CW (CNN)

Konwolucja i "pooling" w CNN (Convolutional Neural Network)

Input (konwolucji) $\boldsymbol{u} = (u(i,j))_{i=1,j=1}^{N,N}, \quad u(i,j) \in \mathbb{R} \quad (1 \le i,j \le N)$
Output (konwolucji) $\boldsymbol{x} = (x(i,j))_{i=1,j=1}^{N,N}, \quad x(i,j) \in \mathbb{R} \quad (1 \le i,j \le N)$
$(N = 5)$

Jądro (*ang.* kernel) $\boldsymbol{w} = (w(i',j'))_{i'=-\infty,j'=-\infty}^{\infty,\infty}$
$w(i',j') \in \mathbb{R} \quad (-\infty < i',j' < \infty)$
Praktycznie weźmy $w(i',j') = 0$ gdy $(i',j') \notin \{-H,\dots,H\} \times \{-H,\dots,H\}$
$(H = 1)$

Konwolucja (splot) $\boldsymbol{u} \mapsto \boldsymbol{x} = \boldsymbol{w} * \boldsymbol{u}$
Konwolucja = ekstrakcja cech obrazów (feature extraction)

$$x(i,j) = f_1\Bigg(\sum_{i'=-\infty}^{\infty} \sum_{j'=-\infty}^{\infty} w(i',j') u(i-i', j-j') \Bigg)$$

f_1 funkcja progowa lub $f_1(x) = x$

Uwaga. $(i-i', j-j') \notin \{1,\dots,N\} \times \{1,\dots,N\} \Rightarrow u(i-i', j-j') = 0$
(Jak to implementować w Pythonie?)

"Pooling" $\boldsymbol{x} \mapsto \boldsymbol{y}$
Pooling (łączenie) = operacja próbkowania przestrzennego
Input ("pooling") $\boldsymbol{x} = (x(i,j))_{i=1,j=1}^{N,N}, \quad x(i,j) \in \mathbb{R} \quad (1 \le i,j \le N)$
$(N = 5)$
Output ("pooling") $\boldsymbol{y} = (y(i,j))_{i=1,j=1}^{N-K,N-K}, \quad y(i,j) \in \mathbb{R} \quad (1 \le i,j \le N-K)$

$$y(i,j) = f_2\Bigg(\frac{1}{(K+1)^2} \sum_{(p,q) \in P_{ij}} x(p,q) \Bigg) \quad (1 \le i,j \le N-K)$$

$P_{ij} = \{(i+k, j+\ell)\}_{k=0,\ell=0}^{K,K}$
f_2 funkcja progowa lub $f_2(x) = x$
$(N = 5, K = 3, N-K = 2)$

Dane dla zadań

$$\boldsymbol{u}_i = \begin{pmatrix} u(1,1) & u(1,2) & u(1,3) & u(1,4) & u(1,5) \\ u(2,1) & u(2,2) & u(2,3) & u(2,4) & u(2,5) \\ u(3,1) & u(3,2) & u(3,3) & u(3,4) & u(3,5) \\ u(4,1) & u(4,2) & u(4,3) & u(4,4) & u(4,5) \\ u(5,1) & u(5,2) & u(5,3) & u(5,4) & u(5,5) \end{pmatrix} \qquad (1 \le i \le 5)$$

$\blacksquare = 1 \in \mathbb{R}$ (float)
$0 \in \mathbb{R}$ (float)

$$\boldsymbol{u}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & \blacksquare & \blacksquare & 0 \\ 0 & \blacksquare & 0 & \blacksquare & 0 \\ 0 & \blacksquare & \blacksquare & \blacksquare & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{u}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & 0 & 0 \\ \blacksquare & 0 & \blacksquare & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{u}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & \blacksquare & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 \end{pmatrix} \quad \boldsymbol{u}_4 = \begin{pmatrix} 0 & 0 & \blacksquare & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & \blacksquare & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{u}_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \blacksquare & \blacksquare & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{w}_j = \begin{pmatrix} w(-1,-1) & w(-1,0) & w(-1,1) \\ w(0,-1) & w(0,0) & w(0,1) \\ w(1,-1) & w(1,0) & w(1,1) \end{pmatrix} \qquad (1 \le j \le 4)$$

$$\boldsymbol{w}_1 = \begin{matrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & 0 & 0 \\ \blacksquare & 0 & 0 \end{matrix} \quad \boldsymbol{w}_2 = \begin{matrix} 0 & 0 & \blacksquare \\ 0 & 0 & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{matrix} \quad \boldsymbol{w}_3 = \begin{matrix} \blacksquare & \blacksquare & 0 \\ 0 & \blacksquare & 0 \\ 0 & \blacksquare & 0 \end{matrix} \quad \boldsymbol{w}_4 = \begin{matrix} 0 & \blacksquare & 0 \\ 0 & \blacksquare & 0 \\ 0 & \blacksquare & 0 \end{matrix}$$

Zadania

(1) (Konwolucja) Niech $\boldsymbol{u} \mapsto \boldsymbol{x} = (x(i,j))_{i=1,j=1}^{5,5} = \boldsymbol{w} * \boldsymbol{u}$ będzie zdefiniowana wzorem

$$x(i,j) = f_1\Bigg(\sum_{i'=-1}^{1} \sum_{j'=-1}^{1} w(i',j')u(i-i',j-j') \Bigg)$$

z funkcją progową

$$f_1(x) = \begin{cases} 0 & \text{gdy } x < 0 \\ 1 & \text{gdy } x \ge 0. \end{cases}$$

Obliczyć i wyświetlić jako obraz $\boldsymbol{w}_j * \boldsymbol{u}_i$ $(1 \le i \le 5,\ 1 \le j \le 4)$.

(2) ("pooling") Niech $\boldsymbol{u} \mapsto \boldsymbol{x} = (x(i,j))_{i=1,j=1}^{5,5} = \boldsymbol{w} * \boldsymbol{u}$ będzie zdefiniowana wzorem

$$x(i,j) = f_1\Bigg(\sum_{i'=-1}^{1} \sum_{j'=-1}^{1} w(i',j')u(i-i',j-j') \Bigg)$$

z funkcją $f_1(x) = x$. Dla tego $\boldsymbol{x}$ zdefiniujemy "pooling" $\boldsymbol{y} = (y(i,j))_{i=1,j=1}^{2,2}$, $y(i,j) \in \mathbb{R}$ $(1 \le i,j \le 2)$ wzorem

$$y(i,j) = f_2\Bigg(\frac{1}{4^2} \sum_{(p,q)\in P_{ij}} x(p,q) \Bigg) \ (1 \le i,j \le 2)$$

z funkcją progową

$$f_2(x) = \begin{cases} 0 & \text{gdy } x < 0 \\ 1 & \text{gdy } x \ge 0 \end{cases}$$

i $P_{ij} = \{(i+k, j+\ell)\}_{k=0,\ell=0}^{3,3}$. Obliczyć i wyświetlić jako obraz "pooling" $\boldsymbol{y}$ dla $\boldsymbol{w}_j * \boldsymbol{u}_i$ $(1 \le i \le 5,\ 1 \le j \le 4)$.