

M1-IRELE

ELEC-H401 Modulation and coding

DVB-C project

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Introduction

This report aims to complete the code that simulates a DVB-C transmission chain in matlab. It provides additional information from the theoretical part of the project.

The first part builds the transmission chain and link the blocks together such that the received signal is the same as the one transmitted (in a noiseless case).

Second part to be explained later

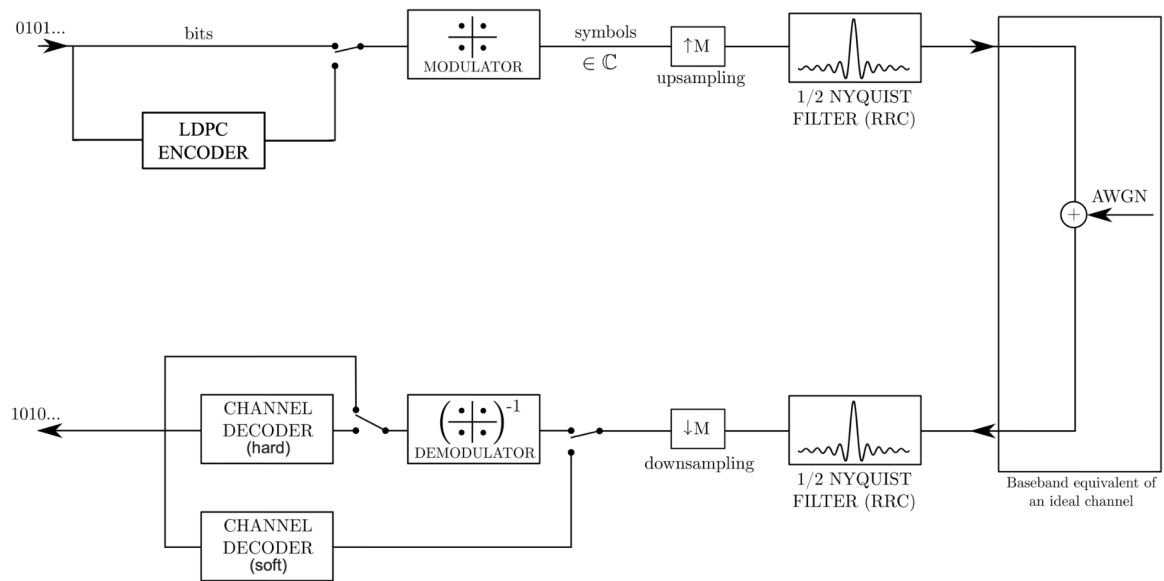


Figure 1: DVB-C transmission chain

Baseband representation

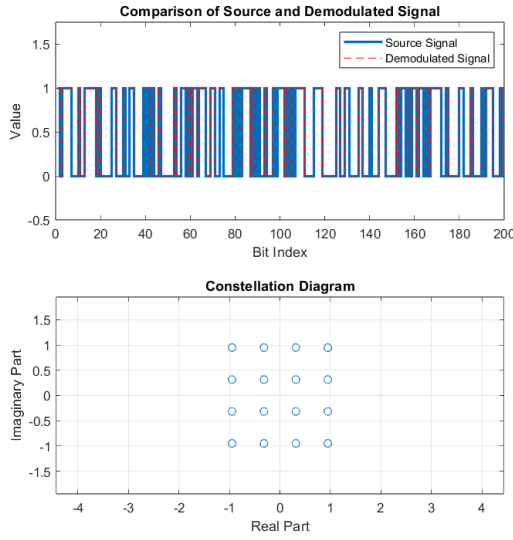
By looking at the block diagram of the transmission chain 1, one can see we never move the baseband signal to the carrier frequency. As the simulation runs on a computer, using the bandpass representation of the signal would require much more samples as the sampling frequency would need to be at least twice the carrier frequency. By simulating the chain in baseband, the minimal sampling frequency is reduced to the symbol rate in order to have at least one sample per symbol.

Because the signal is oversampled, the sampling frequency is then equal to the symbol rate multiplied by the oversampling factor.

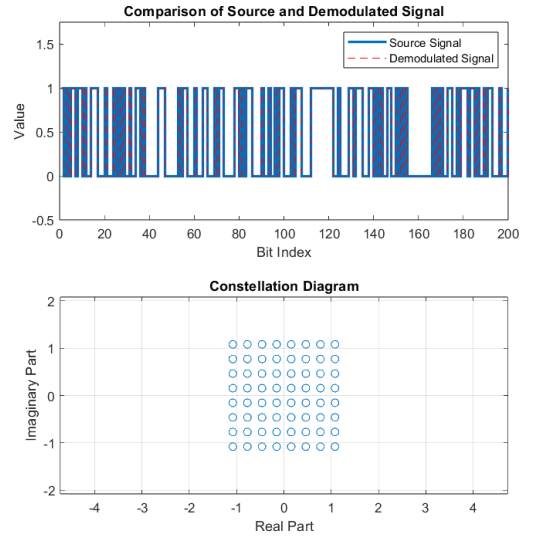
Modulation and Demodulation

After generating N random bits, they are modulated. This allows to send fewer symbols than the number of bits. We chose QAM modulation as it combines ASK and PSK. Depending on the number of bits per symbol (N_{bps}), the number of bits sent (N) had to be chosen such that $N/N_{\text{bps}} \in \mathbb{N}$.

Figure 2 compares the constellation diagrams obtained for QAM-16 and QAM-64. As the constellations points are more spaced on the left, QAM-16 is less prone to a wrong demodulation (when noise will be added). This comes at the cost of a lower bitrate: for the same symbol rate, QAM-64 will send 6 bits while QAM-16 only send 4. It clearly shows a compromise between reliability and capacity.



(a) QAM-16 modulation



(b) QAM-64 modulation

Figure 2: Comparison of QAM modulations, where the mean square error is computed between the transmitted and received bitstream

Optimal demodulator and detector

First, it is important to remind that the transmitted signal is represented by a set of coefficients which results from the projection of the signal on an orthonormal basis related to the chosen modulation. Once transmitted, the signal is affected by noise (AWG noise here). In the general case, this noise moves the signal, resulting in a change in its coefficient in the basis, as can be seen on Figure 3

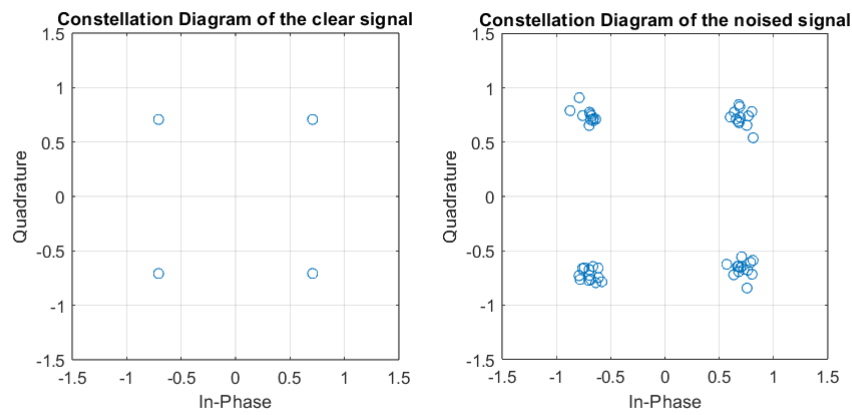


Figure 3: Effect of noise on the constellation diagram

To construct an optimal demodulator, 2 criteria should be taken into account. The first one is the sufficient statistic criteria. It is proven that once the received signal is projected on the sub-space defined by the previous basis functions, the noise component outside of the sub-space is independent from the pro-

jected signal. It means that there is no information loss when projecting the received signal on the original sub-space and the optimal decision can therefore be made using the projection of the incoming signal.

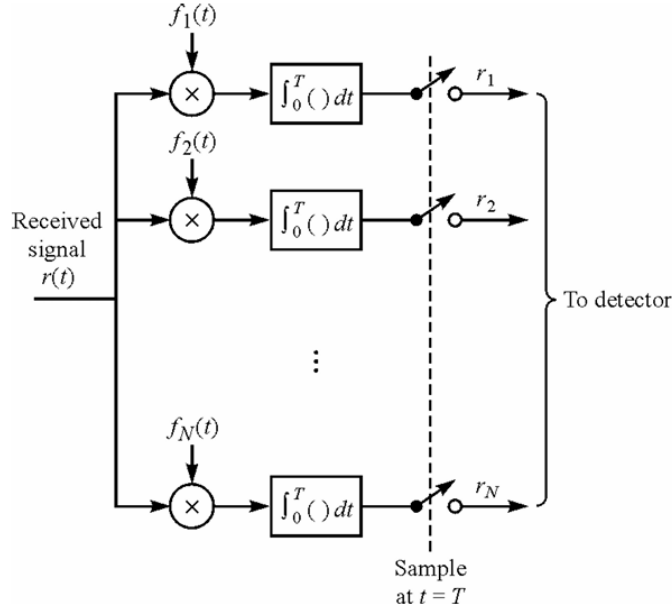


Figure 4: Projection on basis sub-space

$$\begin{aligned}
 E[n'(t)r_k] &= E[n'(t)]s_{mk} + E[n'(t)n_k] \\
 &= E[n'(t)n_k] \\
 &= E\left[\left(n(t) - \sum_{l=1}^K n_l f_l(t)\right)n_k\right] \\
 &= \int_{t'=-\infty}^{\infty} E[n(t)n(t')]f_k(t')dt' - \sum_{l=1}^K E[n_l n_k]f_l(t) \\
 &= \frac{N_0}{2}f_k(t) - \frac{N_0}{2}f_k(t) = 0
 \end{aligned}$$

Figure 5: Criteria of sufficient statistics

The second criteria is the usage of matched filters. The demodulator used to achieve the sufficient statistic property is composed of a bank of correlators (projection on a basis function). Instead of using a bank of correlators, we can use a bank of filters matched to the basis functions of the chosen modulation. It is proven that such filters at the demodulator gives a maximized SNR (minimize the power of the noise at the exit of the demodulator).

