ELEC-H415 Communication Channels

V2V communication project

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Introduction

TODO

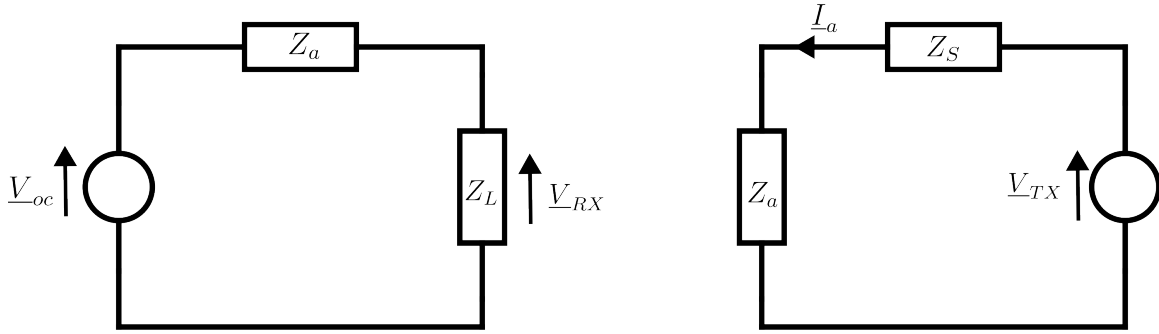## Theoretical answers

2.1   Step 1

Figure 2.1: Equivalent electric circuit of an antenna in RX (left) and TX (right)

Fig 2.1 shows the equivalent electrical circuit at RX and TX where $\underline{V}_{oc}$ is the induced voltage, $\underline{V}_{RX}$ the voltage at the output of the RX antenna, $\underline{V}_{TX}$ at the input of the TX antenna and $\underline{I}_a$ the current entering the TX antenna.

As both transmitting and receiving antenna are vertical $\lambda/2$ dipoles, their equivalent heights can be analytically computed:

$$\vec{h}_e(\theta, \phi) = \frac{\lambda}{\pi} \frac{\cos(\frac{1}{2} \cos \theta)}{\sin^2 \theta} \vec{1_z}$$
$$\vec{h}_{e\perp}(\theta, \phi) = -\frac{\lambda}{\pi} \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \vec{1_\theta}$$

As θ and ϕ are spherical coordinates, the horizontal plane (in which the simulation is done) corresponds to $\theta = \pi/2$. The transverse equivalent height is then reduced to hte following, where it does not depend on the azimuthal angle ϕ:

$$\vec{h}_{e\perp}(\phi) = -\frac{\lambda}{\pi}\vec{1}_\theta$$

The transverse part of the equivalent height gives rise to an expression for the emitted electric field.

$$
\begin{aligned}
\underline{\vec{E}} &= -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\vec{h}_{e\perp}(\theta,\phi) \\
&= j\omega\underline{I}_a\frac{\mu_0\lambda}{4\pi^2}\frac{e^{-j\beta r}}{r}\vec{1}_\theta \\
&= j\frac{\underline{I}_a\mu_0 c}{2\pi}\frac{e^{-j\beta r}}{r}\vec{1}_\theta \\
&= j\frac{\underline{I}_a Z_0}{2\pi}\frac{e^{-j\beta r}}{r}\vec{1}_\theta
\end{aligned}
$$

To make the transmission parameters appear in the electric field expression, β and $\underline{I}_a$ must be replaced with the formulas given below. Z_0, Z_a, Z_S can also be replaced by respectively 120π, $\frac{720\pi}{32}$ and $\frac{720\pi}{32}$ (assuming the impedances are matching).

$$\beta = \frac{2\pi f_c}{c}$$

$$\underline{V}_{TX} = (Z_a + Z_S)\underline{I}_a = \frac{720\pi}{16}\underline{I}_a$$

Wich yields

$$
\begin{aligned}
\underline{\vec{E}} &= j\frac{120\pi\underline{I}_a}{2\pi}\frac{e^{-\frac{j2\pi f_c r}{c}}}{r}\vec{1}_\theta \\
&= j\frac{4\underline{V}_{TX}}{3\pi}\frac{e^{-\frac{j2\pi f_c r}{c}}}{r}\vec{1}_\theta
\end{aligned}
$$

The last step is to replace the travel distance r with $c\tau$ as the wave is propagating at the speed of light in free space. The electric field thus becomes:

$$\underline{\vec{E}} = j\frac{4c\underline{V}_{TX}}{3\pi}\frac{e^{-j2\pi f_c \tau}}{\tau}\vec{1}_\theta$$

The voltage at the output of the antenna $\underline{V}_{RX}$ can be deduced from the equivalent electric circuit 2.1 as it is a simple voltage divider. Assuming a matching between the antenna and the load, we have:

$$\underline{V}_{RX} = \frac{Z_L}{Z_a + Z_L}\underline{V}_{oc} = \frac{1}{2}\underline{V}_{oc}$$

$$\underline{V}_{oc} = -\vec{h}_{e\perp}(\theta,\phi)\Big|_{\theta=\pi/2} \cdot \underline{\vec{E}}_i$$

