Algorithms to Live By

The Computer Science of Human Decisions

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Introduction

Algorithms to Live By

Imagine you're searching for an apartment in San Francisco—arguably the most harrowing American city in which to do so. The booming tech sector and tight zoning laws limiting new construction have conspired to make the city just as expensive as New York, and by many accounts more competitive. New listings go up and come down within minutes, open houses are mobbed, and often the keys end up in the hands of whoever can physically foist a deposit check on the landlord first.

Such a savage market leaves little room for the kind of fact-finding and deliberation that is theoretically supposed to characterize the doings of the rational consumer. Unlike, say, a mall patron or an online shopper, who can compare options before making a decision, the would-be San Franciscan has to decide instantly either way: you can take the apartment you are currently looking at, forsaking all others, or you can walk away, never to return.

Let's assume for a moment, for the sake of simplicity, that you care only about maximizing your chance of getting the very best apartment available. Your goal is reducing the twin, Scylla-and-Charybdis regrets of the "one that got away" and the "stone left unturned" to the absolute minimum. You run into a dilemma right off the bat: How are you to know that an apartment is indeed the best unless you have a baseline to judge it by? And how are you to establish that baseline unless you look at (and *lose*) a number of apartments? The more information you gather, the better you'll know the right opportunity when you see it—but the more likely you are to have already passed it by.

So what do you do? How do you make an informed decision when the very act of informing it jeopardizes the outcome? It's a cruel situation, bordering on paradox.

When presented with this kind of problem, most people will intuitively say something to the effect that it requires some sort of balance between looking and leaping—that you must look at enough apartments to establish a standard, then take whatever satisfies the standard you've established. This notion of balance is, in fact, precisely correct. What most people *don't* say with any certainty is what that balance is. Fortunately, there's an answer.

Thirty-seven percent.

If you want the best odds of getting the best apartment, spend 37% of your apartment hunt (eleven days, if you've given yourself a month for the search) noncommittally exploring options. Leave the checkbook at home; you're just calibrating. But after that point, be prepared to immediately commit—deposit and all—to the very first place you see that beats whatever you've already seen. This is not merely an intuitively satisfying compromise between looking and leaping. It is the *provably optimal* solution.

We know this because finding an apartment belongs to a class of mathematical problems known as "optimal stopping" problems. The 37% rule defines a simple series of steps—what computer scientists call an "algorithm"—for solving these problems. And as it turns out, apartment hunting is just one of the ways that optimal stopping rears its head in daily life. Committing to or forgoing a succession of options is a structure that appears in life again and again, in slightly different incarnations. How many times to circle the block before pulling into a parking space? How far to push your luck with a risky business venture before cashing out? How long to hold out for a better offer on that house or car?

The same challenge also appears in an even more fraught setting: dating. Optimal stopping is the science of serial monogamy.

Simple algorithms offer solutions not only to an apartment hunt but to all such situations in life where we confront the question of optimal stopping. People grapple with these issues every day—although surely poets have spilled more ink on the tribulations of courtship than of parking—and they do so with, in some cases, considerable anguish. But the anguish is unnecessary. Mathematically, at least, these are solved problems.

Every harried renter, driver, and suitor you see around you as you go through a typical week is essentially reinventing the wheel. They don't need a therapist; they need an algorithm. The therapist tells them to find the right, comfortable balance between impulsivity and overthinking.

The algorithm tells them the balance is thirty-seven percent.

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There is a particular set of problems that all people face, problems that are a direct result of the fact that our lives are carried out in finite space and time. What should we do, and leave undone, in a day or in a decade? What degree of mess should we embrace—and how much order is excessive? What balance between *new* experiences and *favored* ones makes for the most fulfilling life?

These might seem like problems unique to humans; they're not. For more than half a century, computer scientists have been grappling with, and in many cases solving, the equivalents of these everyday dilemmas. How should a processor allocate its "attention" to perform all that the user asks of it, with the minimum overhead and in the least amount of time? When should it switch between different tasks, and how many tasks should it take on in the first place? What is the best way for it to use its limited memory resources? Should it collect more data, or take an action based on the data it already has? Seizing the day might be a challenge for humans, but computers all around us are seizing milliseconds with ease. And there's much we can learn from how they do it.

Talking about algorithms for human lives might seem like an odd juxtaposition. For many people, the word "algorithm" evokes the arcane and inscrutable machinations of big data, big government, and big business: increasingly part of the infrastructure of the modern world, but hardly a source of practical wisdom or guidance for human affairs. But an algorithm is just a finite sequence of steps used to solve a problem, and algorithms are much broader—and older by far—than the computer. Long before algorithms were ever used by machines, they were used by people.

The word "algorithm" comes from the name of Persian mathematician al-Khwārizmī, author of a ninth-century book of techniques for doing mathematics by hand. (His book was called *al-Jabr wa'l-Muqābala*—and the "al-jabr" of the title in turn provides the source of our word "algebra.") The earliest known mathematical algorithms, however, predate even al-Khwārizmī's work: a four-thousand-year-old Sumerian clay tablet found near Baghdad describes a scheme for long division.

But algorithms are not confined to mathematics alone. When you cook bread from a recipe, you're following an algorithm. When you knit a sweater from a pattern, you're following an algorithm. When you put a sharp edge on a piece of flint by executing a precise sequence of strikes with the end of an antler—a key step in making fine stone tools—you're following an algorithm. Algorithms have been a part of human technology ever since the Stone Age.

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In this book, we explore the idea of *human algorithm design*—searching for better solutions to the challenges people encounter every day. Applying the lens of computer science to everyday life has consequences at many scales. Most immediately, it offers us practical, concrete suggestions for how to solve specific problems. Optimal stopping tells us when to look and when to leap. The explore/exploit tradeoff tells us how to find the balance between trying new things and enjoying our favorites. Sorting theory tells us how (and whether) to arrange our offices. Caching theory tells us how to fill our closets. Scheduling theory tells us how to fill our time.

At the next level, computer science gives us a vocabulary for understanding the deeper principles at play in each of these domains. As Carl Sagan put it, "Science is a way of thinking much more than it is a body of knowledge." Even in cases where life is too messy for us to expect a strict numerical analysis or a ready answer, using intuitions and concepts honed on the simpler forms of these problems offers us a way to understand the key issues and make progress.

Most broadly, looking through the lens of computer science can teach us about the nature of the human mind, the meaning of rationality, and the oldest question of all: how to live. Examining cognition as a means of solving the fundamentally computational problems posed by our environment can utterly change the way we think about human rationality.

The notion that studying the inner workings of computers might reveal how to think and decide, what to believe and how to behave, might strike many people as not only wildly reductive, but in fact misguided. Even if computer science did have things to say about how to think and how to act, would we want to listen? We look at the AIs and robots of science fiction, and it seems like theirs is not a life any of us would want to live.

