KTH Royal Institute of Technology

Omogen Heap

Simon Lindholm, Johan Sannemo, Mårten Wiman

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Contest (1)

template.cpp — 18 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
#define pb push_back
#define eb emplace_back
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

constexpr int oo = (((unsigned int)-1) >> 1);

int main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

.bashrc — 3 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <>
```

.vimrc — 6 lines

```
set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on   |   im jk <esc>   |   im kj <esc>   |   no ; :
" Select region and then type :Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
 \| md5sum \| cut -c-6
```

hash.sh — 3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

troubleshoot.txt — 52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Theory (2)

2.1 General Math

2.1.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{array}{l} ax + by = e \\ cx + dy = f \end{array} \Rightarrow \begin{array}{l} x = \dfrac{ed - bf}{ad - bc} \\ y = \dfrac{af - ec}{ad - bc} \end{array}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.1.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.1.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.1.4 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \qquad \int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.1.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.1.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.1.7 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

2.2 Geometry

2.2.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$s_a = \sqrt{bc\left[1 - \left(\dfrac{a}{b+c}\right)^2\right]}$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

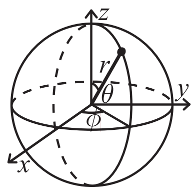
2.2.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.2.3 Spherical coordinates



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \mathrm{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \mathrm{atan2}(y, x)$$

2.3 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.3.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \; \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \; k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \; \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda, \; \sigma^2 = \lambda$$

2.3.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \; \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \; \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu,\sigma^2)$, $\sigma>0$.

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1\sim\mathcal{N}(\mu_1,\sigma_1^2)$ and $X_2\sim\mathcal{N}(\mu_2,\sigma_2^2)$ then

$$aX_1+bX_2+c\sim\mathcal{N}(\mu_1+\mu_2+c,a^2\sigma_1^2+b^2\sigma_2^2)$$

2.3.3 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1,X_2,\dots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P}=(p_{ij})$, with $p_{ij}=\Pr(X_n=i|X_{n-1}=j)$, and $\mathbf{p}^{(n)}=\mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)}=\Pr(X_n=i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi=\pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i=\frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty}\mathbf{P}^k=\mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii}=1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i\in\mathbf{A}$, when the initial state is j, is $a_{ij}=p_{ij}+\sum_{k\in\mathbf{G}}a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i=1+\sum_{k\in\mathbf{G}}p_{ki}t_k$.

2.4 Combinatorics

2.4.1 Permutations

Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty}g_S(n)\frac{x^n}{n!}=\exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n)=(n-1)(D(n-1)+D(n-2))=nD(n-1)+(-1)^n=\left\lfloor\frac{n!}{e}\right\rceil$$

Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x=x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G=\mathbb{Z}_n$ to get

$$g(n)=\frac{1}{n}\sum_{k=0}^{n-1}f(\gcd(n,k))=\frac{1}{n}\sum_{k|n}f(k)\phi(n/k).$$

2.4.2 Partitions and subsets

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0)=1,\ p(n)=\sum_{k\in\mathbb{Z}\setminus\{0\}}(-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n)\sim 0.145/n\cdot\exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

Lucas' Theorem

Let n,m be non-negative integers and p a prime. Write $n=n_kp^k+...+n_1p+n_0$ and $m=m_kp^k+...+m_1p+m_0$. Then $\binom{n}{m}\equiv\prod_{i=0}^{k}\binom{n_i}{m_i}\pmod p$.

2.4.3 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t)=\frac{t}{e^t-1}$ (FFT-able). $B[0,\dots]=[1,-\frac{1}{2},\frac{1}{6},0,-\frac{1}{30},0,\frac{1}{42},\dots]$

Sums of powers:

$$\sum_{i=1}^{n}n^m=\frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty}f(i)=\int_m^{\infty}f(x)dx-\sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx\int_m^{\infty}f(x)dx+\frac{f(m)}{2}-\frac{f'(m)}{12}+\frac{f'''(m)}{720}+O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k)=c(n-1,k-1)+(n-1)c(n-1,k),\ c(0,0)=1$$
$$\textstyle\sum_{k=0}^{n}c(n,k)x^k=x(x+1)\dots(x+n-1)$$

$c(8,k)=8,0,5040,13068,13132,6769,1960,322,28,1$
$c(n,2)=0,0,1,3,11,50,274,1764,13068,109584,\dots$

Eulerian numbers

Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j:s s.t. $\pi(j)\geq j$, k j:s s.t. $\pi(j)>j$.

$$E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$$

$$E(n,0)=E(n,n-1)=1$$

$$E(n,k)=\sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k)=S(n-1,k-1)+kS(n-1,k)$$

$$S(n,1)=S(n,n)=1$$

$$S(n,k)=\frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n)=1,1,2,5,15,52,203,877,4140,21147,\dots$. For p prime,

$$B(p^m+n)\equiv mB(n)+B(n+1)\pmod p$$

Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1n_2\cdots n_kn^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

Catalan numbers

$$C_n=\frac{1}{n+1}\binom{2n}{n}=\binom{2n}{n}-\binom{2n}{n+1}=\frac{(2n)!}{(n+1)!n!}$$

$$C_0=1,\ C_{n+1}=\frac{2(2n+1)}{n+2}C_n,\ C_{n+1}=\sum C_iC_{n-i}$$

$C_n=1,1,2,5,14,42,132,429,1430,4862,16796,58786,\dots$

- sub-diagonal monotone paths in an $n\times n$ grid.

- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

2.5 Number Theory

2.5.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

2.5.2 Highly composite numbers

Up to: number of divisors (number itself)

10^2 : 12(60) 10^3 : 32(840) 10^4 : 64(7560) 10^5 : 128(83160)
10^6 : 240(720720) 10^7 : 448(8648640) 10^8 : 768(73513440)
10^9 : 1344(735134400) 10^{10} : 2304(6983776800)
10^{11} : 4032(97772875200) 10^{12} : 6720(963761198400)
10^{13} : 10752(9316358251200) 10^{14} : 17280(97821761637600)
10^{15} : 26880(866421317361600) 10^{16} : 41472(8086598962041600)
10^{17} : 64512(74801040398884800)
10^{18} : 103680(897612484786617600)

2.5.3 Primes

$p = 962592769$ is such that $2^{21} \mid p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

2.5.4 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

2.5.5 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

2.6 Graphs

2.6.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

2.6.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \ge \cdots \ge d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Math (3)

3.1 Misc

3.1.1 Combinatorics

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$.

a0a312, 5 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i]) c = c * ++m / (j+1);
  return c;
} // a0a312
```

Combinatorics.h

Description: combinatorics structure
Memory: $\mathcal{O}(mxn)$
Time: $\mathcal{O}(mxn)$

"../math/ModPow.h" d4ae0e, 10 lines

```
int mul(int a, int b) {return (int)((ll)a * b % mod);}
struct Combinatorics{
  vi f, fi;
  Combinatorics(int mxn):f(mxn),fi(mxn){
    f[0] = 1; rep(i, 1, mxn)f[i]=mul(f[i-1],i);
    fi[mxn-1] = modpow(f[mxn-1], mod-2);
    for(int i=mxn-1;i>0;i--)fi[i-1] = mul(fi[i],i);
  } // 5e6a1f
  int choose(int n, int k){return mul(f[n],mul(fi[k],fi[n-k]));
      }
}; // ffee09
```

3.1.2 Polynomial Calculus

Polynomial.h

c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  } // ae76f3
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  } // afcaea
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  } // 3f874a
}; // c9b7b0
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h" b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      } // b69f41
      ret.push_back((l + h) / 2);
    } // fc22f0
  } // d15986
  return ret;
} // b00bfe
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \ldots n-1$.
Time: $\mathcal{O}(n^2)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  } // 4c74fe
  return res;
} // 285367
```

3.1.3 Recurrences and Sums

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

"../math/ModPow.h" 96548b, 17 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1; ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  } // 8c2376
  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
} // 96548b
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 21 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);
  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  }; // 55c8ab
  Poly pol(n + 1), e(pol); pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  } // 8137be
  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
} // 5948dc
```

FloorModSum.h

Description: Floor sum and Mod sum.
`modsum(to, c, k, m)` $= \sum_{i=0}^{\text{to}-1} (ki+c)\%m$. `divsum` is similar but for floored division.
Time: $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
} // 78bfc8

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
} // 5daf3e
```

3.2 Optimization

3.2.1 Fractions and Real Numbers

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0,1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.
Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
Time: $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      } // cacde6
    } // d6d2f6
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  } // 7df851
  return dir ? hi : lo;
} // ef9d52
```

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a,b]$ assuming f is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else { // 4513d0
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    } // 2fe74a
  return a;
} // 31d45b
```

TernarySearch.h

Description: Find the smallest i in $[a,b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  } // ce7859
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
} // 5d6373
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions.

8eeeaf, 14 lines

```
typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
  pair<double, P> cur(f(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
      P p = cur.second;
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(f(p), p));
    } // cc6436
  } // 8d9318
  return cur;
} // 75cdd9
```

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}(\log N)$

dd6c5e, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    } // 3abeb0
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    } // f1df8b
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  } // 543b7b
```

```
} // ec1f58
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

4756fc, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
} // ddcce2
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.

Usage: `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`

92dd79, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
} // 720738
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
} // 1e3820
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`

Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

aa8530, 68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    } // 6ff8e9

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    } // ca4460
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  } // 9cd0a8

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      } // 46853f
      if (r == -1) return false;
      pivot(r, s);
    } // 7d839b
  } // f15644

  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      } // 683310
    } // b6553f
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  } // 396a95
}; // c57b35
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.

Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];},`
`[&](int lo, int hi, T val){...});`

Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

753a4c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else { // 956f3f
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  } // effcac
} // fb5eee
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
} // 8bf818
```

3.3 Matrices

3.3.1 General

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.

Time: $\mathcal{O}\left(N^3\right)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    } // 4ec6a2
  } // ee1466
  return res;
} // bd5cec
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.

