A Housing Deposit Mechanism for Monetary Policy Transmission

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Outline

- Motivation
- Research Question
- Simple Model
- Solution Method
- Complicated Model

Motivation

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Toronto

GTA home prices up 28% from last year as supply remains hampered, TRREB says

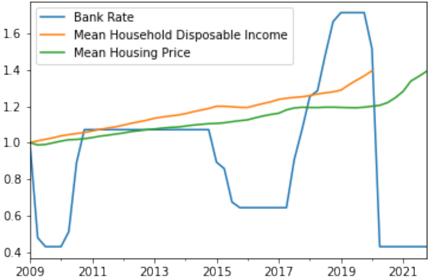
Average selling price for a home surpassed $1.3 million last month

Tara Deschamps · The Canadian Press · Posted: Mar 03, 2022 12:10 PM ET | Last Updated: March 3

Source: (Deschamps, 2022)

Motivation

Average housing prices in Canada have increased 16% since the beginning of the pandemic. This period also coincided with aggressive monetary and fiscal stimulus.

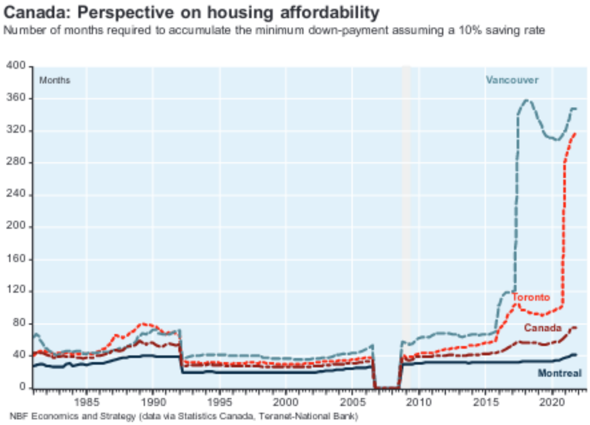


Note: Values shown are indices (Q1 2009 = 1).
Source: ("Statistics Canada: Canada's National Statistical Agency", 2022)

Motivation

Source: (Dahms & Duchame, 2022)

Motivation



Source: (Dahms & Duchame, 2022)

Question

- What effect does monetary policy have on the affordability of housing and in particular, the affordability of housing deposits?
- Can housing affordability feed back into the effectiveness of monetary policy action in the future?

Model

- I will consider a model where a minimum down-payment must be made in order to enter the housing market.
  - Can interpret this as meaning there is a minimum size of house that is available for purchase.
- Results in two types of agents: homeowners and renters.
- Will use an OLG structure in order to have interesting transition between the two types.
- First start with a very simple model: endowment economy, two periods, supply of housing and consumption goods fixed at 1 (partial equilibrium).

Setup

- Agents live for two periods: $t = 0, 1$ and in each period a unit mass of agents are born.
- In period 0 agents obtain an endowment drawn from $U(0, 1)$. In period 1 their only income is their savings from period 0.
- Renters can save only cash and are thus subject to uninsurable inflation risk. On the other hand homeowners have access to saving in both housing and bonds. Therefore, being a homeowner is a strictly better state for an agent.
- No arbitrage implies that the return on housing is equal to the return on bonds $(1 + i_t)$. It is assumed that in equilibrium the homeowners actually hold no bonds.
- Agents become homeowners if their endowment is greater than the minimum down-payment ϕp_t^a. Where ϕ is a constant and p_t^a is the housing price.

Renter Problem

$$
\max_{c^r_{t|t}, c^r_{t+1|t}, h^r_{t|t}, h^r_{t+1|t}, \omega_{t+1|t}} ln(c^r_{t|t}) + ln(h^r_{t|t}) + \beta[ln(c^r_{t+1|t}) + ln(h^r_{t+1|t})]
$$

$$
\text{s.t. } \omega_{t|t} = c^r_{t|t} + r^h_t h^r_{t|t} + \omega_{t+1|t}
$$

$$
\omega_{t+1|t} = \mathbb{E}_t(p_{t+1}) c^r_{t+1|t} + \mathbb{E}_t(r^h_{t+1}) h^r_{t+1|t} \tag{1}
$$

Homeowner Problem

$$\max_{c^o_{t|t}, c^o_{t+1|t}, h_{t|t}, h_{t+1|t}, a_{t|t}, b_{t|t}} ln(c^o_{t|t}) + ln(h_{t|t}) + \beta[ln(c^o_{t+1|t}) + ln(h_{t+1|t})]$$

