ASHESI

Discrete Structures and Theory (Spring 2023)
Revision for Mid-Semester Exam
Date: 01/03/2023

Exercise 1:

Use a direct proof to show that the sum of two odd integers is even.

Exercise 2:

Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
a) a proof by contraposition.
b) a proof by contradiction.

Exercise 3:

Determine which of the following statements are true, and which are false.

a) $a \in \{a\}$
b) $\{3,5\} = \{5,3\}$
c) $[3,4) \subseteq (3,4]$
d) $\emptyset = \{\emptyset\}$
e) $\emptyset = \{\,\}$
f) $\{\{b\}\} \subseteq \{\{a\},b,c\}$
g) $\{a\} \in \mathcal{P}(\{\{a\},b,c\})$

Exercise 4:

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

Exercise 5:

For each of these arguments, explain which rules of inference are used for each step.

a) "Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket."
b) "There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre."

Exercise 6:

Express each of these statements using predicates with two variables and nested quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) Someone in this class has visited every country in the world.
b) No one has climbed every mountain in the Himalayas.
c) Every faculty member at your school has mentored a child from Berekuso Basic School.

Exercise 7:

Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

a) $Q(1, 1)$ b) $Q(2, 0)$ c) $\forall y Q(1, y)$ d) $\exists x Q(x, 2)$ e) $\exists x \exists y Q(x, y)$
f) $\forall x \exists y Q(x, y)$ g) $\exists y \forall x Q(x, y)$ h) $\forall y \exists x Q(x, y)$ i) $\forall x \forall y Q(x, y)$

Exercise 8:

Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is $1, 2$, or 3 and y is $1, 2$, or 3. Write out these propositions using disjunctions and conjunctions.

a) $\forall x \forall y P\ (x, y)$ b) $\exists x \exists y P\ (x, y)$ c) $\exists x \forall y \neg P\ (x, y)$ d) $\forall y \exists x P\ (x, y)$

Exercise 9:

Let $W\ (x, y)$ mean that student x has visited website y, where the domain for x consists of all students in your school and the domain for y consists of all websites. Express each of these statements by a simple English sentence.

a) $W(\text{Sarah Smith, www. att. com})$
b) $\exists x W(x, \text{www. imdb. org})$
c) $\exists y W(\text{José Orez}, y)$
d) $\exists y (W(\text{Ashok Puri}, y) \wedge W(\text{Cindy Yoon}, y))$
e) $\exists y \forall z (y \neq (David\ Belcher) \wedge (W(David\ Belcher, z) \rightarrow W\ (y, z)))$
f) $\exists x \exists y \forall z ((x \neq y) \wedge (W\ (x, z) \leftrightarrow W\ (y, z)))$

Exercise 10:

Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

Exercise 11:

Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

Exercise 12:

Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

Exercise 13:

Let p and q be the propositions
p : You drive over 65 miles per hour.
q : You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).

a) You do not drive over 65 miles per hour.
b) You drive over 65 miles per hour, but you do not get a speeding ticket.
c) You will get a speeding ticket if you drive over 65 miles per hour.
d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
f) You get a speeding ticket, but you do not drive over 65 miles per hour.
g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Exercise 14:

Write each of these statements in the form "if p, then q" in English.

a) I will remember to send you the address only if you send me an e-mail message.
b) To be a citizen of this country, it is sufficient that you were born in the United States.
c) If you keep your textbook, it will be a useful reference in your future courses.
d) The Red Wings will win the Stanley Cup if their goalie plays well.
e) That you get the job implies that you had the best credentials.
f) The beach erodes whenever there is a storm.
g) It is necessary to have a valid password to log on to the server.
h) You will reach the summit unless you begin your climb too late.

Exercise 15:

State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.