Middle East Technical University — Department of Computer Engineering

CENG 223

Discrete Computational Structures

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Homework 3

Student Name and Surname: Ersel Hengirmen
Student Number: 2468015

Question 1

$(2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110})\text{mod } 11 \equiv ?$
Since 11 is prime and does not divide 2,4,6,8 and 10, by fermat's little theorem:
$(2^{11-1})\text{mod } 1 \equiv 1$
$(4^{11-1})\text{mod } 11 \equiv 1$
$(6^{11-1})\text{mod } 11 \equiv 1$
$(8^{11-1})\text{mod } 11 \equiv 1$
$(10^{11-1})\text{mod } 11 \equiv 1$

$$\begin{aligned}
&(2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110})\text{mod } 11\\
&\equiv (2^{10*2+2} + 4^{10*4+4} + 6^{10*6+6} + 8^{10*8} + 10^{10*11})\text{mod } 11\\
&\equiv (1^2 * 2^2 + 1^4 * 4^4 + 1^6 * 6^6 + 1^8 + 1^{11})\text{mod } 11\\
&\equiv (4 + 256 + 46656 + 1 + 1)\text{mod } 11 \equiv (4 + 3 + 5 + 1 + 1)\text{mod } 11 \equiv (14)\text{mod } 11\\
&\equiv 3
\end{aligned}$$

Question 2

	gcd(5n+3,7n+4)	
Higher	**Lower**	**Combination**
7n+4	5n+3	7n+4=1*(5n+3)+(2n+1)
5n+3	2n+1	5n+3=2*(2n+1)+(n+1)
2n+1	n+1	2n+1=1*(n+1)+n
n+1	n	n+1=1*(n)+1
n	1	n=n*(1)
1	0	

By Euclid's algorithm gcd(5n + 3, 7n + 4)=1

Question 3

$m^2 = n^2 + kx$
For this problem we know that m,n and k are integer numbers and x is a prime number.
$m^2 = n^2 + kx$

Now we will extract n^2 from both sides
$m^2 - n^2 = kx$

by the definition of divisibility $x|m^2 - n^2$ is clear but to make it more clear
since $m^2 - n^2 = (m + n) * (m - n)$ we can change the above equation to:
$(m + n) * (m - n) = kx$

Now we will divide every side by x:
$\frac{(m + n) * (m - n)}{x} = k$

We know that m and n are integers which means m+n and m-n must also be integers.
we can easily see that $x|m^2 - n^2$ since k is also an integer
since x is prime it can not be written as multiplication of 2 prime or composite numbers
because of that either m+n or m-n must be divisible to x. Otherwise k wouldnt be and integer.

Question 4

I will show that for all n such that n $\geq$ 1 the following is true:
$1 + 4 + 7 + ... + (3n - 2) = \frac{n(3n - 1)}{2}$
Note: I will use P(k) as the validity of above equality for an arbitrary integer k$\geq$1
Base Step:
for n =1
1=1*(3-1)/2=1
so it is valid for our base case n=1

Inductive Step: Assume that P(k) is valid for an arbitrary integer k$\leq$1
now we will prove 1+4+7...+(3k-2)+(3(k+1)-2)=$\frac{(k + 1)(3(k + 1) - 1)}{2}$ is valid (P(k+1))i:

1+4+7...+(3k-2)+(3(k+1)-2)
=(1+4+7+.....(3k-2))+(3(k+1)-2)
Since we assumed that P(k) is valid $(1 + 4 + 7 + ...(3k - 2)) = \frac{k(3k - 1)}{2}$ so using that

$= \frac{k(3k - 1)}{2} + 3k + 1$

$= \frac{k(3k - 1) + 6k + 2}{2}$

$= \frac{k(3k - 1) + (3k - 1) + 3 * (k + 1)}{2}$

$= \frac{(k + 1)(3k - 1) + 3 * (k + 1)}{2}$

$$= \frac{(k+1)(3k+2)}{2}$$
$$= \frac{(k+1)(3(k+1)-1)}{2}$$
From these we saw that if P(k) is valid P(k+1) must also be valid
This completes the inductive step

By mathematical induction $1+4+7+...+(3n-2) = \frac{n(3n-1)}{2}$ is true for all integers n≥1