In conclusion, by using a bank of filters matched on the orthonormal basis function set by the choice of the modulation, we can construct a optimal demodulator which will ensure an optimal decision based on the received signal and ensure a maximum SNR at the output of this demodulator.

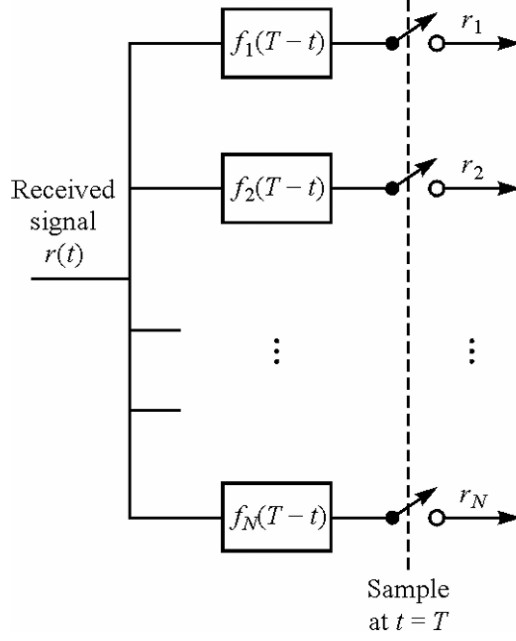


Figure 6: Bank of filters matching basis functions

$$\text{SNR} := \frac{\left[\int_{\tau=-\infty}^{\infty} s(\tau) h(-\tau) d\tau \right]^2}{\frac{N_0}{2} \int_{\tau=-\infty}^{\infty} h^2(-\tau) d\tau}$$

Cauchy-Schwartz inequality:

$$\left[\int_{t=-\infty}^{\infty} g_1(t) g_2(t) dt \right]^2 \leq \int_{t=-\infty}^{\infty} g_1^2(t) dt \int_{t=-\infty}^{\infty} g_2^2(t) dt$$

Equality if $g_1(t) = k g_2(t)$ for any arbitrary constant k

Figure 7: Maximum SNR demonstration

At the output of the demodulator, we still need to make sure the optimal choice of the M possible $s_m(t)$ is made based on the received signal. To achieve this, the maximum likelihood criteria is used. It is equivalent to the maximum a posteriori criteria (general criteria) in the case of equiprobable symbols $s_m(t)$. The criteria leads to the following result : the optimizal $s_m(t)$ choice is found by taking the minimum euclidian distance between the observable received signal $r(t)$ and all the possible modulated signal $s_m(t)$.

Pulse shaping

With modulation only, the bandwidth of the transmitted signal is infinite. This is problematic as it could interfere with neighboring channels. A filtering is applied to resolve this but the chosen filter must respect two other constraints: it must cancel inter-symbol interference (ISI) and must maximize the SNR.

The raised cosine filter is chosen as it limits the bandwidth and cancels ISI. To maximize the SNR, it is applied as a matched filter by using the square root of it at the transmitter and at the receiver.

The time domain and frequency domain representation of the raised cosine filter is shown in Figure 8. Figure 9 shows how the signal is shaped in the time domain and how there is indeed no ISI. Finally, the power spectral density ¹ of the transmitted signal is plotted in figure 10 where the frequency band is limited to $[-3, 3]$ MHz.

