ASHESI

Discrete Structures and Theory (Spring 2023)
Discussion 3
Date: 03/02/2023

Exercise 1:
Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?
a) $P(orange)$ b) $P(lemon)$ c) $P(true)$ d) $P(false)$

Exercise 2:
Let $N(x)$ be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.
a) $\exists x N(x)$ b) $\forall x N(x)$ c) $\neg\exists x N(x)$ d) $\exists x \neg N(x)$ e) $\neg\forall x N(x)$ f) $\forall x \neg N(x)$

Exercise 3:
Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.
a) $\forall x(C(x) \rightarrow F(x))$ b) $\forall x(C(x) \wedge F(x))$
c) $\exists x(C(x) \rightarrow F(x))$ d) $\exists x(C(x) \wedge F(x))$

Exercise 4:
Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
a) $\exists x(x^2 = 2)$ b) $\exists x(x^2 = -1)$
c) $\forall x(x^2 + 2 \geq 1)$ d) $\forall x(x^2 = x)$

Exercise 5:
Suppose that the domain of the propositional function $P(x)$ consists of the integers $0, 1, 2, 3$, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.
a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$ e) $\neg\exists x P(x)$ f) $\neg\forall x P(x)$

Exercise 6:
Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi.
b) Everyone in your class is friendly.
c) There is a person in your class who was not born in California.
d) A student in your class has been in a movie.
e) No student in your class has taken a course in logic programming.

Exercise 7:

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

a) There is a horse that can add.

b) Every koala can climb.

c) No monkey can speak French.

d) Some lions do not drink coffee.

e) There exists a pig that can swim and catch fish.

Exercise 8:

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x(x^2 \geq x)$ b) $\forall x(x > 0 \lor x < 0)$ c) $\forall x(x = 1)$