ASHESI

Discrete Structures and Theory (Spring 2023)
Week 5 Discussion
Date: 17/02/2023

Exercise 1:
Show that the premises "If you send me an e-mail message, then I will finish writing the program", "If you do not send me an e-mail message, then I will go to sleep early", and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

Exercise 2:
Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \land \exists x Q(x)$ is true then $\exists x (P(x) \land Q(x))$ is true.

1.	$\exists x P(x) \land \exists x Q(x)$	Premise
2.	$\exists x P(x)$	Simplification from (1)
3.	$P(c)$	Existential instantiation from (2)
4.	$\exists x Q(x)$	Simplification from (1)
5.	$Q(c)$	Existential instantiation from (4)
6.	$P(c) \land Q(c)$	Conjunction from (3) and (5)
7.	$\exists x (P(x) \land Q(x))$	Existential generalization

Exercise 3:
Explain which rules of inference are used in the following argument:
"Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

Exercise 4:
Explain which rules of inference are used in the following argument:
"Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

Exercise 5:
Explain which rules of inference are used in the following argument:
"Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

Exercise 6:

Justify the rule of **universal transitivity**, which states that if $\forall x(P\ (x)\ \rightarrow\ Q(x))$ and $\forall x(Q(x)\ \rightarrow\ R(x))$ are true, then $\forall x(P\ (x)\ \rightarrow\ R(x))$ is true, where the domains of all quantifiers are the same.

Exercise 7:

Use rules of inference to show that if $\forall x(P\ (x)\ \rightarrow\ (Q(x)\ \wedge\ S(x)))$ and $\forall x(P\ (x)\ \wedge\ R(x))$ are true, then $\forall x(R(x)\ \wedge\ S(x))$ is true.