

1.

a. True

This biconditional is true because both propositions are true and the implication is true in both directions, i.e. $p \rightarrow q$ is true and $q \rightarrow p$ is true. True and True is True.

b. False

This biconditional is false because the second proposition is false, making the first implication false. False and True is False.

c. True

This biconditional is true because both individual propositions are false. This makes the implication in both directions true. True and True is True.

d. False

This biconditional is false because the implication in the second direction is false. True and False is False.

2.

$$\begin{aligned}(p \rightarrow r) \vee (q \rightarrow r) &\equiv (p \wedge q) \rightarrow r \\(p \rightarrow r) \vee (q \rightarrow r) & \\ \equiv (\neg p \vee r) \vee (\neg q \vee r) &\quad \text{applying Conditional-Disjunction Equivalence Law} \\ \equiv \neg p \vee r \vee \neg q \vee r &\quad \text{applying 1st Commutative Law} \\ \equiv \neg p \vee \neg q \vee r \vee r &\quad \text{applying 1st Associative Law} \\ \equiv \neg p \vee \neg q \vee (r \vee r) &\quad \text{applying 1st Idempotent Law} \\ \equiv \neg p \vee \neg q \vee r & \\ \equiv (\neg p \vee \neg q) \vee r &\quad \text{applying 1st Associative Law} \\ \equiv \neg(p \wedge q) \vee r &\quad \text{applying 1st De Morgan's Law} \\ \equiv (p \wedge q) \rightarrow r &\quad \text{applying Conditional-Disjunction Equivalence Law}\end{aligned}$$

Therefore, $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$.

3.

$Y(x)$: "x is a youtuber."

U = Ashesi students

a) $\exists x Y(x)$:

"Some Ashesi students are youtubers."

b) $\forall x Y(x)$:

"All Ashesi students are youtubers."

c) $\neg \exists x Y(x) \equiv \forall x \neg Y(x)$:

"All Ashesi students are not youtubers."

d) $\exists x \neg Y(x)$:

"Some Ashesi students are not youtubers."

e) $\neg \forall x Y(x) \equiv \exists x \neg Y(x)$:

"Some Ashesi students are not youtubers."

f) $\forall x \neg Y(x)$:

"All Ashesi students are not youtubers."