ASHESI

Discrete Structures and Theory- Spring 2023
Homework 5
Deadline: 24/02/2023

1. (4 POINTS) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

2. (3 POINTS) Use a direct proof to show that if n is even, then $(n+3)^2$ is odd.

3. (2 POINTS) Suppose that *Prolog* facts are used to define the predicates $mother(M,Y)$ and $father(F,X)$, which represent that M is the mother of Y and F is the father of X, respectively. Give a *Prolog* rule to define the predicate $grandfather(X,Y)$, which represents that X is the grandfather of Y . [Hint: You can write a disjunction in Prolog either by using a semicolon to separate predicates or by putting these predicates on separate lines.]
   Do not forget to explain your answer.