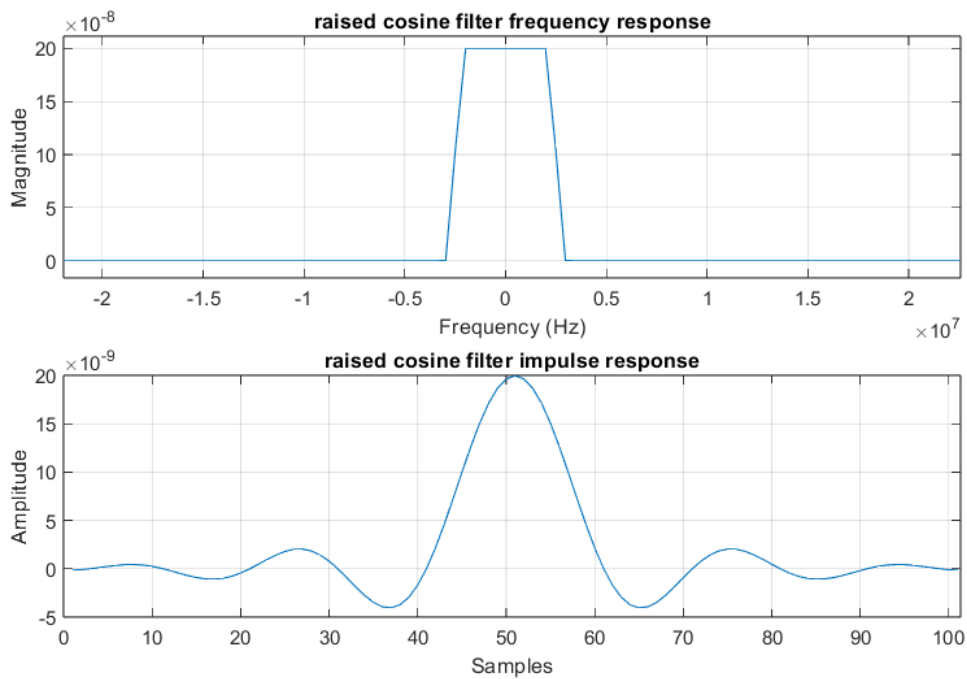


Figure 8: Time and frequency domain representation of the raised cosine filter

¹PSD as the signal is a random variable

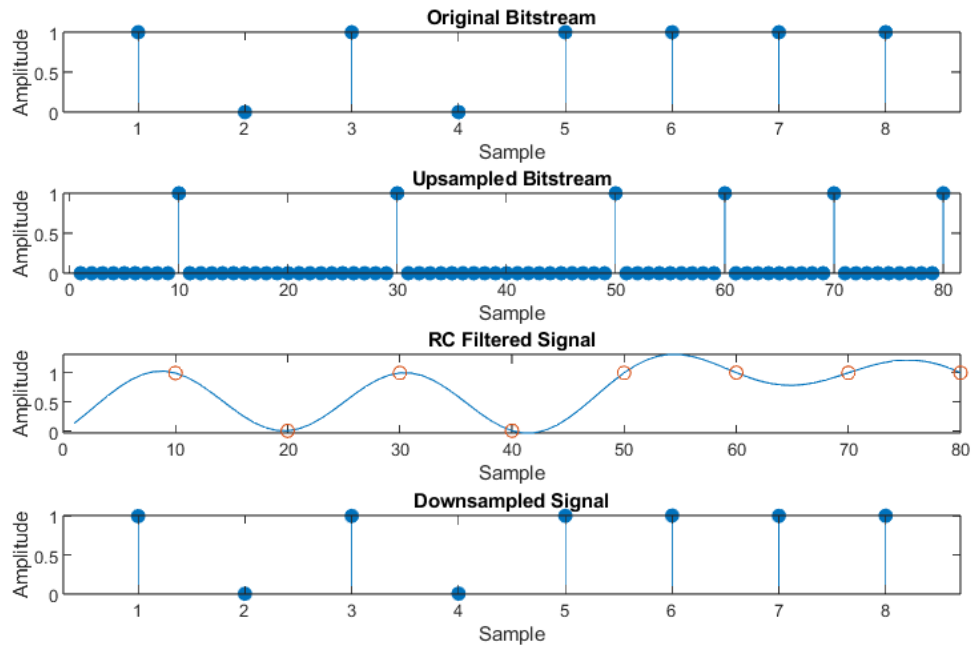


Figure 9: Pulse shaping with a raised cosine filter

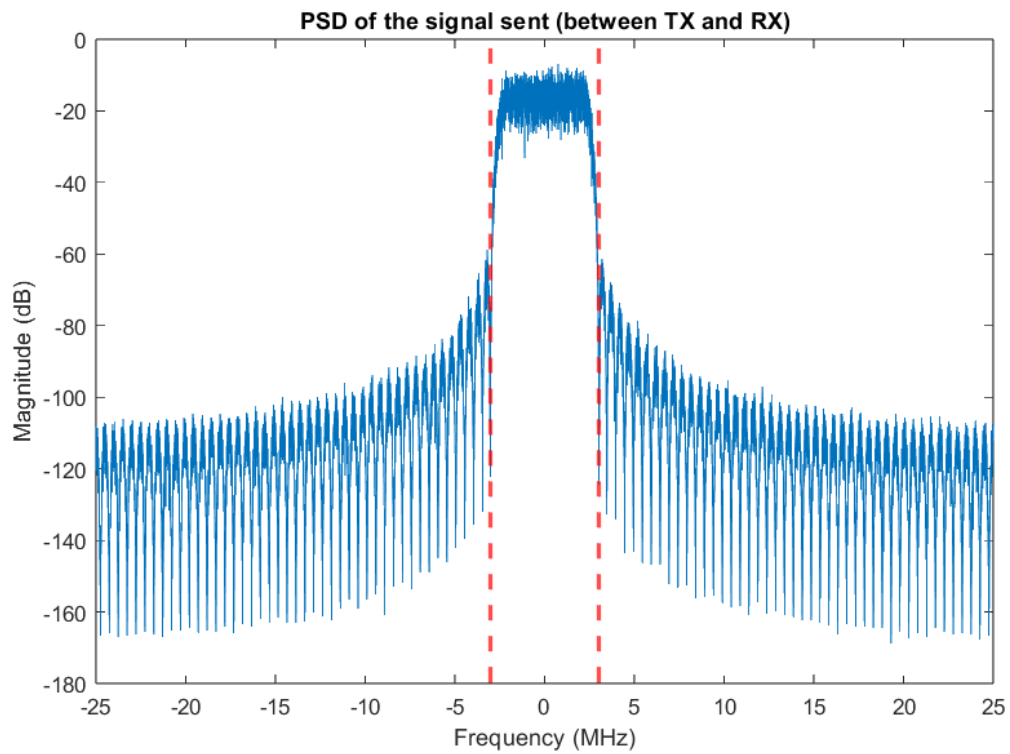


Figure 10: PSD of the transmitted signal after pulse shaping

Noise addition

The last building block is a noise source. It generates additive white Gaussian noise in baseband. When the signal is too much corrupted, the demodulation can fail. The BER curves are plotted in figure 11 and they show the impact of the noise power N_0 on the bit error rate. The compromise between reliability and capacity is again visible: in the same conditions (same E_b/N_0), a modulation with lower capacity will have a smaller BER.

The theoretical BER curves are plotted on figure 11 and are compared with the simulation results. They stay close to each other until the BER reaches 10^{-4} . This limit could go even lower by increasing the number of bits sent but we limited it to 10^6 in order for the code to run quite fast.

To impose a value of E_b/N_0 , we start by computing the energy of the transmitted signal before adding the noise. The power of the noise is then chosen as $N_0 = E_b / (E_b/N_0)_{\text{desired}}$.

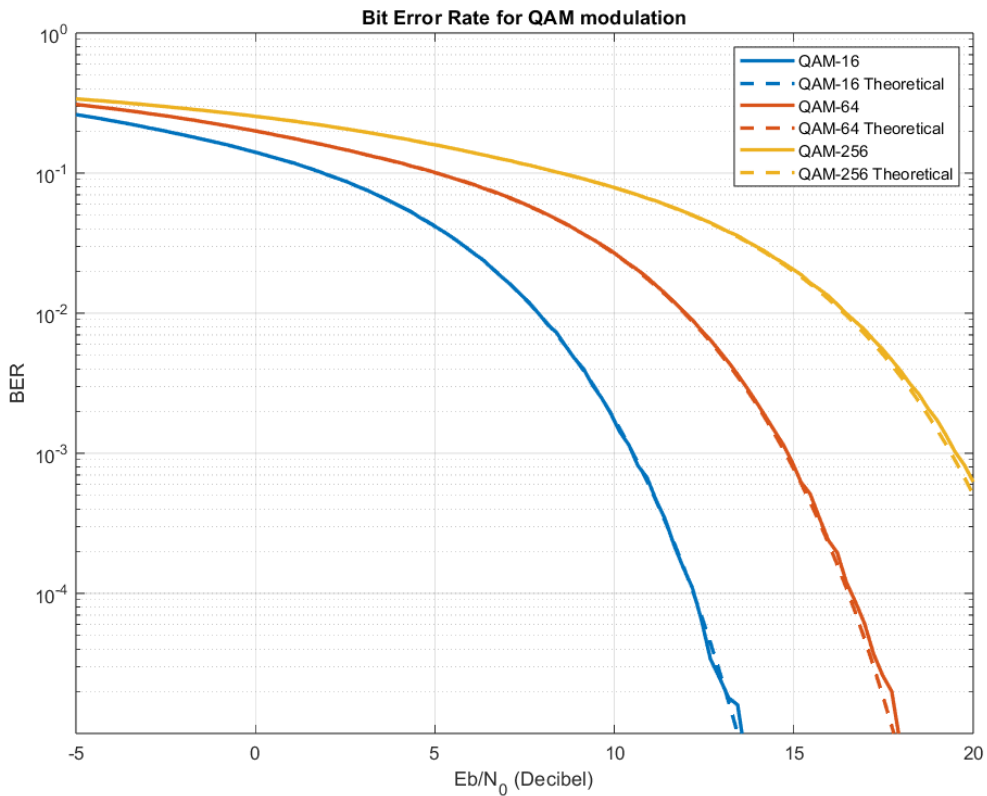


Figure 11: BER curves for different QAM modulations

Bit rate

Considering the following characteristics :

- (a) Physical bandwidth of $B_{\text{physical}} = 6 \text{ MHz}$
- (b) Roll-off factor $\beta = 0.2$
- (c) QAM 16 modulation $\rightarrow \text{Nbps} = 4$

We can derive the symbol rate from the physical bandwidth

$$f_{\text{symbol}} = \frac{B_{\text{physical}}}{1 + \beta} = 5 \text{ MHz}$$

$$\text{Bit-rate} = f_{\text{symbol}} * \text{Nbps} = 5 \text{ MHz} * 4 = 20 \text{ MBps}$$

Description

Because the receiver and transmitter are not at the same location, the carrier frequencies and the samplers at TX and RX will have a different phase and due to the inaccuracies of the oscillator, the frequencies will also be slightly different.

This is summarized in 4 effects:

- **Carrier frequency offset (CFO):** The difference in the carrier frequencies at TX and RX ($= \Delta\omega$). It will add ISI as the RRC are not anymore matched and a linearly increasing phase shift will appear.
- **Phase offset:** The difference between the phase of the carrier signal at TX and RX.
- **Sampling frequency offset (SFO):** The difference in the sampling frequencies at TX and RX.
- **Time shift:** The timing offset of the sampling at TX and RX.

Implementation

CFO

The CFO implementation is done by multiplying the signal with a complex exponential $e^{j2\pi\phi_{\text{ppm}}f_c t}$. The phase offset is added to the CFO. It is defined in ppm (part per million) where the ppm value is $\frac{\Delta\omega}{f_c} 10^{-6}$.

Figure 12 shows the BER curves with different CFO values. In order to have useful results, the linear phase shift is removed right after the second RRC filter. This allows to only keep the effect of ISI on the BER curve.

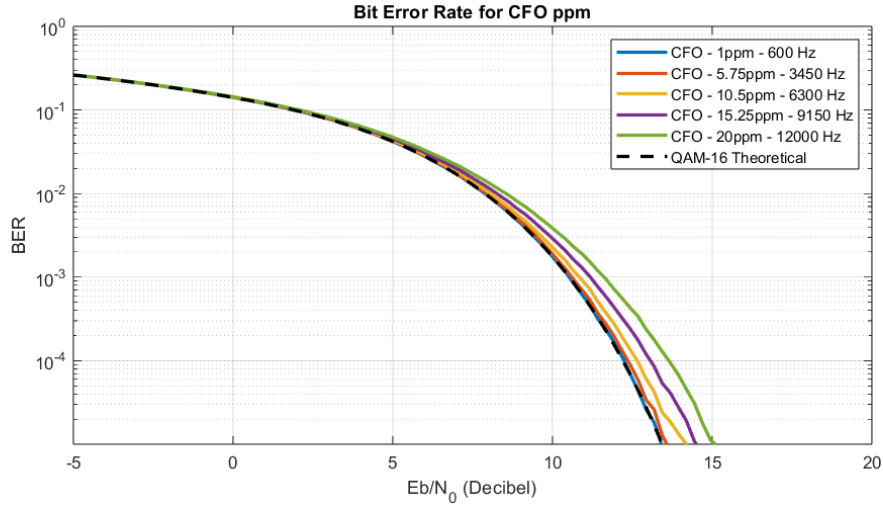


Figure 12: BER with different CFO values

Figure 13 shows the effect of CFO on the symbol constellation for QAM-16. The linear increase of the phase due to CFO makes the received symbols to form circles in the constellation diagram.

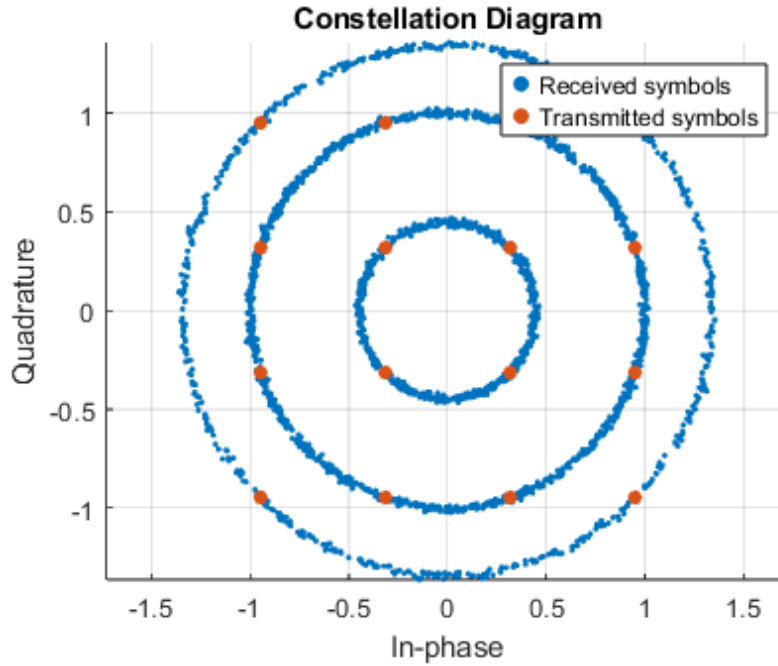


Figure 13: Constellation before and after CFO

Phase offset

The same is done for the phase offset where the exponential is simply $e^{j\phi}$ where ϕ is chosen once at the beginning of the simulation.