Time: $\mathcal{O}\left(N^3\right)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      } // e81b29
    } // 30d1b2
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  } // f39a45
  return (ans + mod) % mod;
} // 5e85f0
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}\left(n^3\right)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
```

```
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    } // ebeea3
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  } // 26d90b

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  } // 03ae0c

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
} // ebfff6
```

3.3.2 Linear Systems

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    } // c92205
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    } // 881860
    rank++;
  } // 0f0f0f

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  } // ed1d08
  return rank; // (multiple solutions if rank < m)
} // ade67b
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; } // 87878c
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2 m\right)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    } // 80687c
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    } // b44a9b
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    } // 87192e
    rank++;
  } // fe9281

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  } // 8fdba1
  return rank; // (multiple solutions if rank < m)
} // 26d73e
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else { // 464c09
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    } // 62de5a
  } // ed9cce
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else { // 0543e4
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    } // aa91c6
  } // 28af28
  return b;
} // 06d549
```

3.4 Convolutions

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ (~1s for $N = 2^{22}$)

00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
```

```
  } // a8a74e
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    } // 577e9c
} // 01fdd0
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
} // 873509
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 22 lines

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  vl res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
  vector<C> L(n), R(n), outs(n), outl(n);
  rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
  rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
  fft(L), fft(R);
  rep(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
  } // cb3017
  fft(outl), fft(outs);
  rep(i,0,sz(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  } // 58fa4f
  return res;
} // c1f2f1
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

"../math/ModPow.h" ced03d, 35 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  } // f39127
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    } // 9a8565
} // 3b763b
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
      n = 1 << B;
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  rep(i,0,n)
    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
} // 3876bf
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

464cf3, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    } // 7b7e71
  } // faec61
  if (inv) for (int& x : a) x /= sz(a); // XOR only
} // 57eeaf
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
} // 3cbd18
```

3.5 Number Theory

3.5.1 Modular Arithmetic

CRT.h

Description: Chinese Remainder Theorem.
`crt(a, m, b, n)` computes x such that $x \equiv a \pmod{m}$, $x \equiv b \pmod{n}$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
} // 04d93a
```

ModPow.h

b83e45, 7 lines

```
const ll mod = 1000000007; // faster if const
ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
} // d1ec54
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \le a, b \le c \le 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
} // e9309c
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
} // 100b91
```

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod{b}$ in the range $[0, 2b)$.

751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  } // f67e7e
}; // 38ea39
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod{m}$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.
Time: $\mathcal{O}(\sqrt{m})$

c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
} // c040b8
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod{p}$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 19a793, 20 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
```

```
  } // 5b6623
  for (int i : pr) isPrime[i] = 1;
  return pr;
} // 8ee6d2
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0) ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m) t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p; x = x * gs % p; b = b * g % p;
  } // e3aa6f
} // 19a793
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  } // edfaf1
  return 1;
} // 60dcd1
```

PollardRho.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d8d98d, 18 lines

```
ull pollard(ull n) {
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  auto f = [&](ull x) { return modmul(x, x, n) + i; };
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  } // 989d40
  return __gcd(prd, n);
} // cd2ac3
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
} // d54ba8
```

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
} // 33ba8f
```

phiFunction.h

Description: *Euler's* ϕ function up to LIM. Note that $\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Time: $\mathcal{O}(n \log \log n)$

cf7d6d, 7 lines

```
const int LIM = 5000000;
int phi[LIM];
void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
} // 04349b
```

Geometry (4)

4.1 Geometric primitives

Point.h

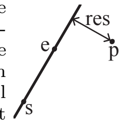
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); } // 4822a3
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; } // 9a9c95
}; // d2d691
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

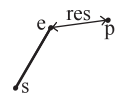


"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
} // 00891c
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



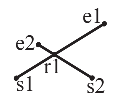
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
} // ae751a
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;

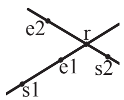
"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
```

```
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
} // 9d57f2
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
```
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```
"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
} // 47279a
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: `bool left = sideOf(p1,p2,q)==1;`
"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
} // 33fa03
```

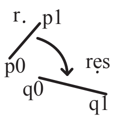
OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.
"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
} // c597e8
```

linearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
} // 45ea01
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
```
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively
oriented triangles with vertices at 0 and i
```
0f0602, 35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  } // c935fb
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
}; // e258c0
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
} // ce5ed3

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
} // 5eac29
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
} // 3d8073
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
} // ba3082
```

4.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.
"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
} // c64785
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.
"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  } // e25263
  if (h2 == 0) out.pop_back();
  return out;
} // 4835b9
```

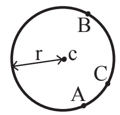
CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$
"../../content/geometry/Point.h" 19add1, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = q + d * (t-1);
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  }; // a526fe
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
} // f082e0
```

circumcircle.h

Description:
The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
} // 607d98
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} // 79372e
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      } // 64802f
    } // 7b0ecf
  } // dcfd0e
  return {o, r};
} // 09dd0a
```

4.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  } // 1b9961
  return cnt;
} // c7225e
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
} // 6939b3
```

PolygonCenter.h

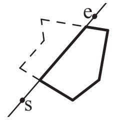
Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  } // 307102
  return res / A / 3;
} // 0d0d84
```

PolygonCut.h



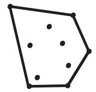
Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h" d07181, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    auto a = s.cross(e, cur), b = s.cross(e, prev);
    if ((a < 0) != (b < 0))
      res.push_back(cur + (prev - cur) * (a / (a - b)));
    if (a < 0)
      res.push_back(cur);
  } // 757c0d
  return res;
} // 42c993
```

ConvexHull.h



Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    } // bf0344
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
} // ec85f8
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    } // 56cc40
  return res.second;
} // 5f726b
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  } // b265ab
  return sgn(l[a].cross(l[b], p)) < r;
} // c74639
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  } // 68a24c
  return lo;
} // 7f0477

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    } // 52528c
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  } // c05c70
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    } // 8fa383
  return res;
} // 36fc8e
```

PolygonContainmentTree.h

Description: building tree of polygon containment

Memory: $\mathcal{O}(N)$

Time: $\mathcal{O}(N \log N)$

59c16e, 44 lines

```
struct P { ll x, y; };

int current_x;
struct Segment {
  int idx; P p1, p2; bool is_upper;
  Segment(P p, P q, int i): idx(i), p1(p), p2(q), is_upper(p2.x
        < p1.x) { if (is_upper)swap(p1, p2); }
  ld get_y(ll x) const { return (ld) (p2.y - p1.y) / (p2.x - p1
      .x) * (x - p1.x) + p1.y; }
  tuple<ld, bool, int> get_comp() const { return {get_y(
      current_x), is_upper, p2.x}; }
  bool operator<(const Segment & o) const { return get_comp() <
        o.get_comp(); }
}; // fc8b4f

vector<int> build(vector<vector<P>>& polygons) {
  int n = sz(polygons);
  vector<tuple<int, int, int, Segment>> edges; // polygon edges
  rep(idx, 0, n) {
    const auto & v = polygons[idx];
    rep(i, 0, sz(v)) {
      int j = (i + 1) % sz(v);
      if (v[i].x == v[j].x)continue; // ignores vertical edges
      Segment seg = Segment(v[i], v[j], idx);
      edges.eb(seg.p1.x, 0, -seg.p1.y, seg);
      edges.eb(seg.p2.x, 1, -seg.p2.y, seg);
    } // b28b76
  } // 2603da
  sort(edges.begin(), edges.end());
  set&lt;Segment&gt; s;
  vector pai(n+1, n), vis(n, 0);
  for (auto [l, t, y, seg]: edges) {
    current_x = l;
    int i = seg.idx;
    if (t == 0) {
      if (not vis[i]) {
        vis[i] = true;
        auto it = s.upper_bound(seg);
        if (it == s.end())pai[i] = n+q;
        else if (it->is_upper)pai[i] = it->idx;
        else pai[i] = pai[it->idx];
      } // a8607f
      s.insert(seg);
    } // 3aede6
    else s.erase(seg);
  } // f96148
  return pai;
} // e3cb8b
```

4.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  } // 5b096c
  return ret.second;
} // bf22c6
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  } // ca4da5

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    } // 31010d
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    } // 66e741
  } // 2044ae
}; // a77e97

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    } // 1199af

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  } // 74c273

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  } // 94cda0
}; // 6f5c51
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H; // 18059e

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
} // 6aff7b
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
} // b3b5b1
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
} // 86ce01
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
} // 4a4fc2

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  } // c9e598

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
```

```
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    } // a2e9b5
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  } // fcf7ef
  return { ra, rb };
} // 7cf639

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); } // 43
      e195 // 43e195
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
} // a02307
```

4.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
} // fca9df
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); } // 8eef6b
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); } // bd6a08
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  } // a77b7e
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  } // 73af70
}; // 8058ae
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
}; // cf7c9e

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  }; // d73a06
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      } // 5cd5dc
    } // 220067
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    } // 248ed4
  } // 47289c
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
}; // be2ca2
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
} // 4fa19e
```

Graph (5)

5.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.

Time: $\mathcal{O}(VE)$

830a8f, 23 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int s) {
  nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
  rep(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);
    } // 452019
  } // 75a370
  rep(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;
  } // 1d7315
} // fa39de
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] =$ `inf` if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.

Time: $\mathcal{O}\left(N^3\right)$

531245, 12 lines

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);
```

```
  } // f38e9e
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
} // f12f13
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V|+|E|)$ d678d8, 8 lines

```
vi topoSort(const vector<vi>& gr) {
  vi indeg(sz(gr)), q;
  for (auto& li : gr) for (int x : li) indeg[x]++;
  rep(i,0,sz(gr)) if (indeg[i] == 0) q.push_back(i);
  rep(j,0,sz(q)) for (int x : gr[q[j]])
    if (--indeg[x] == 0) q.push_back(x);
  return q;
} // d678d8
```

FunctGraph.h

Description: Functional Graph
Memory: $\mathcal{O}(n)$
Time: $\mathcal{O}(n)$ 152fc5, 25 lines

```
struct FunctGraph{
  int n;
  vi head, comp;
  vector<vi> gr, cycles;

  FunctGraph(vi& fn):
    n(sz(fn)), head(n, -1), comp(n), gr(n) {
    rep(i, 0, n)gr[fn[i]].pb(i);
    vi visited(n, 0);
    auto dfs = [&](auto rec, int v, int c) -> void{
      head[v] = c; visited[v] = 1;
      for(int f : gr[v])if (head[f]!=f)rec(rec, f, c);
    }; // e1fa06
    rep(i, 0, n){
      if (visited[i])continue;
      int l=fn[i], r=fn[fn[i]];
      while(l!=r) l=fn[l], r=fn[fn[r]];
      vi cur = {r};
      for(l=fn[l]; l!=r; l=fn[l]) cur.pb(l);
      for(int x : cur) head[x] = x, comp[x] = sz(cycles);
      cycles.pb(cur);
      for(int x : cur) dfs(dfs, x, x);
    } // 01a153
  } // 0a2937
}; // 152fc5
```

5.2 Network flow

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.
Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ 0ae1d4, 48 lines

