MIDDLE EAST TECHNICAL UNIVERSITY

CENG222 - Statistical Methods for Computer Engineering

Homework 2

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Contents

1 Q1. (40 pts.) **2**

1.1 a) Write the probability density function f(x). 2

1.2 b) Determine the mean, variance and standard deviation. . . . 2

1.3 c) What is the probability that the student will take between 90 and 120 minutes to finish the homework? 2

1.4 d) What is the probability that the student will take more than 150 minutes, given that he always takes more than 120 minutes to finish any CENG222 homework? 3

2 Q2. (40 pts.) **3**

2.1 a) Calculate the mean and standard deviation of CP 4

2.2 b) What is the probability that there are less than 8 CP supporters in this sample? 4

2.3 c) What is the probability that there are more than 15 CP supporters in this sample? 4

2.4 d) What is the probability that number of CP supporters will be between 7 and 14? ($7 \leq N \leq 14$) 5

3 Q3. (20 pts.) **5**

3.1 a) What is the probability that there will not be a strike within one year given that the building was hit today? 5

3.2 b) What is the probability that there will not be a strike within one year given that the building has not been hit in a year? . 6

1 Q1. (40 pts.)

The amount of time that it takes a student to complete a CENG222 homework is uniformly distributed between 60 and 180 minutes.

1.1 a) Write the probability density function f(x).

Since the distribution is uniform it means that the time it takes for a CENG222 homework to be completed can take any value between 60-120 minutes with the same probability.

so since their probability is the same and the probability of it taking any value is 1 (P$\{60 \leq X \leq 180\}$=1) we can easily say that P(X=a) for a an arbitrary number between 60-180 is 1/(180-60)=1/120 so:

$$f(x) = 1/120 \quad for\ 60 \leq x \leq 180$$
$$f(x) = 0 \quad otherwise$$

1.2 b) Determine the mean, variance and standard deviation.

so for any continues distribution its mean is $\int_a^b x.f(x)\,dx$ which means:

$$E(x) = \int_{60}^{180} x\frac{1}{120}\,dx = \frac{x^2}{2}\frac{1}{120}\Big|_{60}^{180} = \frac{180*180-60*60}{240} = \frac{240*120}{240} = 120$$

or we could have done it like this

$$for\ b = 180, a = 60$$

$$E(x) = \int_a^b x\frac{1}{b-a}\,dx = \frac{x^2}{2}\frac{1}{b-a}\Big|_a^b = \frac{b^2-a^2}{2*(b-a)} = \frac{b+a}{2} = 120$$

1.3 c) What is the probability that the student will take between 90 and 120 minutes to finish the homework?

We can sat its $\frac{120-90}{180-60}$ but thats not fun so lets calculate it from the pdf(since pdf is derivative of cdf we are taking its integral between given values to find the probability between those limits. This workd for every continous distribution):

$$\int_{90}^{120} \frac{1}{120} = x\frac{1}{120}\Big|_{90}^{120} = \frac{30}{120}$$

1.4 d) What is the probability that the student will take more than 150 minutes, given that he always takes more than 120 minutes to finish any CENG222 homework?

The problem wants us to calculate PA—B for A being taking more than 150 minutes,B being taking more then 120 minutes and lastly PA—B being that the student will take more than 150 minutes, given that he always takes more than 120 minutes to finish any CENG222 homework.

We know that the dependant conditional probability of A given B is equal to probability of the event we want divided by the whole probability(in this case it is $\frac{P(A \cap B)}{P(B)}$):

$$\frac{P(X > 150 \cap X > 120)}{P(X > 120)} = \frac{P(X > 150)}{P(X > 120)} = \frac{P(180 \geq X > 150)}{P(180 \geq X > 120)} = \frac{\int_{150}^{180} \frac{1}{120}}{\int_{120}^{180} \frac{1}{120}} = \frac{30}{60} = \frac{1}{2}$$

Note: we took the integrals to 180 because after 180 pdf is 0

2 Q2. (40 pts.)

In a certain country, there are 400.000 registered voters. 2% of these voters support the "Communist Party". A random sample of 500 people are selected to conduct a poll.

Solve the following questions using Normal approximation to Binomial distribution.

Normal approximation to Binomial distribution uses:

$$Binomial(n, p) \approx Normal\Big(\mu = np, \sigma = \sqrt{npq}\Big)$$

Note: Normally Normal approximation to Binomial distribution can not be used here since p is less then 0.05.

2.1 a) Calculate the mean and standard deviation of CP

for a,b,c parts of the problem mean and standard deviation becomes:

$$\mu = 500 * 0.02 = 10$$

$$\sigma = \sqrt{500 * 0.02 * 0.98} = \sqrt{9.8} = 3.1304951685$$

2.2 b) What is the probability that there are less than 8 CP supporters in this sample?

the question becomes P(X<8) we need to first change it to adhere continuity correction(since 8 shouldn't be taken) so it becomes P(X<7.5). now we can calculate it:

$$P\{X < 7.5\} = P\{Z < \frac{7.5 - 10}{3.1304951685}\} = P\{Z < -0.79859570625\}$$

$$= \phi(-0.79859570625) = 0.21226$$

2.3 c) What is the probability that there are more than 15 CP supporters in this sample?

the question becomes P(X≤15) we need to first change it to adhere continuity correction(since 15 can be taken) so it becomes P(X≤15.5). now we can calculate it:

$$P\{X \leq 15.5\} = P\{Z < \frac{15.5 - 10}{3.1304951685}\} = P\{Z < 1.75691055375\}$$

$$= \phi(1.75691055375) = 0.960533$$

2.4 d) What is the probability that number of CP supporters will be between 7 and 14? ($7 \leq N \leq 14$)

The question is asked for the population not the sample. And we know that 0.02 of 400000 people are CP supporters which comes up to 8000. which mean probability of CP supporters being 8000 is P(X=8000)=1 and beause of that **P(7≤X≤14)=0**

But if the question was asked for the sample we would have calculated it like this:

The question becomes P(7≤X≤14) we need to first change it to adhere continuity correction(since 7 and 14 are part of it) so it becomes P(6.5<X<14.5). now we can calculate it:

$$P\{6.5 < X < 14.5\} = P\{\frac{6.5-10}{3.1304951685} < Z < \frac{14.5-10}{3.1304951685}\}$$

$$= P\{-1.11803398875 < Z < 1.43747227125\}$$

$$= \phi(1.43747227125) - \phi(-1.11803398875) = 0.92471 - 0.1317 = 0.79293$$

3 Q3. (20 pts.)

Let's say lightning strikes a skyscraper about once a year ($\lambda = 1$)

3.1 a) What is the probability that there will not be a strike within one year given that the building was hit today?

The question asks us the probability that there will not be a strike within one year P(X<1) or we can say it is the probablity of first event happening after this year P(T>1). Which is an exponential distribution(P(T>1)=1-P(T≤1)=1-F(1))

For exponential distribution, cdf is calculated by $1 - e^{-\lambda t}$ so the result of our problem will be:

$$1 - (1 - e^{-1*1}) = e^{-1} = 0.36787944117$$

3.2 b) What is the probability that there will not be a strike within one year given that the building has not been hit in a year?

It will be 0.36787944117, the same as part a because exponential distribution is memoryless. The events are not dependant on each other, an event is dependant on the fixed period of time(and λ of course).