The effect of the phase offset is only visible on the constellation plot (figure 14) where every point is rotated by a fixed angle (whereas CFO rotated the symbols linearly with time).

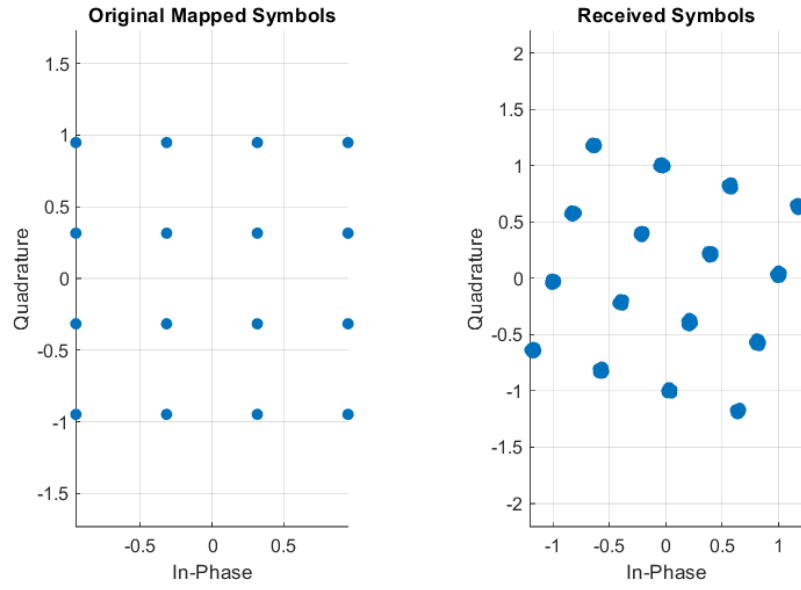


Figure 14: Constellation before and after phase offset

On a BER curve (figure 15), the phase is not visible as from the errors originating from the phase offset are either on every symbol or on none and this is why the error does not depend anymore on E_b/N_0 .

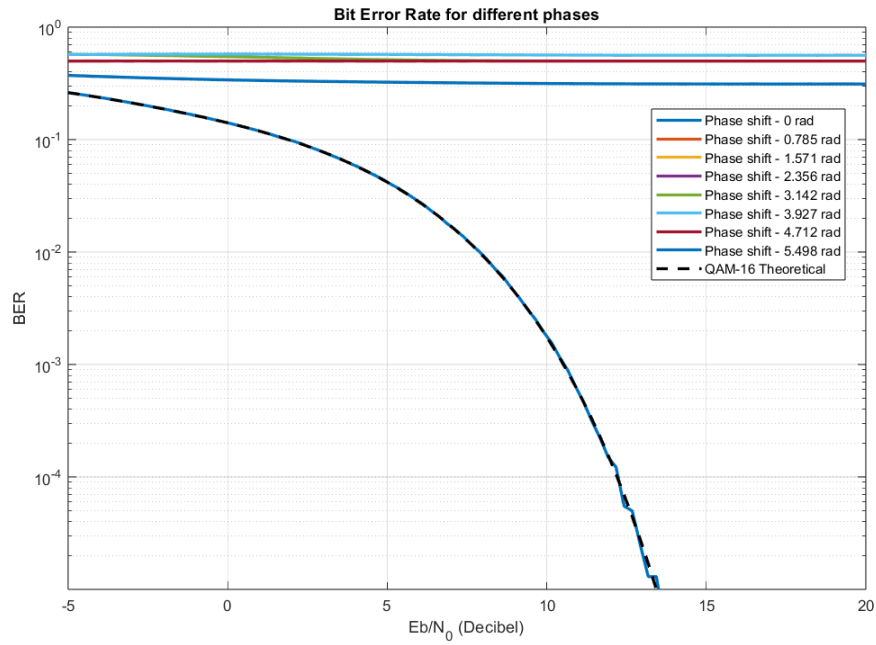


Figure 15: BER with phase offset

SFO

The SFO is neglected in the simulation as it would need some interpolation and more complex computations.

Time shift

The time shift is implemented by simply shifting the samples in the array with an oversampling factor that is large enough.

A larger time shift will increase the BER as the samples will be taken at the wrong time. For sufficiently low values, it will still behave as a "classical" BER curve but from some point, there is just no more correlation between the measured sample and the received one and the BER tends to a 0.5 line, as shown in figure 16.

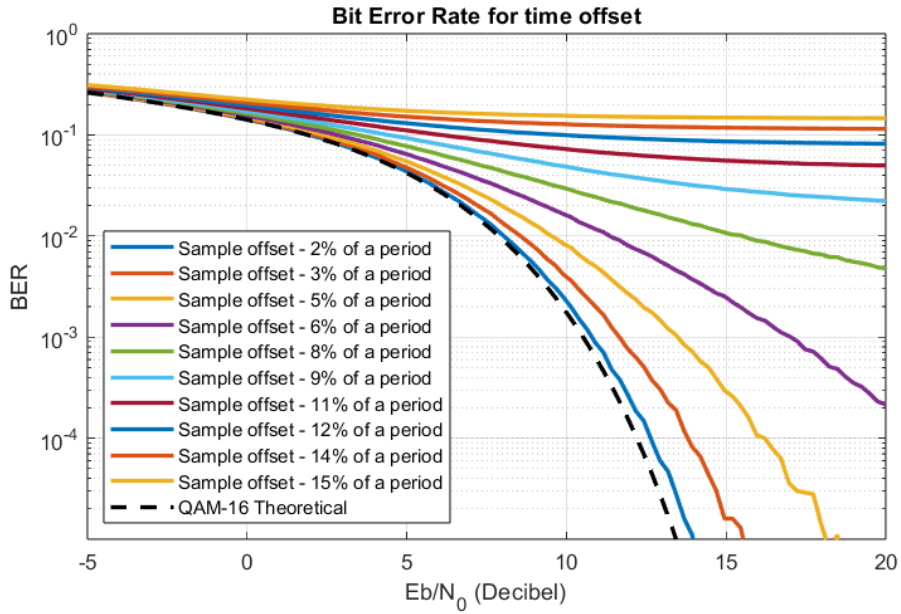


Figure 16: BER with time shift

As the phase offset makes the receiver sample the signal between true symbols, the constellation at its output has points that are spread around the initial symbols, as shown in figure 17.

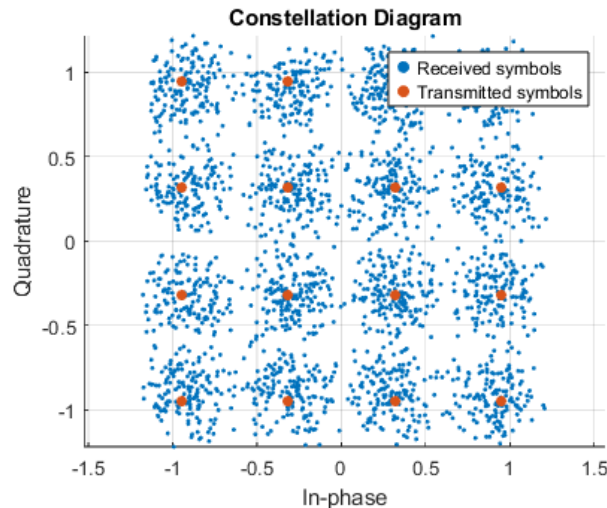


Figure 17: Constellation diagram with a phase offset of 15% of the symbol period

Choice of E_b/N_o

Securing a SNR high enough to ensure a remaining acceptable time error after Gardner algorithm implementation (2 percents of the symbole rate) is a necessary condition to fulfill.

The impact on the BER can be seen on the graph below - figure 18. In our case, for a symbol rate of 5MHz, the SNR should be above 4dB - 5dB and will be used in our simulation. Moreover, this SNR allows us to achieve, once the frame synchronisation implemented, a remaining carrier frequency offset of 2 ppm which is very close from the BER performance we can get without CFO - figure 19.

