

CENG 223

Discrete Computational Structures

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Homework 3

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Question 1

$$(2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110}) \bmod 11 \equiv ?$$

Since 11 is prime and does not divide 2,4,6,8 and 10, by fermat's little theorem:

$$(2^{11-1}) \bmod 11 \equiv 1$$

$$(4^{11-1}) \bmod 11 \equiv 1$$

$$(6^{11-1}) \bmod 11 \equiv 1$$

$$(8^{11-1}) \bmod 11 \equiv 1$$

$$(10^{11-1}) \bmod 11 \equiv 1$$

$$\begin{aligned} & (2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110}) \bmod 11 \\ \equiv & (2^{10 \cdot 2 + 2} + 4^{10 \cdot 4 + 4} + 6^{10 \cdot 6 + 6} + 8^{10 \cdot 8} + 10^{10 \cdot 11}) \bmod 11 \\ \equiv & (1^2 \cdot 2^2 + 1^4 \cdot 4^4 + 1^6 \cdot 6^6 + 1^8 + 1^{11}) \bmod 11 \\ \equiv & (4 + 256 + 46656 + 1 + 1) \bmod 11 \equiv (4 + 3 + 5 + 1 + 1) \bmod 11 \equiv (14) \bmod 11 \\ \equiv & 3 \end{aligned}$$

Question 2

$\gcd(5n+3, 7n+4)$		
Higher	Lower	Combination
$7n+4$	$5n+3$	$7n+4 = 1 \cdot (5n+3) + (2n+1)$
$5n+3$	$2n+1$	$5n+3 = 2 \cdot (2n+1) + (n+1)$
$2n+1$	$n+1$	$2n+1 = 1 \cdot (n+1) + n$
$n+1$	n	$n+1 = 1 \cdot (n) + 1$
n	1	$n = n \cdot (1)$
1	0	

By Euclid's algorithm $\gcd(5n + 3, 7n + 4) = 1$

Question 3

$$m^2 = n^2 + kx$$

For this problem we know that m,n and k are integer numbers and x is a prime number.

$$m^2 = n^2 + kx$$

Now we will extract n^2 from both sides

$$m^2 - n^2 = kx$$

by the definition of divisibility $x|m^2 - n^2$ is clear but to make it more clear

since $m^2 - n^2 = (m + n) * (m - n)$ we can change the above equation to:

$$(m + n) * (m - n) = kx$$

Now we will divide every side by x:

$$\frac{(m + n) * (m - n)}{x} = k$$

We know that m and n are integers which means m+n and m-n must also be integers.

we can easily see that $x|m^2 - n^2$ since k is also an integer

since x is prime it can not be written as multiplication of 2 prime or composite numbers

because of that either m+n or m-n must be divisible to x(proof of this is below). Otherwise k wouldnt be and integer.

Proof:

Firstly dont forget x is prime.

We will prove that if $x|(m+n)(m-n)$ than $(x|(m+n)) \vee (x|(m-n))$

suppose that $\gcd(x, (m+n))=1$ (if its not then x divides (m+n) since x is prime and we proved $((x|(m+n)) \vee (x|(m-n)))$)

By Bezout's Lemma there are integers a,b such that $xa+(m+n)b=1$

$$xa+(m+n)b=1$$

$$(m-n)xa+(m+n)(m-n)b=(m-n)$$

$(m-n)xa$ has x so it is divisible by x

$(m+n)(m-n)b$ has $(m+n)(m-n)$ so it is divisible by x

$(m-n)=(m-n)xa+(m+n)(m-n)b$ both of them are divisible by x so $(m-n)$ is divisible by x

and my proof is complete

Question 4

I will show that for all n such that $n \geq 1$ the following is true:

$$1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n - 1)}{2}$$

Note: I will use P(k) as the validity of above equality for an arbitrary integer $k \geq 1$

Base Step:

for $n = 1$

$$1 = 1 * (3 - 1) / 2 = 1$$

so it is valid for our base case $n=1$

Inductive Step: Assume that P(k) is valid for an arbitrary integer $k \leq 1$

now we will prove $1+4+7\dots+(3k-2)+(3(k+1)-2)=\frac{(k+1)(3(k+1)-1)}{2}$ is valid (P(k+1))i:

$$1+4+7\ldots+(3k-2)+(3(k+1)-2)$$

$$=(1+4+7+\ldots(3k-2))+(3(k+1)-2)$$

Since we assumed that $P(k)$ is valid $(1+4+7+\ldots(3k-2)) = \frac{k(3k-1)}{2}$ so using that

$$= \frac{k(3k-1)}{2} + 3k + 1$$

$$= \frac{k(3k-1) + 6k + 2}{2}$$

$$= \frac{k(3k-1) + (3k-1) + 3 * (k+1)}{2}$$

$$= \frac{(k+1)(3k-1) + 3 * (k+1)}{2}$$

$$= \frac{(k+1)(3k+2)}{2}$$

$$= \frac{(k+1)(3(k+1)-1)}{2}$$

From these we saw that if $P(k)$ is valid $P(k+1)$ must also be valid
This completes the inductive step

By mathematical induction $1+4+7+\ldots+(3n-2) = \frac{n(3n-1)}{2}$ is true for all integers $n \geq 1$