ASHESI

Discrete Structures and Theory (Spring 2023)
Discussion 10
Date: 05/04/2023

Exercise 1:
Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = (n+1)\ (2n+1)(2n+3)/3$ whenever n is a nonnegative integer.

Exercise 2:
Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + \cdots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$ whenever n is a nonnegative integer.

Exercise 3:
Prove that $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4$ whenever n is a nonnegative integer.

Exercise 4:
a) Find a formula for

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

by examining the values of this expression for small values of n.

b) Prove the formula you conjectured in part (a).

Exercise 5:
Use mathematical induction to prove that $n^3 - n$ is divisible by 3, for every positive integer n.

Exercise 6:
Prove that whenever n is a positive integer,
21 divides $4^{n+1} + 5^{2n-1}$

Exercise 7:
Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes, where these pieces cover three squares at a time, as shown in the picture below: