A Simple Biosphere Model

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1 Introduction

The mean carbon uptake (assimilation, A) and evapotranspiration (E) are investigated under three different conditions:

1. Plants are not limited by soil water,
2. Soil water limits stomatal conductance of plants,
3. Soil water limits photosynthesis in plants.

The limitation of either stomatal conductance or net assimilation is described by adding a multiplication factor, f_s, to g_s or A, respectively,

$$f_s = \begin{cases} 1 & S > S_c \\ \frac{S-S_w}{S_c-S_w} & \text{else} \\ 0 & S < S_w \end{cases}$$

with the critical soil moisture $S_c = 0.4S_m$ and the permanent wilting point $S_w = 0.1S_m$

To investigate how these 3 different conditions affect A and E, A and E are simulated over 1 year, with a time step of 1 day, for each of the conditions. The code (written in Python) which was used to simulate this biospheric model is given in Appendix 1. Several parts of the implementation of the code are explained in Section 2.

2 Implementation of the code

First, an array of 1 to 365 is created to represent the days in the year. Therefore, each of the variables calculated by the model (e.g. top pf the atmosphere radiation, temperature, precipitation, etc.) are calculated for each day of the year.

Soil moisture, S, is described by the differential equation,

$$\frac{dS}{dt} = P - E. \tag{1}$$

S is simulated in the model by using the following numerical approximation,

$$\frac{S[t+1] - S[t]}{\Delta t} = P[t] - E[t], \tag{2}$$

where Δt is a time step of 1 day, t is the current time step, and $t+1$ is the next time step. $S[t]$, $P[t]$, and $E[t]$ are the soil moisture, precipitation, and evapotranspiration, respectively, at the current time step, and $S[t+1]$ is the soil moisture at the next time step. Equation 2 is calculated inside a for loop, and therefore a value of S for each day of the year is obtained.

The multiplication factor, f_s is calculated within the for loop which calculates S, and therefore E, A, and g_s can be calculated for each day of the year. A different for loop is used for each condition under investigation.

3 Results and Discussion

Figure 1 shows stochastic rain simulated over one year. There are two events in the middle of the year with no precipitation.

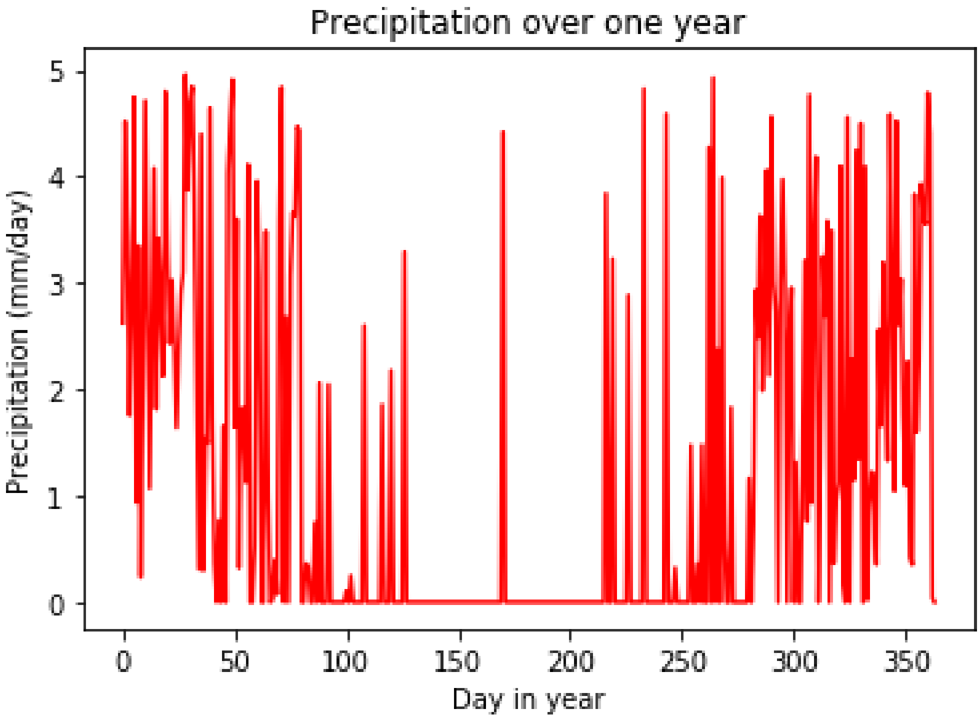

Figure 1: Stochastic rain simulated over one year

The evolution of assimilation, A, throughout one year is shown in Figure 2. The three cases all follow each other at the start and end of the year, however the case where soil moisture limits A diverges during two periods in the middle of the year. The cases where soil moisture limits g_s and where soil moisture does not limit the plant have the same rate of A throughout the whole year because the equation for assimilation used in the model does not consider stomatal conductance. However, if photosynthesis is limited by soil moisture (green line), assimilation of CO_2 is significantly reduced during two periods of reduced precipitation (Fig. 1). These drier periods will cause a decrease in soil moisture, resulting directly in a reduction in A in this case.

