Algorithm for an American put option

Input: $x_{min}, x_{max}, M, N, K, T$ and the parameters of the model

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$\delta\tau = \frac{\sigma^2 T}{2N}, \qquad \delta x = \frac{x_{max} - x_{min}}{M}$

Calculate $\tau_\nu, \nu = 0, 1, \ldots, N$, and $x_i, i = 0, 1, \ldots, M$

For $i = 0, 1, \ldots, M$

$\qquad w_{i,0} = e^{-rT}(K - e^{x_i})^+$

For $\nu = 0, 1, \ldots, N - 1$

$\qquad w_{0,\nu+1} = Ke^{-r(T - \frac{2\tau_{\nu+1}}{\sigma^2})}$

$\qquad w_{M,\nu+1} = 0$

$\qquad$ For $i = 1, 2, \ldots, M - 1$

$\qquad\qquad u_1 = \lambda w_{i+1,\nu} + (1 - 2\lambda)w_{i,\nu} + \lambda w_{i-1,\nu}$

$\qquad\qquad u_2 = e^{-r(T - \frac{2\tau_{\nu+1}}{\sigma^2})}\left(K - e^{x_i - (\frac{2r}{\sigma^2} - 1)\tau_{\nu+1}}\right)^+$

$\qquad\qquad w_{i,\nu+1} = \max(u_1, u_2)$

Output: $w_{i,\nu}$ for $i = 0, 1, \ldots, M, \quad \nu = 0, 1, \ldots, N$

Figure 1: Algorithm - American BS

```
# Runge-Kutta4 coeficients
k1 = self.dt * R
k2 = self.dt * (R + 0.5 * k1)
k3 = self.dt * (R + 0.5 * k2)
k4 = self.dt * (R + k3)

# Deriviated function in time domain
self.U_st[s_i, t_i] = self.U_st[s_i, t_i - 1] + (1.0 / 6.0) * (k1 + 2.0 * k2 + 2.0 * k3 + k4)
```

Figure 2: European Option computation

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```
# Runge-Kutta4 coeficients
k1 = self.dt * R
k2 = self.dt * (R + 0.5 * k1)
k3 = self.dt * (R + 0.5 * k2)
k4 = self.dt * (R + k3)

# Deriviated function in time domain
o = Option(s, self.k, 1, self.r, self.sigma)
actual_price = o.euro_vanilla_call()
calculated_price = self.U_st[s_i, t_i - 1] + (1.0 / 6.0) * (k1 + 2.0 * k2 + 2.0 * k3 + k4)

if actual_price >= calculated_price:
    self.U_st[s_i, t_i] = actual_price
else:
    self.U_st[s_i, t_i] = calculated_price
```
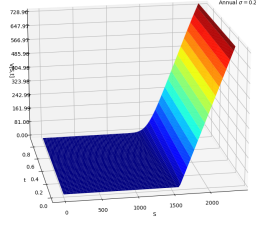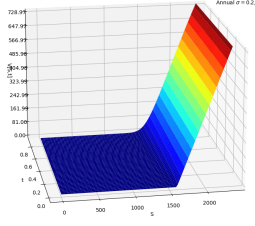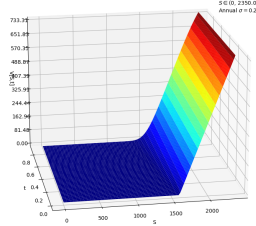
Figure 3: American Option computation

European	American
1.324	1.325
1.671	1.676
3.395	3.400

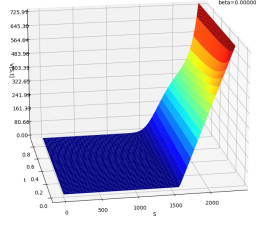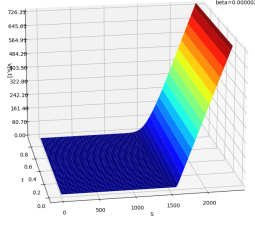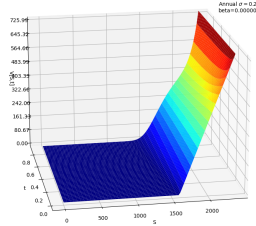Table 1: Option pricing for European and American style using linear Black-Scholes model. $S_0 = 1500$ and $K = 1600$
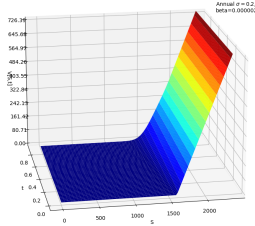
European	American
1.392	1.392
1.907	1.912
3.570	3.575

Table 2: Option pricing for European and American style using non-linear Black-Scholes model. $S_0 = 1500$ and $K = 1600$

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