Bulk viscous FWR with time varying constants revisited

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We study a full causal bulk viscous cosmological model with flat FRW symmetries and where the "constants" G, c and Λ vary. We take into account the possible effects of a $c-$variable into the curvature tensor in order to outline the field equations. Using the Lie method we find the possible forms of the "constants" G and c that make integrable the field equations as well as the equation of state for the viscous parameter. It is found that G, c and Λ follow a power law solution verifying the relationship $G/c^2 = \kappa$. Once these possible forms have been obtained we calculate the thermodynamical quantities of the model in order to determine the possible values of the parameters that govern the quantities, finding that only a growing G and c are possible while Λ behaves as a negative decreasing function.

I. INTRODUCTION

In a recent paper (see [**?**]) we study a cosmological model with flat FRW symmetries filled by a perfect fluid and where the "constants" G, c and Λ were considerate as function on time t. By different reasons exposed in such paper, we took into account the possible effects of a $c-$variable into the curvature tensor in order to outline the field equations. Through the Lie group method we studied the possible forms of the functions G and c that make integrable the field equations. In this way we were finding that G and c follow a power law solution verifying the relationship $G/c^2 = \kappa$. But unfortunately we were not able to determine if G and c are growing or decreasing functions on time t.

In order to determine if G and c are growing or decreasing functions we try to take into account thermodynamical considerations, that is to say, we hope that thermodynamical restrictions help us to determine the behaviour of such functions. For this purpose, in this paper we consider a cosmological model with flat FRW symmetries filled by a full causal bulk viscous fluid and where the constants G, c and Λ are considered as functions on time t. Once we have outlined the field equations (taking into account the possible effects of a $c-$variable into the curvature tensor) we rewrite them in order to obtain a second order differential equation in order to apply the standard Lie procedure. The study of this ode through the Lie group method allows us to obtain the precise form of the functions G and c that make integrable the field equations as well as the equation of state for the bulk viscous parameter ξ.

As we will see in section 3 the field equations only admit scaling symmetries (note that we are working with flat FRW symmetries) i.e. we are studying a self-similar model. This fact obligates that G and c follow a power law solution, $c = c_0 t^{K_1}$, verifying the relationship $G/c^2 = \kappa$, i.e. G and c are functions on time t but in such a way that this relationship must be verified. We find another restriction under this symmetry. The bulk viscosity ξ, must follow the law $\xi = k_\gamma \rho^{1/2}$, i.e. we have found a concrete equation of state for the viscous parameter $\gamma = 1/2$. All these results are in agreement with our previous paper ([**?**]).

(1) *Context:* Vasja had always had the book *Two captains* by Kaverin; he had never given it to anyone. One day he accidentally left the book at Masha's place…

a. # I togda Maša opjat' {otdala / otpravila / vernula} Vase knigu.
and then Masha again gave sent returned Vasja.DAT book.ACC

Intended: 'And then Masha gave / sent / returned Vasja the book, and Vasja had had the book before.'

b. # I togda Maša opjat' {otdala / otpravila / vernula} knigu Vase.
and then Masha again gave / sent / returned book.ACC Vasja.DAT

Intended: 'And then Masha gave / sent / returned the book to Vasja, and Vasja had had the book before.'

Once we have found the possible forms of the functions G and c we calculate the energy density finding in a first approach that the only physical solution imply that G and c must be a growing functions on time t since $K_1 > 0$.

In section 4 we will calculate all the physical quantities in order to complete the solution for our model. In particular we are interested in calculating the entropy in order to obtain some restrictions for the physical parameters as K_1

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and to elucidate if G and c are growing or decreasing functions. In this way we find an exact solution for Λ which behaves as a negative decreasing function.

We end summarizing all these results in the last section.

II. THE MODEL

Following Maartens [?], we consider a Friedmann-Robertson-Walker (FRW) Universe with a line element

$$ds^2 = c^2(t)dt^2 - f^2(t)\left(dx^2 + dy^2 + dz^2\right), \tag{1}$$

filled with a bulk viscous cosmological fluid with the following energy-momentum tensor:

$$T_i^k = (\rho + p + \Pi)\, u_i u^k - (p + \Pi)\, \delta_i^k, \tag{2}$$

where ρ is the energy density, p the thermodynamic pressure, Π is the bulk viscous pressure and u_i is the four velocity satisfying the condition $u_i u^i = 1$.

The gravitational field equations with variable G, c and Λ are:

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi G(t)}{c^4\,(t)}T_{ik} + \Lambda(t)g_{ik}. \tag{3}$$

Applying the covariance divergence to the second member of equation (3) we get:

$$div\left(\frac{G}{c^4}T_i^j + \delta_i^j\Lambda\right) = 0, \tag{4}$$

$$T_{i;j}^j = \left(\frac{4c_{,j}}{c} - \frac{G_{,j}}{G}\right)T_i^j - \frac{c^4\delta_i^j\Lambda_{,j}}{8\pi G}, \tag{5}$$

that simplifies to:

$$\dot{\rho} + 3\,(\rho + p)\,H + 3H\Pi = -\frac{\dot{\Lambda}c^4}{8\pi G} - \rho\frac{\dot{G}}{G} - 4\rho\frac{\dot{c}}{c}, \tag{6}$$

where H stands for the Hubble parameter ($H = \dot{f}/f$). Therefore, our model (with FRW symmetries) is described by the following equations:

$$2\dot{H} - 2\frac{\dot{c}}{c}H + 3H^2 = -\frac{8\pi G}{c^2}\,(p + \Pi) + \Lambda c^2, \tag{7}$$

$$3H^2 = \frac{8\pi G}{c^2}\rho + \Lambda c^2, \tag{8}$$

$$\dot{\rho} + 3\,(\rho + p + \Pi)\,H = -\frac{\dot{\Lambda}c^4}{8\pi G} - \rho\frac{\dot{G}}{G} + 4\rho\frac{\dot{c}}{c}, \tag{9}$$

$$\tau\dot{\Pi} + \Pi = -3\xi H - \frac{\epsilon}{2}\tau\Pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right). \tag{10}$$

We would like to emphasize that deriving (8) and taking into account (7) it is obtained (9), that is to say, the conservation equation (9) could be deduced from the field equation as in the standard case where the "constants" G, c and Λ are true constants (see [?]).

In order to close the system of equations (7-10) we have to give the equation of state for p and specify T, ξ and τ. As usual, we assume the following phenomenological (ad hoc) laws [?]:

$$p = \omega\rho, \quad \xi = k_\gamma\rho^\gamma, \quad T = D_\delta\rho^\delta, \quad \tau = \xi\rho^{-1} = k_\gamma\rho^{\gamma-1}, \tag{11}$$

where $0 \leq \omega \leq 1$, and $k_\gamma \geq 0$, $D_\delta \geq 0$ are dimensional constants, $\gamma \geq 0$ and $\delta \geq 0$ ($\delta = \frac{\omega}{\omega+1}$ so that $0 \leq \delta \leq 1/2$ for $0 \leq \omega \leq 1$) are numerical constants. Eqs. (11) are standard in cosmological models whereas the equation for τ is a simple procedure to ensure that the speed of viscous pulses does not exceed the speed of light. These are without