```
struct PushRelabel {
  struct Edge {
    int dest, back;
    ll f, c;
  }; // 571434
  vector<vector<Edge>> g;
  vector<ll> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}

  void addEdge(int s, int t, ll cap, ll rcap=0) {
    if (s == t) return;
    g[s].push_back({t, sz(g[t]), 0, cap});
    g[t].push_back({s, sz(g[s])-1, 0, rcap});
  } // 817b95

  void addFlow(Edge& e, ll f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  } // 340b4e
  ll calc(int s, int t) {
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;
    rep(i,0,v) cur[i] = g[i].data();
    for (Edge& e : g[s]) addFlow(e, e.c);

    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
          for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest]+1)
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            rep(i,0,v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];
        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1) //
            aafe8e
          addFlow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
    } // 1206ba
  } // 291fbf
  bool leftOfMinCut(int a) { return H[a] >= sz(g); }
}; // 0ae1d4
```

MinCostMaxFlow.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi. 58385b, 79 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;

struct MCMF {
  struct edge {
    int from, to, rev;
    ll cap, cost, flow;
  }; // 092ff8
  int N;
  vector<vector<edge>> ed;
  vi seen;
  vector<ll> dist, pi;
  vector<edge*> par;

  MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    if (from == to) return;
    ed[from].push_back(edge{ from,to,sz(ed[to]),cap,cost,0 });
    ed[to].push_back(edge{ to,from,sz(ed[from])-1,0,-cost,0 });
  } // c71528

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({ 0, s });

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (edge& e : ed[s]) if (!seen[e.to]) {
        ll val = di - pi[e.to] + e.cost;
        if (e.cap - e.flow > 0 && val < dist[e.to]) {
          dist[e.to] = val;
          par[e.to] = &e;
          if (its[e.to] == q.end())
            its[e.to] = q.push({ -dist[e.to], e.to });
          else
            q.modify(its[e.to], { -dist[e.to], e.to });
        } // ca07f4
      } // 4cd18f
    } // 062b8f
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  } // 7e4cbe

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (edge* x = par[t]; x; x = par[x->from])
        fl = min(fl, x->cap - x->flow);

      totflow += fl;
      for (edge* x = par[t]; x; x = par[x->from]) {
        x->flow += fl;
        ed[x->to][x->rev].flow -= fl;
      } // 3bfaf3
    } // 8d9a6a
    rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost * e.flow;
    return {totflow, totcost/2};
  } // 24f5a0

  // If some costs can be negative, call this before maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        for (edge& e : ed[i]) if (e.cap)
          if ((v = pi[i] + e.cost) < pi[e.to])
            pi[e.to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  } // 6847d8
}; // b3692f
```

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only. 482fe0, 36 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T>>&
    graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;
```

```
  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;

    rep(i,0,ptr) {
      int x = q[i];
      for (auto e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        } // 3a4373
      } // 6e8ea0
    } // 56e958
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    } // 548c55
  } // ff82bd
} // 261f29
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    } // ec93df
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  } // ca0062
  return best;
} // 8b0e19
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}(V)$ Flow Computations

"PushRelabel.h" 0418b3, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    PushRelabel D(N); // Dinic also works
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
    tree.push_back({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
  } // 93c5ff
  return tree;
} // 65c0c2
```

5.3 Matching

HopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and r should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $r[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Time: $\mathcal{O}\left(E\sqrt{V}\right)$

731cfb, 20 lines

```
int hopcroftKarp(vector<vi>& g, vi& r) {
  int n = sz(g), res = 0;
  vi l(n, -1), q(n), d(n);
  auto dfs = [&](auto f, int u) -> bool {
    int t = exchange(d[u], 0) + 1;
    for (int v : g[u])
      if (r[v] == -1 || (d[r[v]] == t && f(f, r[v])))
        return l[u] = v, r[v] = u, 1;
    return 0;
  }; // a95e38
  for (int t = 0, f = 0;; t = f = 0, d.assign(n, 0)) {
    rep(i,0,n) if (l[i] == -1) q[t++] = i, d[i] = 1;
    rep(i,0,t) for (int v : g[q[i]]) {
      if (r[v] == -1) f = 1;
      else if (!d[r[v]]) d[r[v]] = d[q[i]] + 1, q[t++] = r[v];
    } // 64af74
    if (!f) return res;
    rep(i,0,n) if (l[i] == -1) res += dfs(dfs, i);
  } // cdf3b2
} // 731cfb
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}(VE)$

522b98, 22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di;
      return 1;
    } // 6ba49a
  return 0;
} // d13a81
int dfsMatching(vector<vi>& g, vi& btoa) {
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i])
      if (find(j, g, btoa, vis)) {
        btoa[j] = i;
        break;
      } // 829ce5
  } // df282b
  return sz(btoa) - (int)count(all(btoa), -1);
} // f24825
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" da4196, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    } // 46e035
  } // 069994
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
} // da4196
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
Time: $\mathcal{O}\left(N^2M\right)$

1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      } // b7c105
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      } // 8c9ba2
      j0 = j1;
    } while (p[j0]); // 546805
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    } // f55064
```

```
  } // 1f3f03
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
} // 1e0fe9
```

GeneralMatching.h

Description: Matching for general graphs. 1-indexed vertices.
Memory: $\mathcal{O}(n+m)$
Time: $\mathcal{O}\left(\sqrt{n}m \log_{\max\{2,1+m/n\}} n\right)$

9437b5, 314 lines

```
class MaximumMatching {
  public:
   struct Edge {int from, to;};
   static constexpr int Inf = 1 << 30;

  private:
   enum Label {
       kInner = -1,  // should be < 0
       kFree = 0     // should be 0
   }; // c09e7e
   struct Link {int from, to;};
   struct Log {int v, par;};
   struct LinkedList {
       LinkedList() {}
       LinkedList(int N, int M) : N(N), next(M) { clear(); }
       void clear() { head.assign(N, -1); }
       void push(int h, int u) { next[u] = head[h], head[h] =
            u; }
       int N; vector<int> head, next;
   }; // 1a51c7
   template <typename T> struct Queue {
       Queue() {}
       Queue(int N) : qh(0), qt(0), data(N) {}
       T operator[](int i) const { return data[i]; }
       void enqueue(int u) { data[qt++] = u; }
       int dequeue() { return data[qh++]; }
       bool empty() const { return qh == qt; }
       void clear() { qh = qt = 0; }
       int size() const { return qt; }
       int qh, qt; vector<T> data;
   }; // 4bfceb
   struct DisjointSetUnion {
       DisjointSetUnion() {}
       DisjointSetUnion(int N) : par(N) { for (int i = 0; i <
            N; ++i) par[i] = i; }
       int find(int u) { return par[u] == u ? u : (par[u] =
            find(par[u])); }
       void unite(int u, int v) {
           u = find(u), v = find(v);
           if (u != v) par[v] = u;
       } // 461e55
       vector<int> par;
   }; // a21f68
  public:
   MaximumMatching(int N, const vector<Edge> &in)
       : N(N), NH(N >> 1), ofs(N + 2, 0), edges(in.size() * 2)
            {
       for (auto &e : in) ofs[e.from + 1] += 1, ofs[e.to + 1]
            += 1;
       for (int i = 1; i <= N + 1; ++i) ofs[i] += ofs[i - 1];
       for (auto &e : in) {
           edges[ofs[e.from]++] = e;
           edges[ofs[e.to]++] = {e.to, e.from};
       } // 36bba8
       for (int i = N + 1; i > 0; --i) ofs[i] = ofs[i - 1];
       ofs[0] = 0;
   } // a78156
   int maximum_matching() {
       initialize(); int match = 0;
       while (match * 2 + 1 < N) {
           reset_count();
           bool has_augmenting_path = do_edmonds_search();
           if (!has_augmenting_path) break;
           match += find_maximal();
           clear();
       } // 8c0a2c
       return match;
   } // da7b99
   vector<Edge> get_edges() {
       vector<Edge> ans;
       for (int i = 1; i <= N; ++i) if (mate[i] > i) ans.
            push_back(Edge{i, mate[i]});
       return ans;
   } // f8586b
  private:
   void reset_count() {
       time_current_ = 0;
       time_augment_ = Inf;
       contract_count_ = 0;
       outer_id_ = 1;
       dsu_changelog_size_ = dsu_changelog_last_ = 0;
   } // 6c73d5
   void clear() {
       que.clear();
       for (int u = 1; u <= N; ++u) potential[u] = 1;
       for (int u = 1; u <= N; ++u) dsu.par[u] = u;
       for (int t = time_current_; t <= N / 2; ++t) list.head[
            t] = -1;
       for (int u = 1; u <= N; ++u) blossom.head[u] = -1;
   } // 6e020f
   inline void grow(int x, int y, int z) {
       label[y] = kInner;
       potential[y] = time_current_;  // visited time
       link[z] = {x, y};
       label[z] = label[x];
       potential[z] = time_current_ + 1;
       que.enqueue(z);
   } // 5a596b
   void contract(int x, int y) {
       int bx = dsu.find(x), by = dsu.find(y);
       const int h = -(++contract_count_) + kInner;
       label[mate[bx]] = label[mate[by]] = h;
       int lca = -1;
       while (1) {
           if (mate[by] != 0) swap(bx, by);
           bx = lca = dsu.find(link[bx].from);
           if (label[mate[bx]] == h) break;
           label[mate[bx]] = h;
       } // d03cf2
       for (auto bv : {dsu.par[x], dsu.par[y]}) {
           for (; bv != lca; bv = dsu.par[link[bv].from]) {
               int mv = mate[bv];
               link[mv] = {x, y};
               label[mv] = label[x];
               potential[mv] =
                   1 + (time_current_ - potential[mv]) +
                        time_current_;
               que.enqueue(mv);
               dsu.par[bv] = dsu.par[mv] = lca;
               dsu_changelog[dsu_changelog_last_++] = {bv, lca
                    };
               dsu_changelog[dsu_changelog_last_++] = {mv, lca
                    };
           } // 8fef02
       } // 2b0c42
   } // a9547e
   bool find_augmenting_path() {
       while (!que.empty()) {
           int x = que.dequeue(), lx = label[x], px =
                potential[x],
               bx = dsu.find(x);
           for (int eid = ofs[x]; eid < ofs[x + 1]; ++eid) {
               int y = edges[eid].to;
               if (label[y] > 0) {  // outer blossom/vertex
                   int time_next = (px + potential[y]) >> 1;
                   if (lx != label[y]) {
                       if (time_next == time_current_) return
                            true;
                       time_augment_ = min(time_next,
                            time_augment_);
                   } else { // a6d4c5
                       if (bx == dsu.find(y)) continue;
                       if (time_next == time_current_) {
                           contract(x, y);
                           bx = dsu.find(x);
                       } else if (time_next <= NH) // 1efde3
                           list.push(time_next, eid);
                   } // 1666ad
               } else if (label[y] == kFree) {  // free vertex
                    // c17e44
                   int time_next = px + 1;
                   if (time_next == time_current_)
                       grow(x, y, mate[y]);
                   else if (time_next <= NH)
                       list.push(time_next, eid);
               } // 48780d
           } // 30c821
       } // 1f7dd0
       return false;
   } // 5f1707
   bool adjust_dual_variables() {
       const int time_lim = min(NH + 1, time_augment_);
       for (++time_current_; time_current_ <= time_lim; ++
            time_current_) {
           dsu_changelog_size_ = dsu_changelog_last_;
           if (time_current_ == time_lim) break;
           bool updated = false;
           for (int h = list.head[time_current_]; h >= 0; h =
                list.next[h]) {
               auto &e = edges[h];
               int x = e.from, y = e.to;
               if (label[y] > 0) {
                   if (potential[x] + potential[y] != (
                        time_current_ << 1))
                       continue;
                   if (dsu.find(x) == dsu.find(y)) continue;
                   if (label[x] != label[y]) {
                       time_augment_ = time_current_;
                       return false;
                   } // e7c864
                   contract(x, y);
                   updated = true;
               } else if (label[y] == kFree) { // 1a7dca
                   grow(x, y, mate[y]);
                   updated = true;
               } // ebac0d
           } // 7bf910
           list.head[time_current_] = -1;
           if (updated) return false;
       } // a2963f
       return time_current_ > NH;
   } // 361891
   bool do_edmonds_search() {
       label[0] = kFree;
       for (int u = 1; u <= N; ++u) {
           if (mate[u] == 0) {
               que.enqueue(u);
```

