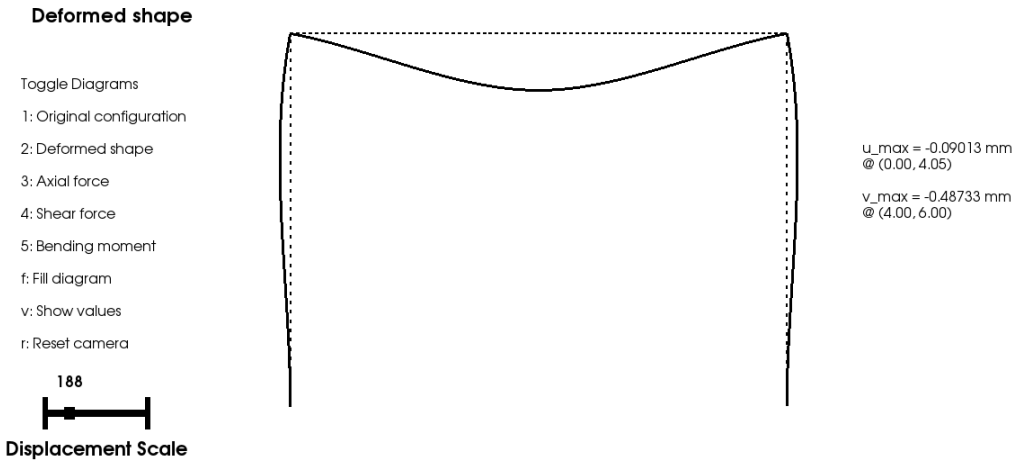

Figure 58: FR2, Load Case 1: Deformed Shape

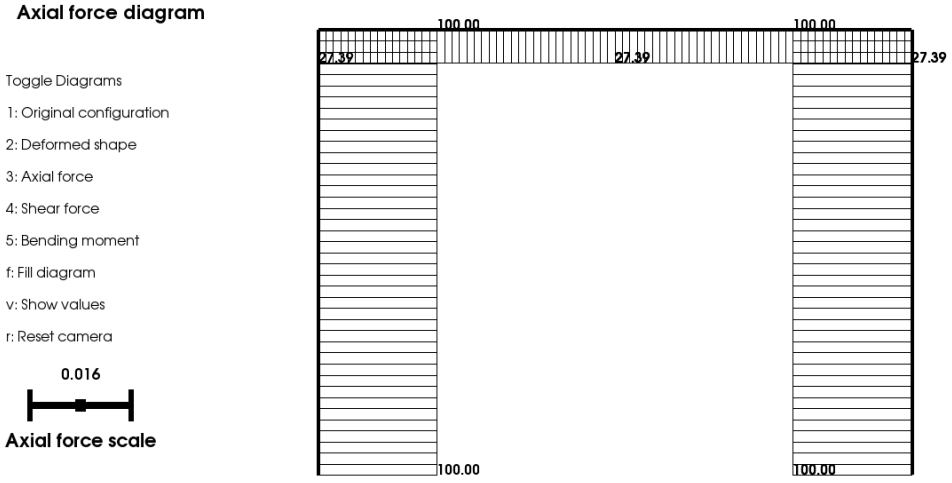

Figure 59: FR2, Load Case 1: Axial Force Diagram

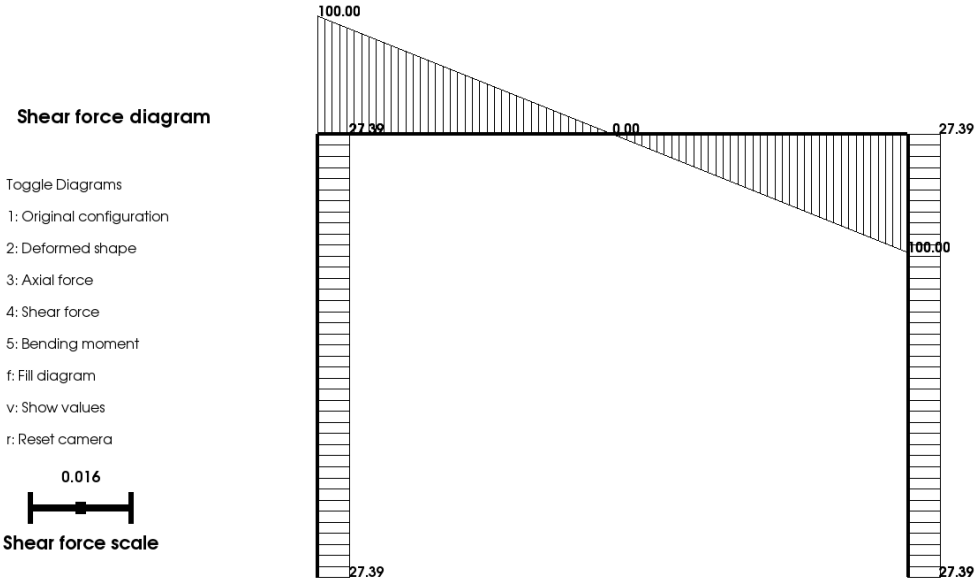

Figure 60: FR2, Load Case 1: Shear Force Diagram

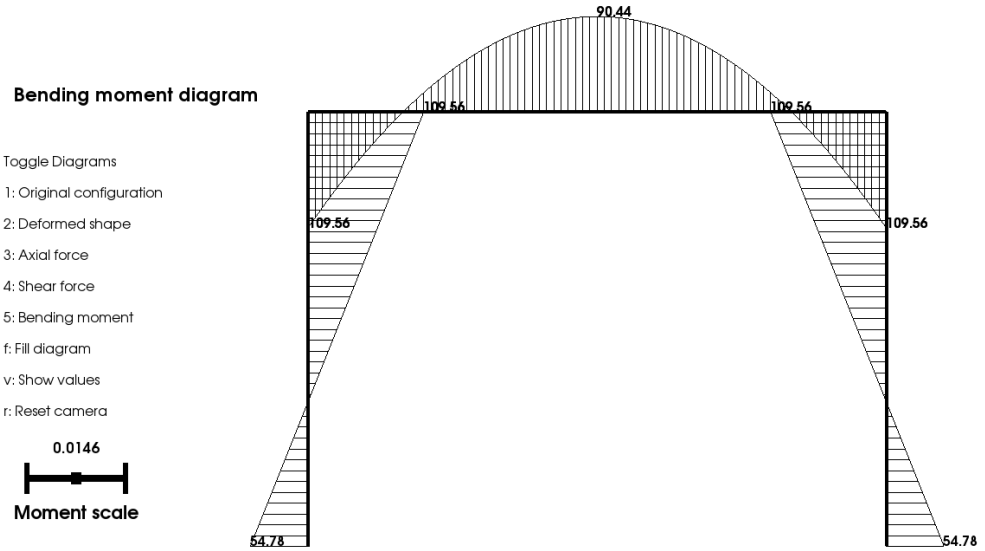

Figure 61: FR2, Load Case 1: Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_A = M_D$	$\frac{\omega l^2}{12(k+2)}$	0.54779E+02	0.54779E+02	0.0%
$M_B = M_C$	$-2M_A$	-0.10956E+03	-0.10956E+03	0.0%
M_{max}	$\frac{\omega l^2}{8} + M_B$	0.90442E+02	0.90442E+02	0.0%
$V_A = V_D$	$\frac{\omega l}{2}$	0.10000E+03	0.10000E+03	0.0%
$H_A = H_D$	$\frac{3M_A}{h}$	0.27389E+02	0.27389E+03	0.0%

1.3.2 Load Case 2

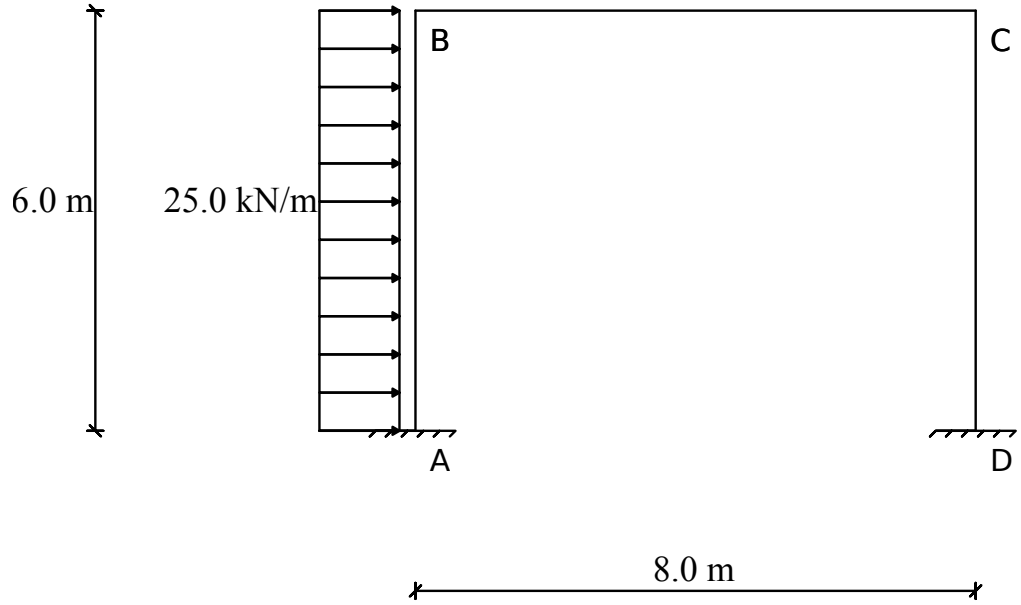

Figure 62: Problem 1: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
4
0.0 0.0
0.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
3
1 2
2 3
3 4
# SECTIONS <number of sections> <A, I, h, X>
2
0.48828125D+03 0.31250D-02 0.5 0
0.15000000D+04 0.54000D-02 0.6 0
# SECTION INCIDENCES
2
1
2
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
```

```
1 2
1 3
4 1
4 2
4 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
0
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
1
1 0.0 -25.0 -25.0 0.0 0.0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            0.00        6.00
  3            8.00        6.00
  4            8.00        0.00

# SECTIONS
Sec.No             A                  I                  h                  X
  1            .4883D+03          .3125D-02          0.5000             0.0000
  2            .1500D+04          .5400D-02          0.6000             0.0000

# MATERIALS
Mat.No             E                  v              alpha              gamma
  1            .3000D+08            0.20           .0000D+00          0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              2             1
  2            2             3              1             1
  3            3             4              2             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    1                3
  4                    4                1
  5                    4                2
  6                    4                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
```

```
Node         Direction         Magnitude

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom
  1          0.00      -25.00      -25.00        0.00        0.00

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00     0.0000000000D+00     0.0000000000D+00
  2      0.6478814510D-02     0.1806358088D-08    -0.4854082024D-03
  3      0.6478798237D-02    -0.1806358088D-08    -0.1056018269D-02
  4      0.0000000000D+00     0.0000000000D+00     0.0000000000D+00


