The structure of non-spacelike curves from a spacetime singularity

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We investigate here the causal structure of spacetime in the vicinity of a spacetime singularity. The particle and energy emission from such ultra-dense regions forming in gravitational collapse of a massive matter cloud is governed by the nature of non-spacelike paths near the same. These trajectories are examined to show that if a null geodesic comes out from the singularity, then there exist families of future-directed non-spacelike curves which also necessarily escape from the naked singularity. The existence of such families is crucial to the physical visibility of the singularity. We do not assume any underlying symmetries for the spacetime, and earlier considerations on the nature of causal trajectories emerging from a naked singularity are generalized and clarified.

PACS numbers: 04.20.Dw, 04.70.-s, 04.70.Bw

One of the most important questions in gravity physics today is whether spacetime singularities could be visible to faraway observers in the universe. The occurrence of singularities is a generic feature of Einstein's theory of gravity, as shown by the singularity theorems in general relativity [**?**]. These could occur in cosmology at the origin of the universe, or in the continual gravitational collapse of a massive star at the end of its life cycle.

Spacetime singularities are the regions where the physical conditions such as densities and curvatures are at their extreme. While the big bang singularity of cosmology is visible in principle, which gave rise to the universe as a whole, we cannot actually see it. On the other hand, when a massive star collapses continually under gravity, the eventual spacetime singularity can be either hidden within an event horizon of a black hole, or it could be visible to outside observers depending on how the collapse of the cloud dynamically evolves and the causal structure in the vicinity of the singularity. The visibility or otherwise of singularities in gravitational collapse is an issue basic to the foundations of black hole physics. A visible naked singularity forming in collapse could provide an opportunity for the physical effects taking place in these extreme gravity regions to be observable to outside observers in the universe.

It is some times argued that extreme physical conditions prevail closer to spacetime singularities, where quantum effects must be important and must be incorporated into considerations. This is plausible and in the very late stages of gravitational collapse quantum effects have to be taken into account, which could possibly resolve a classical naked singularity (see e.g. [**?**]). The point, however, is that in such a case also the causal structure of spacetime in the vicinity of the super-ultradense regions forming in collapse would be such that these would be visible in principle to an external observer. In other words, the quantum effects taking place in the regions with arbitrarily high matter densities and curvatures could be seen by the external observers. Then, even though the naked singularity itself may be resolved, this provides an interesting physical scenario to possibly observe quantum gravity effects. The communication of physical effects from such extreme regions would be again governed by the existence of families of non-spacelike paths from the vicinity of the singularity. It is therefore important to understand the structure of such families within a gravitational collapse framework.

(1) a. *[Mittlerweile ist anerkannt,] [dass es zur Sicherung von Beschäftigung vor allem auf Flexibilität ankommt.]* (GO_SPEECH_007)

b. *[It has now been recognized] [that flexibility is the most important factor] [when it comes] [to safeguarding jobs.]* (ETrans_SPEECH_007)

(2) a. *[Die Staats- und Regierungschefs der Europäischen Union haben in Göteborg erneut ihre Bereitschaft bekräftigt,] [die in Kyoto eingegangenen Verpflichtungen zur Verminderung der Treibhausgase zu erfüllen.]* (GO_SPEECH_001)

b. *[In Gothenburg the EU heads of state and government reaffirmed their willingness] [to fulfil the commitments] [they made in Kyoto] [to reduce greenhouse gases.]* (ETrans_SPEECH_001)

The formation and existence of naked singularity in a gravitationally collapsing matter cloud has been extensively investigated in past decade or so (see e.g. [**?**] for some reviews, and references therein). This is typically deduced based on the causal structure of spacetime and the nature of trapped region in the vicinity of the singularity. Let the continual gravitational collapse develop from a regular initial data from an initial spacelike surface, and result into the development of a spacetime singularity. If there is an outgoing future directed null geodesic, which terminates in the past at the singularity, then the singularity is at least locally visible. Such a locally naked singularity will be globally visible provided the outgoing null geodesic would come out of the boundary of the collapsing cloud to reach a faraway observer in the spacetime. On the other hand, if the collapse terminates into a black hole final state, no non-spacelike curves would escape from the singularity. It is typically seen that the nature of initial density and pressure profiles and the dynamical evolutions as allowed by the Einstein equations actually determine the final state of the cloud in terms of either a black hole or a naked singularity.