```
            label[u] = u;  // component id
        } else // 52e61c
            label[u] = kFree;
    } // 585bc4
    while (1) {
        if (find_augmenting_path()) break;
        bool maximum = adjust_dual_variables();
        if (maximum) return false;
        if (time_current_ == time_augment_) break;
    } // 3b82c1
    for (int u = 1; u <= N; ++u) {
        if (label[u] > 0)
            potential[u] -= time_current_;
        else if (label[u] < 0)
            potential[u] = 1 + (time_current_ - potential[u
                ]);
    } // c77cf7
    return true;
} // e1fc96
void rematch(int v, int w) {
    int t = mate[v];
    mate[v] = w;
    if (mate[t] != v) return;
    if (link[v].to == dsu.find(link[v].to)) {
        mate[t] = link[v].from;
        rematch(mate[t], t);
    } else { // 45eff2
        int x = link[v].from, y = link[v].to;
        rematch(x, y);
        rematch(y, x);
    } // 4a404f
} // c4e2e9
bool dfs_augment(int x, int bx) {
    int px = potential[x], lx = label[bx];
    for (int eid = ofs[x]; eid < ofs[x + 1]; ++eid) {
        int y = edges[eid].to;
        if (px + potential[y] != 0) continue;
        int by = dsu.find(y), ly = label[by];
        if (ly > 0) {  // outer
            if (lx >= ly) continue;
            int stack_beg = stack_last_;
            for (int bv = by; bv != bx; bv = dsu.find(link[
                bv].from)) {
                int bw = dsu.find(mate[bv]);
                stack[stack_last_++] = bw;
                link[bw] = {x, y};
                dsu.par[bv] = dsu.par[bw] = bx;
            } // 211d3e
            while (stack_last_ > stack_beg) {
                int bv = stack[--stack_last_];
                for (int v = blossom.head[bv]; v >= 0;
                    v = blossom.next[v]) {
                    if (!dfs_augment(v, bx)) continue;
                    stack_last_ = stack_beg;
                    return true;
                } // 9ded5e
            } // 476c04
        } else if (ly == kFree) { // e5357f
            label[by] = kInner;
            int z = mate[by];
            if (z == 0) {
                rematch(x, y);
                rematch(y, x);
                return true;
            } // 781371
            int bz = dsu.find(z);
            link[bz] = {x, y};
            label[bz] = outer_id_++;
            for (int v = blossom.head[bz]; v >= 0; v =
                blossom.next[v]) {
                if (dfs_augment(v, bz)) return true;
            } // 5ea9e4
        } // 1c508f
    } // 7f2ab3
    return false;
} // 44c41a
int find_maximal() {
    for (int u = 1; u <= N; ++u) dsu.par[u] = u;
    for (int i = 0; i < dsu_changelog_size_; ++i) {
        dsu.par[dsu_changelog[i].v] = dsu_changelog[i].par;
    } // 384510
    for (int u = 1; u <= N; ++u) {
        label[u] = kFree;
        blossom.push(dsu.find(u), u);
    } // 2a02fd
    int ret = 0;
    for (int u = 1; u <= N; ++u)
        if (!mate[u]) {
            int bu = dsu.par[u];
            if (label[bu] != kFree) continue;
            label[bu] = outer_id_++;
            for (int v = blossom.head[bu]; v >= 0; v =
                blossom.next[v]) {
                if (!dfs_augment(v, bu)) continue;
                ret += 1;
                break;
            } // e4fb6a
        } // 4db9e3
    assert(ret >= 1); return ret;
} // 500171
void initialize() {
    que = Queue<int>(N);
    mate.assign(N + 1, 0);
    potential.assign(N + 1, 1);
    label.assign(N + 1, kFree);
    link.assign(N + 1, {0, 0});
    dsu_changelog.resize(N);
    dsu = DisjointSetUnion(N + 1);
    list = LinkedList(NH + 1, edges.size());
    blossom = LinkedList(N + 1, N + 1);
    stack.resize(N);
    stack_last_ = 0;
} // be011c
private:
const int N, NH;
vector<int> ofs;
vector<Edge> edges;
Queue<int> que;
vector<int> mate, potential;
vector<int> label;
vector<Link> link;
vector<Log> dsu_changelog;
int dsu_changelog_last_, dsu_changelog_size_;
DisjointSetUnion dsu;
LinkedList list, blossom;
vector<int> stack;
int stack_last_;
int time_current_, time_augment_;
int contract_count_, outer_id_;
}; // b7724d
using Edge = MaximumMatching::Edge;
void example() { // Graph of Love problem
    int n; cin >> n;
    vector<Edge> edges(n, {0, 0});
    for (int i = 1, j; i <= n; i++)cin >> j, edges[i-1] = {i, j
        };
    auto M = MaximumMatching(n, edges);
    cout << M.maximum_matching() << '\n';
} // 0523ec
```

OnlineMatching.h

Description: Modified khun developed for specific question able to run $2 * 10^6$ queries, in $2 * 10^6$ x 10^6 graph in 3 seconds codeforces

Time: $\mathcal{O}(confia)$

6ac539, 42 lines

```
struct OnlineMatching {
    int n = 0, m = 0;
    vector<int> vis, match, dist;
    vector<vector<int>> g;
  vector<int> last;
  int t = 0;

    OnlineMatching(int n_, int m_) : n(n_), m(m_),
    vis(n, 0), match(m, -1), dist(n, n+1), g(n), last(n, -1)
    {}

    void add(int a, int b) {
        g[a].pb(b);
    } // 746097

    bool kuhn(int a) {
    vis[a] = t;
    for(int b: g[a]) {
        int c = match[b];
        if (c == -1) {
        match[b] = a;
        return true;
        } // 38b210
        if (last[c] != t || (dist[a] + 1 < dist[c]))
        dist[c] = dist[a] + 1, last[c] = t;
    } // d30675
    for (int b: g[a]) {
      int c = match[b];
      if (dist[a] + 1 == dist[c] && vis[c] != t && kuhn(c)) {
        match[b] = a;
        return true;
      } // 2dac75
    } // e58bd5
        return false;
    } // b533ee
  bool can_match(int a) {
    t++;
    last[a] = t;
    dist[a] = 0;
    return kuhn(a);
  } // 32302b
}; // 6ac539
```

5.4 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.

Time: $\mathcal{O}(E + V)$

76b5c9, 24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F& f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  for (auto e : g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));
```

```
  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j); // ae85bd
    f(cont); cont.clear();
    ncomps++;
  } // 64c1b9
  return val[j] = low;
} // 3513bd
template<class G, class F> void scc(G& g, F f) {
  int n = sz(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
} // 56b050
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph. In a biconnected component there are at least two internally disjoint paths between any two nodes a cycle exists through them. Note that a node can be in several components. Every edge is in a single component. Nodes without edges will not be in any components

Time: $\mathcal{O}(E+V)$ 61b438, 53 lines

```
vector<pair<int, int>> edges; // edges
vector<vector<pair<int, int>>> g; // [b, edge idx]
vi tin, st, art;
int dfstime = 0;
vector<vi> bcc;

int dfs(int a, int p) {
  int top = tin[a] = ++dfstime;
  bool child = (p != -1);
  for (auto [b, e]: g[a]) {
    if (tin[b]) {
      top = min(top, tin[b]);
      if (tin[b] < tin[a]) st.pb(e);
    } // 0df373
    else {
      int si = sz(st);
      int up = dfs(b, e);
      top = min(top, up);
      if (up > tin[a]) { /*e is a bridge */}
      if (up >= tin[a]) {
        st.pb(e);
        bcc.eb(st.begin() + si, st.end());
        st.resize(si);
        art[a] += child;
        child = true;
      } // 46103a
      else if (up < tin[a]) st.pb(e);
    } // 38ea8a
  } // 53a60e
  return top;
} // 17df2f

void bicomps() {
  int n = sz(g);
  tin.assign(n, 0), art.assign(n, 0);
  rep(i,0,n) if (!tin[i]) dfs(i, -1);
} // 600614

vi comp;
vector<vi> tree;
void build_tree() { // Optional
  int n = sz(g);
  comp.resize(n), tree.resize(n + sz(bcc));
  rep(i, 0, sz(bcc)) {
    for (int eid: bcc[i]) {
      auto [a, b] = edges[eid];
      if (art[a] && (empty(tree[a]) || tree[a].back() != n+i))
          tree[a].pb(n+i), tree[n+i].pb(a);
      if (art[b] && (empty(tree[b]) || tree[b].back() != n+i))
          tree[b].pb(n+i), tree[n+i].pb(b);
      comp[a] = comp[b] = n + i;
    } // 2e7ad5
  } // 8e4c66
  rep(i, 0, n) if (art[i]) comp[i] = i;
} // b645e9
```

TwoCC.h

Description: Finds all two edge connected components in an undirected graph.

Time: $\mathcal{O}(E+V)$ af5478, 44 lines

