ISTA 421/521 – Homework 1

Due: Friday, September 5, 5pm (to the dropbox)
8 points total

STUDENT NAME

Undergraduate / Graduate

All written portions of the homework must be submitted to the D2L dropbox as a .pdf.

In the following, FCMA refers to the course text: Simon Rogers and Mark Girolami (2012), *A First Course in Machine Learning*. (All questions from the book are reproduced here in full, so you do not need the book to proceed.)

For general notes on using latex to typeset math, see: `http://en.wikibooks.org/wiki/LaTeX/Mathematics`

1. **Exercise 1.1** from FCMA p.35 [0.5 pts]

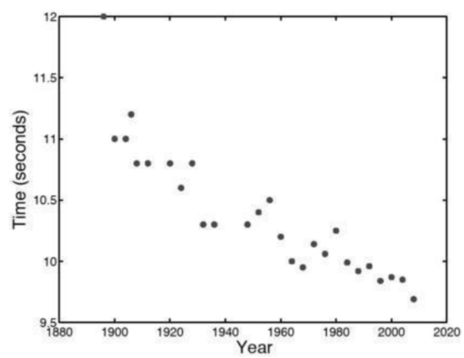

Figure 1: Reproduction of figure 1.1, Olympic men's 100m data

By examining Figure 1.1 [from p. 2 of FCMA, reproduced here], estimate (by hand / in your head) the kind of values we should expect for w_0 and w_1 (e.g., High? Low? Positive? Negative?).

Solution. <Solution goes here.>

NOTE: The following three exercises (2, 3, and 4) review basic linear algebra concepts; we will review these briefly in Lecture 3.

Notation conventions:

- Script variables, such as x_{n2} and w_1 represent scalar values
- Lowercase bold-face variables, such as $\mathbf{x}_n$ and $\mathbf{w}$, represent vectors
- Uppercase bold-face variables, such as $\mathbf{X}$, represent n (rows) $\times$ m (columns) matrices
- Note that because all indexes in the following are either a value between 0, 1, ..., 9, or a scalar, n, I am representing multiple dimension indexes without a comma, as it is unambiguous; e.g., x_{32} is the element scalar value of $\mathbf{X}$ at row 3, column 2. When we have to refer to specific index values greater than 9, we'll use commas, such as $x_{32,3}$ is the scalar value in the 32nd row and 3rd column.
- '$\top$' in expressions like $\mathbf{w}^\top$ indicates the transpose operator.

2. **Exercise 1.3** from FCMA p.35 [1pt]

   Show that:

$$\mathbf{w}^\top \mathbf{X}^\top \mathbf{X}\mathbf{w} = w_0^2 \left(\sum_{n=1}^{N} x_{n1}^2 \right) + 2w_0 w_1 \left(\sum_{n=1}^{N} x_{n1} x_{n2} \right) + w_1^2 \left(\sum_{n=1}^{N} x_{n2}^2 \right),$$

   where

$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ \vdots & \vdots \\ x_{N1} & x_{N2} \end{bmatrix}.$$

   (Hint – it's probably easiest to do the $\mathbf{X}^\top \mathbf{X}$ first!)

   Solution. <Solution goes here.>

3. **Exercise 1.4** from FCMA p.35 [1pt]

   Using $\mathbf{w}$ and $\mathbf{X}$ as defined in the previous exercise, show that $(\mathbf{X}\mathbf{w})^\top = \mathbf{w}^\top \mathbf{X}^\top$ by multiplying out both sides.

