Assignment 5: Time Series Models

ECO 321

DUE: Monday, May 14 in class

Instructions: You need to write a Stata do-file to answer question 2. Your do-file must produce a log file that contains your Stata output. You must submit your log file along with your answers to the assignment. Assignments that do not contain log files will be marked down.

1. We want to forecast the unemployment rate in the United States (UrateUS) using quarterly data from the first quarter of 1980 to the fourth quarter of 2016 ($T = 36$ **years**$\times$4 **quarters**= 144).

a) The following table presents the first four autocorrelations for the United States unemployment rate and its changes over the time period from Q1:1980 to Q4:2016. Explain briefly what the autocorrelations in the unemployment rate, Y_t, measure.

Table 1: **First Four Autocorrelations of the U.S. Unemployment Rate and Its Change, 1980:Q1 – 2016:Q4**

Lag	Unemployment Rate Y_t	Change in Unemployment Rate ΔY_t
1	0.97	0.62
2	0.92	0.32
3	0.83	0.12
4	0.75	-0.07

The first four autocorrelations in the unemployment rate measure how the unemployment rate is correlated with itself over time. All of the autocorrelations are positive, so the unemployment rate series exhibits positive autocorrelation. Positive autocorrelation means that if the series is above (below) its mean in the last period (first lag), then it will be above (below) its mean this period. The same can be said the for the value of the unemployment rate two, three, and four periods ago and the value today.

b) The accompanying table gives the United States unemployment rate for the period 2015:Q1-2016:Q1 in levels, and the lagged unemployment rate for 2015:Q1. Fill in the blanks for the missing lagged values and the changes in the unemployment rate.

Table 2: **Changes in Unemployment Rates in the U.S., Q1:2015 to Q1:2016**

Quarter	Unemployment Rate Y_t	First Lag	Change in Unemployment Rate ΔY_t
Q1:2015	4.3	4.4	-0.1
Q2:2015	4.3	4.3	0.0
Q3:2015	4.2	4.3	-0.1
Q4:2015	4.1	4.2	-0.1
Q1:2016	4.0	4.1	-0.1

(c) You decide to test whether the unemployment rate is nonstationary. The result is as follows:

$$\Delta \hat{UrateUS_t} = \underset{(0.020)}{-0.004} - \underset{(0.15)}{0.25} UrateUS_{t-1}$$

$$R^2 = 0.13 \quad SER = 0.55$$

Is the unemployment rate series nonstationary? Why or why not?

This regression represents the Dickey-Fuller test for nonstationarity. The null hypothesis is: $H_0 : \gamma_1 = 0$ against the alternative that $H_1 : \gamma_1 < 0$. The t-statistic on $\gamma_1 = \frac{-0.25}{0.15} = -1.6 > -2.86$, so we fail to reject the null hypothesis of nonstationarity. Therefore, we conclude that the unemployment rate series is nonstationary and violates one of our OLS assumptions for unbiased/consistent forecasts. $H_0 : \gamma_1 = 0$ = means non stat. we assume nonstat

(d) You decide to estimate an AR(1) model using changes (i.e., first-differences) in the US unemployment rate. The result is as follows:

$$\Delta \hat{UrateUS_t} = \underset{(0.022)}{-0.003} + \underset{(0.106)}{0.621} \Delta UrateUS_{t-1}$$

$$R^2 = 0.39 \quad SER = 0.255$$

If most of the forecast error comes from uncertainty about the future error terms instead of sampling error from estimating the coefficients in the AR(1) model, then what is your best guess of the RMSFE here? root mean square forecast error.

The best guess of the RMSFE is the $SER = 0.255$ when our sample size is large and/or sampling error associated with estimating β_0 and β_1 is very small.

(e) The actual unemployment rate during the fourth quarter of 2015 is 4.1 percent, and it decreased from the third quarter to the fourth quarter by 0.1 percent. Using the AR(1) model's results, what is your forecast for the unemployment rate level in the first quarter of 2016?