An important physical issue then would be whether such a naked singularity forming in gravitational collapse could radiate away energy and particles. The physical visibility, or otherwise, of the singularity developing towards the end of continual collapse of the matter cloud depends crucially on the existence and structure of families of non-spacelike trajectories coming out from its vicinity. For example, if only a single photon could escape from the singularity, it will be non-visible to an external observer for all practical purposes. On the other hand, existence of families of future directed non-spacelike paths could make the singularity visible for outside observers. Also, any material particles would escape from the vicinity of the singularity only if there are timelike curves escaping away from these ultra-dense regions.

As pointed out above, a standard technique to deduce the development of a naked singularity in gravitational collapse has been to examine the existence of a future directed outgoing radial null geodesic from the same. Radial null geodesics are, however, somewhat special paths. The actual physical appearance of these ultra-dense regions will be typically determined by the non-radial null trajectories from the same, and the energy emission, if any will be governed by the timelike curves and other non-spacelike trajectories escaping away from the singularity. For this reason, several authors have considered the possibility of non-radial null geodesics coming out from a naked singularity in the context of spherically symmetric dust collapse models [**?**]. Also, families of non-spacelike and timelike geodesics have been worked out in the self-similar perfect fluid collapse, and the Vaidya radiation collapse models [**?**]. Most of these considerations have been in the framework of spherically symmetric spacetimes, at times together with other symmetry conditions imposed such as self-similarity of the models, and within the framework of a specific matter model.

A general consideration, however, on the nature of non-spacelike trajectories near a naked singularity will be of interest from such a perspective. We examine here the non-spacelike trajectories from a naked singularity in a general manner, and show that if a radial null geodesic comes out, then large families of non-spacelike curves also necessarily come out from the singularity. In other words, it is seen that the existence of a radial null geodesic is sufficient to ensure the existence of families of timelike and non-spacelike trajectories escaping, and in this sense a single photon escaping in a radial direction from the singularity is never an isolated phenomenon. This generalizes and clarifies earlier considerations in this direction such as [**?**], without assuming any symmetry conditions on the underlying spacetime or any specific matter model for the collapse. As we point out below, the naked singularity generates an indecomposable future set, the null boundary of which gives all the null generators, including the null geodesics, coming out from the singularity.

Let us consider a continual gravitational collapse which ends in a naked singularity, that is, the causal structure near the singularity is such that a null geodesic trajectory γ comes out from the same. Specifically, γ is future-directed, which in the past terminates at the singularity, and is therefore a past-incomplete null geodesic. To examine in general the possible existence and nature of non-spacelike curves coming out of this naked singularity, we use here the causal boundary construction as developed by Geroch, Kronheimer and Penrose [**?**]. (Our notations are same as given by [**?**] and the spacetime $\mathcal{M}$ is taken to satisfy a suitable causality condition such as strong causality, which rules out existence of closed timelike curves.) In this procedure, a boundary is attached to the regular spacetime manifold, which includes spacetime singularities as well as the points at infinity [**?**]. An open set W in the spacetime is called a *future set* if it contains its own future, that is, we have $I^+(W) \subset W$. Further, a future set W is called an *indecomposable future set (IF)* if it cannot be expressed as the union of two proper subsets which are themselves future sets. Indecomposable past sets (IPs) are similarly defined. The idea of the causal boundary construction is to divide the collection of IFs and IPs into two classes, namely the one representing regular points of the spacetime, and the other class giving all its boundary points or the ideal points, which include spacetime singularities as well as points at infinity. The collection of IFs (or IPs) can be divided into the two parts as follows. For a set W which is an IF, if there exists an event in the spacetime $p \in \mathcal{M}$ such that $W = I^+(p)$, then W is called a *proper IF* or a PIF. All other IFs are called *terminal IFs* or TIFs, which represent spacetime singularities and the points at infinity.