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1 -0.13548D+02  0.12020D+03  0.22372D+03  0.13548D+02  0.29797D+02  0.47504D+02
 2  0.29797D+02 -0.13548D+02 -0.47504D+02 -0.29797D+02  0.13548D+02 -0.60878D+02
 3  0.13548D+02  0.29797D+02  0.60878D+02 -0.13548D+02 -0.29797D+02  0.11790D+03
================================================================================
```

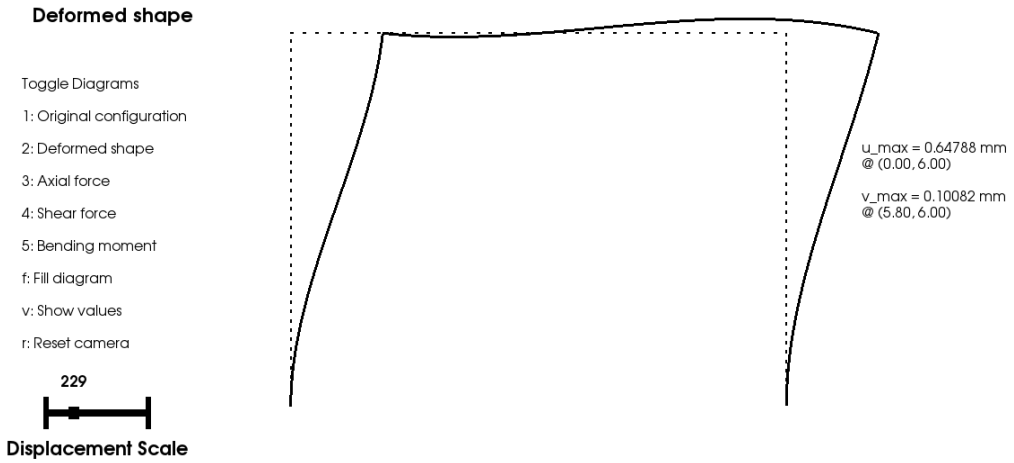

Figure 63: FR2, Load Case 2: Deformed Shape

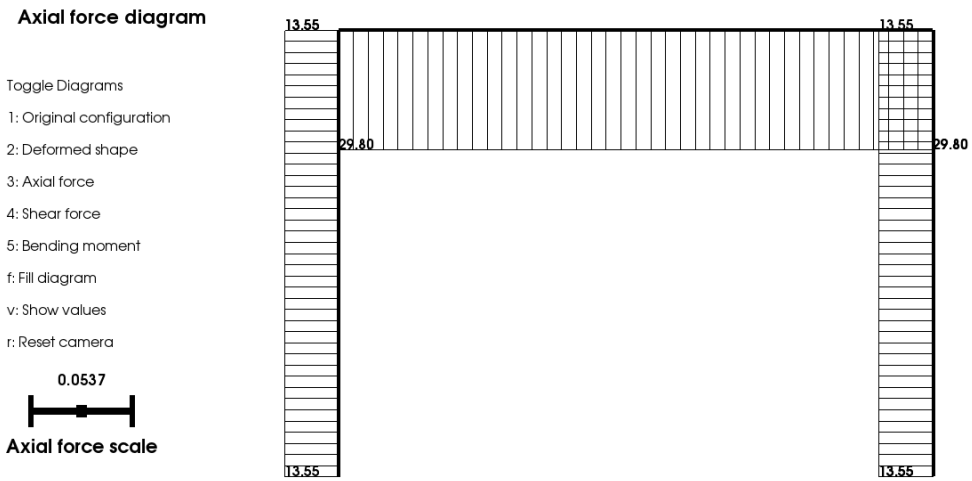

Figure 64: FR2, Load Case 2: Axial Force Diagram

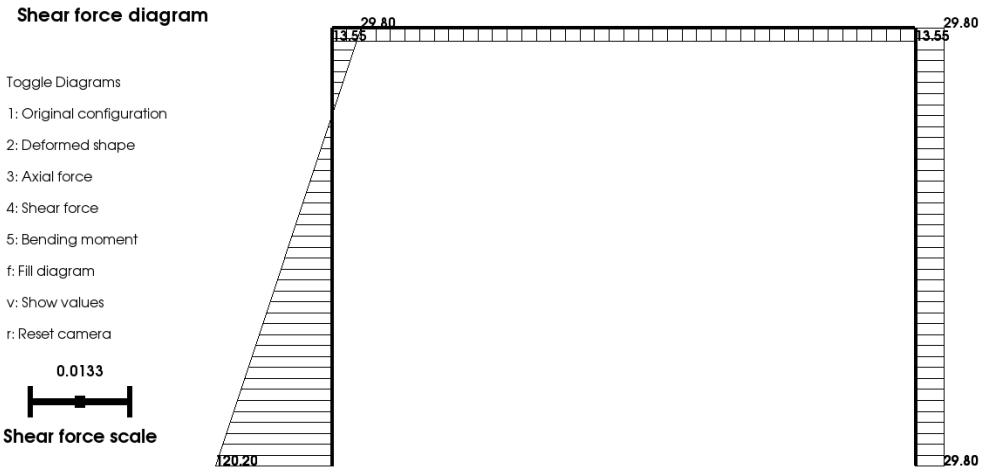

Figure 65: FR2, Load Case 2: Shear Force Diagram

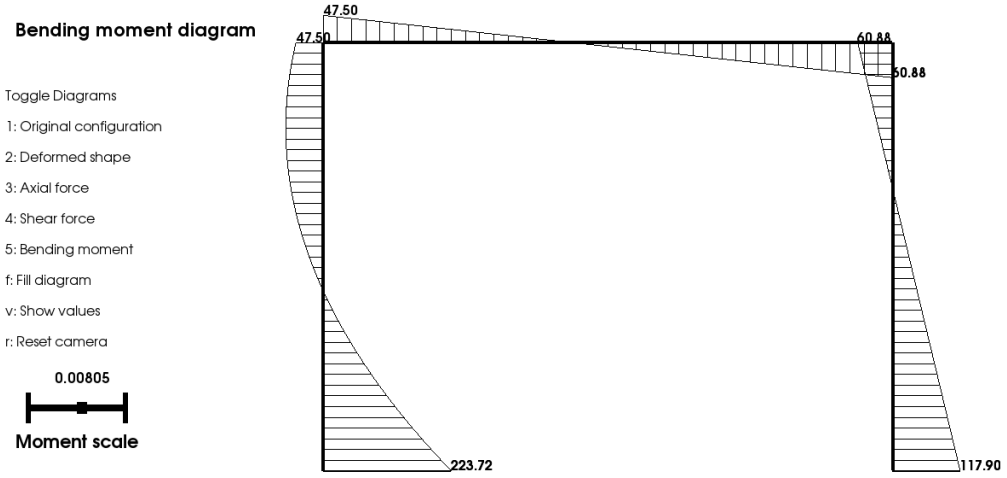

Figure 66: FR2, Load Case 2: Bending Moment Diagram

	Exact Expression	Exact Value	Computed Value	% RE
M_A	$\frac{\omega h^2}{4}[-\frac{k+3}{6(k+2)} - \frac{4k+1}{6k+1}]$	-0.22372E+03	-0.22372E+03	0.0%
M_D	$\frac{\omega h^2}{4}[-\frac{k+3}{6(k+2)} + \frac{4k+1}{6k+1}]$	0.11790E+03	0.11790E+03	0.0%
M_B	$\frac{\omega h^2}{4}[-\frac{k}{6(k+2)} + \frac{2k}{6k+1}]$	0.47504E+02	0.47504E+02	0.0%
M_C	$\frac{\omega h^2}{4}[-\frac{k}{6(k+2)} - \frac{2k}{6k+1}]$	-0.60878E+02	0.60878E+02	0.0%
H_D	$\omega h \frac{2k+3}{8(k+2}$	-0.29797E+02	-0.29797E+02	0.0%
H_A	$-(\omega h - H_D)$	-0.12020E+03	-0.12020E+03	0.0%

1.3.3 Load Case 3

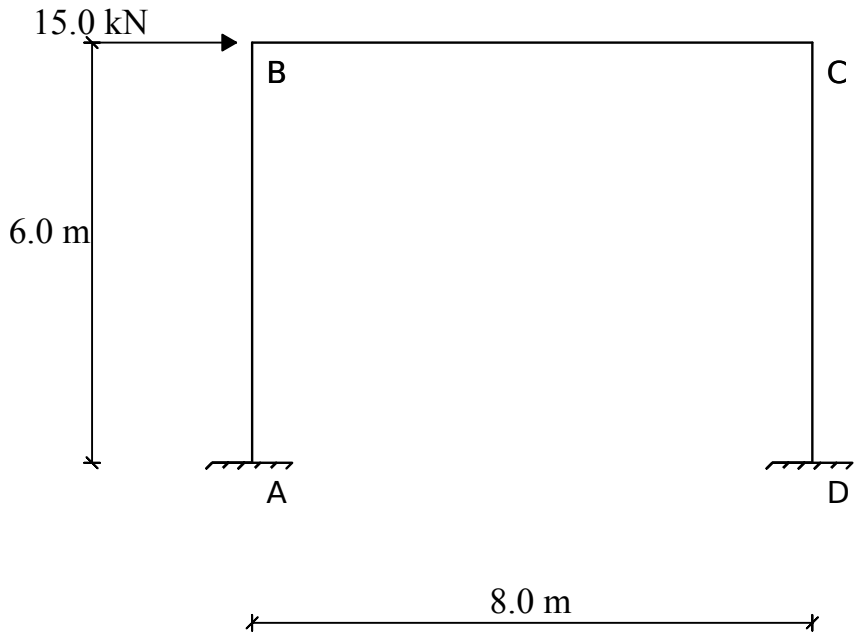

Figure 67: Problem 1: Loading, geometry and supports

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
4
0.0 0.0
0.0 6.0
8.0 6.0
8.0 0.0
# ELEMENTS <number of elements> <start node, end node>
3
1 2
2 3
3 4
# SECTIONS <number of sections> <A, I, h, X>
2
0.48828125D+03 0.31250D-02 0.5 0
0.15000000D+04 0.54000D-02 0.6 0
# SECTION INCIDENCES
2
1
2
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
30.e6 0.2 0.0 0.0
# MATERIAL INCIDENCES
1
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
```

```
1 1
1 2
1 3
4 1
4 2
4 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 1 50