$\hat{\beta}_0$ $\hat{\beta}_1$ $\Delta \hat{Y}_t$ $\Delta \hat{Y}_{t+1|t}$

Forecasted Change$= -0.003 + 0.621(-0.10) = -0.003 - 0.0621 = -0.0624$

Since the unemployment rate was 4.1 percent in Q4:2015, the forecast for the unemployment rate in Q1:2016 is $4.1 - 0.0624 = 4.0386$.

The forecast error in Q1:2016 is $4.0 - 4.0386 = -0.0386$. amt of "error" we're off by.

actual − forecast

(f) You want to see how sensitive your forecast is to changes in the specification, so you consider an AR(4) model. The results are as follows:

$$\begin{aligned}
\Delta Ur\hat{a}teUS_t &= \underset{(0.022)}{-0.005} + \underset{(0.125)}{0.663}\Delta UrateUS_{t-1} - \underset{(0.139)}{0.082}\Delta UrateUS_{t-2} \\
&+ \underset{(0.117)}{0.106}\Delta UrateUS_{t-3} - \underset{(0.091)}{0.176}\Delta UrateUS_{t-4}
\end{aligned}$$

$$R^2 = 0.416 \quad SER = 0.253$$

What is your forecast for the unemployment rate level in 2016:Q1? Compare the forecast error of the AR(4) model with the forecast error of the AR(1) model.

Change in Forecast$= -0.005 + 0.663(-0.10) - 0.082(-0.10) + 0.106(0) - 0.176(-0.10) = -0.005 - 0.0663 + 0.0082 + 0.0176 = -0.0455$

forecast

Since the unemployment rate was 4.1 percent in Q4:2015, the forecast for the unemployment rate is $4.1 - 0.0455 = 4.0545$ in Q1:2016.

actual - forecast

The forecast error is $4.0 - 4.0545 = -0.0545$ in Q1:2016. The forecast error in the AR(4) model is larger than what we obtained from the AR(1) model.

(g) Given the information below, which model should you use for forecasting?

We choose the AR(p) model with the smallest BIC and AIC. We do not pay much attention to the R^2 because it will always be increasing as we add lags to the model. In this example, the BIC and AIC are minimized when $p = 1$. So we choose an AR(1) model.

smallest value in columns.

p	BIC	AIC	R^2
0	0.604	0.624	0.000
1	0.158	0.118	0.393
2	0.185	0.125	0.397
3	0.217	0.138	0.400
4	0.218	0.119	0.416
5	0.277	0.139	0.420

2. The data set macro.dta contains time series data for U.S. real aggregate personal consumption (*cons*), real gross domestic investment (*invest*), interest rates on 3 month Treasury Bills (*intrate*), real M1 money stock (*m1*), and real GDP (*gdp*) for 1947:Q1 - 1988:Q1. *cons*, *invest*, *m1* and *gdp* are all measured in billions of 1982 dollars.

a) Estimate autocorrelations for log(*cons*), log(*gdp*), log(*m1*) and *intrate* and their first differences. What can you say about the stationarity or non-stationarity of each series?

The autocorrelations for the variables in logs suggest that these variables exhibit nonstationarity because the $\hat{\beta}_1$s (i.e., first-order autocorrelations) are close to 1 and the 95% confidence intervals include 1 for 3 out of 4 of the variables. If we cannot reject the hypothesis that $\beta_1 = 1$ in an AR(1) model, then that suggests the series is nonstationary.

The autocorrelations for the variables in first differences suggest the first-differenced series might be stationary because the β_1s are not close to 1 and the 95% confidence intervals do not include 1 for any of the 4 variables. However, we cannot be sure that these variables are stationary until we use the Dickey-Fuller test.

b) Conduct the Dickey-Fuller test for unit roots in each of the variables listed in part (a). Use 8 lags in each of the Dickey-Fuller regressions. What do you conclude for each series?

We cannot reject the null hypothesis of nonstationarity for all 4 of the variables measured in logs (i.e., levels).

We can reject the null hypothesis of nonstationarity for all 4 of the variables measured in first differences.

c) The Keynesian consumption function is $\log(cons) = \beta_0 + \beta_1 \log(gdp) + u$. Estimate this regression as specified and in first-differenced form. Interpret the effect of *gdp* on *cons*. Does your interpretation change substantively in the two versions of the model (i.e., in levels vs. first differences)? Which estimates are more reliable?

