**A Simple Biosphere Model**

*"Life is woven out of air by light."*

*Jacob Moleschott,*
*1822–1893*

Plants take up carbon dioxide ($CO_2$) in the process called photosynthesis. This process needs energy in form of light that is absorbed, for example, by the pigment chlorophyll. Photosynthesis needs also energy for the transport of phosphorus and other tasks, which leads to simultaneous use of stored carbohydrates, releasing energy. This process is called leaf respiration.

$CO_2$ enters the leaves through tiny holes called stomata, which can open or close depending on environmental conditions. Water leaves the plant also through the stomata, which is transpiration. The plant aims to maximise carbon gain while minimizing water loss. That means they actively control the stomata, only opening as much as needed to get the $CO_2$ for photosynthesis. If there is not enough soil water available to the plant, stomata close and photosynthesis declines.

There are two competing theories about the process behind the closed stomata and reduced photosynthesis when soil water gets low: 1. Plants will close the stomata if soil water availability drops below a certain threshold. 2. Plants will alter the photosynthesis system if soil water drops below the threshold so that the plants make less photosynthesis; stomatal conductance then decreases as a consequence due to less photosynthesis (see below).

We want to code a simple biosphere and test the different hypothesis about soil water limitation.

Given the parameters: $\alpha = 0.09$ $\mu$mol($CO_2$)/mol(photons)
$\beta = 20$ $\mu$mol($CO_2$)/m$^2$s
$c_{Rd} = 0.11$
$E_{Rd} = 50967$ J/mol($CO_2$)
$R = 8.314$ J/mol(air)K
$m = 9$
$b = 0.001$
$C_a = 400$ $\mu$mol($CO_2$)/mol(air)
$S_m = 5$ mm
$S_0 = 4$ mm
$lat = 50\,^\circ\mathrm{N} = 50\pi/180$ radians
$n_{day} = 365$

Top of the atmosphere radiation $R_g$ [$\mu$mol(photons)/m$^2$s ] can be calculated on day-of-year $d$ as:

$$d_r = 1 + 0.033\cos\left(2\pi\frac{d}{n_{day}}\right) \quad (1)$$

$$d_d = 0.409\sin\left(2\pi\frac{d}{n_{day}} - 1.39\right) \quad (2)$$

$$\omega = \arccos\left(-\tan(lat)\tan(d_d)\right) \quad (3)$$

$$R_g = 2.2\frac{1367}{2\pi}d_r\left[\omega\sin(lat)\sin(d_d) + cos(lat)\cos(d_d)\sin(\omega)\right]. \quad (4)$$

We assume air temperatures proportional to radiation:

$$T = \frac{R_g}{0.25 \cdot 2.2 \cdot 1367} 30 - 5. \tag{5}$$

Saturated vapour pressure at a given temperature $T$ in kPa is:

$$e_s = 0.6108 \exp\left\{\frac{17.270T}{T + 237.3}\right\}. \tag{6}$$

Relative humidity is given by absolute $e_a$ versus saturated humidity $e_s$:

$$h = \frac{e_a}{e_s}. \tag{7}$$

We assume $e_a = 0.7e_s$ throughout the whole year.

We produce stochastic rain in two steps:

1. Sample a uniform random number between 0 and 1.
2. If the number is greater than $1 - 0.5[\cos(2\pi d/n_{day}) + 1]$:
   - Sample another uniform random number between 0 and 1.
   - $P$ [mm/day] is then the random number times 5.

   else $P = 0$.

Leaf respiration $R_d$ is temperature dependent [$\mu$mol($CO_2$)/$m^2$s]:

$$R_d = c_{Rd}\beta \exp\left\{\frac{E_{Rd}(T - 25)}{298R(T + 273)}\right\}. \tag{8}$$

So net assimilation $A$ [$\mu$mol($CO_2$)/$m^2$s] can be described by a hyperbolic equation based on Michaelis-Menten kinetics:

$$A = \frac{\alpha\beta R_g}{\alpha R_g + \beta} - R_d. \tag{9}$$

Stomata and net assimilation are linked so that stomatal conductance $g_s$ can be calculated with the so-called Ball-Berry equation:

$$g_s = m\frac{A}{h \cdot C_a} + b. \tag{10}$$

Transpiration $E$ by the plants is then in mm($H_2O$)/day:

$$E = 1.5552 g_s(e_s - e_a). \tag{11}$$

Transpiration takes water from a soil pool $S$ of a maximum size $S_m$. The balance equation for the soil is:

$$\frac{dS}{dt} = P - E. \tag{12}$$

The limitation of either stomal conductance or net assimilation can be described by a multiplication factor to $g_s$ or $A$, respectively:

$$f_s = \begin{cases} 1 & S > S_c \\ \frac{S-S_w}{S_c-S_w} & \text{else} \\ 0 & S < S_w \end{cases} \tag{13}$$

with the critical soil moisture $S_c = 0.4\ S_m$ and the permanent wilting point $S_w = 0.1\ S_m$.

Please investigate how mean carbon uptake and total evapotranspiration change if plants are not limited by soil water, soil water limits stomatal conductance or soil water limits photosynthesis. Please simulate the development of the biospheric states and fluxes over one year with daily time step. Output the mean carbon uptake as well as total evapotranspiration and plot the time courses of $A$, $E$, $g_s$, and $S$.