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            0.00        6.00
  3            8.00        6.00
  4            8.00        0.00

# SECTIONS
Sec.No             A                  I                  h                  X
  1            .4883D+03          .3125D-02          0.5000             0.0000
  2            .1500D+04          .5400D-02          0.6000             0.0000

# MATERIALS
Mat.No             E                  v              alpha              gamma
  1            .3000D+08            0.20           .0000D+00          0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              2             1
  2            2             3              1             1
  3            3             4              2             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    1                3
  4                    4                1
  5                    4                2
  6                    4                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K
```

```
# NODAL LOADS
Node         Direction         Magnitude
  2              1              50.00

# ELEMENT LOADS
Element       Px          Py1          Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1           em           e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                    DY                    PHI

  1      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00
  2      0.5089924312D-02      0.1806358088D-08     -0.7707146381D-03
  3      0.5089910658D-02     -0.1806358088D-08     -0.7707118334D-03
  4      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1 -0.13548D+02  0.25000D+02  0.95809D+02  0.13548D+02 -0.25000D+02  0.54191D+02
 2  0.25000D+02 -0.13548D+02 -0.54191D+02 -0.25000D+02  0.13548D+02 -0.54191D+02
 3  0.13548D+02  0.25000D+02  0.54191D+02 -0.13548D+02 -0.25000D+02  0.95809D+02
================================================================================
```