```
vector<vector<pair<int, int>>> g;
vector<pair<int, int>> edges;
vi tin, st, bridges;
int dfstime = 0;
vector<vi> twocc;

int dfs(int a, int p) {
  int top = tin[a] = ++dfstime;
  int si = st.size();
  st.pb(a);
  for (auto [b, e]: g[a]) if (e != p) {
    if (tin[b]) top = min(top, tin[b]);
    else {
      int up = dfs(b, e);
      top = min(top, up);
      if (up > tin[a]) bridges.pb(e);
    } // 1dd281
  } // ada038
  if (top == tin[a]) {
    twocc.eb(st.begin() + si, st.end());
    st.resize(si);
  } // 0f03f3
  return top;
} // 55bb17

void twocomps() {
  int n = sz(g);
  tin.assign(n, 0);
  rep(i, 0, n) if (!tin[i]) dfs(i, -1);
} // 9a2d22

vi comp;
vector<vi> tree;
void build_tree() { // Optional
  int n = sz(g);
  tree.resize(sz(twocc)); comp.resize(n);
  rep(i, 0, sz(twocc))
    for (int a: twocc[i]) comp[a] = i;

  for (int eid: bridges) {
    auto [a, b] = edges[eid];
    tree[comp[a]].pb(comp[b]), tree[comp[b]].pb(comp[a]);
  } // 1a7e1f
} // c306f0
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (∼x).

Usage: `TwoSat ts(number of boolean variables);`
```
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses. 5f9706, 56 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false , 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  } // 7b5f84

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  } // 516db0
  void setValue(int x) { either(x, x); }

  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    } // 8d3782
    either(cur, ~li[1]);
  } // 10f2ea

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i); // b15351
    return val[i] = low;
  } // ef583a

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  } // 2bb76d
}; // 5f9706
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$ 780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }} // 22a87a // 94de26
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()};
} // 780b64
```

DominatorTree.h
Description: Dominator Tree, creates the graph `tree`, where all ancestors of a `u` in the tree are necessary in the path from the `root` to `u`
Memory: $\mathcal{O}(n)$
Time: $\mathcal{O}((n+m)log(n))$ build 69af96, 57 lines

```
struct DominatorTree {
  int n;
  vector<vector<int>> g, gt, tree, bucket, down;
  vector<int> S;
  vector<int> dsu, label, sdom, idom, id;
  int dfstime =0;

  DominatorTree(vector<vector<int>> & _g, int root)
    : n(sz(_g)), g(_g), gt(n), tree(n), bucket(n), down(n),
    S(n), dsu(n), label(n), sdom(n), idom(n), id(n) {
    prep(root); reverse(S.begin(), S.begin() + dfstime);
    for(int u : S) {
      for(int v : gt[u]) {
        int w = fnd(v);
        if(id[ sdom[w] ] < id[ sdom[u] ])
          sdom[u] = sdom[w];
      } // e059b2
      gt[u].clear();
      if(u != root) bucket[ sdom[u] ].push_back(u);
      for(int v : bucket[u]) {
        int w = fnd(v);
        if(sdom[w] == sdom[v]) idom[v] = sdom[v];
        else idom[v] = w;
      } // 72077b
      bucket[u].clear();
      for(int v : down[u]) dsu[v] = u;
      down[u].clear();
    } // 3197c4
    reverse(S.begin(), S.begin() + dfstime);
    for(int u : S) if(u != root) {
      if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
      tree[ idom[u] ].push_back(u);
    } // 96e582
    idom[root] = root;
  } // b239ba
  void prep(int u){
    S[dfstime] = u;
    id[u] = ++dfstime;
    label[u] = sdom[u] = dsu[u] = u;

    for(int v : g[u]){
      if(!id[v])
        prep(v), down[u].push_back(v);
      gt[v].push_back(u);
    } // 4d7944
  } // 4351b9

  int fnd(int u, int flag = 0){
    if(u == dsu[u]) return u;
    int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
    if(id[ sdom[b] ] < id[ sdom[ label[u] ] ])
      label[u] = b;
    dsu[u] = v;
    return flag ? v : label[u];
  } // d64927

}; // 69af96
```

5.5 Coloring

EdgeColoring.h
Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$ e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    } // 316eb7
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  } // fdc6d3
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
} // e210e2
```

5.6 Heuristics

MaximalCliques.h
Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  } // 181f8f
} // c9dc5f
```

MaximumClique.h
Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs. f7c0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  } // 7c428e
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        } // 3221ac
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q; // 2a0537
      q.pop_back(), R.pop_back();
    } // 87639b
  } // f0a49d
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  } // 36accb
}; // b63641
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

5.7 Trees

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.

Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 7347c3, 26 lines

```
struct LCA {
  vi time;
  int lca(int a, int b) { return -1; }
}; // 32d408

typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b));
  } // 677c62
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  } // 5efe90
  return ret;
} // 83c9a2
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

Time: $\mathcal{O}\left((\log N)^2\right)$

"../data-structures/LazySegmentTree.h" 9547af, 46 lines

```
template <bool VALS_EDGES> struct HLD {
  int N, tim = 0;
  vector<vi> adj;
  vi par, siz, rt, pos;
  Node *tree;
  HLD(vector<vi> adj_)
    : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1),
      rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0); dfsHld(0); }
  void dfsSz(int v) {
    for (int& u : adj[v]) {
      adj[u].erase(find(all(adj[u]), v));
      par[u] = v;
      dfsSz(u);
      siz[v] += siz[u];
      if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
    } // 9f610f
  } // db817b
  void dfsHld(int v) {
    pos[v] = tim++;
    for (int u : adj[v]) {
      rt[u] = (u == adj[v][0] ? rt[v] : u);
      dfsHld(u);
    } // ee65b7
  } // 044fde
  template <class B> void process(int u, int v, B op) {
    for (;; v = par[rt[v]]) {
      if (pos[u] > pos[v]) swap(u, v);
      if (rt[u] == rt[v]) break;
      op(pos[rt[v]], pos[v] + 1);
    } // 00190c
    op(pos[u] + VALS_EDGES, pos[v] + 1);
  } // 431b66
  void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tree->add(l, r, val); });
  } // a181b8
  int queryPath(int u, int v) { // Modify depending on problem
    int res = -1e9;
    process(u, v, [&](int l, int r) {
        res = max(res, tree->query(l, r));
    }); // b1dde7
    return res;
  } // 1a6944
  int querySubtree(int v) { // modifySubtree is similar
    return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);
  } // e86b89
}; // 9547af
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.

Time: All operations take amortized $\mathcal{O}(\log N)$.

0fb462, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  } // 454758
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  } // 0cc949
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      y->c[h ^ 1] = x;
    } // 1a82cf
    z->c[i ^ 1] = this;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  } // 1cf643
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    } // e639f4
  } // bfb1f7
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
  } // 67f9a1
}; // 225109

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  } // 60799e
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    } // 8acbe8
  } // a58ec7
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  } // b80a22
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    } // 586a65
  } // 74c908
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; } // 1ccdfc
      pp->c[1] = u; pp->fix(); u = pp;
    } // b10f33
    return u;
  } // 4ac291
}; // ceab83
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
```

```cpp
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  } // 0d348f
  Edge top() { prop(); return key; }
}; // ab4902
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
} // c5109e
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      } // 00a339
    } // c8f0da
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  } // fa3c2c

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  } // 4f9b56
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
} // efa3a4
```

Data structures (6)

6.1 General

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```cpp
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
}; // cdd37e
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});
```

OrderedSet.h

Description: ordered/indexed set and multiset Bad constant factor, works only in Linux
Memory: $\mathcal{O}(N)$
Time: $\mathcal{O}(\log N)$ operations

<bits/extc++.h> //include it before any defines — b8c30a, 15 lines

```cpp
using namespace __gnu_pbds;
template<class T, class B = null_type> using ordered_set = tree
    <T, B, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

template<class T>
struct ordered_multiset{
  ordered_set<pair<T, int>> o; int c;
  ordered_multiset():c(0){}
  unsigned order_of_key(T x){return o.order_of_key({x, -1});}
  const T* find_by_order(int p){return &(*o.find_by_order(p)).
      first;}
  void insert(T x){o.insert({x, c++});}
  void erase(T x){o.erase(o.lower_bound({x, 0}));}
  unsigned size(){return o.size();}
  const T* lower_bound(T x){return &(*o.lower_bound({x, 0})).
      first;}
  const T* upper_bound(T x){return &(*o.upper_bound({x, c})).
      first;}
}; // d4e54f
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: `int t = uf.time(); ...; uf.rollback(t);`
Time: $\mathcal{O}(\log(N))$

de4ad0, 21 lines

```cpp
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  } // 30bb61
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  } // 6c709f
}; // de4ad0
```

StaticCHT.h

Description: static CHT - add must be ordered by slope (a), queries by x.
Time: amortized $\mathcal{O}(1)$.

da7e40, 28 lines

```cpp
struct CHT {
  int it; vector<ll> a, b;
  CHT():it(0){}
  ll eval(int i, ll x){return a[i]*x + b[i];}
  bool useless(){
    int sz = a.size();
    int r = sz-1, m = sz-2, l = sz-3;
#warning careful with overflow!
    return (b[l] - b[r])*(a[m] - a[l]) <
      (b[l] - b[m])*(a[r] - a[l]);
  } // c37135
  void add(ll A, ll B){
    a.push_back(A); b.push_back(B);
    while (!a.empty()){
      if ((a.size() < 3) || !useless()) break;
      a.erase(a.end() - 2);
      b.erase(b.end() - 2);
    } // b21fc8
    it = min(it, int(a.size()) - 1);
  } // 6df532
  ll get(ll x){
    while (it+1 < a.size()){
      if (eval(it+1, x) > eval(it, x)) it++;
      else break;
    } // fe9dba
    return eval(it, x);
  } // b44949
}; // da7e40
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming (“convex hull trick”).
Time: $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```cpp
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
}; // 7e3ecf

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); } // 10f081
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  } // 2fac86
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  } // 08625f
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  } // d21e2f
}; // 5771f0
```

6.2 Range Queries

Spec.h

Description: Algebraic structures for RQDS. Includes Ruan's version.
Time: varies

6260e7, 67 lines