In part, that's because when we think about computers, we think about coldly mechanical, deterministic systems: machines applying rigid deductive logic, making decisions by exhaustively enumerating the options, and grinding out the exact right answer no matter how long and hard they have to think. Indeed, the person who first imagined computers had something essentially like this in mind. Alan Turing defined the very notion of computation by an analogy to a human mathematician who carefully works through the steps of a lengthy calculation, yielding an unmistakably right answer.

So it might come as a surprise that this is not what modern computers are actually doing when they face a difficult problem. Straightforward arithmetic, of course, isn't particularly challenging for a modern computer. Rather, it's tasks like conversing with people, fixing a corrupted file, or winning a game of Go—problems where the rules aren't clear, some of the required information is missing, or finding exactly the right answer would require considering an astronomical number of possibilities—that now pose the biggest challenges in computer science. And the algorithms that researchers have developed to solve the hardest classes of problems have moved computers away from an extreme reliance on exhaustive calculation. Instead, tackling real-world tasks requires being comfortable with chance, trading off time with accuracy, and using approximations.

As computers become better tuned to real-world problems, they provide not only algorithms that people can borrow for their own lives, but a better standard against which to compare human cognition itself. Over the past decade or two, behavioral economics has told a very particular story about human beings: that we are irrational and error-prone, owing in large part to the buggy, idiosyncratic hardware of the brain. This self-deprecating story has become increasingly familiar, but certain questions remain vexing. Why are four-year-olds, for instance, still better than million-dollar supercomputers at a host of cognitive tasks, including vision, language, and causal reasoning?

The solutions to everyday problems that come from computer science tell a different story about the human mind. Life is full of problems that are, quite simply, *hard*. And the mistakes made by people often say more about the intrinsic difficulties of the problem than about the fallibility of human brains. Thinking algorithmically about the world, learning about the fundamental structures of the problems we face and about the properties of their solutions, can help us see how good we actually are, and better understand the errors that we make.

In fact, human beings turn out to consistently confront some of the hardest cases of the problems studied by computer scientists. Often, people need to make decisions while dealing with uncertainty, time constraints, partial information, and a rapidly changing world. In some of those cases, even cutting-edge computer science has not yet come up with efficient, always-right algorithms. For certain situations it appears that such algorithms might not exist at all.

Even where perfect algorithms haven't been found, however, the battle between generations of computer scientists and the most intractable real-world problems has yielded a series of insights. These hard-won precepts are at odds with our intuitions about rationality, and they don't sound anything like the narrow prescriptions of a mathematician trying to force the world into clean, formal lines. They say: Don't always consider all your options. Don't necessarily go for the outcome that seems best every time. Make a mess on occasion. Travel light. Let things wait. Trust your instincts and don't think too long. Relax. Toss a coin. Forgive, but don't forget. To thine own self be true.

Living by the wisdom of computer science doesn't sound so bad after all. And unlike most advice, it's backed up by proofs.

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Just as designing algorithms for computers was originally a subject that fell into the cracks between disciplines—an odd hybrid of mathematics and engineering—so, too, designing algorithms for humans is a topic that doesn't have a natural disciplinary home. Today, algorithm design draws not only on computer science, math, and engineering but on kindred fields like statistics and operations research. And as we consider how algorithms designed for machines might relate to human minds, we also need to look to cognitive science, psychology, economics, and beyond.

We, your authors, are familiar with this interdisciplinary territory. Brian studied computer science and philosophy before going on to graduate work in English and a career at the intersection of the three. Tom studied psychology and statistics before becoming a professor at UC Berkeley, where he spends most of his time thinking about the relationship between human cognition and computation. But nobody can be an expert in all of the fields that are relevant to designing better algorithms for humans. So as part of our quest for algorithms to live by, we talked to the people who came up with some of the most famous algorithms of the last fifty years. And we asked them, some of the smartest people in the world, how their research influenced the way they approached their own lives—from finding their spouses to sorting their socks.

The next pages begin our journey through some of the biggest challenges faced by computers and human minds alike: how to manage finite space, finite time, limited attention, unknown unknowns, incomplete information, and an unforeseeable future; how to do so with grace and confidence; and how to do so in a community with others who are all simultaneously trying to do the same. We will learn about the fundamental mathematical structure of these challenges and about how computers are engineered—sometimes counter to what we imagine—to make the most of them. And we will learn about how the mind works, about its distinct but deeply related ways of tackling the same set of issues and coping with the same constraints. Ultimately, what we can gain is not only a set of concrete takeaways for the problems around us, not only a new way to see the elegant structures behind even the hairiest human dilemmas, not only a recognition of the travails of humans and computers as deeply conjoined, but something even more profound: a new vocabulary for the world around us, and a chance to learn something truly new about ourselves.

1 Optimal Stopping

When to Stop Looking

> *Though all Christians start a wedding invitation by solemnly declaring their marriage is due to special Divine arrangement, I, as a philosopher, would like to talk in greater detail about this . . .*
>
> —JOHANNES KEPLER

> *If you prefer Mr. Martin to every other person; if you think him the most agreeable man you have ever been in company with, why should you hesitate?*
>
> —JANE AUSTEN, *EMMA*

It's such a common phenomenon that college guidance counselors even have a slang term for it: the "turkey drop." High-school sweethearts come home for Thanksgiving of their freshman year of college and, four days later, return to campus single.

An angst-ridden Brian went to his own college guidance counselor his freshman year. His high-school girlfriend had gone to a different college several states away, and they struggled with the distance. They also struggled with a stranger and more philosophical question: how good a relationship did they have? They had no real benchmark of other relationships by which to judge it. Brian's counselor recognized theirs as a classic freshman-year dilemma, and was surprisingly nonchalant in her advice: "Gather data."

The nature of serial monogamy, writ large, is that its practitioners are confronted with a fundamental, unavoidable problem. When have you met enough people to know who your best match is? And what if acquiring the data costs you that very match? It seems the ultimate Catch-22 of the heart.

As we have seen, this Catch-22, this angsty freshman cri de coeur, is what mathematicians call an "optimal stopping" problem, and it may actually have an answer: 37%.

Of course, it all depends on the assumptions you're willing to make about love.

The Secretary Problem

In any optimal stopping problem, the crucial dilemma is not which option to *pick*, but how many options to even *consider*. These problems turn out to have implications not only for lovers and renters, but also for drivers, homeowners, burglars, and beyond.