Consider now the set $I^+(\gamma)$ where γ is any null geodesic curve coming out of the singularity. It is then a future set, because for any $p \in I^+(\gamma)$, we have $I^+(p) \subset I^+(\gamma)$. We can see that $I^+(\gamma)$ is an indecomposable future set, or an IF. To show this, we use a somewhat modified version of the proof of Theorem 2.1 of [**?**]. Suppose $I^+(\gamma) = A \cup B$ with A and B both being future sets. If neither A is fully contained in B or vice-versa, we can then find two events x, y such that $x \in A - B$ and $y \in B - A$. We have both $x, y \in I^+(\gamma)$ so there are points $x', y' \in \gamma$ such that $x \in I^+(x')$ and $y \in I^+(y')$. But x' and y' are causally related so suppose now that x' is in the past of y' on γ. Then there is a null geodesic from x' to y', and there is a timelike curve from y' to y as above. This implies that there must be a timelike curve from x' to y (see e.g. [?], p.183). It follows that $y \in I^+(x')$. As we already have $x \in I^+(x')$, this implies that $x, y \in I^+(x')$. Therefore, x' lies in the intersection of the sets $I^-(x)$ and $I^-(y)$, which is an open set and so contains a neighbourhood N of x'. Let z be an event in $I^+(x') \cap N$, then $z \in I^+(\gamma)$ and so has to be in one of the future sets A or B. Suppose it is in A, then since there are future directed timelike curves from z to both x and y, it follows that both $x, y \in A$, which is a contradiction. Hence it follows that $I^+(\gamma)$ has to be an IF. Since γ is a past-incomplete null geodesic, there is no regular point $p \in \mathcal{M}$ such that $I^+(p) = I^+(\gamma)$, and hence it follows that $I^+(\gamma)$ is necessarily a TIF.

The TIF set $I^+(\gamma)$ here represents a boundary point of the spacetime which is the naked singularity. While the naked singularity formation as endstate of a continual gravitational collapse has been investigated extensively in past decade or so (especially within the framework of spherically symmetric collapse and for certain non-spherical examples), the main technique there has been to show that there exists a radial null geodesic coming out in the future and which terminates in the past at the spacetime singularity [?] It is thus seen that the gravitational collapse from regular initial matter profiles could result into either of the black hole or naked singularity endstates, depending on the nature of the initial data from which it evolves and the dynamical evolutions of the collapsing cloud as allowed by the Einstein equations.

However, as remarked above, if one is to examine the visibility and other related physical characteristics of a naked singularity that formed in gravitational collapse, then it is important to examine and understand the structure of families of non-spacelike curves from the same. Again, if non-spacelike curves come out of the singularity but do not go out of the boundary of the collapsing cloud then the singularity will be only locally visible but the outside observers would not be able to see the same. It is necessary therefore to understand the structure of non-spacelike curves from a naked singularity in general.

It is now possible to do this as below. Since the set $S = I^+(\gamma)$ is a TIF, it follows from [?] that in this case there must exist a past-inextendible timelike curve λ such that $S = I^+(\lambda)$. In the case of collapse ending in a naked singularity and a radial null geodesic γ escaping from the same, we thus see that the set $I^+(\gamma)$ *is* a TIF, and so by the above result, there is a timelike curve λ generating this TIF, in the sense that $S = I^+(\lambda)$. Since both the non-spacelike trajectories γ and λ represent the same ideal or boundary point of the spacetime which is the naked singularity, and since $I^+(\gamma) = I^+(\lambda)$ by definition, it follows that the future-directed timelike curve λ must terminate in the past at the naked singularity. In other words, we have shown that there exists is a timelike curve λ which escapes away to future and which terminates in the past at the naked singularity.

It follows that if $p \in \lambda$ and q is any other event such that $q \in I^+(p)$, then there are timelike curves from the naked singularity to q. This proves the existence of families of infinitely many future-directed non-spacelike trajectories escaping away from the naked singularity. In general, if λ' is any other future directed non-spacelike curve such that $I^+(\lambda') = I^+(\lambda)$, then it follows that they all represent the same TIF, which is the naked singularity, and that λ' terminates in the past at this singularity. Thus there is an infinity of future going non-spacelike curves which emerge from the singularity.

We thus see that the usual method employed to show the existence of a naked singularity in collapse, which establishes the existence of a radial null geodesic escaping away from the same, is sufficient to lead to the existence of infinite families of future going non-spacelike curves from the naked singularity as seen here. In the present consideration, we no longer need any special symmetry assumptions on the spacetime such as spherical symmetry, self-similarity etc, or any specific form of matter model such as dust equation of state which are usually assumed in such discussions.