```cpp
struct Group { //sum
  using T = int;
  static constexpr T id = 0;
  static T op(T a, T b){return a+b;}
```

```
  static T inv(T a){return -a;}
}; // 3e6767

struct LazySpecRuan { //sum
  using S = int;
  using K = int;
  static S op(S a, S b) { return max(a, b); }
  static S update(K f, S a) { return f + a; }
  static K compose(const K f, const K g) { return f + g; }
  static S id() { return 0; }
}; // 69c889

struct LazySpecArthur { //set add sum
  using T = ll;
  using L = pair<int, ll>;
  static constexpr T id = 0;
  static T op(T a, T b){return a + b;}
  static T ch(T past, L upd, int lx, int rx){
    ll s = rx-lx;
    auto [t, x] = upd;
    if (t)return s*x;
    else return past+s*x;
  } // 442906
  static L cmp(L cur, L upd){
    auto [t1, x1] = cur;
    auto [t2, x2] = upd;
    if (t2)return upd;
    else return {t1, x1+x2};
  } // e78ccc
}; // c4da38

struct node{int max1,max2,maxc; ll sum;};
struct BeatsSpec{ //chmin sum
  using T = node; using L = int;
  static constexpr T id = node{-oo,-oo,0,0};
  static T op(T a, T b){
    node n;
    if (a.max1 > b.max1){
      n.max1 = a.max1;
      n.max2 = max(a.max2, b.max1);
      n.maxc = a.maxc;
    } // 60044a
    else if (a.max1 == b.max1){
      n.max1 = a.max1;
      n.max2 = max(a.max2, b.max2);
      n.maxc = a.maxc+b.maxc;
    } // 89a062
    else{
      n.max1 = b.max1;
      n.max2 = max(b.max2, a.max1);
      n.maxc = b.maxc;
    } // dbf86a
    n.sum=a.sum+b.sum;
    return n;
  } // 7211b8
  static T ch(T a, L b, int l, int r){
    if (a.max2 <= b)a.sum -= (ll)(a.max1-b)*a.maxc, a.max1 = b;
    return a;
  } // 6f5ec4
  static L cmp(L a, L b){return min(a,b);}
  static bool brk(L a, T b){return b.max1 <= a;}
  static bool tag(L a, T b){return b.max2 < a;}
}; // 0a5678
```

Treap.h

Description: Lazy treap.
Time: $\mathcal{O}(\log N)$

103a81, 53 lines

```
#define TT template<typename S>
#define NN node<S>*
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count());
TT
struct node {
  node *l=0, *r=0;
  int y, c=1;
  S::S val, acc;
  S::K lz;
  bool hlz=0;
  node(S::S aval) :  y(uniform_int_distribution<int>(0,(int)1e9
      )(rng)), val(aval), acc(aval) {}
  void rc();
}; // 1328e4
TT
int cnt(S n) {return n ? n-> c : 0;}
TT
void prop(NN t) {
  if (!t or !t->hlz) return;
  t->hlz=0;
  t->val = S::update(t->lz, 1, t->val);
  t->acc = S::update(t->lz, t->c, t->acc);
  if (t->l) t->l->lz = t->l->hlz ? S::compose(t->l->lz, t->lz)
       : t->lz, t->l->hlz=1;
  if (t->r) t->r->lz = t->r->hlz ? S::compose(t->r->lz, t->lz)
       : t->lz, t->r->hlz=1;
} // 27acca
TT
void node<S>::rc() {
  c = cnt(l) + cnt(r) + 1;
  prop(l); prop(r);
  acc = S::op(S::op(l ? l->acc : S::id(), val), r ? r->acc : S
      ::id());
} // d7b5cc
TT
pair<NN,NN> split(NN t, int k) {
  if (k == 0) return {0, t};
  if (cnt(t->l) >= k) {
    prop(t->l);
    auto [l, r] = split(t->l, k);
    t->l = r;
    t->rc();
    return {l, t};
  } // b8bc7b
  prop(t->r);
  auto [l,r] = split(t->r, k - cnt(t->l) - 1);
  t->r = l;
  t->rc();
  return {t, r};
} // 81a46c
TT
auto merge(NN l, NN r) {
  if (!l) return r;
  if (!r) return l;
  if (l->y < r->y) return prop(l->r), l->r = merge(l->r, r), l
      ->rc(), l;
  return prop(r->l), r->l = merge(l, r->l), r->rc(), r;
} // 2649b9
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

e62fac, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  } // a388f1
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  } // 6defa0
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    } // 63f005
    return pos;
  } // ea70d8
}; // e62fac
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" 157f07, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  } // 01fc7b
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
  } // d5ca1f
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
        // aee02d
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  } // bb1454
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  } // 8334c3
}; // 157f07
```

MoQueries.h

Description: Answer interval or tree path queries. Includes interval version without deletion. If values are on tree edges, change step to add/remove the edge (a, c) and remove the initial add call (but keep in).
Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

d5377e, 76 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
```

```
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  } // 0f7fae
  return res;
} // e3731f

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  }; // 329c88
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; } //
                      3839ba
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  } // c880be
  return res;
} // ce9c1e

vector mo_no_deletion(vector<pii>& qs, int n){
  int q = sz(qs), sq = (int)sqrt(q)+1, blk = (n+sq+1)/sq;
  vector<vi> o((n+blk-1)/blk);
  rep(i,0,q)o[qs[i].first/blk].pb(i);
  for(auto& vq : o)sort(all(vq), [&](int i, int j){return qs[i
      ].second < qs[j].second;});
  vector<int> ans(q);
  rep(i,0,sz(o)){
    auto& vq = o[i];
    int l = blk*i + blk, r = l-1;
    // prepare to answer queries
    for(int qi : vq){
      auto [ql, qr] = qs[qi];
      if (qr <= l){ //if it does not extrapolate
        rep(j,ql,qr+1)add(j, 1); //solving manually
        continue;
      } // b94913
      while(r < qr)add(++r, 1);
      int ml = l; //we will move l manually
      // prepare checkpoint
      while(ml > ql)add(--ml, 0);
      ans[qi] = calc();
      // revert checkpoint: discard changes made by moving l
    } // 6569ca
  } // 09ff2d
  return ans;
} // fe1e74
```

ColorUpdate.h

Description: Adds intervals and keep information about then
Memory: $\mathcal{O}(Q)$
Time: $\mathcal{O}(Q * log(Q))$

afa378, 34 lines

```
struct ColorUpdate {
  using IT  = pair<pair<int, int>, int>;
  map<int, ll> freq; set<IT> rgs;
  vector<set<IT>::iterator> intersect(int l, int r) {
    // Return all ranges that intersects with [l, r]
    vector<set<IT>::iterator> ans;
    auto it = rgs.lower_bound(pair(pair(r+1, -1), -1));
    while(it != rgs.begin()) {
      it = prev(it);
      auto [lx, rx] = it->first;
      if (rx < l) break;
      ans.pb(it);
    } // dda9d0
    return ans;
  } // 9480c5
  void add(int l, int r, int c) {
    // Adds a range [l, r] with color c
    auto v = intersect(l, r);
    vector<IT> to_add = {{{l, r}, c}};
    for (auto it: intersect(l, r)) {
      // Remove it information
      freq[it->second] -= it->first.second - it->first.first +
          1;
      to_add.pb({{it->first.first, l-1}, it->second});
      to_add.pb({{r+1, it->first.second}, it->second});
      rgs.erase(it);
    } // 00093e
    for (auto [x, c]: to_add) {
      if (x.first > x.second) continue;
      rgs.insert({x, c});
      // Add x c information
      freq[c] += x.second - x.first + 1;
    } // 56edf2
  } // 6fd6a1
}; // afa378
```

MergeSortTree.h

Description: Merge Sort Tree
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(\log^2 N)$

7c1cf9, 38 lines

```
template<class T> struct MGST{
  int n, h; vector<vector<T>> t;
  int lg(int x){return __builtin_clz(1)-__builtin_clz(x);}
  MGST(vector<T> v): n(sz(v)), h(lg(n)){
    if (n != (1<<h))n = 1<<(++h);
    t.assign(h, vector<T>(n));
    rep(i,0,sz(v))t[0][i] = v[i];
    rep(i,sz(v),n)t[0][i] = oo; //non-existent
    rep(k,0,h)for(int i = 0, s = 1<<k; i < n; i += 2*s){
      int p1=0, p2=0;
      rep(p,i,i+2*s){
        if (p1==s)t[k+1][p] = t[k][i+s+p2], p2++;
        else if (p2==s)t[k+1][p] = t[k][i+p1], p1++;
        else if (t[k][i+p1] < t[k][i+s+p2])t[k+1][p] = t[k][i+
            p1], p1++;
        else t[k+1][p] = t[k][i+s+p2], p2++;
      } // 690730
    } // eb4c11
  } // b7e287
  T query_helper(T x, int k, int l){
    auto it = upper_bound(t[k]+l, t[k]+l+(1<<k), x);
    if (it == t[k]+l)return 0;
    else return *prev(it);
  } // ef2397
  T lb(T x, int l, int r){ //biggest <= x in [l, r]
    T ans = 0; r++;
    for(int k = 0; l < r; k++){
      if ((l>>k)&1){
        ans = max(ans, query_helper(x, k, l));
        l += 1<<k;
      } // 1bf09f
      if ((r>>k)&1){
        r -= 1<<k;
        ans = max(ans, query_helper(x, k, r));
      } // aa3bad
    } // b63c49
    return ans;
  } // 8e4027
}; // 7c1cf9
```

MPsum.h

Description: Multidimensional Psum Requires Abelian Group (op, inv, id)
Memory: $\mathcal{O}\left(N^D\right)$
Time: $\mathcal{O}\left(2^D\right)$

3c2845, 20 lines

```
#define MAs template<class... As> //multiple arguments
template<int D, class S> struct Psum{
  using T = typename S::T;
  int n; vector<Psum<D-1, S>> v;
  MAs Psum(int s, As... ds):n(s+1),v(n,Psum<D-1, S>(ds...)){}
  MAs void set(T x, int p, As... ps){v[p+1].set(x, ps...);}
  void push(Psum& p){rep(i, 1, n)v[i].push(p.v[i]);}
  void init(){rep(i, 1, n)v[i].init(),v[i].push(v[i-1]);}
  MAs T query(int l, int r, As... ps){
    return S::op(v[r+1].query(ps...),S::inv(v[l].query(ps...)))
        ;
  } // eac6a8
}; // 4f6c04
template<class S> struct Psum<0, S>{
  using T = typename S::T;
  T val=S::id;
  void set(T x){val=x;}
  void push(Psum& a){val=S::op(a.val,val);}
  void init(){}
  T query(){return val;}
}; // 713694
```

