ASHESI

Discrete Structures and Theory (Spring 2023) Homework 1
Deadline: 27/01/2023
Total: 9 POINTS

1. (3 POINTS) Write each of these statements in the form "if p, then q" in English.

   a) It is sufficient to leave Accra at 6:00AM to get to campus by 8:00AM.

   SOLUTION (1/2 POINT)
   If we leave Accra at 6:00AM, then we will get to campus by 8:00am

   b) Willy gets caught whenever he cheats.

   SOLUTION (1/2 POINT)
   If Willy cheats, then he gets caught.

   c) You can access the website only if you pay a subscription fee.

   SOLUTION (1/2 POINT)
   If you can access the website, then you paid a subscription fee.

   d) Getting elected follows from knowing the right people.

   SOLUTION (1/2 POINT)
   If you know the right people, then you will get elected.

   e) Winds from the south imply a spring thaw.

   SOLUTION (1/2 POINT)
   If there are winds from the south, then there's a spring thaw.

   f) Abena will go to Accra unless she missed the bus.

   SOLUTION (1/2 POINT)
   If Abena does not miss the bus, then she will go to accra.

2. (3 POINTS) Construct a truth table for each of these compound propositions:

a) $p \oplus p$

SOLUTION (1 POINT)

p	$p \oplus p$
T	F
F	F

b) $(p \oplus q) \vee (p \oplus \neg q)$

SOLUTION (2 POINTS)

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

3. (3 POINTS) Use logical equivalences to show that $[\neg p \wedge (p \vee q)] \rightarrow q$ is a tautology.

Steps	***Reasons***
$[\neg p \wedge (p \vee q)] \rightarrow q \equiv \neg[\neg p \wedge (p \vee q)] \vee q$	Conditional-disjunction equivalence (1/4 POINT)
$\equiv (\neg\neg p \vee \neg (p \vee q)) \vee q$	1ST De Morgan's Law (1/4 POINT)
$\equiv (p \vee (\neg p \wedge \neg q)) \vee q$	Double negation law & 2nd De Morgans Law (1 POINT)
$\equiv ((p \vee \neg p) \wedge (p \vee \neg q)) \vee q$	1st Distributive Law (1/4 POINT)
$\equiv (\mathrm{T} \wedge (p \vee \neg q)) \vee q$	1st negation law (1/4 POINT)
$\equiv (p \vee \neg q) \vee q$	1st Identity law (1/4 POINT)

$\equiv p \vee (\neg q \vee q)$	1st associative law (1/4 POINT)
$\equiv (p \vee \mathrm{T})$	1st negation law (1/4 POINT)
$\equiv \mathrm{T}$	1st domination law (1/4 POINT)

OR

$[\neg p \wedge (p \vee q)] \rightarrow q \equiv \neg[\neg p \wedge (p \vee q)] \vee q$	Conditional-disjunction equivalence (1/2 POINT)
$\equiv \neg\neg p \vee \neg (p \vee q) \vee q$	1ST De Morgan's Law (1/2 POINT)
$\equiv (p \vee q) \vee \neg (p \vee q)$	Double negation law & 1st Commutative law (1 POINT)
$\equiv \mathrm{T}$	1st Negation law (1 POINT)