

Seminar Economics and Psychology of Risk & Time

Assignment 1

Group 7

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TASK 1

a)

$$u(x) = x/10 \quad (0.2, 40), (0.6, 50), (0.2, 30)$$

$$EV = 0.2 \times (40/10) + 0.6 \times (50/10) + 0.2 \times (30/10) = 4.4$$

$$CE: m/10 = 4.4$$

$$m = 44$$

The certainty equivalent is 44.

b)

$$u(x) = x/10 \quad w(p) = p^2$$

Probability of outcome q or better:

$$q=40, p=(0.2) + (0.6) = 0.8$$

$$q=50, p = (0.6) \quad q = 30, p = (0.2) + (0.6) = 0.8$$

$$q=30, p=(0.2) + (0.6) = 0.8$$

Probability outcome is strictly better than b:

$$b=40, p=0.6, \quad b=50, p=0, \quad b=30, p=(0.2) + (0.6) = 0.8$$

$$w(p) = p^2$$

$$w(0.8) - w(0.6) = (0.8)^2 - (0.6)^2 = 0.28$$

$$w(0.6) - w(0.0) = (0.6)^2 - (0.0)^2 = 0.36$$

$$w(1) - w(0.8) = (1)^2 - (0.8)^2 = 0.36$$

Expected utility, using these updated probability weights is:

$$EV = 0.28 \times (40/10) + 0.36 \times (50/10) + 0.36 \times (30/10) = 4$$

$$CE: m/10 = 4$$

$$m = 40$$

The certainty equivalent is 40.

#2a)

Under EU, a preference for prospect A over B implies:

$$u(3000) > 0.8(4000) + 0.2(0)$$

dividing both sides by 4 & adding $0.75(0)$

$$0.75(0) + 0.25 u(3000) > 0.2(4000) + 0.05(0) + 0.75(0)$$

A preference for prospect D over C implies

$$0.2(4000) + 0.8(0) > 0.25(3000) + 0.75(0)$$

\Rightarrow Under EU, an individual cannot simultaneously prefer (A and D) or (B and C).

(b)

$$EV(A) = 3000 \quad D(A) = 3000$$

$$EV(B) = EU = 3200$$

$$D(B) = 0.8 \times (4000 + 0.0002 \times (4000 - 3200)^2) + 0.2 \times (0 - 0.0002 \times (0 - 3200)^2) = 3302.4 - 409.6 = 2892.8$$

Hence, $D(A) > D(B)$

$$EV(C) = EU = 750$$

$$D(C) = 0.25 \times (3000 + 0.0002 \times (3000 - 750)^2) + 0.75 \times (0 - 0.0002 \times (0 - 750)^2) = 1003.125 - 84 = 919.75$$

$$EV(D) = EU = 800$$

$$D(D) = 0.2 \times (4000 + 0.0002 \times (4000 - 800)^2) + 0.8 \times (0 - 0.0002 \times (0 - 800)^2) = 1209.6 - 102.4 = 1107.2$$

Hence, $D(D) > D(C)$

(c) Using the parametrization by Tversky & Kahnemann:

CPT(A)

x	p	π	$\pi U(x)$
3000	1	$\omega^+(1) - \omega^+(0) = 1$	<u>1147.80</u>

CPT(B)

x	p	π	$U(x)$	$U(x)$
4000	0.8	$\omega^+(0.8) - \omega^+(0) = 0.607$	63.245	1478.47

$$\pi U(x) = \underline{897.43}$$

CPT(C)

x	p	π	$U(x)$	$\pi U(x)$
3000	0.25	$\omega^+(0.25) = 0.2907$	1147.8	<u>333.67</u>

CPT(D)

x	p	π	$U(x)$	$\pi U(x)$
4000	0.2	$\omega^+(0.2) = 0.261$	1478.47	<u>385.53</u>

$\Rightarrow CPT(A) > CPT(B) \ \& \ CPT(D) > CPT(C)$

3) A) $(0.25, \$75; 0.25, \$50; 0.25, \$25; 0.25, \$0)$

x	p	π	$u(x) = U(x)$	$\pi * U(x)$
75	0.25	$w^+(0.25) - w^+(0) = 0.29074$	44.674	12.989
50	0.25	$w^+(0.50) - w^+(0.25) = 0.12990$	31.268	4.062
25	0.25	$w^+(0.75) - w^+(0.50) = 0.14763$	16.990	2.508
0	0.25	$w^+(1) - w^+(0.75) = 0.43173$	0	0

$$u^+(x) = x^{0.88} \quad \text{if } x \geq 0$$

$$u^-(x) = 2.25 * -(-x)^{0.88} \quad \text{if } x < 0$$

$\lambda = 2.25$

$$w^+(p) = \frac{p^{0.61}}{(p^{0.61} + (1-p)^{0.61})^{1/0.61}}$$

$$w^-(p) = \frac{p^{0.69}}{(p^{0.69} + (1-p)^{0.69})^{1/0.69}}$$

$$w^+(0.25) = \frac{0.25^{0.61}}{(0.25^{0.61} + (0.75)^{0.61})^{1/0.61}} = 0.29074$$