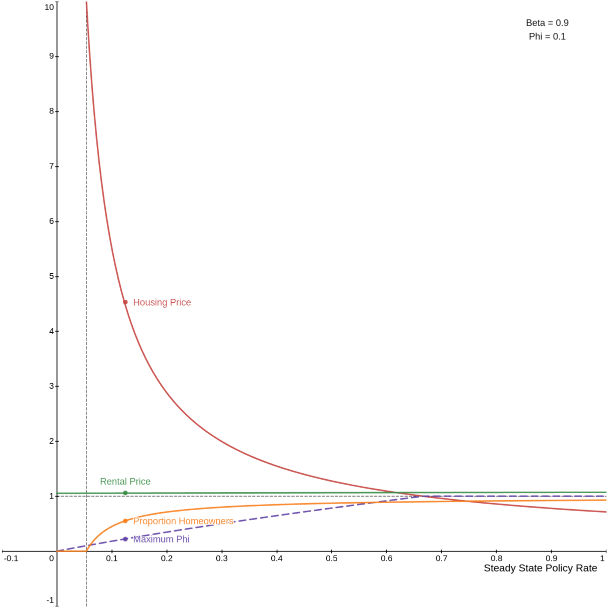
$$\text{s.t. } \omega_{t|t} + r^a_t(a_{t|t} - h_{t|t}) = c^o_{t|t} + p^a_t(1 + i_t)a_{t|t} + b_{t|t}$$

$$(1 + i_t)b_{t|t} + \mathbb{E}_t(p^a_{t+1})a_{t|t} = \mathbb{E}_t(p_{t+1})c^o_{t+1|t} + \mathbb{E}_t(r^a_{t+1})h_{t+1|t} \tag{2}$$

Results

$$
\begin{aligned}
\bar{p}^a =& \frac{1+\bar{i}}{\bar{i}(1+\beta(1+\bar{i}))} && (3) \\
\hat{c}_{t+1} =& (1-\frac{1}{2}\phi\bar{p}^a)(1+\hat{i}-\mathbb{E}_t(\pi_{t+1}))\hat{c}^o_{t|t} \\
&+\frac{\beta\phi\bar{p}^a}{2(1+\beta)}(1-\mathbb{E}_t(\pi_{t+1}))\hat{c}^r_{t|t} \\
&+\frac{1-\frac{1}{2}\phi\bar{p}^a}{1+\beta(1+\bar{i})}\hat{c}^o_{t+1|t+1} \\
&+\frac{\phi\bar{p}^a}{2(1+\beta)}\hat{c}^r_{t+1|t+1} && (4)
\end{aligned}
$$

Results



Beta = 0.9
Phi = 0.1
Housing Price
Rental Price
Proportion Homeowners
Maximum Phi
Steady State Policy Rate
10
9
8
7
6
5
4
3
2
1
-1
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Interpretation

- Interest rates are related to the cost of homeownership.
- Therefore, lower (steady-state) rates raise demand for housing.
- However, since supply is fixed, this translates directly into increased housing prices.
- Due to minimum down-payment higher prices result in lower homeownership.
- Since homeowners are directly affected by monetary policy, and renters are not, lower steady state interest rates result in a weaker response to monetary policy shocks via IS equation.

Problem

- This analysis is only partial equilibrium. In order to see how shocks propagate through the system we need to move to general equilibrium framework.
- Unfortunately, we already have to leave the analytically tractable world at this point, even in steady state.
- In order for supply to respond to demand (IS equation) there has to be endogenous labour supply. But then the agents could manipulate their labour supply to affect their type $\implies$ heterogeneous and non-linear response.
- Because of the high-dimensional and non-linear nature of this problem, traditional simulation techniques may perform poorly (Han et al., 2021).

Solution

Machine Learning!

- I will use the approach of Maliar et al. (2021) and Azinovic et al. (2019) in order to estimate the equilibrium policy functions in a much more complicated (and realistic) version of the baseline model.
- Use neural networks because they are known to be useful for estimating non-linear functions in high-dimensional spaces. Neural networks are known to be able to overcome the curse of dimensionality (Shen et al., 2021).
- Usual downsides (overfitting, interpretability) are less relevant; in this context we are only interested in functional approximation. Indeed, sufficiently large neural networks can approximate any (measurable) function (Hornik et al., 1989).

Machine Learning Solution Method

- Abstractly, the process of estimating a macroeconomic equilibrium can be thought of as the process of estimating the optimal policy functions (control variables) (and in some cases the value function) of agents conditional on (1) states, (2) shocks, and (3) expectations.
- Once policy functions are known the state transition is implied by the resource constraints of the economy. Using this we can observe how shocks propagate (e.g. impulse response functions).
- Instead of perturbation or projection I will parameterize the policy functions as a neural network:

$$\psi(X_t, \epsilon_t; \theta) = \sigma_k(...\sigma_1(W_1\sigma_0(W_0 [X_t, \epsilon_t]^T + b_0) + b_1)... + b_k) \quad (5)$$

- Where σ_i are activation functions for each layer, and W_i and b_i are weight and bias parameters for each layer.

