1.

∀𝑥(𝑃 (𝑥) ∨ 𝑄(𝑥))

∀𝑥((¬𝑃 (𝑥) ∧ 𝑄(𝑥)) → 𝑅(𝑥)

⸫∀𝑥(¬𝑅(𝑥) → 𝑃 (𝑥))

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|  | **Steps** | **Reason** |
| **1** | ∀𝑥(𝑃 (𝑥) ∨ 𝑄(𝑥)) | Premise |
| **2** | ∀𝑥((¬𝑃(𝑥) ∧ 𝑄(𝑥)) → 𝑅(𝑥) | Premise |
| **3** | 𝑃(c) ∨ 𝑄(c) | Universal instantiation on 1 |
| **4** | (¬𝑃(c) ∧ 𝑄(c) ) → 𝑅(c) | Universal instantiation on 2 |
| **5** | ¬ (¬𝑃(c) ∧ 𝑄(c) ) ∨ 𝑅(c) | Conditional disjunction equivalence on 4 |
| **6** | (𝑃(c) ∨ ¬𝑄(c) ) ∨ 𝑅(c) | 1st De Morgan’s Law on 5 |
| **7** | ¬𝑄(c) ∨ (𝑃(c) ∨ 𝑅(c)) | 1st Associativity Law on 6 |
| **8** | 𝑃(c) ∨ (𝑃(c) ∨ 𝑅(c)) | Resolution on 3 and 7 |
| **9** | 𝑃(c) ∨ 𝑅(c) | 1st Idempotent Law on 8 |
| **10** | 𝑅(c) ∨ 𝑃(c) | 1st Commutativity Law on 9 |
| **11** | ¬𝑅(c) → 𝑃(c) | Conditional disjunction equivalence on 10 |
| **12** | ∀𝑥(¬𝑅(𝑥) → 𝑃(𝑥)) | Universal generalization on 11 |

Therefore, ∀𝑥(¬𝑅(𝑥) → 𝑃(𝑥)) is true, if the premises ∀𝑥(𝑃 (𝑥) ∨ 𝑄(𝑥)) and ∀𝑥((¬𝑃(𝑥) ∧ 𝑄(𝑥)) → 𝑅(𝑥) are true.

2.

If 𝑛 is even, then (𝑛 + 3)2 is odd.

If 𝑛 is even, then, by definition, there exists an integer 𝑘 such that 𝑛 = 2𝑘.

We substitute this value of 𝑛 into the expression (𝑛 + 3)2 as follows:

(𝑛 + 3)2 = (2𝑘 + 3)2

= 4𝑘2 + 12𝑘 + 9 (expanding the term)

= 2(2𝑘2 + 6𝑘 + 4) + 1 (factorizing 2 out of the terms with *k*)

= 2*t* + 1, where *t* = 2𝑘2 + 6𝑘 + 4

*t* is an integer because the product of integers is an integer, and the sum of integers is an integer.

An odd number, *r*, can be expressed in the form 2*k* + 1, where *k* is an integer.

Therefore, (𝑛 + 3)2 is an odd number.

3.

grandfather(X, Z) :-

father(X, Y), (mother(Y, Z) ; father(Y, Z)).

This rule is saying that X is the grandfather of Z if:

X is the father of some person Y and

Y is the mother or father of some Z

The , operator is used to combine two conditions, so the first condition is that X is the father of Y, and the second condition is that Y is either the mother or father of Z.

The ; operator is used to combine two possibilities: either Z is the mother of Y, or Z is the father of Y. So, the rule reads as follows: X is the grandfather of Z if there exists some person Y, who is the child of X and the parent of Z.