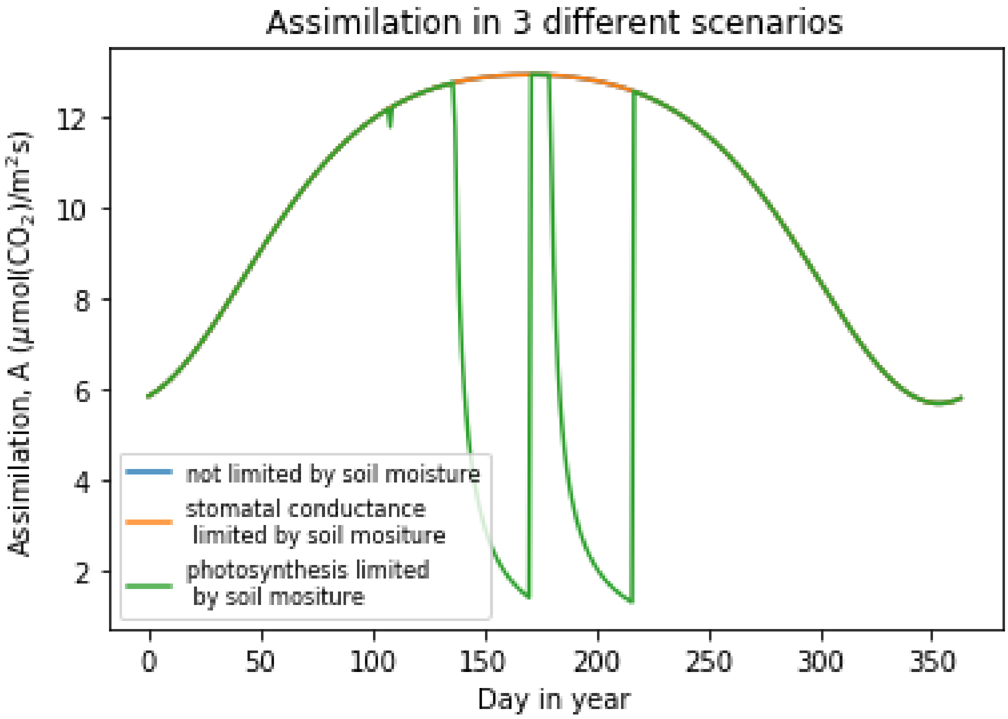

Figure 2: Assimilation, A simulated over one year under three different scenarios, plants are not limited by soil water (blue), soil water limits stomatal conductance (orange), and soil water limits photosynthesis (green).

The evolution of evapotranspiration, E, throughout one year is shown in Figure 3. The two cases which consider soil moisture follow the case where soil moisture doesn't limit the plant well at the start and end of the year, but during the middle of the year, evapotranspiration significantly decreases in the cases where soil moisture limits the plant compared to the case where soil moisture doesn't limit the plant. This can be explained by the reduction in soil moisture due to a reduction in precipitation during two periods during the summer period. Due to the reduced soil moisture, a higher number of stomata on the leaf will be closed in both cases, limiting the amount of water which can be lost from the plant through evapotranspiration. Figure 3 shows that the case where stomatal conductance is limited by soil moisture (orange) has a reduced value of E compared to the case where A is limited by soil moisture (green) in the low precipitation events. This is because in when g_s is limited, stomata will close directly, limiting evapotranspiration, while in the case where A is limited, stomata are closed as a consequence of reduced photosynthesis.