The **37% Rule*** derives from optimal stopping's most famous puzzle, which has come to be known as the "secretary problem." Its setup is much like the apartment hunter's dilemma that we considered earlier. Imagine you're interviewing a set of applicants for a position as a secretary, and your goal is to maximize the chance of hiring the single best applicant in the pool. While you have no idea how to assign scores to individual applicants, you can easily judge which one you prefer. (A mathematician might say you have access only to the *ordinal* numbers—the relative ranks of the applicants compared to each other—but not to the *cardinal* numbers, their ratings on some kind of general scale.) You interview the applicants in random order, one at a time. You can decide to offer the job to an applicant at any point and they are guaranteed to accept, terminating the search. But if you pass over an applicant, deciding not to hire them, they are gone forever.

The secretary problem is widely considered to have made its first appearance in print—sans explicit mention of secretaries—in the February 1960 issue of *Scientific American*, as one of several puzzles posed in Martin Gardner's beloved column on recreational mathematics. But the origins of the problem are surprisingly mysterious. Our own initial search yielded little but speculation, before turning into unexpectedly physical detective work: a road trip down to the archive of Gardner's papers at Stanford, to haul out boxes of his midcentury correspondence. Reading paper

*We use boldface to indicate the algorithms that appear throughout the book.

correspondence is a bit like eavesdropping on someone who's on the phone: you're only hearing one side of the exchange, and must infer the other. In our case, we only had the replies to what was apparently Gardner's own search for the problem's origins fiftysome years ago. The more we read, the more tangled and unclear the story became.

Harvard mathematician Frederick Mosteller recalled hearing about the problem in 1955 from his colleague Andrew Gleason, who had heard about it from somebody else. Leo Moser wrote from the University of Alberta to say that he read about the problem in "some notes" by R. E. Gaskell of Boeing, who himself credited a colleague. Roger Pinkham of Rutgers wrote that he first heard of the problem in 1955 from Duke University mathematician J. Shoenfield, "and I believe he said that he had heard the problem from someone at Michigan."

"Someone at Michigan" was almost certainly someone named Merrill Flood. Though he is largely unheard of outside mathematics, Flood's influence on computer science is almost impossible to avoid. He's credited with popularizing the traveling salesman problem (which we discuss in more detail in chapter 8), devising the prisoner's dilemma (which we discuss in chapter 11), and even with possibly coining the term "software." It's Flood who made the first known discovery of the 37% Rule, in 1958, and he claims to have been considering the problem since 1949—but he himself points back to several other mathematicians.

Suffice it to say that wherever it came from, the secretary problem proved to be a near-perfect mathematical puzzle: simple to explain, devilish to solve, succinct in its answer, and intriguing in its implications. As a result, it moved like wildfire through the mathematical circles of the 1950s, spreading by word of mouth, and thanks to Gardner's column in 1960 came to grip the imagination of the public at large. By the 1980s the problem and its variations had produced so much analysis that it had come to be discussed in papers as a subfield unto itself.

As for secretaries—it's charming to watch each culture put its own anthropological spin on formal systems. We think of chess, for instance, as medieval European in its imagery, but in fact its origins are in eighth-century India; it was heavy-handedly "Europeanized" in the fifteenth century, as its shahs became kings, its viziers turned to queens, and its elephants became bishops. Likewise, optimal stopping problems have had a number of incarnations, each reflecting the predominating concerns of its time. In the nineteenth century such problems were typified by baroque lotteries and by women choosing male suitors; in the early twentieth century by holidaying motorists searching for hotels and by male suitors choosing women; and in the paper-pushing, male-dominated mid-twentieth century, by male bosses choosing female assistants. The first explicit mention of it by name as the "secretary problem" appears to be in a 1964 paper, and somewhere along the way the name stuck.

Whence 37%?

In your search for a secretary, there are two ways you can fail: stopping early and stopping late. When you stop too early, you leave the best applicant undiscovered. When you stop too late, you hold out for a better applicant who doesn't exist. The optimal strategy will clearly require finding the right balance between the two, walking the tightrope between looking too much and not enough.

If your aim is finding the very best applicant, settling for nothing less, it's clear that as you go through the interview process you shouldn't even consider hiring somebody who isn't the best you've seen so far. However, simply being the best yet isn't enough for an offer; the very first applicant, for example, will of course be the best yet by definition. More generally, it stands to reason that the rate at which we encounter "best yet" applicants will go down as we proceed in our interviews. For instance, the second applicant has a 50/50 chance of being the best we've yet seen, but the fifth applicant only has a 1-in-5 chance of being the best so far, the sixth has a 1-in-6 chance, and so on. As a result, best-yet applicants will become steadily more impressive as the search continues (by definition, again, they're better than all those who came before)—but they will also become more and more infrequent.

Okay, so we know that taking the *first* best-yet applicant we encounter (a.k.a. the first applicant, period) is rash. If there are a hundred applicants, it also seems hasty to make an offer to the *next* one who's best-yet, just because she was better than the first. So how do we proceed?

Intuitively, there are a few potential strategies. For instance, making an offer the third time an applicant trumps everyone seen so far—or maybe the fourth time. Or perhaps taking the next best-yet applicant to come along after a long "drought"—a long streak of poor ones.

But as it happens, neither of these relatively sensible strategies comes out on top. Instead, the optimal solution takes the form of what we'll call the **Look-Then-Leap Rule**: You set a predetermined amount of time for "looking"—that is, exploring your options, gathering data—in which you categorically don't choose anyone, no matter how impressive. After that point, you enter the "leap" phase, prepared to instantly commit to anyone who outshines the best applicant you saw in the look phase.

We can see how the Look-Then-Leap Rule emerges by considering how the secretary problem plays out in the smallest applicant pools. With just one applicant the problem is easy to solve—hire her! With two applicants, you have a 50/50 chance of success no matter what you do. You can hire the first applicant (who'll turn out to be the best half the time), or dismiss the first and by default hire the second (who is also best half the time).

Add a third applicant, and all of a sudden things get interesting. The odds if we hire at random are one-third, or 33%. With two applicants we could do no better than chance; with three, can we? It turns out we can, and it all comes down to what we do with the second interviewee. When we see the first applicant, we have no *information*—she'll always appear to be the best yet. When we see the third applicant, we have no *agency*—we have to make an offer to the final applicant, since we've dismissed the others. But when we see the second applicant, we have a little bit of both: we know whether she's better or worse than the first, and we have the freedom to either hire or dismiss her. What happens when we just hire her if she's better than the first applicant, and dismiss her if she's not? This turns out to be the best possible strategy when facing three applicants; using this approach it's possible, surprisingly, to do just as well in the three-applicant problem as with two, choosing the best applicant exactly half the time.*

Enumerating these scenarios for four applicants tells us that we should still begin to leap as soon as the second applicant; with five applicants in the pool, we shouldn't leap before the third.