Since the variables in levels exhibited nonstationarity, we would prefer the model that regresses the first difference in log(consumption) on the first difference in log(gdp). Interpreting the first differences model tells us that as real GDP increases by 1%, consumption increases by 0.37%.

d) The Keynesian money demand function is $\log(m1) = \beta_0 + \beta_1 \log(gdp) + \beta_2 intrate + u$. Estimate this regression as specified and in first-differenced form. Interpret the effects of *gdp* and *intrate* on *m1*. Does your interpretation change substantively in the two versions? Which estimates are more reliable?

Since the variables in logs exhibited nonstationarity, we would prefer the model that regresses the first difference in log(m1) on the first difference in log(gdp) and the first difference in intrate. Interpreting the first differences model tells us that as real GDP increases by 1%, the money supply increases by 0.33%. As the interest rate increases by 1%, the money supply decreases by 0.01%, but the effect is just barely statistically significant at the 5% level. The effect of the interest rate on the money supply decreases by a factor of 10 when using the first-differenced model.

(1)

```
-------------------------------------------------------------------------------
      name:  <unnamed>
       log:  /Users/jessicavanparys/Dropbox/Working_Docs/teaching/ECO321/Assignments/Assignmen
> t_5/vanparys_assignment5_solutions.log
  log type:  text
 opened on:  15 May 2018, 10:58:41

.
. *============================================================================
. *Set a delimiter that allows wrap-around text
. *============================================================================

.
. #delimit;
delimiter now ;
. *============================================================================;
. *============================================================================;
. *============================================================================;
. *Question 2;
. *============================================================================;
. *Open the macro.dta data set;
. *============================================================================;
. clear;

. use "/Users/jessicavanparys/Dropbox/Working_Docs/teaching/ECO321/Assignments/Assignment_5/ma
> cro.dta";

. su;

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
        cons |        165    1416.371    555.2339      658.1     2527.9
      invest |        165    385.4539    150.7757      160.2      741.8
     intrate |        165    4.885115    3.256603    .375667   15.08733
          m1 |        165    492.8857    43.68785   432.8333      634.8
     realgdp |        165    2304.193    843.1918     1012.4     4035.7
-------------+---------------------------------------------------------
        date |        165          30    47.77552        -52        112

. *============================================================================;
. *(a):
> *============================================================================;
. foreach x of varlist cons realgdp m1 intrate {;
  2. ge l`x'=log(`x');
  3. };

. sort date;

. foreach x of varlist cons realgdp m1 intrate {;
```

(2)

```
  2. ge lag_1_l`x'=l`x'[_n-1];
  3. };
(1 missing value generated)
(1 missing value generated)
(1 missing value generated)
(1 missing value generated)

. foreach x of varlist cons realgdp m1 intrate {;
  2. ge fd_l`x'=l`x'-lag_1_l`x';
  3. };
(1 missing value generated)
(1 missing value generated)
(1 missing value generated)
(1 missing value generated)

. foreach x of varlist cons realgdp m1 intrate {;
  2. ge fd_lag_l`x'=fd_l`x'[_n-1];
  3. };
(2 missing values generated)
(2 missing values generated)
(2 missing values generated)
(2 missing values generated)

. su;

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
        cons |        165    1416.371    555.2339      658.1     2527.9
      invest |        165    385.4539    150.7757      160.2      741.8
     intrate |        165    4.885115    3.256603    .375667   15.08733
          m1 |        165    492.8857    43.68785   432.8333      634.8
     realgdp |        165    2304.193    843.1918     1012.4     4035.7
-------------+---------------------------------------------------------
        date |        165          30    47.77552        -52        112
       lcons |        165    7.176085    .4062231   6.489357   7.835144
    lrealgdp |        165    7.671553    .3855437   6.920079   8.302935
         lm1 |        165    6.196584    .0850832   6.070353    6.45331
    lintrate |        165    1.332618    .7710472  -.9790521   2.713855
-------------+---------------------------------------------------------
 lag_1_lcons |        164    7.172066     .404164   6.489357   7.832292
lag_1_lrea~p |        164    7.667703      .38353   6.920079   8.298191
   lag_1_lm1 |        164    6.195051    .0830247   6.070353    6.45331
lag_1_lint~e |        164    1.330067    .7727101  -.9790521   2.713855
    fd_lcons |        164     .008206    .0081564  -.0207052   .0391784
-------------+---------------------------------------------------------
 fd_lrealgdp |        164     .008432    .0117228  -.0291042   .0439367
      fd_lm1 |        164    .0016832    .0124049  -.0493002    .041357
 fd_lintrate |        164    .0166463    .1545368  -.5982901   .6552023
fd_lag_lcons |        163    .0081996    .0081811  -.0207052   .0391784
```