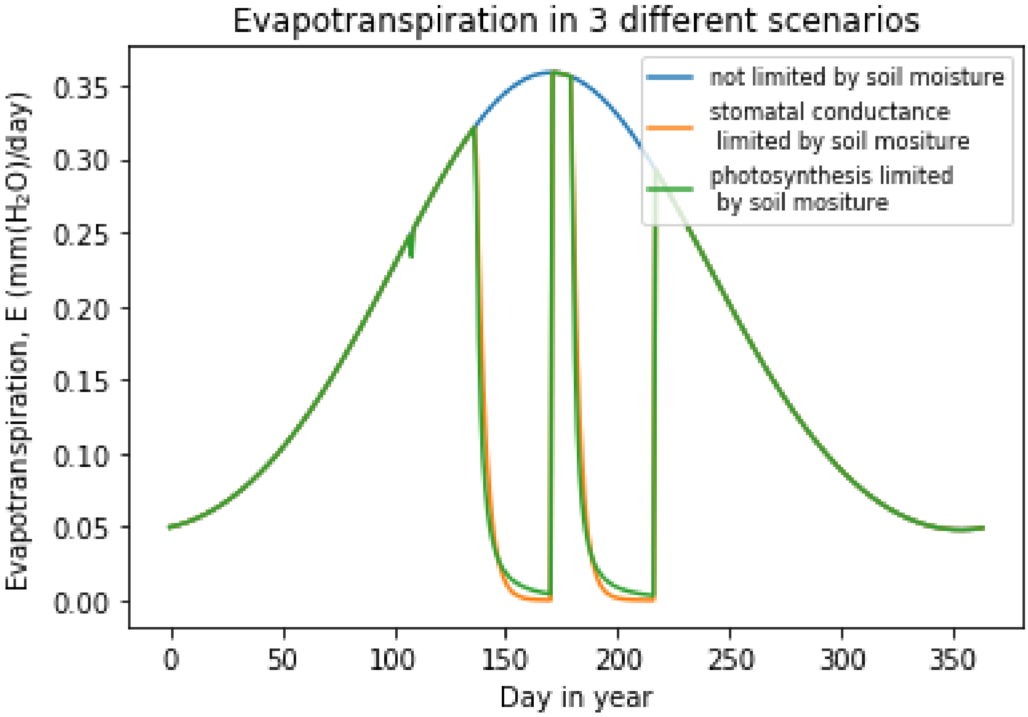

Figure 3: Evapotranspiration, E simulated over one year under three different scenarios, plants are not limited by soil water (blue), soil water limits stomatal conductance (orange), and soil water limits photosynthesis (green).

Figure 4 shows how soil moisture, S, evolves throughout one year under the three cases. The three cases evolve similarly at the start and end of the year. Under the case where soil moisture is not limiting A or g_s, soil moisture decreases more than the other cases in the middle of the year. This is because as precipitation decreases during this time, evapotranspiration remains high in this case. Therefore, the balance determining S ($\frac{dS}{dt} = P - E$) is more negative than the other cases, so the soil will become very dry. Whereas in the cases where soil moisture determines A and g_s, E reduces during periods of low precipitation in the middle of the year (Fig. 3), and therefore the S balance doesn't become as negative, allowing the soil to retain more moisture.

