SBR Physical Modelling

e-Yantra Team

Indian Institute of Technology, Bombay

February 16, 2024

Agenda for Discussion

State Space Modelling

The general state space representation of a continuous time invariant linear dynamical system is

$$\dot{x}(t) = Ax(t) + Bu(t),$$

$$y(t) = Cx(t) + Du(t),$$

where $x(t) \in \mathcal{R}^n$ and $u(t) \in \mathcal{R}^m$, $y(t) \in \mathcal{R}^p$, $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$ and $D \in \mathcal{R}^{p\times m}$

States: Minimum no. of variable required at time $t = 0$ combined with input information to completely define the system behavior for time $t \geq 0$ is known as states.

A typical non linear system in its state space form is represented as

$$\dot{x}(t) = f(t, x(t), u(t)),$$

$$y(t) = h(t, x(t), u(t))$$

Mathematical Modelling: Steps to follow

- Find the Kinetic Energy(KE) & Potential Energy(PE) of the system.
- Find the Lagrangian Function $\mathcal{L} = KE - PE$.
- Identify the position vector denoted by q and velocity vectors denoted as $\dot{q}$.
- Differentiate the Lagrangian function as per the formula known as Euler-Lagrange equation as given below

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = F$$

  where F is non constraint force in case of linear motion and Torque in rotational motion.
- On solving Euler Lagrange Equation, One usually gets the equation which is non-linear in nature as below:

$$\dot{x}(t) = f(t, x(t), u(t))$$

Derivations

Linearization

- Equate $\dot{x} = 0$ to find equilibrium points of the system.
- Find Jacobian of the system using the following formula.

$$J_A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}, \ J_B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}$$

  Note: Equilibrium points are the points at which if the system starts from, it will remain there for all future points.
- Substitute Equilibrium point to achieve matrices A and B to find out linear equations around each equilibrium point.

Derivations contd...

Controllability

- Controllability is the ability of the system to change from any initial state to any desired state in a finite state.

$$C = [B \; AB \; A^2B \; .... \; A^mB]$$

- System is said to be controllable if

$$rank(C) = n, full\ rank$$

where n is the order of the square matrix A ,

$$m = n - 1$$

Stability and Stabilizability

- Find out the eigen values of System Matrix A for each equilibrium point. If all the eigenvalues have negative real part then the system is stable else system will be unstable.
  Note: If there is a single eigenvalue at zero, the system is considered marginally stable, However, we'll consider it unstable because any small disturbance can make the system unstable and hence not desired to be poles/eigenvalues to be positioned on imaginary axis.
- If the system is unstable, we need to assess whether it can be made stable of not.
- System must be Controllable or atleast stabilizable, for us to make the system stable at the concerned equilibrium point.

Thank You

Thank You!