(3)

```
fd_lag_lre~p |        163    .0084547    .0117553  -.0291042   .0439367
-------------+---------------------------------------------------------
  fd_lag_lm1 |        163    .0016838    .0124431  -.0493002    .041357
fd_lag_lin~e |        163    .0170023    .1549456  -.5982901   .6552023

. corrgram lcons, lag(1);

                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1        0.9829   0.9991   162.31  0.0000           |-------           |-------

. corrgram lrealgdp, lag(1);

                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1        0.9795   0.9960   161.21  0.0000           |-------           |-------

. corrgram lm1, lag(1);

                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1        0.9623   1.0169    155.6  0.0000           |-------           |--------

. corrgram lintrate, lag(1);

                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1        0.9515   0.9533   152.12  0.0000           |-------           |-------

. *Autocorrelations for logs;
. reg lcons lag_1_lcons;

      Source |       SS           df       MS      Number of obs   =       164
-------------+----------------------------------   F(1, 162)       >  99999.00
       Model |  26.5775264         1  26.5775264   Prob > F        =    0.0000
    Residual |  .010821885       162  .000066802   R-squared       =    0.9996
-------------+----------------------------------   Adj R-squared   =    0.9996
       Total |  26.5883483       163  .163118701   Root MSE        =    .00817

------------------------------------------------------------------------------
       lcons |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 lag_1_lcons |   .9990928    .001584   630.76   0.000      .995965    1.002221
       _cons |   .0147122   .0113781     1.29   0.198    -.0077564    .0371807
------------------------------------------------------------------------------
```

(4)

```
. reg lrealgdp lag_1_lrealgdp;

      Source |       SS           df       MS      Number of obs   =       164
-------------+----------------------------------   F(1, 162)       >  99999.00
       Model |   23.787425         1   23.787425   Prob > F        =    0.0000
    Residual |  .022025674       162  .000135961   R-squared       =    0.9991
-------------+----------------------------------   Adj R-squared   =    0.9991
       Total |  23.8094507       163   .14607025   Root MSE        =    .01166

--------------------------------------------------------------------------------
      lrealgdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
lag_1_lrealgdp |   .9960486   .0023813   418.28   0.000     .9913462    1.000751
         _cons |   .0387302   .0182818     2.12   0.036     .0026289    .0748315
--------------------------------------------------------------------------------

. reg lm1 lag_1_lm1;

      Source |       SS           df       MS      Number of obs   =       164
-------------+----------------------------------   F(1, 162)       =   7601.14
       Model |  1.16185283         1  1.16185283   Prob > F        =    0.0000
    Residual |  .024762111       162  .000152853   R-squared       =    0.9791
-------------+----------------------------------   Adj R-squared   =    0.9790
       Total |  1.18661494       163  .007279846   Root MSE        =    .01236

------------------------------------------------------------------------------
         lm1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   lag_1_lm1 |   1.016891   .0116637    87.18   0.000     .9938584    1.039923
       _cons |  -.1029566   .0722635    -1.42   0.156    -.2456564    .0397432
------------------------------------------------------------------------------

. reg lintrate lag_1_lintrate;

      Source |       SS           df       MS      Number of obs   =       164
-------------+----------------------------------   F(1, 162)       =   3893.11
       Model |  88.4435339         1  88.4435339   Prob > F        =    0.0000
    Residual |  3.68031323       162  .022717983   R-squared       =    0.9601
-------------+----------------------------------   Adj R-squared   =    0.9598
       Total |  92.1238471       163  .565176976   Root MSE        =    .15072

--------------------------------------------------------------------------------
      lintrate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
lag_1_lintrate |   .9532848   .0152783    62.39   0.000     .9231145     .983455
         _cons |   .0787807   .0234835     3.35   0.001     .0324075    .1251538
--------------------------------------------------------------------------------
```