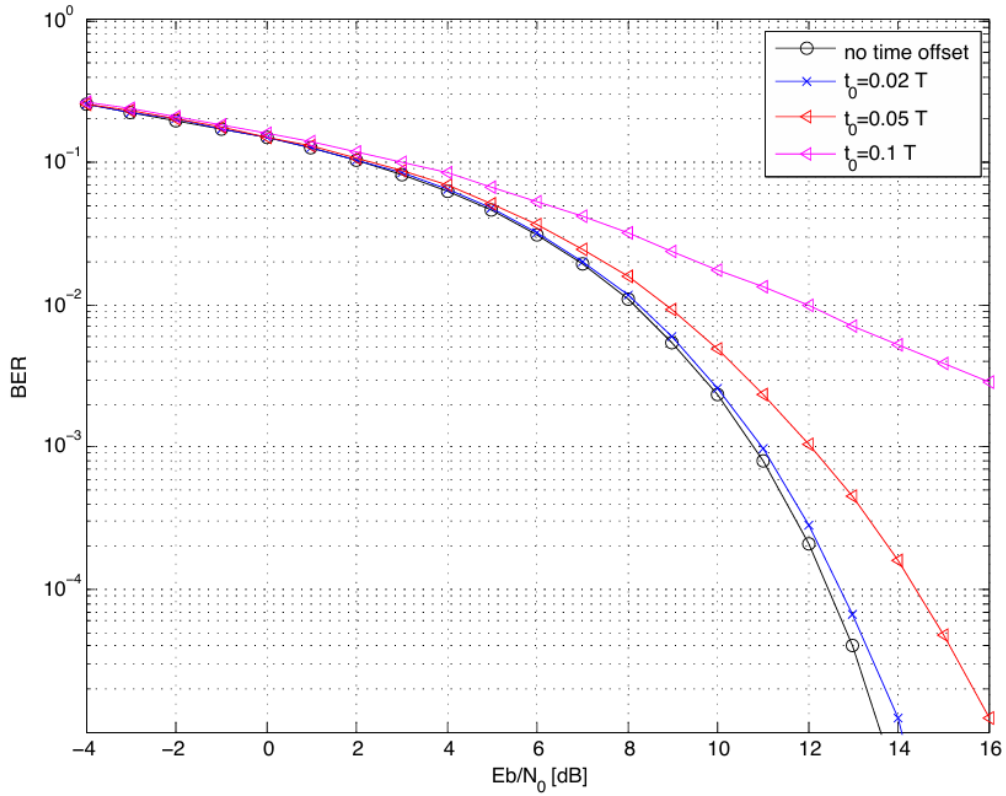


Figure 18: Impact of time shift on bit error rate

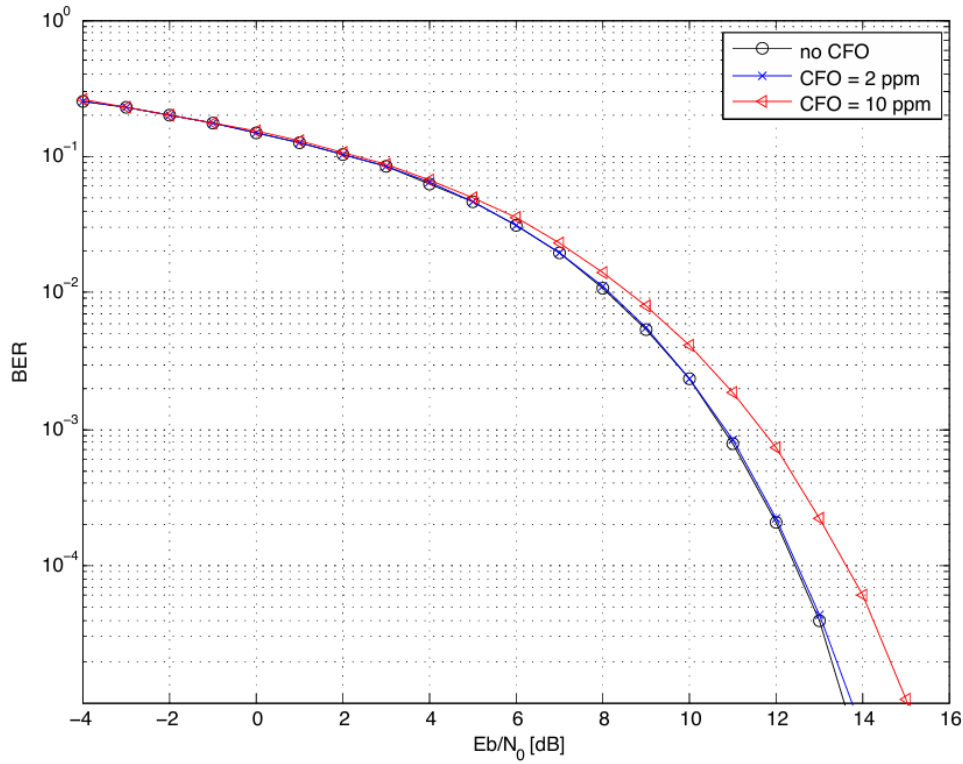


Figure 19: Impact of carrier frequency offset on bit error rate

Length of the pilot and the data sequence

Another condition to successfully implement the frame synchronisation algorithm and secure a remaining CFO of 2 ppm, for a SNR of 4-5dB, is to ensure to get:

- A preamble with a large enough number of symbols ($N > 20$).
- Cross-correlation sub-windows which are enough separated ($K > 8$).

Correction

Synchronisation error correction order

The main bottleneck of the synchronisation error correction is the combination of the time shift error and the carrier frequency error. This error combination leads us to find a algorithm which correct one without being affected by the other. The Gardner algorithm is fulfilling this condition as it is able to correct the time shift error and it is robust to CFO. Once the time shift error is corrected, the second algorithm can be implemented and detect/estimate the frequency shift introduced by the CFO.

The results of the implementation of this Gardner algorithm for different time shifts and different weight coefficients \mathbf{k} are plotted below.

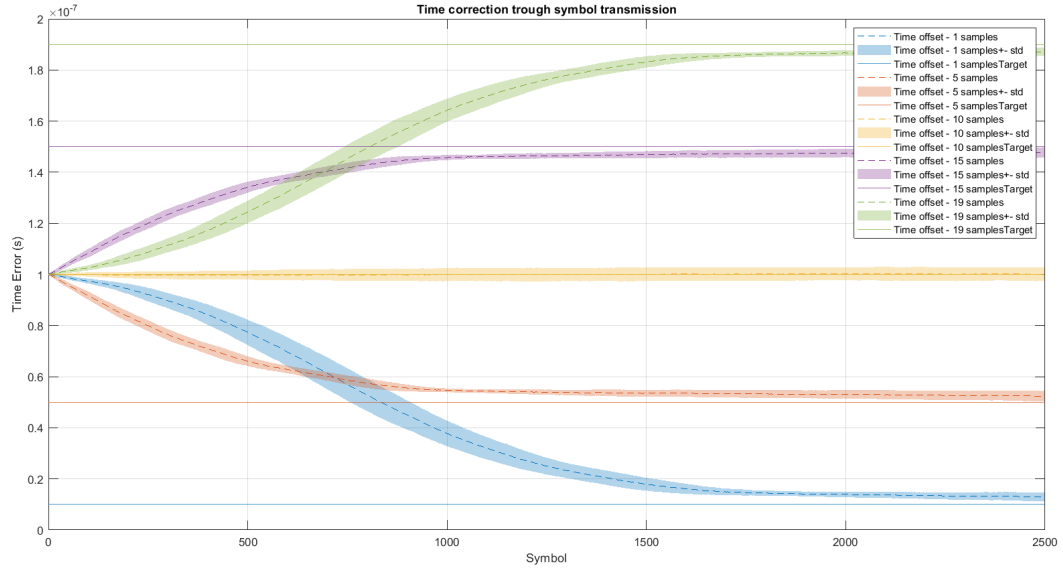


Figure 20: Time offset estimation over the samples with different time shifts

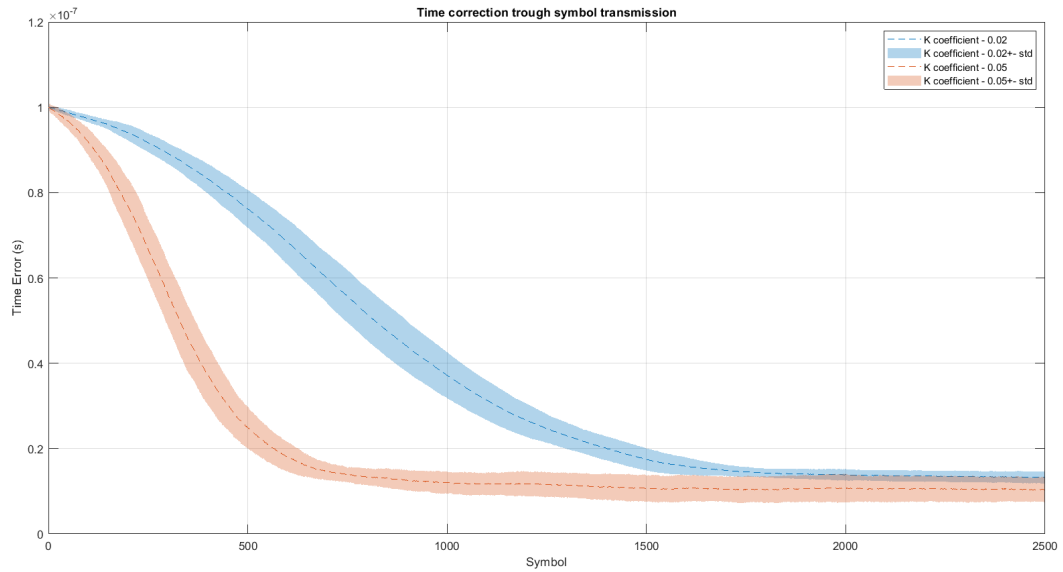


Figure 21: Time offset estimation over the samples with different weight coefficients

It is also interesting to mention that the Gardner algorithm is a **continuous** error correction - which means that the time shift error is corrected along the symbol stream sent.

In the other hand, The frame synchronisation is a correction done by analyzing a **window** of symbols.

