1.

$\forall x(P(x) \vee Q(x))$

$\underline{\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x)}$

$\therefore \forall x(\neg R(x) \rightarrow P(x))$

	Steps	Reason
1	$\forall x(P(x) \vee Q(x))$	Premise
2	$\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x)$	Premise
3	$P(\mathrm{c}) \vee Q(\mathrm{c})$	Universal instantiation on 1
4	$(\neg P(\mathrm{c}) \wedge Q(\mathrm{c})) \rightarrow R(\mathrm{c})$	Universal instantiation on 2
5	$\neg(\neg P(\mathrm{c}) \wedge Q(\mathrm{c})) \vee R(\mathrm{c})$	Conditional disjunction equivalence on 4
6	$(P(\mathrm{c}) \vee \neg Q(\mathrm{c})) \vee R(\mathrm{c})$	1st De Morgan's Law on 5
7	$\neg Q(\mathrm{c}) \vee (P(\mathrm{c}) \vee R(\mathrm{c}))$	1st Associativity Law on 6
8	$P(\mathrm{c}) \vee (P(\mathrm{c}) \vee R(\mathrm{c}))$	Resolution on 3 and 7
9	$P(\mathrm{c}) \vee R(\mathrm{c})$	1st Idempotent Law on 8
10	$R(\mathrm{c}) \vee P(\mathrm{c})$	1st Commutativity Law on 9
11	$\neg R(\mathrm{c}) \rightarrow P(\mathrm{c})$	Conditional disjunction equivalence on 10
12	$\forall x(\neg R(x) \rightarrow P(x))$	Universal generalization on 11

Therefore, $\forall x(\neg R(x) \rightarrow P(x))$ is true, if the premises $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x)$ are true.

2.

If n is even, then $(n+3)^2$ is odd.

If n is even, then, by definition, there exists an integer k such that $n = 2k$.

We substitute this value of n into the expression $(n+3)^2$ as follows:

$(n+3)^2 = (2k+3)^2$

$= 4k^2 + 12k + 9$ (expanding the term)

$= 2(2k^2 + 6k + 4) + 1$ (factorizing 2 out of the terms with k)

$= 2t + 1$, where $t = 2k^2 + 6k + 4$

t is an integer because the product of integers is an integer, and the sum of integers is an integer.

An odd number, r, can be expressed in the form $2k + 1$, where k is an integer.

Therefore, $(n+3)^2$ is an odd number.

3.

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grandfather(X, Z) :-
    father(X, Y), (mother(Y, Z) ; father(Y, Z)).
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This rule is saying that X is the grandfather of Z if:

X is the father of some person Y and

Y is the mother or father of some Z

The , operator is used to combine two conditions, so the first condition is that X is the father of Y, and the second condition is that Y is either the mother or father of Z.

The ; operator is used to combine two possibilities: either Z is the mother of Y, or Z is the father of Y. So, the rule reads as follows: X is the grandfather of Z if there exists some person Y, who is the child of X and the parent of Z.