# Crystallographic Restriction Theorem

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# Chapter 1

# Introduction

The *Crystallographic Restriction Theorem* characterizes exactly which rotation orders are achievable by integer matrices. Specifically, an $N \times N$ integer matrix can have finite order $m$ if and only if $\psi(m) \leq N$, where $\psi$ is a function based on Euler's totient function.

The project repository is `https://github.com/your-username/General_Crystallographic_Restriction`.

# Chapter 2

# The Psi Function and Main Definitions

The main theorem requires the definition of the $\psi$ function, which measures the "arithmetic complexity" of a positive integer $m$. For a prime power $p^k$, we define:

- $\psi_{pp}(p, k) = \varphi(p^k)$ for $p$ odd or $k \geq 2$
- $\psi_{pp}(2, 1) = 0$

For a general integer $m$ with prime factorization $m = \prod_i p_i^{k_i}$, we have:

$$\psi(m) = \sum_i \psi_{pp}(p_i, k_i)$$

## Chapter 3

# Integer Matrix Orders

## Chapter 4

# The Main Theorem

# Chapter 5

# Companion Matrices

## Chapter 6

# Lower Bound on Psi

# Chapter 7

# Rotation Matrices

# Chapter 8

# Crystallographic Restriction Proof