As the applicant pool grows, the exact place to draw the line between looking and leaping settles to 37% of the pool, yielding the 37% Rule: look

*With this strategy we have a 33% risk of dismissing the best applicant and a 16% risk of never meeting her. To elaborate, there are exactly six possible orderings of the three applicants: 1-2-3, 1-3-2, 2-1-3, 2-3-1, 3-1-2, and 3-2-1. The strategy of looking at the first applicant and then leaping for whoever surpasses her will succeed in three of the six cases (2-1-3, 2-3-1, 3-1-2) and will fail in the other three—twice by being overly choosy (1-2-3, 1-3-2) and once by not being choosy enough (3-2-1).

Number of Applicants	Take the Best Applicant After	Chance of Getting the Best
3	1 (33.33%)	50%
4	1 (25%)	45.83%
5	2 (40%)	43.33%
6	2 (33.33%)	42.78%
7	2 (28.57%)	41.43%
8	3 (37.5%)	40.98%
9	3 (33.33%)	40.59%
10	3 (30%)	39.87%
20	7 (35%)	38.42%
30	11 (36.67%)	37.86%
40	15 (37.5%)	37.57%
50	18 (36%)	37.43%
100	37 (37%)	37.10%
1000	369 (36.9%)	36.81%

How to optimally choose a secretary.

at the first 37% of the applicants,* choosing none, then be ready to leap for anyone better than all those you've seen so far.

As it turns out, following this optimal strategy ultimately gives us a 37% chance of hiring the best applicant; it's one of the problem's curious mathematical symmetries that the strategy itself and its chance of success work out to the very same number. The table above shows the optimal strategy for the secretary problem with different numbers of applicants, demonstrating how the chance of success—like the point to switch from looking to leaping—converges on 37% as the number of applicants increases.

A 63% failure rate, when following the *best possible* strategy, is a sobering fact. Even when we act optimally in the secretary problem, we will still fail most of the time—that is, we won't end up with the single best applicant in the pool. This is bad news for those of us who would frame romance as a search for "the one." But here's the silver lining. Intuition would suggest that our chances of picking the single best applicant should steadily

*Just a hair under 37%, actually. To be precise, the mathematically optimal proportion of applicants to look at is $1/e$—the same mathematical constant e, equivalent to 2.71828 . . . , that shows up in calculations of compound interest. But you don't need to worry about knowing e to twelve decimal places: anything between 35% and 40% provides a success rate extremely close to the maximum. For more of the mathematical details, see the notes at the end of the book.

decrease as the applicant pool grows. If we were hiring at random, for instance, then in a pool of a hundred applicants we'd have a 1% chance of success, and in a pool of a million applicants we'd have a 0.0001% chance. Yet remarkably, the math of the secretary problem doesn't change. If you're stopping optimally, your chance of finding the single best applicant in a pool of a hundred is 37%. And in a pool of a million, believe it or not, your chance is still 37%. Thus the bigger the applicant pool gets, the more valuable knowing the optimal algorithm becomes. It's true that you're unlikely to find the needle the majority of the time, but optimal stopping is your best defense against the haystack, no matter how large.

Lover's Leap

> *The passion between the sexes has appeared in every age to be so nearly the same that it may always be considered, in algebraic language, as a given quantity.*
>
> —THOMAS MALTHUS

> *I married the first man I ever kissed. When I tell this to my children they just about throw up.*
>
> —BARBARA BUSH

Before he became a professor of operations research at Carnegie Mellon, Michael Trick was a graduate student, looking for love. "It hit me that the problem has been studied: it is the Secretary Problem! I had a position to fill [and] a series of applicants, and my goal was to pick the best applicant for the position." So he ran the numbers. He didn't know how many women he could expect to meet in his lifetime, but there's a certain flexibility in the 37% Rule: it can be applied to either the number of applicants or the *time* over which one is searching. Assuming that his search would run from ages eighteen to forty, the 37% Rule gave age 26.1 years as the point at which to switch from looking to leaping. A number that, as it happened, was exactly Trick's age at the time. So when he found a woman who was a better match than all those he had dated so far, he knew exactly what to do. He leapt. "I didn't know if she was Perfect (the assumptions of the model don't allow me to determine that), but there was no doubt that she met the qualifications for this step of the algorithm. So I proposed," he writes.

"And she turned me down."

Mathematicians have been having trouble with love since at least the seventeenth century. The legendary astronomer Johannes Kepler is today perhaps best remembered for discovering that planetary orbits are elliptical and for being a crucial part of the "Copernican Revolution" that included Galileo and Newton and upended humanity's sense of its place in the heavens. But Kepler had terrestrial concerns, too. After the death of his first wife in 1611, Kepler embarked on a long and arduous quest to remarry, ultimately courting a total of eleven women. Of the first four, Kepler liked the fourth the best ("because of her tall build and athletic body") but did not cease his search. "It would have been settled," Kepler wrote, "had not both love and reason forced a fifth woman on me. This one won me over with love, humble loyalty, economy of household, diligence, and the love she gave the stepchildren."

"However," he wrote, "I continued."

Kepler's friends and relations went on making introductions for him, and he kept on looking, but halfheartedly. His thoughts remained with number five. After eleven courtships in total, he decided he would search no further. "While preparing to travel to Regensburg, I returned to the fifth woman, declared myself, and was accepted." Kepler and Susanna Reuttinger were wed and had six children together, along with the children from Kepler's first marriage. Biographies describe the rest of Kepler's domestic life as a particularly peaceful and joyous time.

Both Kepler and Trick—in opposite ways—experienced firsthand some of the ways that the secretary problem oversimplifies the search for love. In the classical secretary problem, applicants always accept the position, preventing the rejection experienced by Trick. And they cannot be "recalled" once passed over, contrary to the strategy followed by Kepler.

In the decades since the secretary problem was first introduced, a wide range of variants on the scenario have been studied, with strategies for optimal stopping worked out under a number of different conditions. The possibility of rejection, for instance, has a straightforward mathematical solution: propose early and often. If you have, say, a 50/50 chance of being rejected, then the same kind of mathematical analysis that yielded the 37% Rule says you should start making offers after just a *quarter* of your search. If turned down, keep making offers to every best-yet person you see until somebody accepts. With such a strategy, your chance of overall success—that is, proposing and being accepted by the best applicant in the pool—will also be 25%. Not such terrible odds, perhaps, for a scenario that combines the obstacle of rejection with the general difficulty of establishing one's standards in the first place.

Kepler, for his part, decried the "restlessness and doubtfulness" that pushed him to keep on searching. "Was there no other way for my uneasy heart to be content with its fate," he bemoaned in a letter to a confidante, "than by realizing the impossibility of the fulfillment of so many other desires?" Here, again, optimal stopping theory provides some measure of consolation. Rather than being signs of moral or psychological degeneracy, restlessness and doubtfulness actually turn out to be part of the best strategy for scenarios where second chances are possible. If you can recall previous applicants, the optimal algorithm puts a twist on the familiar Look-Then-Leap Rule: a longer noncommittal period, and a fallback plan.

For example, assume an immediate proposal is a sure thing but belated proposals are rejected half the time. Then the math says you should keep looking noncommittally until you've seen 61% of applicants, and then only leap if someone in the remaining 39% of the pool proves to be the best yet. If you're still single after considering all the possibilities—as Kepler was—then go back to the best one that got away. The symmetry between strategy and outcome holds in this case once again, with your chances of ending up with the best applicant under this second-chances-allowed scenario also being 61%.

For Kepler, the difference between reality and the classical secretary problem brought with it a happy ending. In fact, the twist on the classical problem worked out well for Trick, too. After the rejection, he completed his degree and took a job in Germany. There, he "walked into a bar, fell in love with a beautiful woman, moved in together three weeks later, [and] invited her to live in the United States 'for a while.'" She agreed—and six years later, they were wed.

Knowing a Good Thing When You See It: Full Information

The first set of variants we considered—rejection and recall—altered the classical secretary problem's assumptions that timely proposals are always accepted, and tardy proposals, never. For these variants, the best approach remained the same as in the original: look noncommittally for a time, then be ready to leap.

But there's an even more fundamental assumption of the secretary problem that we might call into question. Namely, in the secretary problem we know *nothing* about the applicants other than how they compare to one another. We don't have an objective or preexisting sense of what makes for a good or a bad applicant; moreover, when we compare two of them, we know which of the two is better, but not by how much. It's this fact that gives rise to the unavoidable "look" phase, in which we risk passing up a superb early applicant while we calibrate our expectations and standards. Mathematicians refer to this genre of optimal stopping problems as "no-information games."

This setup is arguably a far cry from most searches for an apartment, a partner, or even a secretary. Imagine instead that we had some kind of objective criterion—if every secretary, for instance, had taken a typing exam scored by percentile, in the fashion of the SAT or GRE or LSAT. That is, every applicant's score will tell us where they fall among all the typists who took the test: a 51st-percentile typist is just above average, a 75th-percentile typist is better than three test takers out of four, and so on.

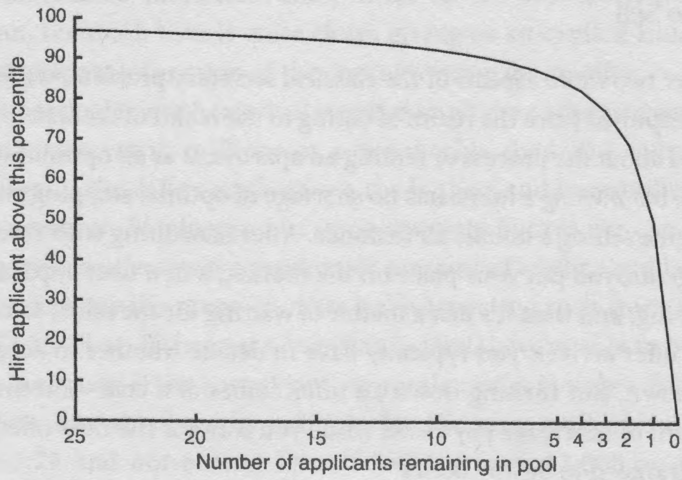
Suppose that our applicant pool is representative of the population at large and isn't skewed or self-selected in any way. Furthermore, suppose we decide that typing speed is the only thing that matters about our applicants. Then we have what mathematicians call "full information," and everything changes. "No buildup of experience is needed to set a standard," as the seminal 1966 paper on the problem put it, "and a profitable choice can sometimes be made immediately." In other words, if a 95th-percentile applicant happens to be the first one we evaluate, we know it instantly and can confidently hire her on the spot—that is, of course, assuming we don't think there's a 96th-percentile applicant in the pool.

And there's the rub. If our goal is, again, to get the single best person for the job, we still need to weigh the likelihood that there's a stronger applicant out there. However, the fact that we have full information gives us everything we need to calculate those odds directly. The chance that our next applicant is in the 96th percentile or higher will always be 1 in 20, for instance. Thus the decision of whether to stop comes down entirely to how many applicants we have left to see. Full information means that we don't need to look before we leap. We can instead use the **Threshold Rule**, where we immediately accept an applicant if she is above a certain percentile. We don't need to look at an initial group of candidates to set this threshold—but we do, however, need to be keenly aware of how much looking remains available.

The math shows that when there are a lot of applicants left in the pool, you should pass up even a very good applicant in the hopes of finding someone still better than that—but as your options dwindle, you should be prepared to hire anyone who's simply better than average. It's a familiar, if not exactly inspiring, message: in the face of slim pickings, lower your standards. It also makes clear the converse: with more fish in the sea, raise them. In both cases, crucially, the math tells you exactly by how much.

The easiest way to understand the numbers for this scenario is to start at the end and think backward. If you're down to the last applicant, of course, you are necessarily forced to choose her. But when looking at the next-to-last applicant, the question becomes: is she above the 50th percentile? If yes, then hire her; if not, it's worth rolling the dice on the last applicant instead, since *her* odds of being above the 50th percentile are 50/50 by definition. Likewise, you should choose the third-to-last applicant if she's above the 69th percentile, the fourth-to-last applicant if she's above the 78th, and so on, being more choosy the more applicants are left. No matter what, never hire someone who's below average unless you're totally out of options. (And since you're still interested only in finding the very best person in the applicant pool, never hire someone who isn't the best you've seen so far.)

The chance of ending up with the single best applicant in this full-information version of the secretary problem comes to 58%—still far from a guarantee, but considerably better than the 37% success rate offered by the



Optimal stopping thresholds in the full-information secretary problem.

37% Rule in the no-information game. If you have all the facts, you can succeed more often than not, even as the applicant pool grows arbitrarily large.

The full-information game thus offers an unexpected and somewhat bizarre takeaway. *Gold digging is more likely to succeed than a quest for love.* If you're evaluating your partners based on any kind of objective criterion—say, their income percentile—then you've got a lot more information at your disposal than if you're after a nebulous emotional response ("love") that might require both experience and comparison to calibrate.

Of course, there's no reason that net worth—or, for that matter, typing speed—needs to be the thing that you're measuring. Any yardstick that provides full information on where an applicant stands relative to the population at large will change the solution from the Look-Then-Leap Rule to the Threshold Rule and will dramatically boost your chances of finding the single best applicant in the group.

There are many more variants of the secretary problem that modify its other assumptions, perhaps bringing it more in line with the real-world challenges of finding love (or a secretary). But the lessons to be learned from optimal stopping aren't limited to dating or hiring. In fact, trying to make the best choice when options only present themselves one by one is also the basic structure of selling a house, parking a car, and quitting when you're ahead. And they're all, to some degree or other, solved problems.