Where $\underline{\vec{E}}_i = -\underline{\vec{E}}$ due to the change of coordinates origin. It is here assumed that there was no reflexion, refraction or transmission through another material. This results in:

$$\underline{V}_{oc} = \frac{\lambda}{\pi} \cdot \underline{E}_i$$

$$= -j\frac{4c\lambda\underline{V}_{TX}}{3\pi^2}\frac{e^{-j2\pi f_c\tau}}{\tau}$$

$$\underline{V}_{RX} = -j\frac{2c\lambda}{3\pi^2}\frac{e^{-j2\pi f_c\tau}}{\tau}\underline{V}_{TX}$$

2.2 Step 2

Assuming the communication takes place via a LOS ray only, the channel impulse response $h(\tau)$ is defined as follows:

$$h(\tau) = \frac{\alpha_1 e^{-j2\pi f_c\tau_1}}{d_1}\delta(\tau - \tau_1)$$

Where τ_1 is the propagation delay and d_1 the propagation distance of the direct ray. α_1 (which might be complex) takes into account a phase change or attenuation due for example to reflections whereas the imaginary exponential next to it corresponds to the phase change due to the propagation delay. In the case of LOS transmission α_1 is equal to 1.

The transfer function $H(f)$ of the channel is found by taking the Fourier transform of $h(\tau)$

$$H(f) = \int_{-\infty}^{\infty} h(\tau)e^{-j2\pi f\tau}d\tau$$

$$= \frac{e^{-j2\pi f_c\tau_1}}{d_1}e^{-j2\pi f\tau_1}$$

$$= \frac{e^{-j2\pi(f+f_c)\tau_1}}{d_1}$$

As we consider a single ray, the narrowband model of the channel h_{NB} (representing the case where the receiver perceives the sum of all propagation path) is simply found by removing the Dirac pulse from $h(\tau)$

$$h_{NB} = \frac{e^{-j2\pi f_c \tau_1}}{d_1}$$

The ratio between the received power P_{RX} and the transmitted power P_{TX} is found with:

$$\frac{P_{\text{received}}}{P_{\text{transmitted}}} = \frac{|h(\tau)|^2}{2}$$
$$= \frac{1}{d_1^2} \tag{2.1}$$

This result can be compared with the Friis formula, given by:

$$P_{RX}(d) = P_{TX} G_{TX}(\theta_{TX}, \phi_{TX}) G_{RX}(\theta_{RX}, \phi_{RX}) \left(\frac{\lambda}{4\pi d}\right)^2 \tag{2.2}$$

The reason for the big difference between the two formulas can be easily explained: in eq 2.1, $P_{\text{transmitted}}$ and P_{received} are the powers of the waves, not the one of the signal before/after passing through the antennas. To correct it, they are replaced by the Poynting vectors $\vec{\mathcal{S}}_{TX}$ and $\vec{\mathcal{S}}_{RX}$:

$$\frac{\left|\vec{\mathcal{S}}_{RX}\right|}{\left|\vec{\mathcal{S}}_{TX}\right|} = \frac{1}{d_1^2} \tag{2.3}$$

To compare the received and the injected power, the Poynting vectors should be replaced by

$$\left|\vec{\mathcal{S}}_{TX}\right| = G_{TX} P_{TX}$$
$$\left|\vec{\mathcal{S}}_{RX}\right| A_{eRX} = P_{RX}$$
$$\text{where} \qquad A_{eRX} = G_{RX} \left(\frac{\lambda}{4\pi}\right)^2$$

When placed back in eq 2.3, the expression matches the Friis formula (eq 2.2).

To further simplify As the antennas are considered to be lossless dipoles, equation 2.5 replaces their gain by $\frac{16}{3\pi}$, the theoretical gain of such antennas.

$$\frac{P_{RX}}{P_{TX}} = G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d_1}\right)^2 \tag{2.4}$$
$$= \frac{16}{9\pi^2} \left(\frac{\lambda}{\pi d_1}\right)^2 \tag{2.5}$$

Two deductions can be made of this result:

- The power loss on the channel depends on the square of the distance between the emitter and the receiver. It implies that doubling the range of an antenna would need a multiplication of the transmitting power by a factor 4.

- A more surprising result is the wavelength squared appearing at the numerator. It shows that there is always a compromise between data rate and power consumption as a larger wavelength will indeed be less attenuated but the maximal data rate will then be lowered in order for the channel to still be considered as narrowband.

2.3 Step 3

2.4 About the simulation

As the road is surrounded by buildings which are assumed to have thick walls, any ray going through a building is considered to be fully attenuated.