$$w^+(0) = \frac{0^{0.61}}{(0^{0.61} + 1^{0.61})^{1/0.61}} = 0$$

$$w^+(0.50) = \frac{0.50^{0.61}}{(0.50^{0.61} + 0.50^{0.61})^{1/0.61}} = 0.42064$$

$$w^+(0.75) = \frac{0.75^{0.61}}{(0.75^{0.61} + 0.25^{0.61})^{1/0.61}} = 0.56827$$

$$w^+(1) = \frac{1^{0.61}}{(1^{0.61} + 0^{0.61})^{1/0.61}} = 1$$

$$u(75) = 75^{0.88} = 44.674$$

$$u(50) = 50^{0.88} = 31.268$$

$$u(25) = 25^{0.88} = 16.990$$

$$u(0) = 0^{0.88} = 0$$

$$0.29074 - 0 = 0.29074$$

$$0.42064 - 0.29074 = 0.12990$$

$$0.56827 - 0.42064 = 0.14763$$

$$1 - 0.56827 = 0.43173$$

$$CPT = 12.989 + 4.062 + 2.508 + 0 = 19.559$$

$$CE^{0.88} = 19.559 \Rightarrow CE = 29.339$$

$$EV = \frac{1}{4}(75) + \frac{1}{4}(50) + \frac{1}{4}(25) + \frac{1}{4}(0) = 37.5$$

$$RP = 37.5 - 29.339 \Rightarrow RP = 8.161$$

B) $(0.25, \$50; 0.25, \$25; 0.25, \$0; 0.25, -\$25)$

X	P	π	$u(x) = U(x)$	$\pi * u(x)$
50	0.25	$w^+(0.25) - w^+(0) = 0.29074$	31.268	9.091
25	0.25	$w^+(0.50) - w^+(0.25) = 0.12990$	16.990	2.207
0	0.25	$w^+(0.75) - w^+(0.50) = 0.14763$	0	0
-25	0.25	$w^-(0.25) - w^-(0) = 0.29352$	-38.227	-11.220

$$w^-(0.25) = \frac{(0.25)^{0.69}}{((0.25)^{0.69} + (0.75)^{0.69})^{1/0.69}} = 0.29352$$

$$w^-(0) = \frac{0^{0.69}}{(0^{0.69} + 1^{0.69})^{1/0.69}} = 0$$

$$u^-(-25) = 2.25 * - (+25)^{0.88} = -38.227$$

$$CPT = 9.091 + 2.207 + 0 + -11.220 = 0.078$$

$$CE^{0.88} = 0.078 \Rightarrow CE = 0.055$$

$$EV = \frac{1}{4}(50) + \frac{1}{4}(25) + \frac{1}{4}(0) + \frac{1}{4}(-25) = 12.5$$

$$RP = 12.5 - 0.055 \Rightarrow RP = 12.445$$

C) $(0.25, \$25; 0.25, \$0; 0.25, -\$25; 0.25, -\$50)$

X	P	π	$u(x) = U(x)$	$\pi * u(x)$
25	0.25	$w^+(0.25) - w^+(0) = 0.29074$	16.990	4.940
0	0.25	$w^+(0.50) - w^+(0.25) = 0.12990$	0	0
-25	0.25	$w^-(0.50) - w^-(0.25) = 0.16047$	-38.227	-6.134
-50	0.25	$w^-(0.25) - w^-(0) = 0.29352$	-70.352	-20.650

$$w^-(0.50) = \frac{(0.50)^{0.69}}{((0.50)^{0.69} + (0.50)^{0.69})^{1/0.69}} = 0.45399$$

$$0.45399 - 0.29352 = 0.16047$$

C)
continued

$$u^{-}(-50) = 2.25 * - (+50)^{0.88} = -70.352$$

$$CPT = 4.940 + 0 + -6.134 + -120.650 = -21.844$$

$$2.25 * - (-CE)^{0.88} = -21.844$$

$$\Rightarrow CE = -13.236$$

$$EV = \frac{1}{4}(25) + \frac{1}{4}(0) + \frac{1}{4}(-25) + \frac{1}{4}(-50) = -12.50$$

$$RP = -12.50 + 13.236 = 0.736$$

$$RP = 0.736$$

D) $(0.25, 0; 0.25, -\$25; 0.25, -\$50; 0.25, -\$75)$

X	P	π	$u(x) = Ux$	$\pi * U(x)$
0	0.25	$w^{+}(0.25) - w^{+}(0) = 0.29074$	0	0
-25	0.25	$w^{-}(0.75) - w^{-}(0.50) = 0.17241$	-38.227	-6.591
-50	0.25	$w^{-}(0.50) - w^{-}(0.25) = 0.16047$	-70.352	-11.289
-75	0.25	$w^{-}(0.25) - w^{-}(0) = 0.29352$	-100.516	-29.503