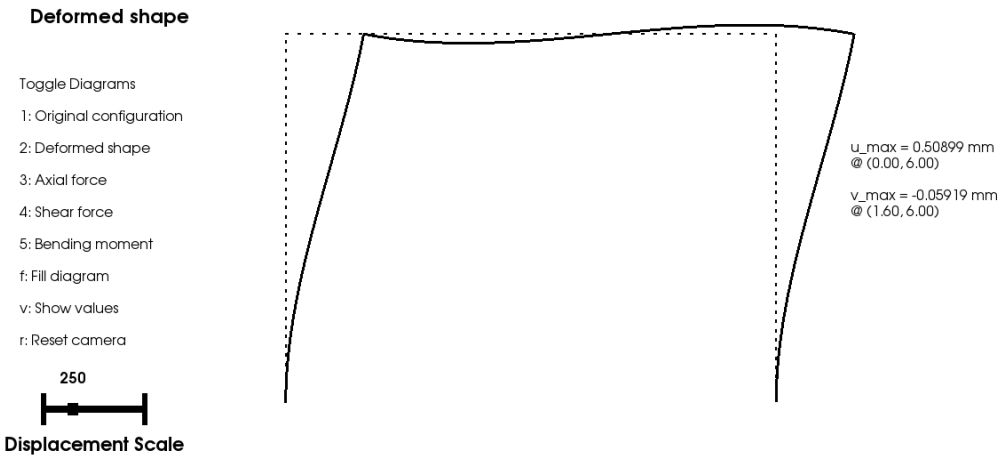

Figure 68: FR2, Load Case 3: Deformed Shape

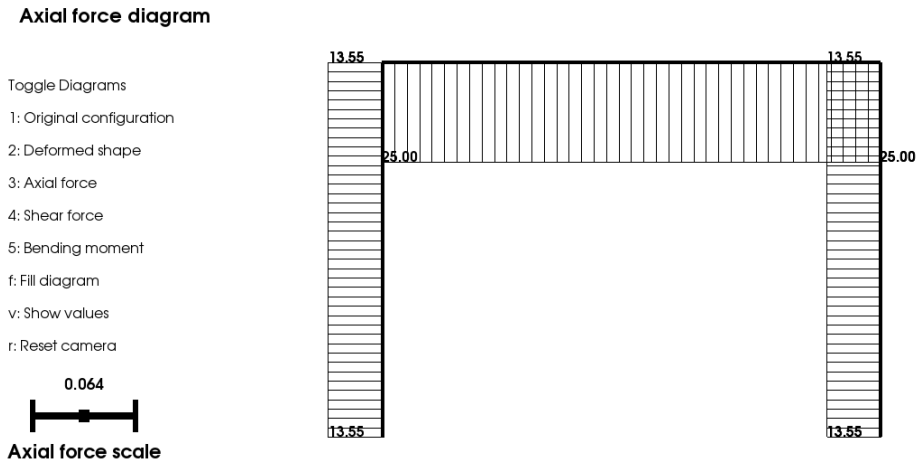

Figure 69: FR2, Load Case 3: Axial Force Diagram

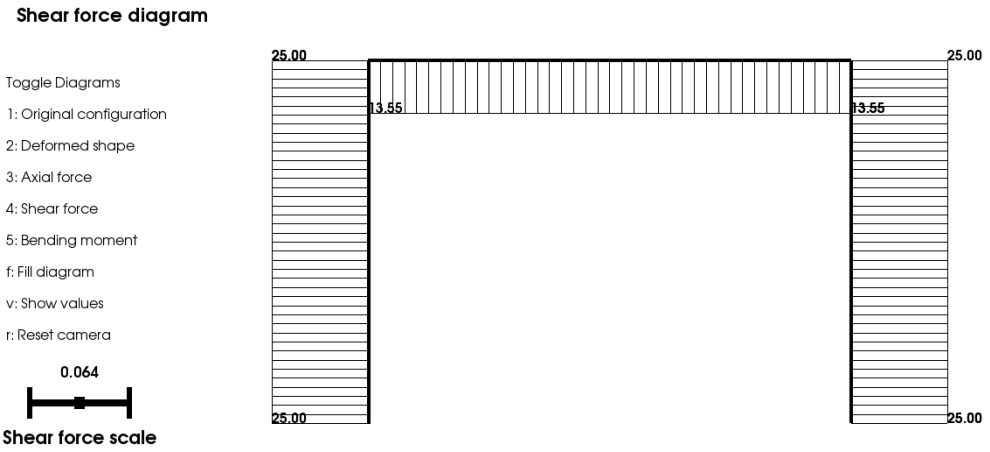

Figure 70: FR2, Load Case 3: Shear Force Diagram

	Exact Expression	Exact Value	Computed Value	% RE
$M_D = -M_A$	$\frac{Ph}{2}[-\frac{3k+1}{6k+1)}$ 0.95809E+02	0.95809E+02	0.0%	
$M_B = -M_C$	$\frac{Ph}{2}[\frac{3k}{6k+1)}$ 0.54191E+02	0.54191E+02	0.0%	
$V_D = -V_A$	$\frac{2M_B}{/}l$	0.13548E+02	0.13548E+02	0.0%
$H_D = H_A$	$P/2$	-0.25000E+02	-0.25000E+02	0.0%