Dist.h

Description: Disjoint Sparse Table Requires Monoid (op, id)
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(\log N)$

2f4715, 21 lines

```
#define repinv(i, a, b) for(int i = (a); i >= (b); i--)
template<class S> struct DiST{
  using T = typename S::T;
  int n, h; vector<vector<T>> t;
  int lg(signed x){return __builtin_clz(1)-__builtin_clz(x);}
  DiST(vector<T> v): n(sz(v)), h(lg(n)){
    if (n != (1<<h))n = 1<<(++h);
    t.assign(h, vector<T>(n)); v.resize(n, S::id);
    for(int d = 0, s = 1; d < h; d++, s *= 2)
    for(int m = s; m < n; m += 2*s){
      t[d][m] = v[m]; t[d][m-1] = v[m-1];
      rep(i, m+1, m+s)t[d][i] = S::op(t[d][i-1], v[i]);
      repinv(i, m-2, m-s)t[d][i] = S::op(v[i], t[d][i+1]);
    } // 3b44fe
  } // 1c2aa0
  T query(int l, int r){
    if (l==r)return t[0][l];
    int k = lg(l^r);
    return S::op(t[k][l], t[k][r]);
  } // 07c10a
}; // 612c6a
```

SparseTable.h
Description: Sparse Table Requires Idempotent Monoid S (op, inv, id)
Memory: $\mathcal{O}(n \log n)$
Time: $\mathcal{O}(1)$ query, $\mathcal{O}(n \log n)$ build
276609, 14 lines

```
template<class S> struct SpTable{
  using T = typename S::T;
  int n; vector<vector<T>> tab;
  int lg(signed x){return __builtin_clz(1)-__builtin_clz(x);}
  SpTable(vector<T> v):n(sz(v)),tab(1+lg(n),vector<T>(n,S::id))
      {
    rep(i,0,n)tab[0][i] = v[i];
    rep(i,0,lg(n))rep(j,0,n-(1<<i))
      tab[i+1][j] = S::op(tab[i][j], tab[i][j+(1<<i)]);
  } // c105d7
  T query(int l, int r){
    int k = lg(++r-l);
    return S::op(tab[k][l], tab[k][r-(1<<k)]);
  } // e06689
}; // 276609
```

SqrtDecomp.h
Description: Sqrt Decompostion
Memory: $\mathcal{O}(n)$
Time: $\mathcal{O}(n)$ build, $\mathcal{O}(sqrt(n))$ queries
f45235, 41 lines

```
struct SqrtDecomp {
  using K = ll; // single element information
  using T = ll; // block information
  int n, bsz, n_block;
  vector<T> v;
  vector<int> id;
  vector<K> block;
  SqrtDecomp(const vector<T> & x): n(sz(x)), v(x), id(n) {
    bsz = sqrt(n) + 1;
    n_block = (n + bsz - 1) / bsz; // ceil(n, bsz)
    rep(i, 0, n) id[i] = i / bsz;
    // Add information to block
    block = vector<K>(n_block, oo);
    rep(i, 0, n) block[id[i]] = min(block[id[i]], v[i]);
  } // 3bc167
  void update(int idx, ll x) { // Update set idx to x
    int bid = id[idx];
    block[bid] = oo;
    v[idx] = x;
    rep(i, bid * bsz, min((bid+1)*bsz, n)) block[bid] = min(
        block[bid], v[i]);
  } // 7aff89
  ll query(int l, int r) { // Query of min in interval [l, r]
    assert(l <= r); // Or return id;
    ll ans = oo;
    auto sblk = [&](int bid, int flag) { // flag [left, right,
        both]
      rep(i, max(l, bid*bsz), min((bid+1)*bsz, r+1)) ans = min(
          ans, v[i]);
    }; // f49504
    auto allblk = [&](int bid) { // Solve entire block
      ans = min(ans, block[bid]);
    }; // 3566fc
    if (id[l] == id[r]) {
      sblk(id[l], 2);
    } // 340382
    else {
      sblk(id[l], 0);
      rep(i, id[l]+1, id[r]) allblk(i);
      sblk(id[r], 1);
    } // e1769a
    return ans;
  } // 7a0d23
}; // f45235
```

6.2.1 Segment Tree

SegmentTree.h
Description: Iterative SegTree Can be changed by modifying Spec
Time: $\mathcal{O}(\log N)$
607842, 18 lines

```
template<typename LS> struct SegTree {
  using S = typename LS::S;
  using K = typename LS::K;
  int n; vector<S> seg;
  SegTree(int _n) : n(_n), seg(2*n, LS::id()) {}
  void update(int no, K val) {
    no += n; seg[no] = LS::update(val, seg[no]);
    while (no > 1) no /= 2, seg[no] = LS::op(seg[no*2], seg[no
        *2+1]);
  } // adca9b
  S query(int l, int r) { // [l, r)
    S vl = LS::id(), vr = LS::id();
    for (l += n, r += n; l < r; l /= 2, r /= 2) {
      if (l & 1) vl = LS::op(vl, seg[l++]);
      if (r & 1) vr = LS::op(seg[--r], vr);
    } // 77c5ac
    return LS::op(vl, vr);
  } // edc68a
}; // 607842
```

LazySegmentTree.h
Description: Lazy Seg (half-open). Can be transformed into Seg Beats by uncommenting conditions.
Time: $\mathcal{O}(\log N * (ch + cmp))$.
fa8696, 30 lines

```
template<class S> struct SegBeats{ // n MUST be a power of 2
  using T = typename S::T; using L = typename S::L;
  int n; vector<T> seg; vector<L> lz; vector<bool> ig;
  SegBeats(int s):n(s),seg(2*n,S::id),lz(2*n),ig(2*n,1){}
  void apply(int p, L v, int l, int r){
    seg[p] = S::ch(seg[p],v,l,r);
    if (r-l>1)lz[p] = ig[p] ? v : S::cmp(lz[p], v), ig[p] = 0;
  } // f453ff
  void prop(int p, int l, int r){
    if (ig[p])return;
    int m = (l+r)/2; ig[p] = 1;
    apply(2*p, lz[p], l, m);
    apply(2*p+1, lz[p], m, r);
  } // f09d8e
  void update(L v, int l, int r){return update(v,l,r,1,0,n);}
  void update(L v, int lq, int rq, int no, int lx, int rx){
    if (rq <= lx or rx <= lq /*or S::brk(v,seg[no])*/)return;
    if (lq <= lx and rx <= rq /*and S::tag(v,seg[no])*/)return
        apply(no, v, lx, rx);
    int mid = (lx+rx)/2; prop(no,lx,rx);
    update(v,lq,rq,2*no,lx,mid); update(v,lq,rq,2*no+1,mid,rx);
    seg[no] = S::op(seg[2*no],seg[2*no+1]);
  } // 04eed2
  T query(int l, int r){return query(l,r,1,0,n);}
  T query(int lq, int rq, int no, int lx, int rx){
    if (rq <= lx or rx <= lq)return S::id;
    if (lq <= lx and rx <= rq)return seg[no];
    int mid = (lx+rx)/2; prop(no,lx,rx);
    return S::op(query(lq,rq,2*no,lx,mid),query(lq,rq,2*no+1,
        mid,rx));
  } // fb0086
}; // fa8696
```

MSegTree.h
Description: Multidimensional SegTree Requires Monoid (op, id)
Memory: $\mathcal{O}\left(N^D\right)$
Time: $\mathcal{O}\left((\log N)^D\right)$
e335e5, 26 lines

```
#define MAs template<class... As> //multiple arguments
template<int D, class S> struct MSegTree{
  using T = typename S::T;
  int n; vector<MSegTree<D-1, S>> seg;
  MAs MSegTree(int s, As... ds):n(s),seg(2*n, MSegTree<D-1, S>(
      ds...)){}
  MAs T get(int p, As... ps){return seg[p+n].get(ps...);}
  MAs void update(T x, int p, As... ps){
    seg[p+=n].update(x, ps...);
    while(p/=2)seg[p].update(S::op(seg[2*p].get(ps...),seg[2*p
        +1].get(ps...)), ps...);
  } // 2c8b52
  MAs T query(int l, int r, As... ps){
    T lv=S::id,rv=S::id;
    for(l+=n,r+=n+1;l<r;l/=2,r/=2){
      if (l&1)lv = S::op(lv,seg[l++].query(ps...));
      if (r&1)rv = S::op(seg[--r].query(ps...),rv);
    } // 1569b6
    return S::op(lv,rv);
  } // bc7474
}; // da06ff
template<class S> struct MSegTree<0, S>{
  using T = typename S::T;
  T val=S::id;
  T get(){return val;}
  void update(T x){val=x;} //set update!
  T query(){return val;}
}; // 50402e
```

LazySparseSeg.h
Description: Lazy Sparse Seg (half-open). Lazy can be removed (prop, lz, ig, ch/cmp)
Time: $\mathcal{O}(\log N * (ch + cmp))$.
d8f4f1, 37 lines

```
template<class I, class S> struct LazySparseSeg{ //I is index
    type
  using T = typename S::T; //value type
  using L = typename S::L; //update type
  struct Node{int lc, rc; T val; L lz; bool ig;};
  I n; vector<Node> v;
  int new_node(){return v.eb(0,0,S::id,L(),1), sz(v)-1;}
  LazySparseSeg(I s): n(s){
    //v.reserve(MXN); //faster node creation
    new_node(); new_node(); //blank and root node
  } // c4ba1b
  void apply(int i, L x, I lx, I rx){
    v[i].val = S::ch(v[i].val, x, lx, rx);
    if (rx-lx>1)v[i].lz = v[i].ig ? x : S::cmp(v[i].lz, x), v[i
        ].ig = 0;
  } // 9ccb90
  void prop(int i, I lx, I rx){
    if (!v[i].lc)v[i].lc = new_node(), v[i].rc = new_node();
    if (v[i].ig)return;
    I mx = (lx+(rx-lx)/2); v[i].ig = 1;
    apply(v[i].lc, v[i].lz, lx, mx); apply(v[i].rc, v[i].lz, mx
        , rx);
  } // a00350
  void update(L x, I l, I r){return update(x, l, r, 1, 0, n-1);
      }
  void update(L x, I l, I r, int i, I lx, I rx){
    if (r <= lx or rx <= l)return;
    if (l <= lx and rx <= r)return apply(i, x, lx, rx);
    I mx = (lx+(rx-lx)/2); prop(i, lx, rx);
    int lc = v[i].lc, rc = v[i].rc;
    update(x, l, r, lc, lx, mx); update(x, l, r, rc, mx, rx);
    v[i].val = S::op(v[lc].val, v[rc].val);
  } // f721de
  T query(I l, I r){return query(l, r, 1, 0, n-1);}
  T query(I l, I r, int i, I lx, I rx){
```

```
    if (r <= lx or rx <= l)return S::id;
    if (l <= lx and rx <= r)return v[i].val;
    I mx = (lx+(rx-lx)/2); prop(i, lx, rx);
    return S::op(query(l, r, v[i].lc, lx, mx), query(l, r, v[i
        ].rc, mx, rx));
  } // 52a49e
}; // d8f4f1
```