(5)

```
. *Autocorrelations for first differences;
. reg fd_lcons fd_lag_lcons;

      Source |       SS           df       MS      Number of obs   =       163
-------------+----------------------------------   F(1, 161)       =      0.40
       Model |  .000026371         1  .000026371   Prob > F        =    0.5304
    Residual |  .010739173       161  .000066703   R-squared       =    0.0024
-------------+----------------------------------   Adj R-squared   =   -0.0037
       Total |  .010765544       162  .000066454   Root MSE        =    .00817

------------------------------------------------------------------------------
    fd_lcons |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
fd_lag_lcons |   .0493169    .078434     0.63   0.530    -.1055752    .2042089
       _cons |   .0077475   .0009071     8.54   0.000     .0059562    .0095389
------------------------------------------------------------------------------

. reg fd_lrealgdp fd_lag_lrealgdp;

      Source |       SS           df       MS      Number of obs   =       163
-------------+----------------------------------   F(1, 161)       =     36.81
       Model |  .004166353         1  .004166353   Prob > F        =    0.0000
    Residual |  .018225125       161    .0001132   R-squared       =    0.1861
-------------+----------------------------------   Adj R-squared   =    0.1810
       Total |  .022391478       162  .000138219   Root MSE        =    .01064

---------------------------------------------------------------------------------
    fd_lrealgdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+----------------------------------------------------------------
fd_lag_lrealgdp |   .4314062     .07111     6.07   0.000     .2909776    .5718349
          _cons |   .0048025   .0010276     4.67   0.000     .0027733    .0068318
---------------------------------------------------------------------------------

. reg fd_lm1 fd_lag_lm1;

      Source |       SS           df       MS      Number of obs   =       163
-------------+----------------------------------   F(1, 161)       =     82.66
       Model |  .008506427         1  .008506427   Prob > F        =    0.0000
    Residual |  .016568963       161  .000102913   R-squared       =    0.3392
-------------+----------------------------------   Adj R-squared   =    0.3351
       Total |   .02507539       162  .000154786   Root MSE        =    .01014

------------------------------------------------------------------------------
      fd_lm1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  fd_lag_lm1 |   .5823536   .0640542     9.09   0.000     .4558588    .7088485
       _cons |   .0006861   .0008019     0.86   0.393    -.0008974    .0022697
------------------------------------------------------------------------------
```

(6)

```
. reg fd_lintrate fd_lag_lintrate;

      Source |       SS           df       MS      Number of obs   =       163
-------------+----------------------------------   F(1, 161)       =     19.79
       Model |  .426018534         1  .426018534   Prob > F        =    0.0000
    Residual |  3.46643683       161  .021530664   R-squared       =    0.1094
-------------+----------------------------------   Adj R-squared   =    0.1039
       Total |  3.89245536       162  .024027502   Root MSE        =    .14673

---------------------------------------------------------------------------------
    fd_lintrate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+----------------------------------------------------------------
fd_lag_lintrate |   .3309615   .0744033     4.45   0.000     .1840294    .4778937
          _cons |   .0111159   .0115624     0.96   0.338    -.0117177    .0339495
---------------------------------------------------------------------------------

. *The autocorrelations for the variables in logs look like these time series exhibit
> nonstationarity because beta_1s are close to 1 and the 95% confidence intervals include
> 1 in 3 out of 4 cases;
. *The autocorrelations for the variables in first differences look stationary
> because the beta_1s are not close to 1 and the 95% confidence intervals do not
> include 1 in any of the 4 cases. However, we cannot be sure that these variables are
> stationary until we use the Dickey-Fuller test;
. *============================================================================;
. *(b):
> *============================================================================;
. *Dickey-Fuller test for log variables;
. dfuller lcons, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       156

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -0.900            -3.491            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.7878

------------------------------------------------------------------------------
     D.lcons |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lcons |
         L1. |  -.0015012   .0016671    -0.90   0.369    -.0047959    .0017936
         LD. |   .0479835   .0812157     0.59   0.556    -.1125267    .2084937
        L2D. |   .2179267   .0815516     2.67   0.008     .0567525    .3791009
        L3D. |   .0215164   .0835698     0.26   0.797    -.1436465    .1866793
        L4D. |  -.0570246   .0831788    -0.69   0.494    -.2214147    .1073655
        L5D. |   -.107475   .0838548    -1.28   0.202     -.273201     .058251
```

```
         L6D. |  -.0821621   .0837815    -0.98   0.328    -.2477432    .0834191
         L7D. |   .1205693   .0821698     1.47   0.144    -.0418265    .2829652
         L8D. |  -.1582006   .0820192    -1.93   0.056    -.3202989    .0038977
              |
        _cons |   .0191378   .0121823     1.57   0.118    -.0049385    .0432142
------------------------------------------------------------------------------

. dfuller lrealgdp, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       156

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.678            -3.491            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.4423

------------------------------------------------------------------------------
  D.lrealgdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lrealgdp |
         L1. |  -.0039282   .0023406    -1.68   0.095     -.008554    .0006976
         LD. |   .3780684   .0797022     4.74   0.000     .2205493    .5355875
        L2D. |   .1787551   .0850693     2.10   0.037     .0106288    .3468815
        L3D. |  -.0694273    .086137    -0.81   0.422    -.2396638    .1008093
        L4D. |  -.1425298   .0857017    -1.66   0.098    -.3119059    .0268463
        L5D. |   -.089938   .0857012    -1.05   0.296    -.2593133    .0794372
        L6D. |   .0665589   .0855502     0.78   0.438     -.102518    .2356357
        L7D. |    .098476   .0841432     1.17   0.244    -.0678201     .264772
        L8D. |  -.1679199   .0788511    -2.13   0.035     -.323757   -.0120828
             |
       _cons |   .0366057   .0182873     2.00   0.047     .0004635    .0727478
------------------------------------------------------------------------------

. dfuller lml, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       156

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -0.876            -3.491            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.7958

------------------------------------------------------------------------------
```

```
       D.lml |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lml |
         L1. |  -.0107245   .0122458    -0.88   0.383    -.0349265    .0134775
         LD. |   .5244796   .0827448     6.34   0.000     .3609474    .6880119
        L2D. |    .008381   .0919475     0.09   0.927    -.1733389     .190101
        L3D. |   .1272002   .0923371     1.38   0.170    -.0552897    .3096902
        L4D. |  -.0105875   .0928957    -0.11   0.909    -.1941815    .1730065
        L5D. |   .0573908   .0924638     0.62   0.536    -.1253497    .2401313
        L6D. |   .0322311   .0926466     0.35   0.728    -.1508707    .2153329
        L7D. |   -.145899    .092612    -1.58   0.117    -.3289323    .0371342
        L8D. |   .0290589   .0856906     0.34   0.735    -.1402953    .1984131
             |
       _cons |    .067268    .075775     0.89   0.376    -.0824896    .2170257
------------------------------------------------------------------------------