Estimation (In General)

- In order to estimate the parameters of the neural network start from some initial guess (θ_0), then iteratively improve the estimate of the parameters by moving the parameters some amount (α_t) against the direction of the gradient. This process is known as Stochastic Gradient Descent (SGD).
- In particular, we use the gradient of a "loss function" (L) which indicates the quality of our estimate.
- In each step we use a different randomly selected sample of data known as a "batch" (b). This is what makes the process "stochastic".

$$\theta_{t+1} = \theta_t - \alpha_t \nabla_\theta \left[\frac{1}{|b|} \sum_{i \in b} L(X_i, \epsilon_i; \theta_t) \right] \tag{6}$$

- Calculation of the gradient is feasible due to a computational technique known as *back-propogation*.

Estimation (Details)

- When solving macroeconomic models the loss is a sum of the (squared) deviations from the optimality conditions of the economic model (first order conditions, Euler equations, Bellman equations, etc).
  - If the first order condition is $U_c(c_t, ...) = \lambda_t$ then the corresponding component of the loss would be $(U_c(\hat{c}_t, ...) - \hat{\lambda}_t)^2$
  - When dealing with a value function we want the LHS and RHS of the Bellman equation to be similar, so we add the penalty: $\left(\frac{U(.)+V_1(.)}{V_0(.)} - 1\right)^2$
- When the optimal choices involve expectations we integrate over the distribution of shocks by replacing the square with the product of the loss components for two independent draws of future shocks (Maliar et al., 2021). By eliminating the covariance term in this way (orthogonality) we maintain the unbiasedness of the batch loss.

Estimation (More Details)

- In high dimensional state spaces it is impractical to draw states randomly as it would take too long for the sampled states to adequately cover the entire state space. This is one cause of the curse of dimensionality (Bellman, 1961).
- Instead, we can use the state transition implied by our current model estimate to sample from the *ergodic set* of the equilibrium. This has the effect of estimating the parameters exactly on the manifold that the states will lie on in equilibrium, which is usually dramatically smaller than the entire state-space (Judd et al., 2011).
- Therefore, the estimation procedure is as follows:
  1. Begin with some initial parameter guess (θ_0) and random sample from the state space (X_0).
  2. Update parameter estimate to θ_1 via SGD over X_0.
  3. Update the sample of states to X_1 using the state transition X_0 implied by $\psi(X_0, \epsilon_0; \theta_0)$ and random draws from the distribution of ϵ. Iterate forward sufficiently that it is plausible that $X_1 \perp\!\!\!\perp X_0$.
  4. Repeat...

Model (Revisited)

- Currently I am working on estimating the following model using these ML techniques.
- Consumption goods supplied by firms in monopolistic competition with Calvo pricing (Calvo, 1983) as is standard in New Keynesian models. Supply of housing still fixed at 1.
- Overlapping generations of agents live for T periods and solve the following maximization problem in each period:

$$\begin{aligned} V(m_{t-1}, a_{t-1}, b_{t-1}, o_{t-1}, t) = & \max\{U(c_t, h_t, 1-n_t) \\ & +\beta \mathbf{1}(.) V(0, a_t, b_t, 1, t+1) \\ & +\beta(1-\mathbf{1}(.))V(m_t, 0, 0, 0, t+1)\} \end{aligned} \tag{7}$$

$$\begin{aligned} \text{s.t. } & c_t + p_t^a(a_t - a_{t-1}) + r_t^h h_t + b_t + m_t \\ & = r_t^h a_{t-1} + \frac{1}{1+\pi_t}((1+i_t)b_{t-1} + m_{t-1}) \end{aligned} \tag{8}$$

$$\mathbf{1}(.) = \mathbf{1}(\{m_{t-1} > \phi p_t^a\} \cup \{o_{t-1} = 1\}) \tag{9}$$

$$a_T = m_T = b_T = 0 \tag{10}$$

Next Steps

- Actually get estimation to produce sensible results for single agent case.
- Add general equilibrium features (production, overlapping generations, estimation of equilibrium prices, stochastic shocks).
- Add bells and whistles (e.g. cashflow constraint forces homeowners to liquidate, homeowners hold cash in equilibrium).
- Calibrate model to try and ascertain how important this mechanism is in reality.

Questions?

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