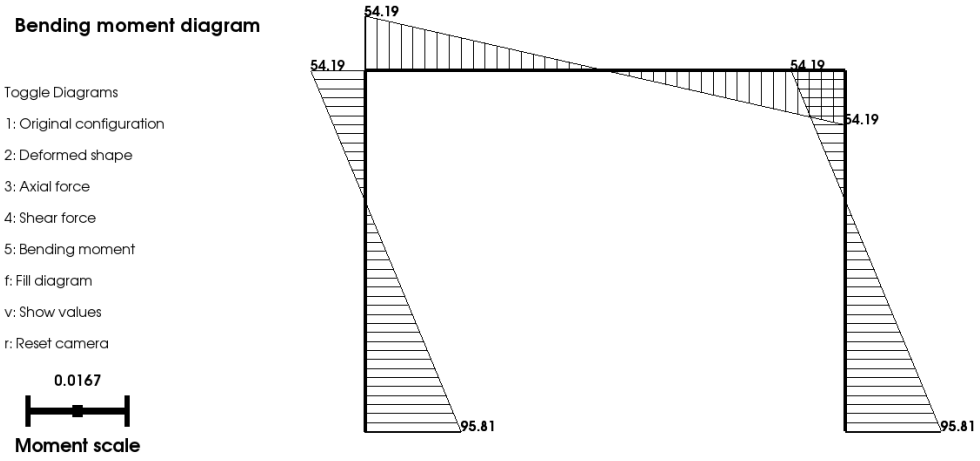

Figure 71: FR2, Load Case 3: Bending Moment Diagram

1.4 Clamped Beam with Nodal Load, Influence of Shear Deformability

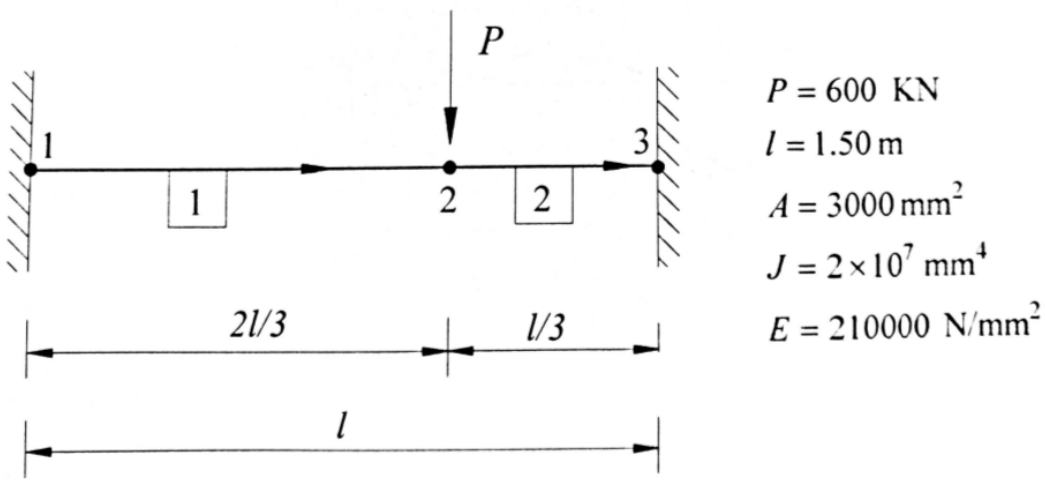

Figure 72: Specialized Problem 1: Loading, geometry and supports

1.4.1 Without Shear Deformability

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
1.0 0.0
1.5 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.30000D-02 0.20000D-04 0.5 0.0
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
0.21000D+09 0.0 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
1 3
3 1
```

```
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 2 -600.00
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================

================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            1.00        0.00
  3            1.50        0.00

# SECTIONS
Sec.No             A                  I                  h                  X
  1            .3000D-02          .2000D-04           0.5000             0.0000

# MATERIALS
Mat.No             E                  v               alpha               gamma
  1            .2100D+09             0.00            .0000D+00             0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    1                3
  4                    3                1
  5                    3                2
  6                    3                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction              K

# ELASTIC LINKS
E. Link. No        Node 1              Node 2           Direction        K

# NODAL LOADS
Node         Direction         Magnitude
  2              2              -600.00

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom
```

```
# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                  DY                  PHI

  1      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00
  2      0.0000000000D+00     -0.1763668430D-02      0.2645502646D-02
  3      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.15556D+03  0.66667D+02  0.00000D+00 -0.15556D+03  0.88889D+02
 2  0.00000D+00 -0.44444D+03 -0.88889D+02  0.00000D+00  0.44444D+03 -0.13333D+03
================================================================================
```

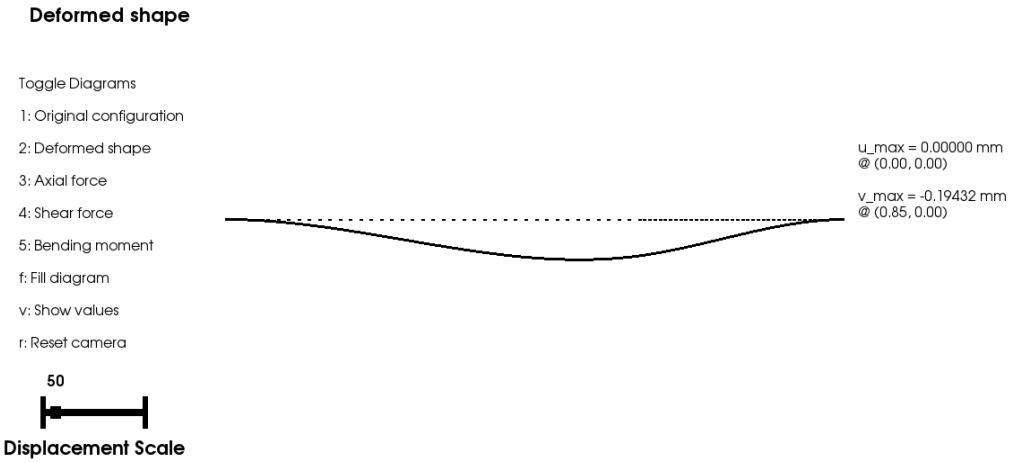

Figure 73: FR1, Load Case 1: Deformed Shape

	Exact Expression	Exact Value	Computed Value	% RE
v_B	$-\frac{P(\frac{2}{3}l)^3(\frac{1}{3}l)^3}{3EJl^3}$	-0.17637E-02	-0.17637E-02	0.0%
φ_B	$-\frac{P(\frac{2}{3}l)^2(\frac{1}{3}l)^3}{2EJl^3}$	0.26455E-02	0.26455E-02	0.0%

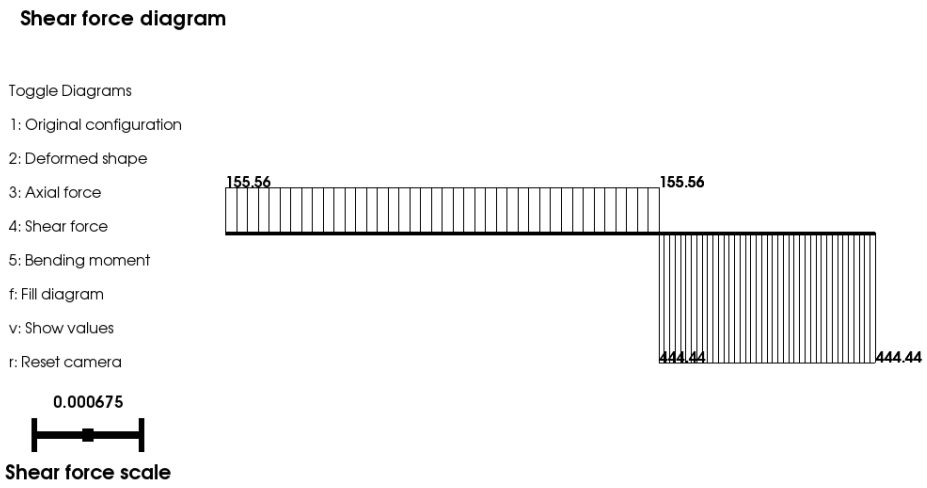

Figure 74: FR1, Load Case 1: Shear Force Diagram

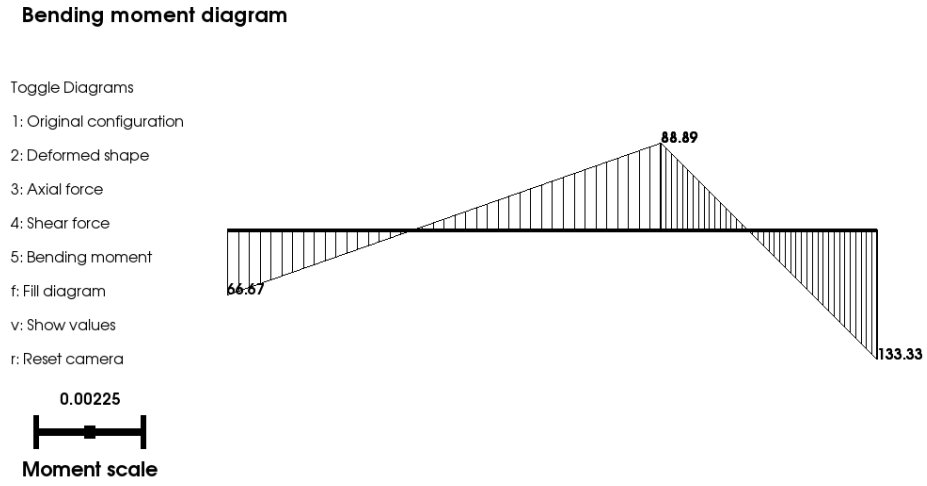

Figure 75: FR1, Load Case 1: Bending Moment Diagram

1.4.2 With Shear Deformability

Shear deformability is introduced by setting $\chi = 2.5$ and $\nu = 0.3$, while all other parameters were held constant. The results are presented below.

```
================================================================================
2DSTIFF - INPUT FILE
[kN/m/C]
================================================================================
# NODES <number of nodes> <x coord, y coord>
3
0.0 0.0
1.0 0.0
1.5 0.0
# ELEMENTS <number of elements> <start node, end node>
2
1 2
```

```
2 3
# SECTIONS <number of sections> <A, I, h, X>
1
0.30000D-02 0.20000D-04 0.5 2.5
# SECTION INCIDENCES
1
1
# MATERIALS <number of materials> <E, v, alpha, gamma>
1
0.21000D+09 0.3 0.0 0.0
# MATERIAL INCIDENCES
1
1
# RESTRAINTS <number of restraints> <node number, direction>
6
1 1
1 2
1 3
3 1
3 2
3 3
# LINKS <number of links> <master node, slave node, direction>
0
# ELASTIC RESTRAINTS <number of el. restraints> <node number, direction, k>
0
# ELASTIC LINKS <number of links> <node1, node2, direction, k>
0
# NODAL LOADS <number of nodal loads> <node, direction, magnitude>
1
2 2 -600.00
# ELEMENT LOADS <number> <Element, Px, Py, Py2, DTtop, DTbottom>
0
# PRESTRESSING <number> <Element, e1, em, e2, P>
0
================================================================================
```

```
================================================================================
2DSTIFF - OUTPUT FILE
[kN/m/C]
================================================================================

# NODE COORDINATES
Node         x-coord     y-coord
  1            0.00        0.00
  2            1.00        0.00
  3            1.50        0.00

# SECTIONS
Sec.No             A                  I                h                  X
  1            .3000D-02          .2000D-04          0.5000             2.5000

# MATERIALS
Mat.No             E                  v               alpha               gamma
  1            .2100D+09            0.30           .0000D+00            0.0000

# ELEMENTS
El.No      Start Node      End Node       Section       Material
  1            1             2              1             1
  2            2             3              1             1

# RESTRAINTS
Restraint No         Node            Direction
  1                    1                1
  2                    1                2
  3                    1                3
  4                    3                1
```

```
  5                   3                2
  6                   3                3

# LINKS
Link No          Master Node         Slave Node         Direction

# ELASTIC RESTRAINTS
E. Restr. No        Node             Direction             K

# ELASTIC LINKS
E. Link. No        Node 1             Node 2           Direction       K

# NODAL LOADS
Node         Direction         Magnitude
  2              2              -600.00

# ELEMENT LOADS
Element       Px         Py1         Py2        DTtop      DTbottom

# PRESTRESSING
Element       e1          em          e2          P

================================================================================
ANALYSIS RESULTS
================================================================================

>> NODAL DISPLACEMENTS

Node            DX                   DY                   PHI

  1      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00
  2      0.0000000000D+00     -0.3868546215D-02      0.2148873990D-02
  3      0.0000000000D+00      0.0000000000D+00      0.0000000000D+00