   Solution. <Solution goes here.>

4. **Exercise 1.5** from FCMA p.35 [1pt]

   When multiplying a scalar by a vector (or matrix), we multiply each element of the vector (or matrix) by that scalar. For $\mathbf{x}_n = [x_{n1}, x_{n2}]^\top$, $\mathbf{t} = [t_1, ..., t_N]^\top$, $\mathbf{w} = [w_0, w_1]^\top$, and

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1{}^\top \\ \mathbf{x}_2{}^\top \\ \vdots \\ \mathbf{x}_N{}^\top \end{bmatrix}$$

   show that

$$\sum_n \mathbf{x}_n t_n = \mathbf{X}^\top \mathbf{t}$$

and

$$\sum_n \mathbf{x}_n \mathbf{x}_n^\top \mathbf{w} = \mathbf{X}^\top \mathbf{X} \mathbf{w}$$

Solution. <Solution goes here.>

5. **Plotting Exercise** – three parts

A. [0.5pt] Use the provided python script `plotlinear.py` and plot three lines (parameters of your choosing). This requires you have you python environment set up. Place the output graphic in your pdf submission and provide a descriptive caption that indicates the intercept and slope values you used to generate the lines. (LaTeX users can use the commented code in the HW latex template for inserting the figure.)

Solution. <Solution goes here.>

B. [0.5pt] Now, try another plot: generate a vector whose entries are the values of $\sin(\mathbf{x})$ for $\mathbf{x}$ in the range $[0, 10]$ in steps of 0.01, and plot it. Label the y-axis 'sin(x)', the x-axis 'x values' and provide a title for the plot, 'Sine Function for x from 0.0 to 10.0'. Include your plot in the pdf submission.

Solution. <Solution goes here.>

C. [0.5pt] Finally, put your script code in the pdf file.

For LaTeX users: you can use the following python code listing environment. The code currently listed here is from the `plotlinear.py` script; replace the code there with the code you wrote for generating the sin function.

Code Listing 1: Example Code Listing: portion of python `plotlinear.py` script

```
import numpy as np
import matplotlib.pyplot as plt

plt.ion()

## Request user input
plt.close();
plt.figure(1)
plt.plot()
print("\nKeeps plotting lines on the current plot until you quit (Ctrl-D) or
                enter a non-number");
while 1:
    intercept = float(raw_input("Enter intercept: "))
    gradient = float(raw_input("Enter gradient (slope): "))
    plt.plot(x, intercept + gradient*x)
    print "\ny = " + str(intercept) + " + " + str(gradient) + " x\n";
```

Solution. <Solution goes here.>

6. **More Programming in Python Practice**

A. [1pt] Write a short script that initializes the random number generator

python: `numpy.random.seed(seed=1)`

Followed by creating two three-dimensional column vectors using

python: `random.rand` (used in the context of code to generate the vectors)

Represent the random variables as $\mathbf{a}$ and $\mathbf{b}$ (be sure you issue the call to set the random seed immediately before creating these variables). Print them at the terminal and copy-and-paste the result here. (If using LaTeX, use the `verbatim` environment to display).

Solution. <Solution goes here.>

B. [2pts] Using the values of $\mathbf{a}$ and $\mathbf{b}$, compute the following and display the result two ways: (1) copy-and-paste the output (from the python interpreter/terminal; again, in LaTeX use the `verbatim` environment), (2) typeset the output (e.g., using the LaTeX math environment).

1. $\mathbf{a} + \mathbf{b} =$?
2. $\mathbf{a} \circ \mathbf{b} =$? (element-wise multiply; Note: the notation $\mathbf{a} \circ \mathbf{b}$ is also known as the Hadamard product, the entrywise product, or the Schur product.)
3. $\mathbf{a}^\top \mathbf{b} =$? (also called the dot-product)

Solution. <Solution goes here.>

Now, set the random seed to 2 and immediately generate a random 3×3 matrix $\mathbf{X}$. In your solution, display the value of $\mathbf{X}$. Using $\mathbf{X}$ and the earlier values of a and b, compute the following in python and typeset the results in two ways, as before.

4. $\mathbf{a}^\top \mathbf{X} =$?
5. $\mathbf{a}^\top \mathbf{X} \mathbf{b} =$?
6. $\mathbf{X}^{-1} =$?

Solution. <Solution goes here.>