When to Sell

If we alter two more aspects of the classical secretary problem, we find ourselves catapulted from the realm of dating to the realm of real estate. Earlier we talked about the process of renting an apartment as an optimal stopping problem, but *owning* a home has no shortage of optimal stopping either.

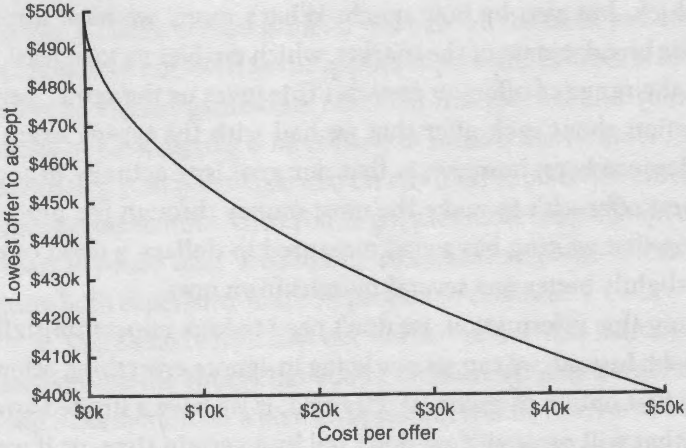
Imagine selling a house, for instance. After consulting with several real estate agents, you put your place on the market; a new coat of paint, some landscaping, and then it's just a matter of waiting for the offers to come in. As each offer arrives, you typically have to decide whether to accept it or turn it down. But turning down an offer comes at a cost—another week (or month) of mortgage payments while you wait for the next offer, which isn't guaranteed to be any better.

Selling a house is similar to the full-information game. We know the objective dollar value of the offers, telling us not only which ones are better than which, but also by how much. What's more, we have information about the broader state of the market, which enables us to at least roughly predict the range of offers to expect. (This gives us the same "percentile" information about each offer that we had with the typing exam above.) The difference here, however, is that our goal isn't actually to secure the single best offer—it's to make the most money through the process overall. Given that waiting has a cost measured in dollars, a good offer today beats a slightly better one several months from now.

Having this information, we don't need to look noncommittally to set a threshold. Instead, we can set one going in, ignore everything below it, and take the first option to exceed it. Granted, if we have a limited amount of savings that will run out if we don't sell by a certain time, or if we expect to get only a limited number of offers and no more interest thereafter, then we should lower our standards as such limits approach. (There's a reason why home buyers look for "motivated" sellers.) But if neither concern leads us to believe that our backs are against the wall, then we can simply focus on a cost-benefit analysis of the waiting game.

Here we'll analyze one of the simplest cases: where we know for certain the price range in which offers will come, and where all offers within that range are equally likely. If we don't have to worry about the offers (or our savings) running out, then we can think purely in terms of what we can expect to gain or lose by waiting for a better deal. If we decline the current offer, will the chance of a better one, multiplied by how *much* better we expect it to be, more than compensate for the cost of the wait? As it turns out, the math here is quite clean, giving us an explicit function for stopping price as a function of the cost of waiting for an offer.

This particular mathematical result doesn't care whether you're selling a mansion worth millions or a ramshackle shed. The only thing it cares about is the difference between the highest and lowest offers you're likely to receive. By plugging in some concrete figures, we can see how this algorithm offers us a considerable amount of explicit guidance. For instance, let's say the range of offers we're expecting runs from $400,000 to $500,000. First, if the cost of waiting is trivial, we're able to be almost infinitely choosy. If the cost of getting another offer is only a dollar, we'll maximize our earnings by waiting for someone willing to offer us $499,552.79 and not a dime less. If waiting costs $2,000 an offer, we should hold out for an even $480,000. In a slow market where waiting costs $10,000 an offer, we should take anything over $455,279. Finally, if



Optimal stopping thresholds in the house-selling problem.

waiting costs half or more of our expected range of offers—in this case, $50,000—then there's no advantage whatsoever to holding out; we'll do best by taking the very first offer that comes along and calling it done. Beggars can't be choosers.

The critical thing to note in this problem is that our threshold depends *only* on the cost of search. Since the chances of the next offer being a good one—and the cost of finding out—never change, our stopping price has no reason to ever get lower as the search goes on, regardless of our luck. We set it once, before we even begin, and then we quite simply hold fast.

The University of Wisconsin–Madison's Laura Albert McLay, an optimization expert, recalls turning to her knowledge of optimal stopping problems when it came time to sell her own house. "The first offer we got was great," she explains, "but it had this huge cost because they wanted us to move out a month before we were ready. There was another competitive offer . . . [but] we just kind of held out until we got the right one." For many sellers, turning down a good offer or two can be a nerve-racking proposition, especially if the ones that immediately follow are no better. But McLay held her ground and stayed cool. "That would have been really, really hard," she admits, "if I didn't know the math was on my side."

This principle applies to any situation where you get a series of offers and pay a cost to seek or wait for the next. As a consequence, it's relevant to cases that go far beyond selling a house. For example, economists have used this algorithm to model how people look for jobs, where it handily explains the otherwise seemingly paradoxical fact of unemployed workers and unfilled vacancies existing at the same time.

In fact, these variations on the optimal stopping problem have another, even more surprising property. As we saw, the ability to "recall" a past opportunity was vital in Kepler's quest for love. But in house selling and job hunting, even if it's possible to reconsider an earlier offer, and even if that offer is guaranteed to still be on the table, you should nonetheless *never* do so. If it wasn't above your threshold then, it won't be above your threshold now. What you've paid to keep searching is a sunk cost. Don't compromise, don't second-guess. And don't look back.

When to Park

> *I find that the three major administrative problems on a campus are sex for the students, athletics for the alumni, and parking for the faculty.*
>
> —CLARK KERR, PRESIDENT OF UC BERKELEY, 1958–1967

Another domain where optimal stopping problems abound—and where looking back is also generally ill-advised—is the car. Motorists feature in some of the earliest literature on the secretary problem, and the framework of constant forward motion makes almost every car-trip decision into a stopping problem: the search for a restaurant; the search for a bathroom; and, most acutely for urban drivers, the search for a parking space. Who better to talk to about the ins and outs of parking than the man described by the *Los Angeles Times* as "the parking rock star," UCLA Distinguished Professor of Urban Planning Donald Shoup? We drove down from Northern California to visit him, reassuring Shoup that we'd be leaving plenty of time for unexpected traffic. "As for planning on 'unexpected traffic,' I think you should plan on expected traffic," he replied. Shoup is perhaps best known for his book *The High Cost of Free Parking*, and he has done much to advance the discussion and understanding of what really happens when someone drives to their destination.