In particular, this also clarifies and generalizes the earlier results on dust collapse and other models mentioned above, which have particularly focussed on non-spacelike null geodesics. The null geodesics of the spacetime have of course a special role to play as far as the visibility of the singularity is concerned. From such a perspective, let us briefly discuss the existence of radial versus the non-radial families of null geodesics from a naked singularity. Suppose a radial null geodesic comes out from the naked singularity S developing in a continual collapse. In that case, as seen above, there exists a timelike curve λ generating the TIF set $I^+(\lambda)$ which represents the boundary point S. All other future-directed non-spacelike curves γ which satisfy $I^+(\lambda) = I^+(\gamma)$ generate the same TIF representing the boundary point S, and they give the families of particle or photon trajectories escaping away from the naked singularity. The boundary of this future set which is a TIF, is a three-dimensional null hypersurface which is ruled by the radial as well as non-radial null geodesics generators γs, which are all incomplete when extended in the past, and which all have the property that $I^+(\lambda) = I^+(\gamma)$. This shows that the existence of a radial null geodesic is sufficient to give families of non-radial null geodesics as well, coming out from the singularity. This generalizes the earlier results on existence of non-radial null geodesics from the singularity for the spherically symmetric dust collapse, when a radial null geodesic came out from the same.

It is thus seen that once a singularity is naked, it gives rise to infinitely many null as well as timelike curves to escape away from the same. In this sense, the emission of paths representing particle or photon trajectories from the visible singularity is a generic phenomena. This is a necessary condition for the singularity to give rise to any physical effects which may possibly be observed by external observers. We have not discussed in the present consideration the global visibility of the singularity, that is, once the families of non-spacelike curves come out of the naked singularity when they will actually cross the boundary of the cloud to escape to an outside observer. It is known, however, in several cases including spherical dust collapse, that whenever a singularity is locally naked then one can choose rest of the free functions in the model so as to make it globally visible [**?**]. A discussion on global visibility in a more general context of perfect fluids is recently given by [**?**].

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[1] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge University Press, Cambridge (1973).

[2] R. Goswami, P. S. Joshi, P. Singh, Phys. Rev. Lett. **96**, 031302 (2006). For a possible quantum resolution of the big bang singularity within the loop quantum gravity framework, see e.g. M. Bojowald, Living Rev. Rel.8:11 (2005); A. Ashtekar, T. Pawlowski, P. Singh, gr-qc/0604013, and references therein.

[3] A. Krolak, Prog. Theor. Phys. Suppl. **136**, p. 45 (1999); P. S. Joshi, Pramana **55**, 529 (2000); R. Giambo', F. Giannoni, G. Magli, P. Piccione, Commun.Math.Phys. 235 (2003) 545-563; T. Harada, H. Iguchi, and K. Nakao, Prog.Theor.Phys. 107 (2002) 449-524.

[4] F. C. Mena and B. Nolan, Class. Quantum Grav. **18**, p.4531 (2001); Class. Quantum Grav. **19**, p.2587 (2002); S. S. Deshingkar, P. S. Joshi, Phys. Rev. **D63**, 024007 (2001); S. S. Deshingkar, P. S. Joshi, I. H. Dwivedi, Phys. Rev. **D65**, 084009 (2002).

[5] P. S. Joshi and I. H. Dwivedi, Commun. Math. Phys. **146**, p.333 (1992); Lett. Math. Phys. **27**, p.235 (1993); I. H. Dwivedi and P. S. Joshi, Class.Quantum Grav. **6**, p.1599 (1989); Class. Quantum Grav. **8**, p.1339 (1991).

[6] R. Geroch, E. Kronheimer, R. Penrose, Proc. R. Soc. Lond. **A327**, p.545 (1972).

[7] A boundary attachment to the spacetime manifold is essential to treat the regular spacetime events together with its singularities and points at infinity in a unified manner. There are different ways to attach a boundary to the spacetime, and they all do not necessarily give the same result. We have used here the approach as given in [**?**] as it depends basically only on the causal structure of spacetime, which is more fundamental as compared to, for example, the differential structure of the spacetime manifold. Also, from a physical point of view, each ideal point here is directly associated with the region of spacetime which it can influence, or which it would be influenced by.

[8] It is known, e.g. in the case of dust collapse, that once the singularity is locally naked, the choice of a suitable behaviour of the mass function (which is a free function, subject only to some physical conditions such as an energy condition and regularity of the initial data) away from the center, allows the null rays to come out from the boundary of the cloud (see [**?**]). It may also be noted that in certain classes of self-similar collapse, once the singularity is locally naked it becomes necessarily globally visible. In any case, as there is no scale in the problem, once the singularity is locally visible, an observer within a large enough black hole will still be able to see it for a long enough time. In such a scenario, the escape of rays outside the boundary of the cloud would not be crucial.

[9] R. Giambo', gr-qc/0603120.