ASHESI

Discrete Structures and Theory (Spring 2023)
Week 4 Discussion
Date: 10/02/2023

Exercise 1:
Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")
a) There is a horse that can add.
b) Every koala can climb.
c) No monkey can speak French.
d) Some lions do not drink coffee.
e) There exists a pig that can swim and catch fish.

Exercise 2:
Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.
a) $\forall x(x^2 \geq x)$ b) $\forall x(x > 0 \vee x < 0)$ c) $\forall x(x = 1)$

Exercise 3:
Let $Q\ (x, y)$ be the statement "Student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.
a) $\exists x \exists y Q(x, y)$ b) $\exists x \forall y Q(x, y)$ c) $\forall x \exists y Q(x, y)$
d) $\exists y \forall x Q(x, y)$ e) $\forall y \exists x Q(x, y)$ f) $\forall x \forall y Q(x, y)$

Exercise 4:
Let $Q(x, y)$ be the statement "student x has been a contestant on quiz show y." Express each of these sentences in terms of $Q(x, y)$, quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.
a) There is a student at your school who has been a contestant on a television quiz show.
b) No student at your school has ever been a contestant on a television quiz show.
c) There is a student at your school who has been a contestant on Jeopardy and on Wheel of Fortune.
d) Every television quiz show has had a student from your school as a contestant.

Exercise 5:

Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is $1, 2$, or 3 and y is $1, 2$, or 3. Write out these propositions using disjunctions and conjunctions.

a) $\forall x \forall y P(x, y)$ b) $\exists x \exists y P(x, y)$

c) $\exists x \forall y P(x, y)$ d) $\forall y \exists x P(x, y)$

Exercise 6:

Let $I(x)$ be the statement "x has an Internet connection" and $C(x, y)$ be the statement "x and y have chatted over the Internet," where the domain for the variables x and y consists of all students in your class.

Use quantifiers to express each of these statements.

a) Jerry does not have an Internet connection.
b) Rachel has not chatted over the Internet with Chelsea.
c) Jan and Sharon have never chatted over the Internet.
d) No one in the class has chatted with Bob.
e) Sanjay has chatted with everyone except Joseph.
f) Someone in your class does not have an Internet connection.
g) Not everyone in your class has an Internet connection.
h) Exactly one student in your class has an Internet connection.
i) Everyone except one student in your class has an Internet connection.
j) Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.
k) Someone in your class has an Internet connection but has not chatted with anyone else in your class.
l) There are two students in your class who have not chatted with each other over the Internet.
m) There is a student in your class who has chatted with everyone in your class over the Internet.
n) There are at least two students in your class who have not chatted with the same person in your class.
o) There are two students in the class who between them have chatted with everyone else in the class.

Exercise 7:

Suppose the variable x represents students, y represents courses, and $T(x, y)$ means "x is taking y".

Match the English statement with all its equivalent symbolic statements in this list:

1. $\exists x \forall y\, T(x, y)$	2. $\exists y \forall x\, T(x, y)$	3. $\forall x \exists y\, T(x, y)$
4. $\neg \exists x \exists y\, T(x, y)$	5. $\exists x \forall y\, \neg T(x, y)$	6. $\forall y \exists x\, T(x, y)$
7. $\exists y \forall x\, \neg T(x, y)$	8. $\neg \forall x \exists y\, T(x, y)$	9. $\neg \exists y \forall x\, T(x, y)$
10. $\neg \forall x \exists y\, \neg T(x, y)$	11. $\neg \forall x \neg \forall y\, \neg T(x, y)$	12. $\forall x \exists y\, \neg T(x, y)$

a) Every course is being taken by at least one student.
b) Some student is taking every course.
c) No student is taking all courses.
d) There is a course that all students are taking.
e) Every student is taking at least one course.
f) There is a course that no students are taking.
g) Some students are taking no courses.
h) No course is being taken by all students.
i) Some courses are being taken by no students.
j) No student is taking any course.