>> MEMBER END FORCES

MEM     N1            T1           M1           N2           T2           M2
 1  0.00000D+00  0.16390D+03  0.72924D+02  0.00000D+00 -0.16390D+03  0.90975D+02
 2  0.00000D+00 -0.43610D+03 -0.90975D+02  0.00000D+00  0.43610D+03 -0.12708D+03
================================================================================
```

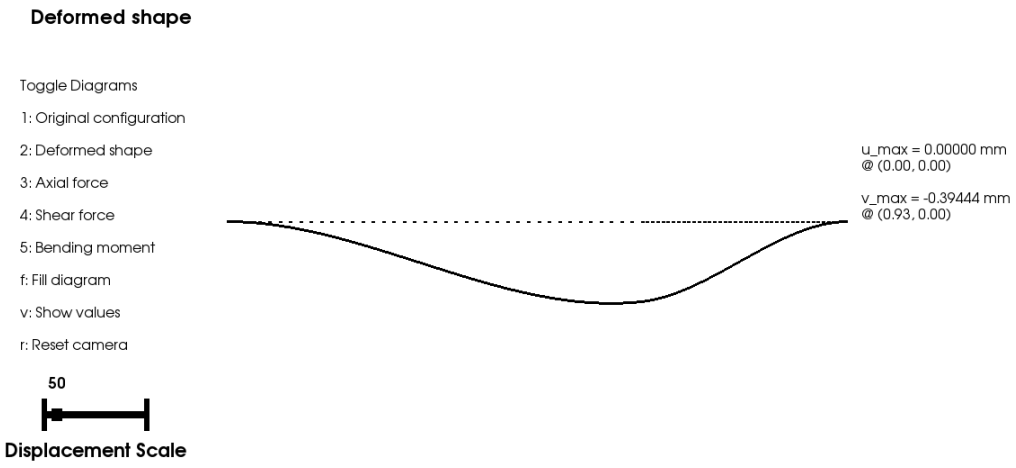

Figure 76: FR1, Load Case 1: Deformed Shape

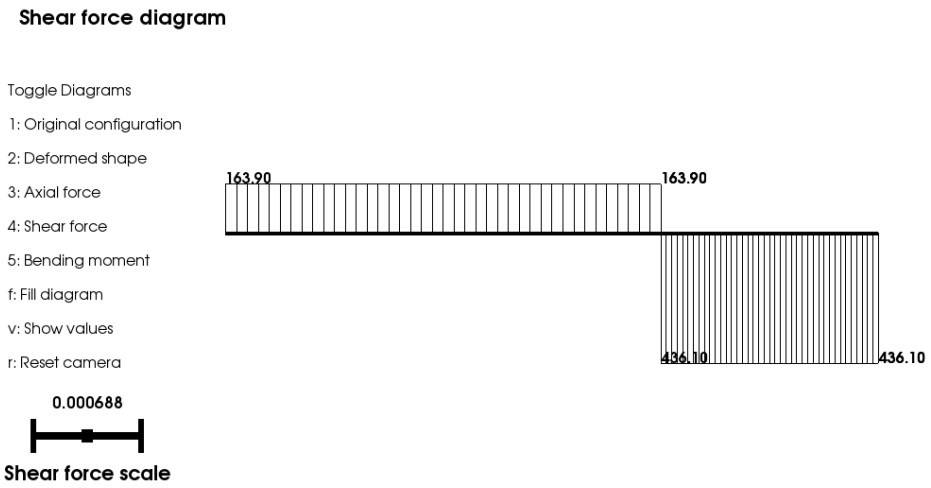

Figure 77: FR1, Load Case 1: Shear Force Diagram

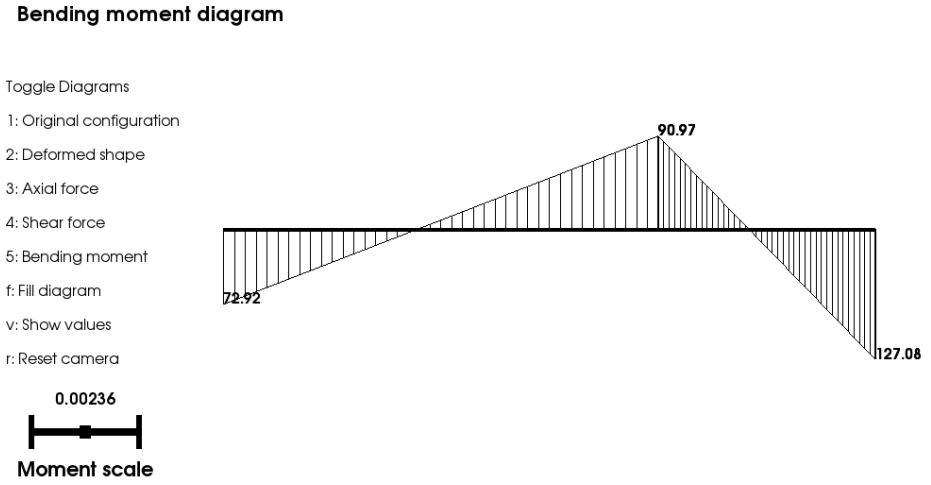

Figure 78: FR1, Load Case 1: Bending Moment Diagram

	Exact Value	Computed Value	% RE
v_B	-0.3869E-02	-0.3869E-02	0.0%
ϕ_B	0.2149E-02	0.2149E-02	0.0%

As can be observed from the presnted values, introducing shear deformability results in a significant variation in nodal displacement and rotation:

$$\frac{\delta v}{v_t} = \frac{v_t - v_m}{v_t} = \frac{|-0.3869E-02-(-0.1764E-02)|}{|-0.3869E-02|} = 54.4\% \frac{\delta \varphi}{\varphi_t} = \frac{\varphi_t - \varphi_m}{\varphi_t} = \frac{|0.2149E-02}{|0.214}$$

However, the variation in the internal force resultants are rather low when compared to that of the displacements, for instance the variation in the moments at A and C:

$$\frac{\delta M_A}{M_{A,t}} = \frac{M_{A,t} - M_{A,m}}{M_{A,t}} = \frac{|-0.72924E+02-(-0.66667E+02)|}{|-0.72924E+02|} = 8.6\% \frac{\delta M_C}{M_{C,t}} = \frac{M_{C,t} - M_{C,m}}{M_{C,t}} =$$