We should pity the poor driver. The ideal parking space, as Shoup models it, is one that optimizes a precise balance between the "sticker price" of the space, the time and inconvenience of walking, the time taken seeking the space (which varies wildly with destination, time of day, etc.), and the gas burned in doing so. The equation changes with the number of passengers in the car, who can split the monetary cost of a space but not the search time or the walk. At the same time, the driver needs to consider that the area with the most parking supply may also be the area with the most demand; parking has a game-theoretic component, as you try to outsmart the other drivers on the road while they in turn are trying to outsmart you.* That said, many of the challenges of parking boil down to a single number: the occupancy rate. This is the proportion of all parking spots that are currently occupied. If the occupancy rate is low, it's easy to find a good parking spot. If it's high, finding anywhere at all to park is a challenge.

Shoup argues that many of the headaches of parking are consequences of cities adopting policies that result in extremely high occupancy rates. If the cost of parking in a particular location is too low (or—horrors!—nothing at all), then there is a high incentive to park there, rather than to park a little farther away and walk. So everybody tries to park there, but most of them find the spaces are already full, and people end up wasting time and burning fossil fuel as they cruise for a spot.

Shoup's solution involves installing digital parking meters that are capable of adaptive prices that rise with demand. (This has now been implemented in downtown San Francisco.) The prices are set with a target occupancy rate in mind, and Shoup argues that this rate should be somewhere around 85%—a radical drop from the nearly 100%-packed curbs of most major cities. As he notes, when occupancy goes from 90% to 95%, it accommodates only 5% more cars but *doubles* the length of everyone's search.

The key impact that occupancy rate has on parking strategy becomes clear once we recognize that parking is an optimal stopping problem. As you drive along the street, every time you see the occasional empty spot you have to make a decision: should you take this spot, or go a little closer to your destination and try your luck?

Assume you're on an infinitely long road, with parking spots evenly spaced, and your goal is to minimize the distance you end up walking to your destination. Then the solution is the Look-Then-Leap Rule. The optimally stopping driver should pass up all vacant spots occurring more than a certain distance from the destination and then take the first space that appears thereafter. And the distance at which to switch from looking to leaping depends on the proportion of spots that are likely to be filled—the occupancy rate. The table on the next page gives the distances for some representative proportions.

*More on the computational perils of game theory in chapter 11.

With this occupancy rate (%)	Wait until this many spaces away, then take the next free spot
0	0
50	1
75	3
80	4
85	5
90	7
95	14
96	17
97	23
98	35
99	69
99.9	693

How to optimally find parking.

If this infinite street has a big-city occupancy rate of 99%, with just 1% of spots vacant, then you should take the first spot you see starting at almost 70 spots—more than a quarter mile—from your destination. But if Shoup has his way and occupancy rates drop to just 85%, you don't need to start seriously looking until you're half a block away.

Most of us don't drive on perfectly straight, infinitely long roads. So as with other optimal stopping problems, researchers have considered a variety of tweaks to this basic scenario. For instance, they have studied the optimal parking strategy for cases where the driver can make U-turns, where fewer parking spaces are available the closer one gets to the destination, and where the driver is in competition against rival drivers also heading to the same destination. But whatever the exact parameters of the problem, more vacant spots are always going to make life easier. It's something of a policy reminder to municipal governments: parking is not as simple as having a resource (spots) and maximizing its utilization (occupancy). Parking is also a *process*—an optimal stopping problem—and it's one that consumes attention, time, and fuel, and generates both pollution and congestion. The right policy addresses the whole problem. And, counterintuitively, empty spots on highly desirable blocks can be the sign that things are working correctly.

We asked Shoup if his research allows him to optimize his own commute, through the Los Angeles traffic to his office at UCLA. Does arguably the world's top expert on parking have some kind of secret weapon?

He does: "I ride my bike."

When to Quit

In 1997, *Forbes* magazine identified Boris Berezovsky as the richest man in Russia, with a fortune of roughly $3 billion. Just ten years earlier he had been living on a mathematician's salary from the USSR Academy of Sciences. He made his billions by drawing on industrial relationships he'd formed through his research to found a company that facilitated interaction between foreign carmakers and the Soviet car manufacturer AvtoVAZ. Berezovky's company then became a large-scale dealer for the cars that AvtoVAZ produced, using a payment installment scheme to take advantage of hyperinflation in the ruble. Using the funds from this partnership he bought partial ownership of AvtoVAZ itself, then the ORT Television network, and finally the Sibneft oil company. Becoming one of a new class of oligarchs, he participated in politics, supporting Boris Yeltsin's reelection in 1996 and the choice of Vladimir Putin as his successor in 1999.

But that's when Berezovsky's luck turned. Shortly after Putin's election, Berezovsky publicly objected to proposed constitutional reforms that would expand the power of the president. His continued public criticism of Putin led to the deterioration of their relationship. In October 2000, when Putin was asked about Berezovsky's criticisms, he replied, "The state has a cudgel in its hands that you use to hit just once, but on the head. We haven't used this cudgel yet. . . . The day we get really angry, we won't hesitate." Berezovsky left Russia permanently the next month, taking up exile in England, where he continued to criticize Putin's regime.

How did Berezovsky decide it was time to leave Russia? Is there a way, perhaps, to think mathematically about the advice to "quit while you're ahead"? Berezovsky in particular might have considered this very question himself, since the topic he had worked on all those years ago as a mathematician was none other than optimal stopping; he authored the first (and, so far, the only) book entirely devoted to the secretary problem.

The problem of quitting while you're ahead has been analyzed under several different guises, but perhaps the most appropriate to Berezovsky's case—with apologies to Russian oligarchs—is known as the "burglar problem." In this problem, a burglar has the opportunity to carry out a sequence of robberies. Each robbery provides some reward, and there's a chance of getting away with it each time. But if the burglar is caught, he gets arrested and loses all his accumulated gains. What algorithm should he follow to maximize his expected take?

The fact that this problem has a solution is bad news for heist movie screenplays: when the team is trying to lure the old burglar out of retirement for one last job, the canny thief need only crunch the numbers. Moreover, the results are pretty intuitive: the number of robberies you should carry out is roughly equal to the chance you get away, divided by the chance you get caught. If you're a skilled burglar and have a 90% chance of pulling off each robbery (and a 10% chance of losing it all), then retire after 90/10 = 9 robberies. A ham-fisted amateur with a 50/50 chance of success? The first time you have nothing to lose, but don't push your luck more than once.