CFO robustness of the Gardner algorithm

To understand why the Gardner algorithm is robust to CFO, we would need to analyze mathematically the algorithm:

$$\epsilon_{n+1} = \epsilon_n - \frac{2k}{T} R[y_{\epsilon_n}[n - \frac{1}{2}](y_{\epsilon_n}^*[n] - y_{\epsilon_{n-1}}^*[n-1]))]$$

By taking a close look to the equation, one can observe that the error is corrected by taking the difference between 2 adjacent symbols (which is why the algorithm is continuous). This specificity limits the effect of a phase shift as this shift will be small between two adjacent symbols.

Differential cross-correlation

Let's have a closer look to the cross correlation equation to grasp the intuition behind the need to use the differential cross-correlation.

$$\sum_l y_{n+l}^* a_l = \sum_l I_{n+l}^* a_l e^{-j\phi_0} e^{-j\Delta w(n+l)T}$$

We can observe that the CFO term is increasing with n resulting to a cross correlation equals to 0 even if the preamble is fitting the window.

Therefore, the idea behind the usage of the differential cross correlation is to remove the dependency of the phase shift with n . This idea leads to modify the equation and multiply the correlation between y and a with its complex conjugate term. Indeed the exponential terms are cancelling and remains an exponential dependent on k , the window division term. Therefore, by computing the differential cross correlation, the calculation will result to a peak once the preamble will match with the synchronisation window, function of $e^{-j\Delta w k}$ and 0 in the other cases.

To reply to the sub-question quoting "Isn't interesting to start the summation at $k = 0$ (no time shift)?", of course the reply is no as explained above.

The robustness of the gardner algorithm implemented in our Matlab code can be seen in 22

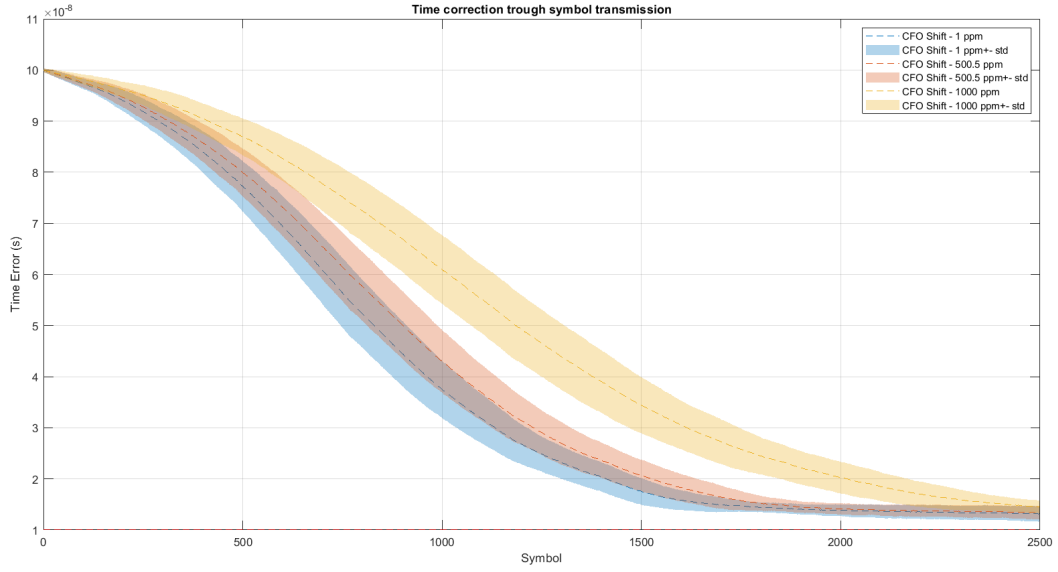


Figure 22: CFO robustness results from our Matlab code

Optimal criteria

To summarize, to have a perfect frame synchronisation which means to have a frame time arrival standard deviation equals to 0 for a realistic CFO error and a remaining time error of 2 percents at the end of Gardner module, a sine qua none condition is to get a large number of samples ($N > 20$) and a large sub-division term ($K > 8$).

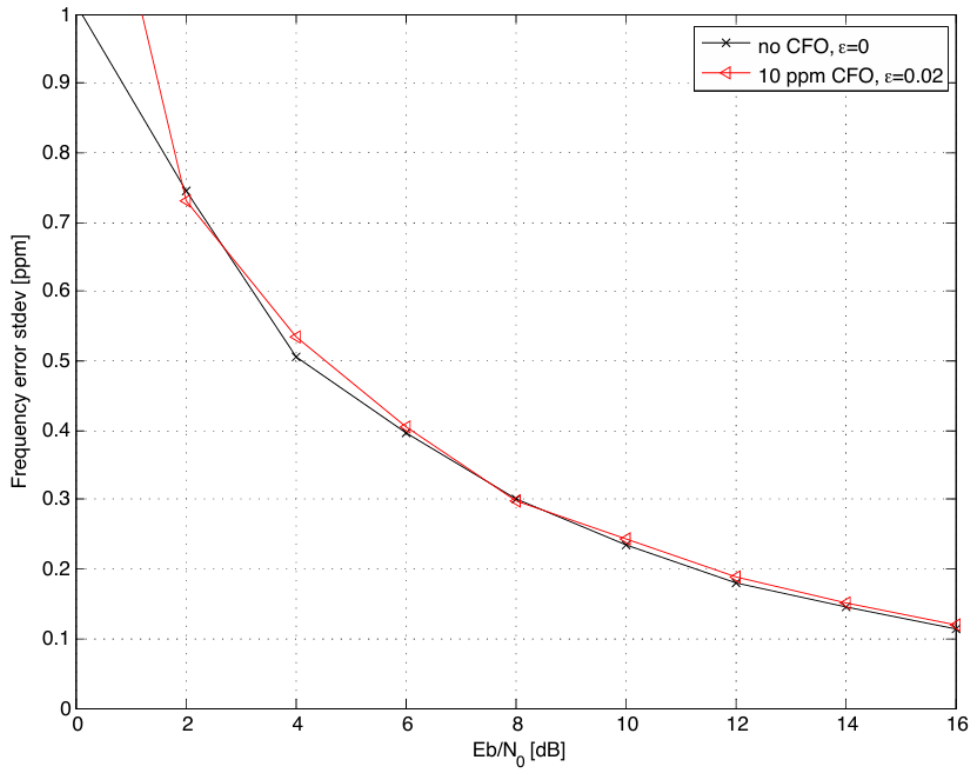


Figure 23: Frame time arrival standard deviation for realistic CFO and sampling time error based on SNR

Frame and frequency acquisition

With the previous correction, the receiver knows when to sample but there still is some uncertainty on the frequency. The way to remove the CFO is to send in the data a known pilot and the first step is to detect when this pilot is received. This is known as frame acquisition and it is performed with the differential cross-correlator:

$$D_k[n] = \frac{1}{N-k} \sum_{l=k}^{N-1} (y^*[n+l]a[l]) (y^*[n+l-k]a[l-k])^*$$

Where a is the known pilot, N its length and k the delay between the two correlations.

$D_k[n]$ is computed for every time index n and for every shift k until its maximum value, K . The estimation of the index of the start of the frame \hat{n} and the estimation of the CFO $\hat{\Delta f}$ are given by:

$$\hat{n} = \arg \max_n \sum_{k=1}^K |D_k[n]|$$

$$\hat{\Delta f} = -\frac{1}{K} \sum_{k=1}^K \frac{\angle D_k[\hat{n}]}{2\pi kT}, \quad T \text{ being the symbol duration.}$$

The implementation of the time of arrival estimator has been done on figure 24 by placing the pilot at the 100th sample and by then plotting $\sum_{k=1}^K |D_k[n]|$. The peak at the 100th sample clearly indicates the time of arrival of the pilot and validates the implementation. A similar test was done for the CFO estimation but it is not shown here.

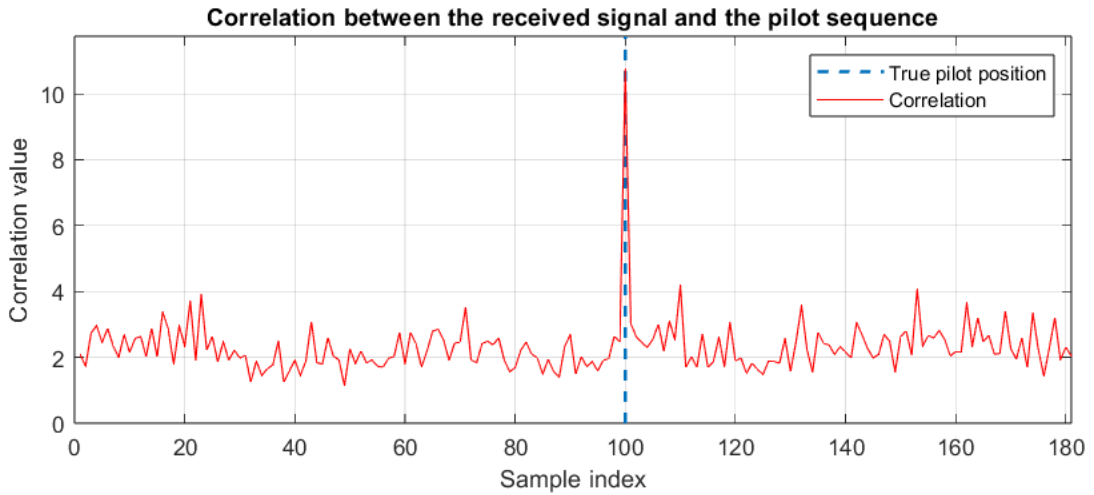


Figure 24: Time of arrival estimation ($N=20$, $K=8$)

The standard deviation of both estimators has been measured on 250 runs for both a varying pilot length N and a varying maximal shift K for an increasing $\frac{E_b}{N_0}$ in figures 25 and 26.