. dfuller lintrate, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       156

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -2.001            -3.491            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.2863

------------------------------------------------------------------------------
  D.lintrate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lintrate |
         L1. |  -.0296331   .0148111    -2.00   0.047     -.058905   -.0003612
         LD. |   .5189852   .0815837     6.36   0.000     .3577475    .6802228
        L2D. |  -.4854138   .0918763    -5.28   0.000    -.6669931   -.3038344
        L3D. |   .2601602   .0995391     2.61   0.010     .0634365     .456884
        L4D. |  -.1256994   .1017053    -1.24   0.218    -.3267043    .0753055
        L5D. |  -.0026333   .1004397    -0.03   0.979    -.2011369    .1958704
        L6D. |  -.1435254   .0979216    -1.47   0.145    -.3370524    .0500015
        L7D. |  -.0877461   .0848281    -1.03   0.303    -.2553957    .0799035
        L8D. |  -.0077366   .0756629    -0.10   0.919    -.1572726    .1417994
             |
       _cons |   .0541495   .0232291     2.33   0.021     .0082409    .1000582
------------------------------------------------------------------------------

. *We cannot reject the null hypothesis of nonstationarity for all 4 of the log variables;
. *Dickey-Fuller test for first-difference variables;
. dfuller fd_lcons, lags(8) regress;
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       155

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -4.531            -3.492            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0002

------------------------------------------------------------------------------
  D.fd_lcons |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    fd_lcons |
         L1. |  -.9647775   .2129385    -4.53   0.000    -1.385642    -.543913
         LD. |   .0240024   .1964317     0.12   0.903    -.3642368    .4122417
        L2D. |    .241586   .1840673     1.31   0.191    -.1222156    .6053877
        L3D. |   .2673619   .1682799     1.59   0.114    -.0652365    .5999602
        L4D. |   .2094889   .1542668     1.36   0.177    -.0954132    .5143909
        L5D. |   .1015515   .1414303     0.72   0.474    -.1779797    .3810828
        L6D. |   .0263036   .1291501     0.20   0.839    -.2289564    .2815636
        L7D. |   .1478296    .114349     1.29   0.198    -.0781766    .3738358
        L8D. |  -.0183776   .0834113    -0.22   0.826    -.1832367    .1464815
             |
       _cons |   .0080161    .001889     4.24   0.000     .0042825    .0117496
------------------------------------------------------------------------------

. dfuller fd_lrealgdp, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       155

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -4.219            -3.492            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0006

------------------------------------------------------------------------------
D.           |
 fd_lrealgdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 fd_lrealgdp |
         L1. |   -.668405   .1584249    -4.22   0.000    -.9815254   -.3552846
         LD. |   .0617177   .1490705     0.41   0.679    -.2329141    .3563496
        L2D. |   .2388672   .1419551     1.68   0.095    -.0417014    .5194358
        L3D. |   .1664662   .1333044     1.25   0.214    -.0970045     .429937
        L4D. |   .0344569   .1198157     0.29   0.774     -.202354    .2712679
```

```
        L5D. |  -.0441873   .1079051    -0.41   0.683    -.2574573    .1690828
        L6D. |   .0343082   .0997212     0.34   0.731    -.1627867    .2314032
        L7D. |   .1259361   .0938336     1.34   0.182    -.0595222    .3113943
        L8D. |  -.0564516   .0807278    -0.70   0.485    -.2160068    .1031035
             |
       _cons |   .0056834   .0015717     3.62   0.000      .002577    .0087897
------------------------------------------------------------------------------

. dfuller fd_lml, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       155

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -3.271            -3.492            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0162

------------------------------------------------------------------------------
    D.fd_lml |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      fd_lml |
         L1. |   -.388057    .118627    -3.27   0.001    -.6225184   -.1535956
         LD. |  -.0952366   .1212468    -0.79   0.433     -.334876    .1444028
        L2D. |  -.0756921   .1143383    -0.66   0.509    -.3016771    .1502929
        L3D. |   .0413639   .1118878     0.37   0.712    -.1797778    .2625055
        L4D. |   .0153082   .1104946     0.14   0.890      -.20308    .2336963
        L5D. |   .0668988   .1082171     0.62   0.537    -.1469879    .2807855
        L6D. |   .0749165   .1043861     0.72   0.474    -.1313984    .2812314
        L7D. |  -.0782253   .0956172    -0.82   0.415    -.2672088    .1107581
        L8D. |  -.1209633   .0845747    -1.43   0.155    -.2881217    .0461951
             |
       _cons |   .0008565   .0008394     1.02   0.309    -.0008026    .0025156
------------------------------------------------------------------------------

. dfuller fd_lintrate, lags(8) regress;

Augmented Dickey-Fuller test for unit root         Number of obs   =       155

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.272            -3.492            -2.886            -2.576
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