LazyPersistentSeg.h

Description: Persistent Lazy Sparse Segment Tree Can be changed by modifying Spec
Time: $\mathcal{O}(\log N * (ch + cmp))$

b43ed0, 39 lines

```
template<class I, class S> struct LazyPersistentSeg{ //I is
    index type
  using T = typename S::T; //value type
  using L = typename S::L; //lazy type
  struct Node{int lc, rc; T val; L lz; bool ig;};
  I n; vector<Node> v;
  int new_node(int l=0, int r=0){return v.eb(l,r,S::op(v[l].val
      ,v[r].val),L(),1), sz(v)-1;}
  LazyPersistentSeg(){ //only creates object, should be "init"
      ed to get root
    //v.reserve(MXN); //faster node creation
    v.eb(0,0,S::id,L(),1); //blank node
  } // 00247f
  int init(I s){return n = s, new_node();}
  int lazy_clone(int i, L lz, I lx, I rx){
    int ni = new_node(v[i].lc, v[i].rc);
    v[ni].lz = v[i].ig ? lz : S::cmp(v[i].lz, lz);
    v[ni].ig = 0; v[ni].val = S::ch(v[i].val, lz, lx, rx);
    return ni;
  } // 4c0efb
  void prop(int i, I lx, I rx){
    if (v[i].ig)return;
    int mx = lx + (rx - lx) / 2; v[i].ig = 1;
    if (lx < rx)
      v[i].lc = lazy_clone(v[i].lc, v[i].lz, lx, mx),
      v[i].rc = lazy_clone(v[i].rc, v[i].lz, mx, rx);
  } // 551bde
  int update(L lz, I l, I r, int root){return update(lz, l, r,
      root, 0, n);}
  int update(L lz, I l, I r, int i, I lx, I rx){
    if (r <= lx or rx <= l)return i;
    if (l <= lx and rx <= r)return lazy_clone(i, lz, lx, rx);
    I mx = lx + (rx - lx) / 2; prop(i, lx, rx);
    return new_node(update(lz, l, r, v[i].lc, lx, mx), update(
        lz, l, r, v[i].rc, mx, rx));
  } // ca252e
  T query(I l, I r, int root){return query(l, r, root, 0, n);}
  T query(I l, I r, int i, I lx, I rx){
    if (r <= lx or rx <= l)return S::id;
    if (l <= lx and rx <= r)return v[i].val;
    I mx = lx + (rx - lx) / 2; prop(i, lx, rx);
    return S::op(query(l, r, v[i].lc, lx, mx), query(l, r, v[i
        ].rc, mx, rx));
  } // 893689
}; // b43ed0
```

Strings (7)

Hashing.h

Description: String hashing (multiple mods and 2^{32})
Memory: $\mathcal{O}(n)$
Time: $\mathcal{O}(1)$ query, $\mathcal{O}(n)$ build

c40ee3, 23 lines

```
typedef uint64_t ull;
template<int M, class B> struct A {
  int x; B b; A(int x=0) : x(x), b(x) {}
  A(int x, B b) : x(x), b(b) {}
  A operator+(A o) const {int y = x+o.x; return{y - (y>=M)*M, b
      +o.b};}
  A operator-(A o) const {int y = x-o.x; return{y + (y< 0)*M, b
      -o.b};}
  A operator*(A o) const { return {(int)((ll)x*o.x % M), b*o.b}
      ; }
  explicit operator ull() const { return x ^ (ull) b << 21; }
}; // e03276
typedef A<1000000007, A<1000000009, unsigned>> H;
static int C; // initialize to a number less than MOD or random
struct HashInterval {
  int n; vector<H> ha, pw;
  template<typename S>
  HashInterval(const S & str) : n(sz(str)), ha(n+1), pw(n+1) {
    pw[0] = 1;
    rep(i,0,n)
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  } // 185a86
  H query(int a, int b) { return ha[b] - ha[a] * pw[b - a]; }
  H queryI(int a, int b) { return query(n - b, n - a); }
}; // 434a8c
```

KMP.h

Description: KMP automaton
Memory: $\mathcal{O}(N)$
Time: $\mathcal{O}(N)$ build, $\mathcal{O}(1)$ query (amortized)

8f8450, 19 lines

```
struct KMP {
  string P; int n; vector<int> nb;
  KMP(string& p) : P(p), n((int)P.size()), nb(n+1) {
    for(int k = 1; k < n; k++) nb[k+1] = nxt(nb[k], P[k]);
  } // ca6dc8
  int nxt(int i, char c){
    for(; i; i = nb[i])if (i < n and P[i]==c)return i+1;
    return P[0]==c;
  } // 2b99e2
  vector<vector<int>> dfa;
    void build_dfa(){
    dfa.assign(n+1, vector<int>(26));
        dfa[0][P[0]-'a'] = 1; //only way to advance at 0
        for(int k = 1; k <= n; k++)
            for(int c = 0; c < 26; c++)
                if (k < n and P[k] == 'a'+c) dfa[k][c] = k+1;
                else dfa[k][c] = dfa[nb[k]][c];
    } // f47d83
}; // 8f8450
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

ee09e2, 9 lines

```
vi Z(const string& S) {
  vi z(sz(S)); int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]]) z[i]++;
    if (i + z[i] > r) l = i, r = i + z[i];
  } // 44be47
  return z;
} // ee09e2
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

e7ad79, 11 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s); array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1]) p[z][i]++, L--, R
        ++;
    if (R>r) l=L, r=R;
  } // a843d3
  return p;
} // e7ad79
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  } // 9374b1
  return a;
} // d07a42
```

Aho.h

Description: Aho automaton
Memory: $\mathcal{O}(alphabetsize * n)$
Time: $\mathcal{O}(alphabetsize * n)$ build, $\mathcal{O}(1)$ query

7fe61a, 35 lines

```
#define vvi vector<vi>
struct Aho {
  int n=1, si; char in;
  vvi tran, nxt;
  vi lnk, term, h;
  Aho(char ain='a', int asi=26) : in(ain), si(asi) { tran.eb(si
      ,-1); term.pb(0); }
  void add(string& s) {
    int cur=0;
    rep(i,0,s.size()) {
      int& nxt= tran[cur][s[i]-in];
      if (nxt != -1) cur=nxt;
      else nxt=cur=n++, term.pb(0),tran.eb(si,-1);
    } // 8426b9
    term[cur]+=1;
  } // f31f2a
  void init() {
    lnk.assign(n,0);
    nxt.assign(n, vi(si));
    h.assign(n,0);
    queue<int> q; q.push(0);
    while (!q.empty()) {
      int a=q.front(); q.pop();
      rep(c,0,si) {
        int& b=nxt[a][c];
        int fail=nxt[lnk[a]][c];
        if (tran[a][c] != -1) {
          b = tran[a][c];
          lnk[b] = a ? fail : 0;
          q.push(b);
          h[b]=h[a]+1;
        } else b=fail; // a1bc18
      } // 83b11a
    } // 494c02
  } // 7f7bf2
}; // 483f6b
```

Automaton.h

Description: Suffix automaton
Memory: $\mathcal{O}(n * 26)$
Time: $\mathcal{O}(n)$ build

7479ba, 35 lines

```
struct Automata {
  int saID = 1, last = 1, n;
  vector<int> len, lnk;
  vector<array<int,27>> to;
  vector<int> occ, fpos;
  vector<int> states;
  Automata(const string & s, const char a = 'a')
    : n(s.size()), len(2*n+2), lnk(2*n+2), to(2*n+2, {0}), occ
        (2*n+2), fpos(2*n+2) {
    for (const auto & c: s) push(c-a);
    states.assign(saID, 0);
    iota(all(states), 1);
    sort(all(states), [&](const auto & u, const auto & v) {
        return len[u] > len[v]; });
    for (auto st: states) occ[lnk[st]] += occ[st];
  } // a267f8
  void push(int c) {
    int a = ++saID, p = last;
    last = a;
    len[a] = len[p] + 1;
    occ[a] = 1;
    fpos[a] = len[a] - 1;
    for (; p > 0 && !to[p][c]; p = lnk[p]) to[p][c] = a;
    int q = to[p][c];
    if (p == 0) lnk[a] = 1;
    else if (len[p] + 1 == len[q]) lnk[a] = q;
    else {
      int clone = ++saID;
      lnk[clone] = lnk[q];
      to[clone] = to[q];
      fpos[clone] = fpos[q];
      len[clone] = len[p] + 1;
      lnk[a] = lnk[q] = clone;
      for (; to[p][c] == q; p = lnk[p]) to[p][c] = clone;
    } // d4d0c5
  } // 070b4e
}; // 7479ba
```

DP (8)

8.1 Optimizations

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \le k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

d38d2b, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  } // 541151
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
}; // d38d2b
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  } // ac5d5a
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
} // b20ccc
```

Various (9)

9.1 Random

RNG.h

Description: RNGs
Time: $\mathcal{O}(1)$

a6861c, 14 lines

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count()); // mt19937_64
uniform_int_distribution<int> distribution(1,n);

num = distribution(rng); // num no range [1, n]
shuffle(vec.begin(), vec.end(), rng); // shuffle

using ull = unsigned long long;
ull mix(ull o){
    o+=0x9e3779b97f4a7c15;
    o=(o^(o>>30))*0xbf58476d1ce4e5b9;
    o=(o^(o>>27))*0x94d049bb133111eb;
    return o^(o>>31);
} // bc6211
ull hash(pii a) {return mix(a.first ^ mix(a.second));}
```

9.2 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

9.3 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

9.3.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

9.3.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("bmi,bmi2,lzcnt,popcnt")` are good for bitsets.

Techniques (A)

techniques.txt

159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```