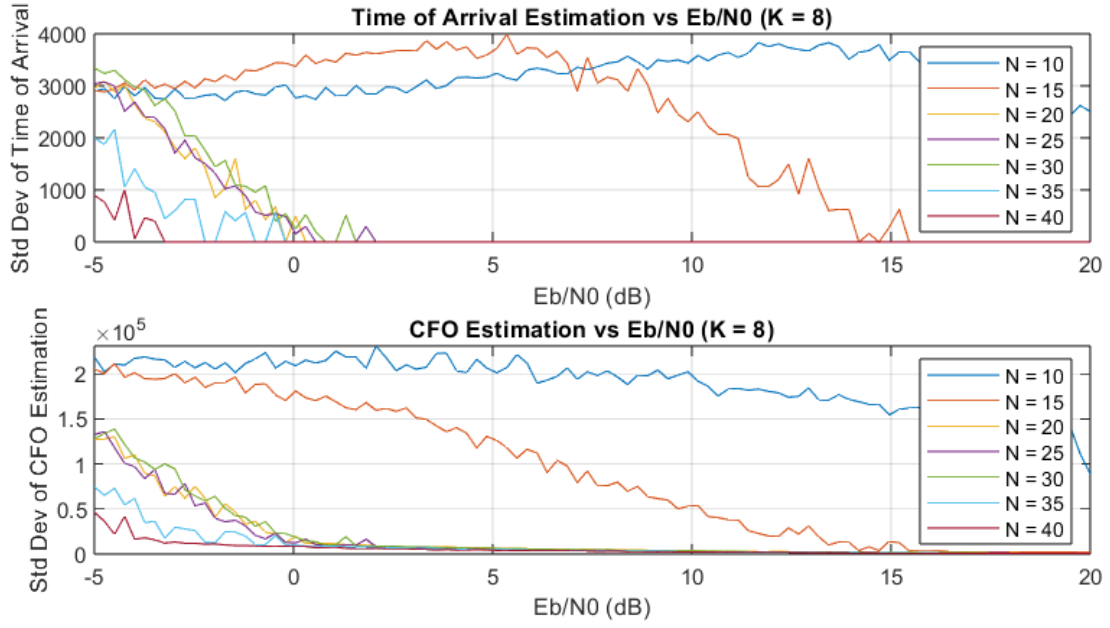


Figure 25: Standard deviation of the CFO estimator with varying pilot length

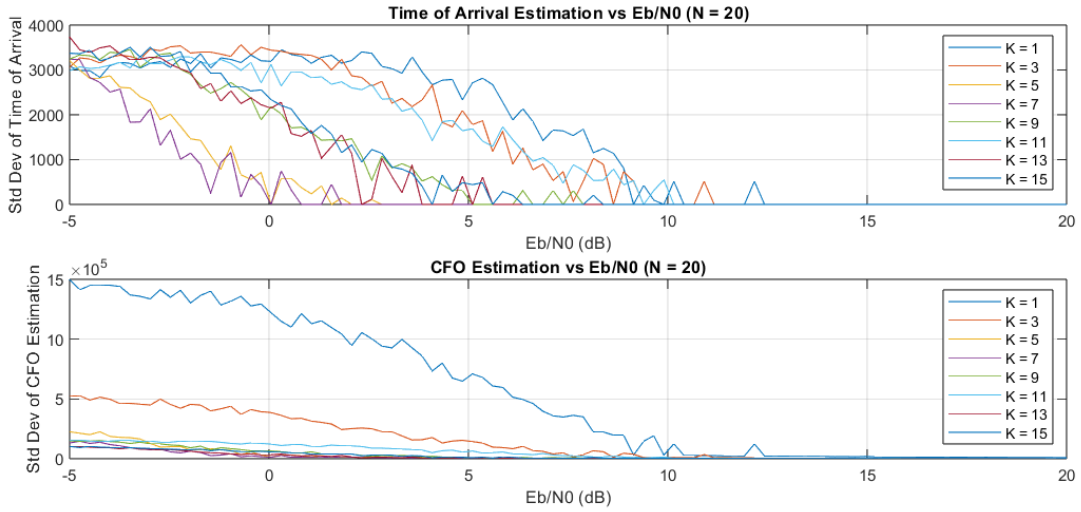


Figure 26: Standard deviation of the CFO estimator with varying maximal shift

As expected, a longer pilot and a larger shift will lower the uncertainty of the estimation and the same goes by for an increasing SNR. The fact that the line corresponding to $N = 10$ on figure 25 does not seem to decrease with $\frac{E_b}{N_0}$ is explained by the fact that N and K are too close to each other.

The robustness of the frame acquisition is also tested against a varying CFO by measuring the standard deviation of the time of arrival estimator for a varying CFO. The results are shown in figure 27 and it can be seen that the CFO has no impact on the time of arrival estimator. This is because the used algorithm is a differential cross-correlator and not a classical correlator.

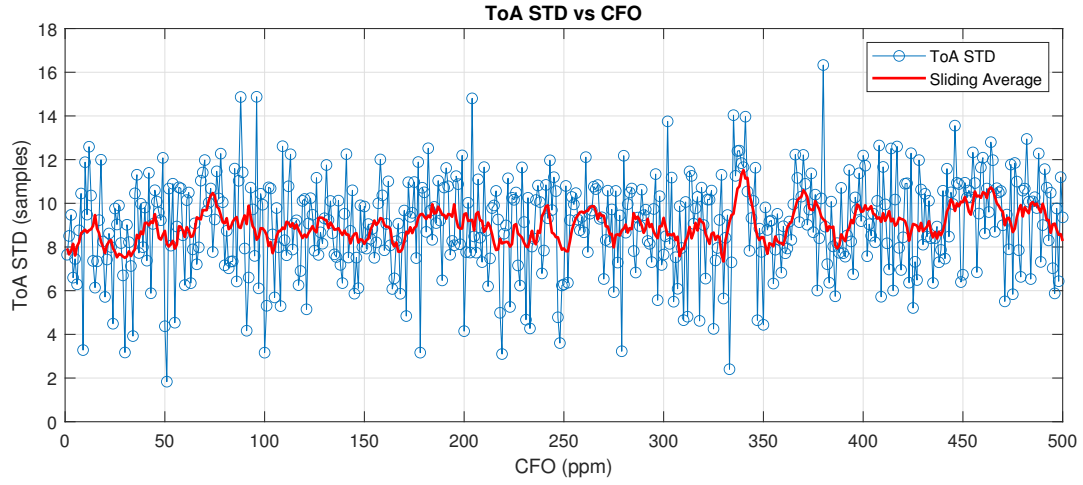


Figure 27: Standard deviation of the ToA estimator with varying CFO (N=20, K=8)

A similar test was done for the CFO estimator and the results are shown in figure 28. The CFO estimator is as robust as it is based on the time of arrival estimator, which was robust to CFO. Its estimation starts to get further away from the real CFO only for unrealistic values of the CFO. The standard deviation is pretty high, which means that a few pilots would be needed to average the estimation and have a value close to the real CFO.

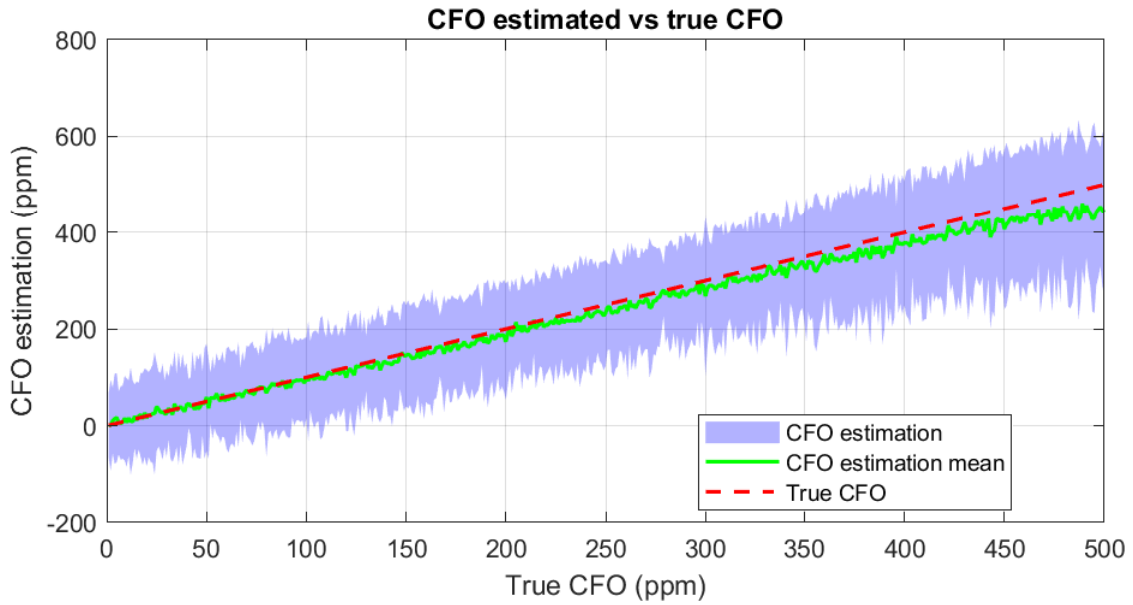


Figure 28: CFO estimation with varying CFO (N=20, K=8)

To avoid needing a lot of pilots, the same plot was done for a higher size of the pilot and a larger shift. The results are shown in figure 29. The standard deviation is now much lower and the estimation is still not affected by the CFO.