$$w^{-}(0.75) = \frac{0.75^{0.69}}{(0.75^{0.69} + 0.25^{0.69})^{1/0.69}} = 0.62640$$

$$0.62640 - 0.45399 = 0.17241$$

$$u^{-}(-75) = 2.25 * - (+75)^{0.88} = -100.516$$

$$CPT = 0 + -6.591 + -11.289 + -29.503 = -47.383$$

$$2.25 * - (-CE)^{0.88} = -47.383$$

$$\Rightarrow CE = -31.909$$

$$EV = \frac{1}{4}(0) + \frac{1}{4}(-25) + \frac{1}{4}(-50) + \frac{1}{4}(-75) = -37.5$$

$$RP = -37.5 + 31.909 = -5.591$$

$$RP = -5.591$$

#4)

	stolen (10%)	accident (0 6%)	nothing (84%)
full ins.	30 K	30 K	30 K
theft ins.	32 K	7 K	32 K
no ins.	10 K	10 K	35 K

$$(a) EU(\text{full ins.}) : \cancel{0.1 \times 5 \times 30^{0.2}} + 1 \times 5 \times 30^{0.2} = 9.872$$

$$EU(\text{theft ins.}) : 0.94 \times 5 \times 32^{0.2} + 0.06 \times 5 \times 7^{0.2} =$$

$$= 9.4 + 0.4427 = 9.8427$$

$$EU(\text{no ins.}) : 0.10 \times 5 \times 10^{0.2} + 0.84 \times 5 \times 35^{0.2}$$

$$= 1.2679 + 8.5519 = 9.8198$$

\Rightarrow Alan will take full insurance !

(b)

$$RDU(\text{full ins.}) = u(30) \times [w(1) - w(0)]$$

$$= 9.87175 \times 1$$

$$RDU(\text{theft ins.}) = u(32) \times [w(0.94)] + u(7) \times [w(1) - w(0.94)]$$

$$= 10 \times 0.9146 + 7.3789 \times 0.0854$$

$$= 9.146 + 0.63 = 9.7762$$

$$RDU(\text{no ins.}) = u(10) \times [w(1) - w(0.84)] + u(35) \times [w(0.84)]$$

$$= 7.9245 \times (1 - 0.8053) + 10.1808 \times 0.8053$$

$$= 1.5429 + 8.1986 = 9.7415$$

\Rightarrow Alan will take full insurance !

$$Q5 \quad E = (0.5: -3000, 0.5: 4500) \quad F = (0.25: -6000; 0.75: 3000)$$

$$G = (0.5: -1500; 0.5: 4500) \quad H = (0.25: -3000; 0.75: 3000)$$

$$W^+ = WP = p^2 \quad u^+(x) = x^5 \quad u^-(x) = -2x^5$$

$$CPT(E) = W(0.5)u^-(-3000) + W^+(0.5)u^+(4500) =$$

$$0.25 \times (-2) \times 3000^5 + 0.25 \times 4500^5 = 4.6 \times 10^{22}$$

$$CPT(F) = W(0.25)u^-(-6000) + W^+(0.75)u^+(3000) =$$

$$0.0625 \times (-2) \times 6000^5 + 0.75^2 \times 3000^5 = -8.35 \times 10^{17}$$

$$CPT(E) > CPT(F)$$

$$CPT(G) = W(0.5)u^-(-1500) + W^+(0.5)u^+(4500)$$

$$= 0.25 \times (-2) \times 1500^5 + 0.25 \times 4500^5 = 4.58 \times 10^{17}$$

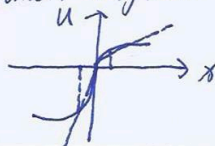
$$CPT(H) = W(0.25)u^-(-3000) + W^+(0.75)u^+(3000) =$$

$$0.0625 \times (-2) \times 3000^5 + 0.75^2 \times 3000^5 = 1.06 \times 10^{17}$$

$$CPT(G) > CPT(H)$$

using the example from the slides, with S-shaped utility (CPT people's choice doesn't invalid the S-shaped utility) CPT.

The intuition is that losing x hurts you more than gaining x brings you pleasure. although people should prefer $0.75: 3000$ than $0.5: 4500$, the possible loss of 6000 with probability of 0.25 hurts too much which outweighs the gain (extra utility) from $0.75: 3000$.



A second possible explanation that comes to the same conclusion that we do not agree with the statement by Levy and Levy (2002) is the following:

Looking at the probability weighting of cumulative prospect theory, shows that small probabilities are overweighted and moderate and large probabilities are underweighted.

Why do people prefer E over F? In prospect F there is a small probability of a very large negative outcome. Since it is a small probability and a negative outcome, people are pessimistic according to CPT. Prospect E on the other hand has equal moderate probabilities and is hence preferred over a prospect with a small overweighted outcome of losing 6000 with probability 0.25.

The same logic holds for prospects G and H. In Prospect H there is a small probability of a very large negative outcome. Since it is a small probability and a negative outcome, people are pessimistic according to CPT and hence prefer prospect G over prospect H.