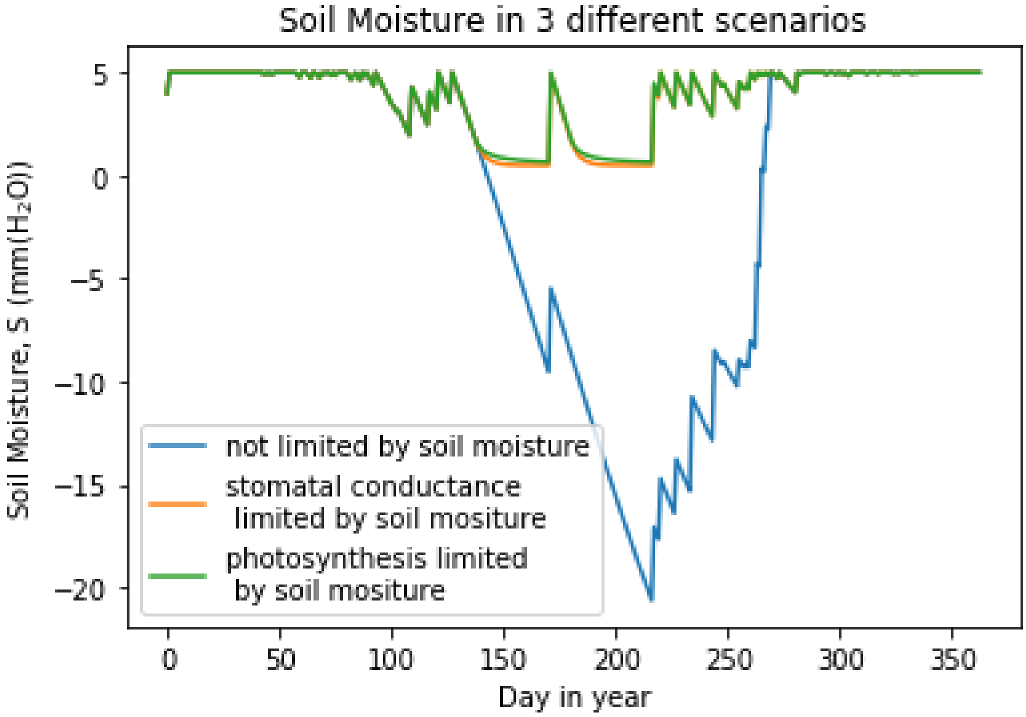

Figure 4: Soil moisture, S simulated over one year under three different scenarios, plants are not limited by soil water (blue), soil water limits stomatal conductance (orange), and soil water limits photosynthesis (green).

Stomatal conductance for the three cases are shown in Figure 5. The cases diverge from each other during 2 periods in the middle of the year, when precipitation has decreased. In the case where soil moisture doesn't limit g_s or A, the stomatal conductance remains high during periods of low precipitation, as the low level of soil moisture doesn't change stomatal conductance, directly or indirectly. However there is a decrease in stomatal conductance during levels of low precipitation in the cases where soil moisture limits g_s and where soil moisture limits A, due to drier conditions triggering the closure of stomata. The stomatal conductance decreases more in the case where soil moisture limits g_s, as this case causes stomata to close directly as a result of soil moisture decrease, while stomata only close indirectly, after a reduction in photosynthesis, in the case where soil moisture limits A.

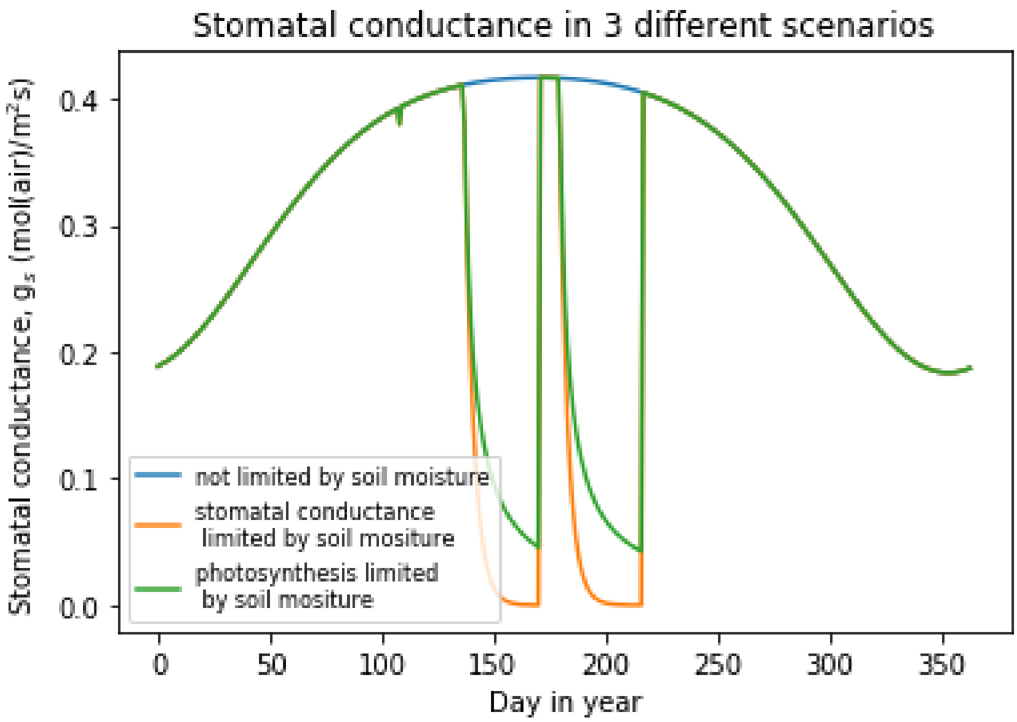

Figure 5: Stomatal conductance, g_s simulated over one year under three different scenarios, plants are not limited by soil water (blue), soil water limits stomatal conductance (orange), and soil water limits photosynthesis (green).