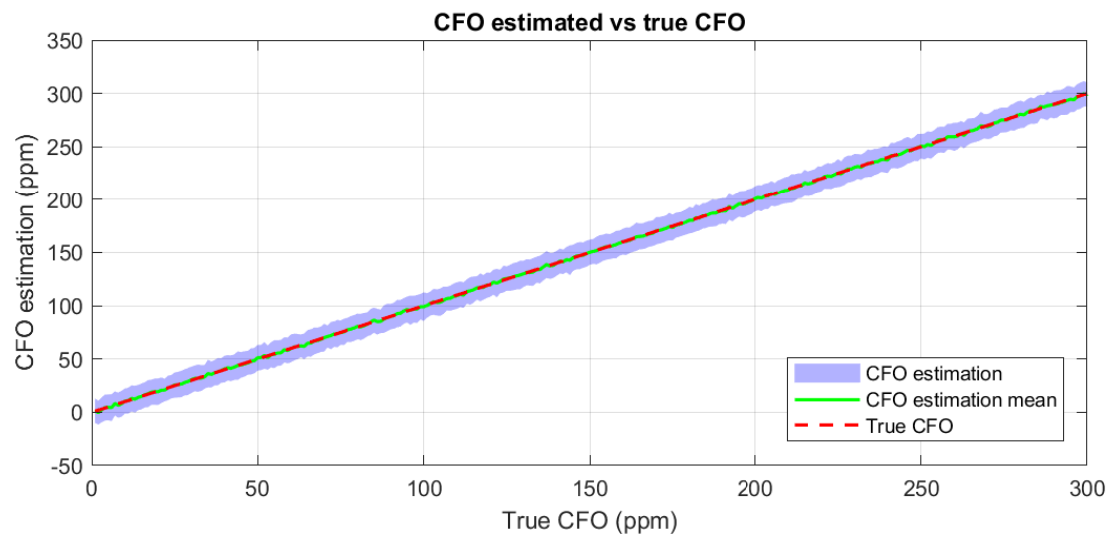


Figure 29: CFO estimation with varying CFO (N=50, K=12)

Structure

Now that every block of the transmission channel has been built and tested separately, the full channel can be simulated. It's structure is the following:

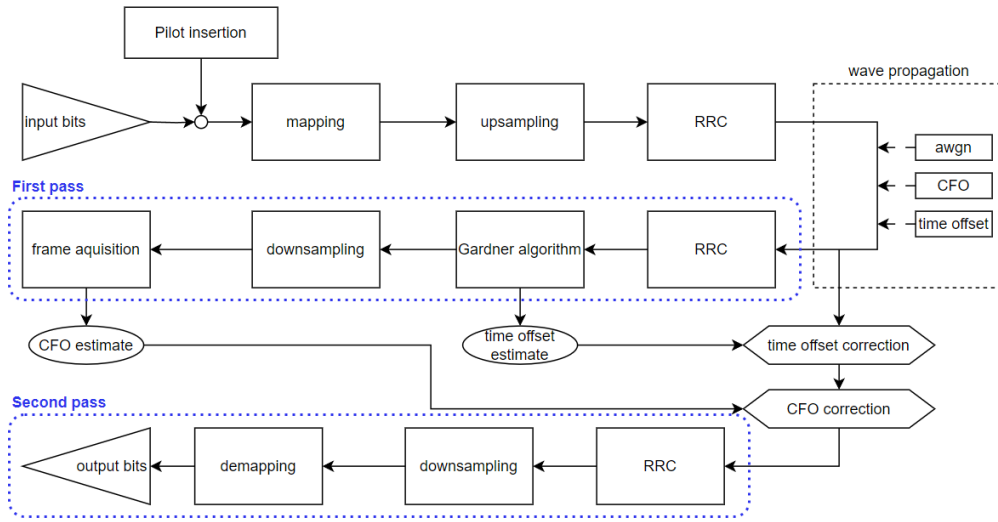


Figure 30: Full channel structure

Single pilot

The following figure shows the error between the input and the output bits of the channel and the position of the pilot:

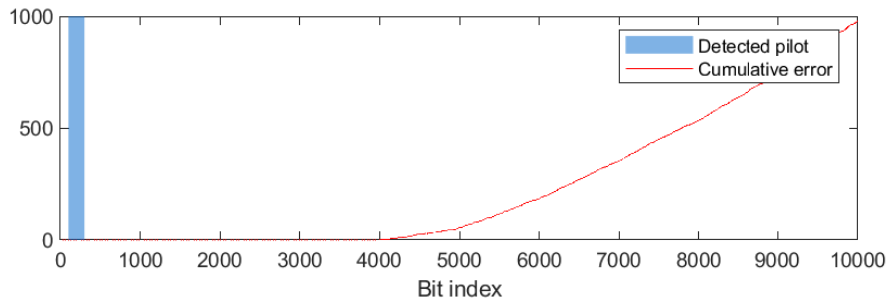


Figure 31: Error between input and output bits for a single pilot ($N = 50$, $K = 12$)

As expected, the error is very low close at the beginning of the packet. This proves that the Gardner

algorithm has successfully estimated the time offset and the CFO estimation was close to the real value. Errors start to appear after 4000 bits due to the linearly increasing phase added by the CFO. This simple example showed the importance of having multiple pilots in a packet.

Multiple pilots

As shown on figure 29, the standard deviation of the CFO estimation is lowered for a longer pilot but it still remains different from zero. This means that multiple pilots should be sent in order to have a better estimation of the CFO.

This is what has been done in the following simulation. A total of 10 pilots of $N = 50$ bits have been sent in a packet of 10.000 bits. The CFO is computed based on each pilot separately and the final estimation is the average of the individual estimations. The estimated CFO of the case in figure 32 is 3.098 ppm for an actual value of 3 ppm, which is quite close.

Because the channel is fully simulated, awgn is present and the E_b/N_0 is set to 10 dB.

Once again, most of the errors are located closer to the end of the packet because the phase shift due to uncorrected CFO is linearly increasing with the bit index.

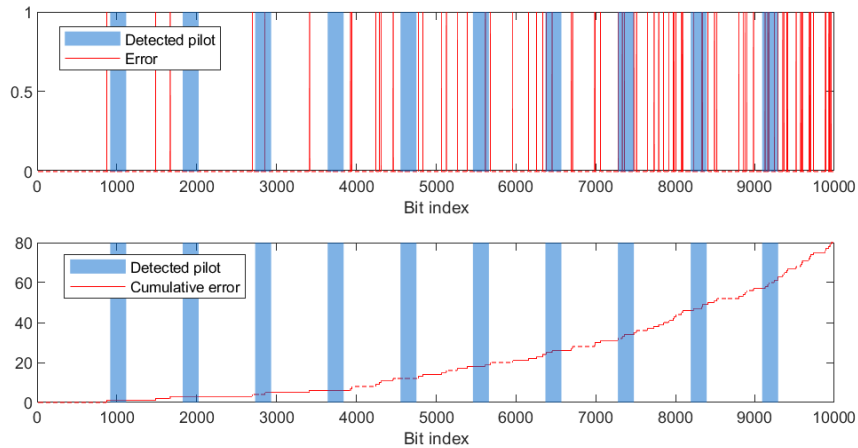


Figure 32: Error between input and output bits for multiple pilots ($N = 50$, $K = 30$)

For this same case, the constellation diagrams of different signals are shown in figure 33:

- The one in the top left corner is the input signal.
- Below it, the output signal if no CFO compensation and no time shift corrector are applied. It is chaotic and no information can be retrieved from it.
- The top right corner shows the output signal after the Gardner algorithm has been applied. The time shift is corrected but the CFO is still present. The constellation is now made of 2 circles, which already shows that the modulation used is QAM-16. Those circles are the result of the CFO (as shown previously in figure 13)

- The bottom right corner shows the output signal after the Gardner algorithm and the CFO compensation have been applied. The points are now grouped around the initial constellation points. They are not perfectly aligned because the CFO estimation is not perfect and because of the noise added to the signal.

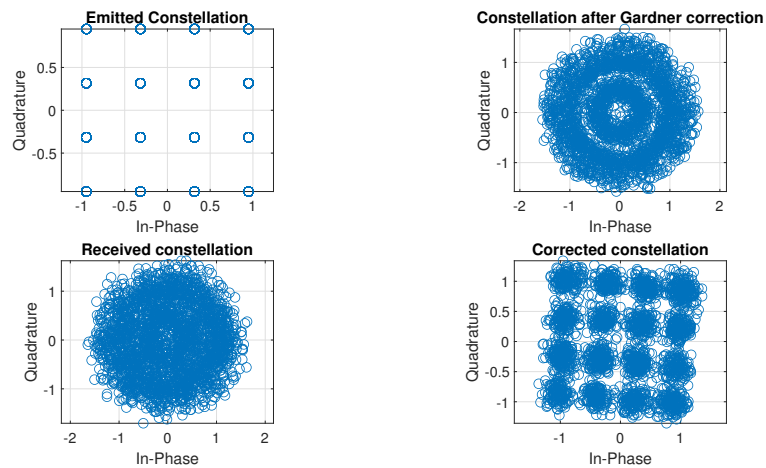


Figure 33: Constellation diagrams for multiple pilots ($N = 50$, $K = 30$)