CW (Maszyna Boltzmanna, MB)

Algorytm MB ($n = 25$ w zadaniu poniżej podanym)

Ustalić stałą tempuraturę $T > 0$.

<u>Warunek początkowy $x(0) = (x_1(0), \ldots, x_n(0))$</u>

$t = 0$, $i = 1, \ldots, n$

$$x_i(0) = \begin{cases} 0 & \text{z prawdopodowieństwem } \frac{1}{2} \quad (\text{rand}()\%2 = 0) \\ 1 & \text{z prawdopodowieństwem } \frac{1}{2} \quad (\text{rand}()\%2 = 1) \end{cases}$$

<u>Przejście z $x(t) = (x_1(t), \ldots, x_n(t))$ na $x(t+1) = (x_1(t+1), \ldots, x_n(t+1))$</u>

$t = 0, 1, 2, \ldots,\quad i = 1, \ldots, n$
Dla każdego $i = 1, \ldots, n$, losować $\beta_i \in [0, 1]$ z jednostajnym prawdopodobieństwem:
np. $\beta_i = \text{rand}()/\text{Rand\_Max}$, $\beta_i = (\text{rand}()\%N)/(N-1)$ $(N = 100, 1000, \ldots)$.

$$x_i(t+1) = \begin{cases} 1 & \text{gdy } 0 \le \beta_i \le f(u_i(t)) \\ 0 & \text{gdy } f(u_i(t)) \le \beta_i \le 1 \end{cases}$$

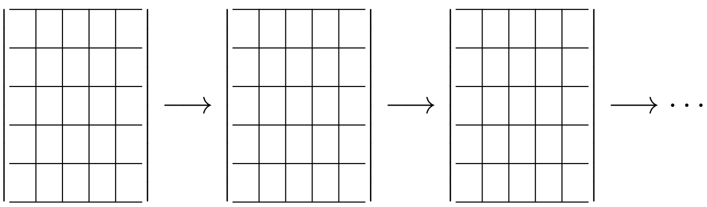
gdzie

$$u_i(t) = \left\{ \sum_{j=1}^{n} w_{ij} x_j(t) \right\} - \theta_i,$$

$$f(u_i(t)) = \frac{1}{1 + e^{-\frac{u_i(t)}{T}}}.$$

Zadanie. Implementować algorytm MB i wyświetlić $x(t) = (x_1(t), \ldots, x_n(t))$ $(t = 0, 1, 2, \ldots)$.

$$x(0) \longrightarrow x(1) \longrightarrow x(2) \longrightarrow \cdots$$



• Stałe parametry w_{ij} są podane następująco.
$\square = 0.0$, $\blacksquare = 1.0$

$$z = (z_i)_{i=1}^{25} = (z_1, \ldots, z_{25}) = \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & \blacksquare & \blacksquare & \square & \square \\ \hline \square & \square & \blacksquare & \square & \square \\ \hline \square & \square & \blacksquare & \square & \square \\ \hline \square & \square & \blacksquare & \square & \square \\ \hline \end{array} \in \mathbb{R}^{25}$$

$1 \le i, j \le 25$

$$c_{ij} = \begin{cases} (z_i - \frac{1}{2})(z_j - \frac{1}{2}) & \text{gdy } i \ne j \\ 0 & \text{gdy } i = j \end{cases}$$

$$w_{ij} = 2c_{ij}, \qquad \theta_i = \sum_{j=1}^{25} c_{ij}$$

Notacja. (Propozycja)
$x_i(t) \rightsquigarrow x[i],\ u_i(t) \rightsquigarrow u[i]$ (Nie prowadzić kroków t!)
$z_i \rightsquigarrow z[i]$
$c_{ij} \rightsquigarrow c[i][j]$
$w_{ij} \rightsquigarrow w[i][j],\ \theta_i \rightsquigarrow theta[i]$

Wskazówki do analizy dla Zadania.
(1) Próbować różne (stałe) temperatury T (wysokie, niskie) i zbadać zależność od tego parametra.
(2) Zauważmy, że

$$\operatorname*{argmin}_x E = \left|\begin{array}{c|c|c|c|c} \hline & & & & \\ \hline & \blacksquare & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline \end{array}\right| \quad \text{oraz} \quad \left|\begin{array}{c|c|c|c|c} \hline \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \hline \blacksquare & & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \end{array}\right| .$$

Sprawdzić czy obrazy

$$\left|\begin{array}{c|c|c|c|c} \hline & & & & \\ \hline & \blacksquare & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline \end{array}\right| \quad \text{i} \quad \left|\begin{array}{c|c|c|c|c} \hline \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \hline \blacksquare & & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \end{array}\right| .$$

pojawiają się w **jednym toku** dla pewnej (stałej) tempuratury.

$$\left|\begin{array}{c|c|c|c|c} \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array}\right| \longrightarrow \left|\begin{array}{c|c|c|c|c} \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array}\right| \longrightarrow \cdots \longrightarrow \left|\begin{array}{c|c|c|c|c} \hline & & & & \\ \hline & \blacksquare & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline & & \blacksquare & & \\ \hline \end{array}\right| \longrightarrow \cdots \longrightarrow \left|\begin{array}{c|c|c|c|c} \hline \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \hline \blacksquare & & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare & & \blacksquare & \blacksquare \\ \hline \end{array}\right| \longrightarrow \cdots$$