```
------------------------------------------------------------------------------
D.           |
 fd_lintrate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 fd_lintrate |
         L1. |  -1.130745   .2144878    -5.27   0.000    -1.554672   -.7068184
         LD. |   .6442852   .1940369     3.32   0.001     .2607791    1.027791
        L2D. |   .1419649   .1791366     0.79   0.429    -.2120914    .4960212
        L3D. |   .3957581   .1594268     2.48   0.014     .0806575    .7108588
        L4D. |   .2636239   .1466103     1.80   0.074    -.0261455    .5533933
        L5D. |   .2434877   .1296368     1.88   0.062    -.0127342    .4997096
        L6D. |   .1057212   .1175573     0.90   0.370    -.1266261    .3380684
        L7D. |    -.00683   .0867566    -0.08   0.937    -.1783009    .1646409
        L8D. |   .0040002   .0767121     0.05   0.958    -.1476181    .1556185
             |
       _cons |   .0122462   .0106069     1.15   0.250     -.008718    .0332104
------------------------------------------------------------------------------

. *We can reject the null hypothesis of nonstationarity for all 4 of the first-difference
> variables;
. *---------------------------------------------------------------------------------------;
. *(c):
> *---------------------------------------------------------------------------------------;
. reg lcons lrealgdp, r;

Linear regression                                      Number of obs   =       165
                                                       F(1, 163)       =  10963.35
                                                       Prob > F        =    0.0000
                                                       R-squared       =    0.9921
                                                       Root MSE        =    .03632

------------------------------------------------------------------------------
             |               Robust
       lcons |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lrealgdp |   1.049442   .0100227   104.71   0.000     1.029651    1.069233
       _cons |  -.8747651   .0786494   -11.12   0.000    -1.030068    -.719462
------------------------------------------------------------------------------

. reg fd_lcons fd_lrealgdp, r;

Linear regression                                      Number of obs   =       164
                                                       F(1, 162)       =     35.51
                                                       Prob > F        =    0.0000
                                                       R-squared       =    0.2920
                                                       Root MSE        =    .00688

------------------------------------------------------------------------------
             |               Robust
```

```
    fd_lcons |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 fd_lrealgdp |   .3759713   .0630969     5.96   0.000     .2513729    .5005698
       _cons |   .0050358   .0006812     7.39   0.000     .0036907    .0063809
------------------------------------------------------------------------------

. *---------------------------------------------------------------------------------------;
. *(d):
> *---------------------------------------------------------------------------------------;
. reg lml lrealgdp lintrate, r;

Linear regression                                      Number of obs   =       165
                                                       F(2, 162)       =    133.30
                                                       Prob > F        =    0.0000
                                                       R-squared       =    0.6164
                                                       Root MSE        =    .05302

------------------------------------------------------------------------------
             |               Robust
         lml |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lrealgdp |   .3279549   .0299266    10.96   0.000     .2688584    .3870514
    lintrate |  -.0960165   .0159211    -6.03   0.000    -.1274562   -.0645768
       _cons |   3.808614   .2092415    18.20   0.000     3.395422    4.221807
------------------------------------------------------------------------------

. reg fd_lml fd_lrealgdp fd_lintrate, r;

Linear regression                                      Number of obs   =       164
                                                       F(2, 161)       =      7.46
                                                       Prob > F        =    0.0008
                                                       R-squared       =    0.0839
                                                       Root MSE        =    .01195

------------------------------------------------------------------------------
             |               Robust
      fd_lml |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 fd_lrealgdp |   .3252979   .0981851     3.31   0.001     .1314012    .5191947
 fd_lintrate |  -.0108141   .0053355    -2.03   0.044    -.0213507   -.0002775
       _cons |  -.0008797   .0013299    -0.66   0.509     -.003506    .0017466
------------------------------------------------------------------------------

. *---------------------------------------------------------------------------------------;
. *---------------------------------------------------------------------------------------;
. *---------------------------------------------------------------------------------------;
. clear;
```