4 Conclusion

When we allow coupling of soil moisture to plant processes like photosynthesis and stomatal conductance, we should see more accurate representations of assimilation and evapotranspiration. To investigate which of the cases are more realistic, soil moisture limiting g_s or soil moisture limiting A, it would be necessary to know how assimilation is affected by lower moisture. If assimilation is reduced in a reduced soil moisture case, then that would indicate that the case in which soil moisture limits A more closely models reality. We can see that allowing the coupling of soil moisture to plant processes can also regulate soil moisture better, as not considering this coupling can result in much drier soil conditions.

Appendix: Code

```
import matplotlib.pyplot as plt
import numpy as np

# parameters:

alpha = 0.09
beta = 20
c_rd = 0.11
E_rd = 50967
R = 8.314
m = 9
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```
b = 0.001
C_a = 400
S_m = 5 #[mm]
S_0 = 4 #[mm]
lat = 50*np.pi/180 #radians
n_day = 365
d = 100 #day of the year

S_c = 0.4*S_m # critical soil moisture
S_w = 0.1*S_m # permanent wilting point soil moisture

# make d an array from 1 to 365 - for the whole year:
d_array = np.arange(1,365,1)

# max no. of time steps = number of elements in d_array
tmax = len(d_array)

# calculating top of the atmosphere radiatin R_g

d_r = 1+0.033*np.cos(2*np.pi*(d_array/n_day))
d_d = 0.409*np.sin(2*np.pi*(d_array/n_day)-1.39)
w = np.arccos(-np.tan(lat)*np.tan(d_d))

R_g = 2.2*(1367/(2*np.pi))*d_r*(w*np.sin(lat)*np.sin(d_d)+np.cos(lat)*np.cos(d_d)*np.

# temperature:
T = (R_g/(0.25*2.2*1367))*30-5

# saturated vapour pressure:
e_s = 0.6108*np.exp(17.270*T/(T+237.3))

# absolute humidity:
e_a = 0.7*e_s

# relative humidity:
h = e_a/e_s

#### stochastic rain #####

# 1. sample a uniform random number between 0 and 1:
rand_num = np.random.rand(len(d_array))

# 2. if the number is greater than x = 1 - 0.5*(np.cos(2*np.pi*d/n_day)+1)

#create empty arrays:
x = np.zeros(len(d_array))
rand_num_2 = np.zeros(len(d_array))
P = np.zeros(len(d_array))

for i in range(0,len(d_array)-1):

    x[i] = 1 - 0.5*(np.cos(2*np.pi*d_array[i]/n_day)+1)

    if rand_num[i] > x[i]:
        rand_num_2 = np.random.rand()
```

```
        P[i] = rand_num_2*5
    else:
        P[i] = 0

# leaf respiration:
R_d = c_rd*beta*np.exp(E_rd*(T-25)/(298*R*(T+273)))

# assimilation:
A = alpha*beta*R_g/(alpha*R_g+beta)-R_d

# stomatal conductance:
g_s = m*(A/(h*C_a))+b

# transpiration:
E = 1.5552*g_s*(e_s-e_a)

###### Soil water limits stomatal conductance - E has to change with each iteration #

# creating empty arrays for the variables to be put through the time loop:
time = np.zeros((tmax))
S = np.zeros((tmax))
# multiplication factor:
f_s = np.zeros(tmax)
# have to make an empty array of E, as E has to be fed back into the for loop of char
E_lim_gs = np.zeros(tmax)
dS = np.zeros(tmax)
A_lim_A = np.zeros(tmax)
g_s_lim_A = np.zeros(tmax)
E_lim_A = np.zeros(tmax)
g_s_lim_gs = np.zeros(tmax)
dt = 1

# initial conditions:
t = 0
time[t] = 0
S[t] = S_0 # initial soil moisture
f_s[t] = 1
dS[t] = dt*(P[0]-E[0]) # the step S takes with each iteration
g_s_lim_gs[t] = g_s[0] # gs when soil moisture limits stomatal conductance
E_lim_gs[t] = E[0] # E when soil moisture limits stomatal conductance
E_lim_A[t] = E[0] # E when soil moisture limits photosynthesis
A_lim_A[t] = A[0] # A when soil moisture limits photosynthesis
g_s_lim_A[t] = g_s[0] # gs when soil moisture limits photosynthesis

# creating empty arrays for soil moisutre under the different scenarios, so that soil
S_no_lim = []
S_lim_gs = []
S_lim_A = []

# first for loop is to simulate S in the scenario of soil moisture not limiting
for t in range(0,tmax-1):
    dS[t] = dt*(P[t]-E[t])
    S[t+1] = S[t]+dS[t]
    time[t+1] = time[t]+dt
```

```
    if(S[t+1]>S_m):
        S[t+1] = S_m
    else:
        S[t+1]=S[t+1]

    if S[t+1]>S_c:
        f_s[t+1] = 1
    elif S[t+1]<S_w:
        f_s[t+1] = 0
    else:
        f_s[t+1] = (S[t+1]-S_w)/(S_c-S_w)

    S_no_lim.append(S[t])

# second for loop is to simulate S, E, A and gs in the scenario of soil moisture limi
for t in range(0,tmax-1):
    dS[t] = dt*(P[t]-E_lim_gs[t])
    S[t+1] = S[t]+dS[t]
    time[t+1] = time[t]+dt

    if(S[t+1]>S_m):
        S[t+1] = S_m
    else:
        S[t+1]=S[t+1]

    if S[t+1]>S_c:
        f_s[t+1] = 1
    elif S[t+1]<S_w:
        f_s[t+1] = 0
    else:
        f_s[t+1] = (S[t+1]-S_w)/(S_c-S_w)

    S_lim_gs.append(S[t])

    # soil moisutre limits stomatal conductance - only E is changed
    g_s_lim_gs[t+1] = g_s[t+1]*f_s[t+1]
    E_lim_gs[t+1] = 1.5552*g_s_lim_gs[t+1]*(e_s[t+1]-e_a[t+1])
plt.plot(P-E_lim_gs)
plt.plot(P-E)

plt.show()
# third for loop is to simulate S, E, A and gs in the scenario of soil moisture limit
for t in range(0,tmax-1):
    dS[t] = dt*(P[t]-E_lim_A[t])
    S[t+1] = S[t]+dS[t]
    time[t+1] = time[t]+dt

    if(S[t+1]>S_m):
        S[t+1] = S_m
    else:
        S[t+1]=S[t+1]

    if S[t+1]>S_c:
        f_s[t+1] = 1
    elif S[t+1]<S_w:
```

```
        f_s[t+1] = 0
    else:
        f_s[t+1] = (S[t+1]-S_w)/(S_c-S_w)

    S_lim_A.append(S[t])

    # soil moisture limits photosynthesis - both E and A change
    A_lim_A[t+1] = f_s[t+1]*A[t+1]
    g_s_lim_A[t+1] = m*(A_lim_A[t+1]/(h[t+1]*C_a))+b
    E_lim_A[t+1] = 1.5552*g_s_lim_A[t+1]*f_s[t+1]*(e_s[t+1]-e_a[t+1])

# soil moisture limits gs - does not change A
A_lim_gs = A

plt.plot(E, label="not limited by soil moisture")
plt.plot(E_lim_gs, label="stomatal conductance \n limited by soil mositure")
plt.plot(E_lim_A, label="photosynthesis limited \n by soil mositure")
plt.legend(prop={'size': 8})
plt.xlabel("Day in year")
plt.ylabel("Evapotranspiration, E (mm(H$_2$O)/day)")
plt.title("Evapotranspiration in 3 different scenarios")
plt.show()

plt.plot(A, label="not limited by soil moisture")
plt.plot(A_lim_gs, label="stomatal conductance \n limited by soil mositure")
plt.plot(A_lim_A, label="photosynthesis limited \n by soil mositure")
plt.legend(loc='lower left', prop={'size': 8})
plt.xlabel("Day in year")
plt.ylabel("Assimilation, A ($\mu$mol(CO$_2$)/m$^2$s)")
plt.title("Assimilation in 3 different scenarios")
plt.show()

plt.plot(g_s, label="not limited by soil moisture")
plt.plot(g_s_lim_gs, label="stomatal conductance \n limited by soil mositure")
plt.plot(g_s_lim_A, label="photosynthesis limited \n by soil mositure")
plt.legend(loc='lower left', prop={'size': 8})
plt.xlabel("Day in year")
plt.ylabel("Stomatal conductance, g$_s$ (mol(air)/m$^2$s)")
plt.title("Stomatal conductance in 3 different scenarios")
plt.show()

plt.plot(S_no_lim, label="not limited by soil moisture")
plt.plot(S_lim_gs, label="stomatal conductance \n limited by soil mositure")
plt.plot(S_lim_A, label="photosynthesis limited \n by soil mositure")
plt.legend()
plt.xlabel("Day in year")
plt.ylabel("Soil Moisture, S (mm(H$_2$O))")
plt.title("Soil Moisture in 3 different scenarios")
plt.show()
```