Despite his expertise in optimal stopping, Berezovsky's story ends sadly. He died in March 2013, found by a bodyguard in the locked bathroom of his house in Berkshire with a ligature around his neck. The official conclusion of a postmortem examination was that he had committed suicide, hanging himself after losing much of his wealth through a series of high-profile legal cases involving his enemies in Russia. Perhaps he should have stopped sooner—amassing just a few tens of millions of dollars, say, and not getting into politics. But, alas, that was not his style. One of his mathematician friends, Leonid Boguslavsky, told a story about Berezovsky from when they were both young researchers: on a water-skiing trip to a lake near Moscow, the boat they had planned to use broke down. Here's how David Hoffman tells it in his book *The Oligarchs*:

> While their friends went to the beach and lit a bonfire, Boguslavsky and Berezovsky headed to the dock to try to repair the motor. . . . Three hours later, they had taken apart and reassembled the motor. It was still dead. They had missed most of the party, yet Berezovsky insisted they *had* to keep trying. "We tried this and that," Boguslavsky recalled. Berezovsky would not give up.

Surprisingly, not giving up—ever—also makes an appearance in the optimal stopping literature. It might not seem like it from the wide range of problems we have discussed, but there are sequential decision-making problems for which there *is* no optimal stopping rule. A simple example is the game of "triple or nothing." Imagine you have $1.00, and can play the following game as many times as you want: bet all your money, and have a 50% chance of receiving triple the amount and a 50% chance of losing your entire stake. How many times should you play? Despite its simplicity, there is no optimal stopping rule for this problem, since each time you play, your average gains are a little higher. Starting with $1.00, you will get $3.00 half the time and $0.00 half the time, so on average you expect to end the first round with $1.50 in your pocket. Then, if you were lucky in the first round, the two possibilities from the $3.00 you've just won are $9.00 and $0.00—for an average return of $4.50 from the second bet. The math shows that you should *always* keep playing. But if you follow this strategy, you will eventually lose everything. Some problems are better avoided than solved.

Always Be Stopping

I expect to pass through this world but once. Any good therefore that I can do, or any kindness that I can show to any fellow creature, let me do it now. Let me not defer or neglect it, for I shall not pass this way again.

—STEPHEN GRELLET

Spend the afternoon. You can't take it with you.

—ANNIE DILLARD

We've looked at specific cases of people confronting stopping problems in their lives, and it's clear that most of us encounter these kinds of problems, in one form or another, daily. Whether it involves secretaries, fiancé(e)s, or apartments, life is full of optimal stopping. So the irresistible question is whether—by evolution or education or intuition—we actually do follow the best strategies.

At first glance, the answer is no. About a dozen studies have produced the same result: people tend to stop early, leaving better applicants unseen. To get a better sense for these findings, we talked to UC Riverside's Amnon Rapoport, who has been running optimal stopping experiments in the laboratory for more than forty years.

The study that most closely follows the classical secretary problem was run in the 1990s by Rapoport and his collaborator Darryl Seale. In this study people went through numerous repetitions of the secretary problem, with either 40 or 80 applicants each time. The overall rate at which people found the best possible applicant was pretty good: about 31%, not far from the optimal 37%. Most people acted in a way that was consistent with the Look-Then-Leap Rule, but they leapt sooner than they should have more than four-fifths of the time.

Rapoport told us that he keeps this in mind when solving optimal stopping problems in his own life. In searching for an apartment, for instance, he fights his own urge to commit quickly. "Despite the fact that by nature I am very impatient and I want to take the first apartment, I try to control myself!"

But that impatience suggests another consideration that isn't taken into account in the classical secretary problem: the role of time. After all, the whole time you're searching for a secretary, you don't have a secretary. What's more, you're spending the day conducting interviews instead of getting your own work done.

This type of cost offers a potential explanation for why people stop early when solving a secretary problem in the lab. Seale and Rapoport showed that if the cost of seeing each applicant is imagined to be, for instance, 1% of the value of finding the best secretary, then the optimal strategy would perfectly align with where people actually switched from looking to leaping in their experiment.

The mystery is that in Seale and Rapoport's study, there wasn't a cost for search. So why might people in the laboratory be acting like there was one?

Because for people there's *always* a time cost. It doesn't come from the design of the experiment. It comes from people's lives.

The "endogenous" time costs of searching, which aren't usually captured by optimal stopping models, might thus provide an explanation for why human decision-making routinely diverges from the prescriptions of those models. As optimal stopping researcher Neil Bearden puts it, "After searching for a while, we humans just tend to get bored. It's not irrational to get bored, but it's hard to model that rigorously."

But this doesn't make optimal stopping problems less important; it actually makes them more important, because the flow of time turns *all* decision-making into optimal stopping.

"The theory of optimal stopping is concerned with the problem of choosing a time to take a given action," opens the definitive textbook on optimal stopping, and it's hard to think of a more concise description of the human condition. We decide the right time to buy stocks and the right time to sell them, sure; but also the right time to open the bottle of wine we've been keeping around for a special occasion, the right moment to interrupt someone, the right moment to kiss them.

Viewed this way, the secretary problem's most fundamental yet most unbelievable assumption—its strict seriality, its inexorable one-way march—is revealed to be the nature of time itself. As such, the explicit premise of the optimal stopping problem is the implicit premise of what it is to be alive. It's this that forces us to decide based on possibilities we've not yet seen, this that forces us to embrace high rates of failure even when acting optimally. No choice recurs. We may get *similar* choices again, but never that exact one. Hesitation—inaction—is just as irrevocable as action. What the motorist, locked on the one-way road, is to space, we are to the fourth dimension: we truly pass this way but once.

Intuitively, we think that rational decision-making means exhaustively enumerating our options, weighing each one carefully, and then selecting the best. But in practice, when the clock—or the ticker—is ticking, few aspects of decision-making (or of thinking more generally) are as important as this one: when to stop.

Notes

INTRODUCTION

3 ***al-Jabr wa'l-Muqābala*****:** *Al-Jabr wa'l-Muqābala* brought with it a truly disruptive technology—the Indian decimal system—and the fact that we refer to this system somewhat erroneously as *Arabic* numerals is testament to the book's influence. The introduction of Arabic numerals, and the algorithms they support, kicked off a medieval showdown between the advocates of this newfangled math (the "algorists") and more traditional accountants who favored Roman numerals backed up by an abacus (the "abacists"). It got pretty intense: the city of Florence passed a law in 1399 that banned the use of Arabic numerals by banks. Ironically, Roman numerals were themselves a controversial innovation when they were offered as an alternative to just writing out numbers with words, being declared "unfitted for showing a sum, since names have been invented for that purpose." See Murray, *Chapters in the History of Bookkeeping.*

3 **four-thousand-year-old Sumerian clay tablet:** A detailed analysis appears in Knuth, "Ancient Babylonian Algorithms." Further information on the history of algorithms, with an emphasis on mathematical algorithms, appears in Chabert, Barbin, and Weeks, *A History of Algorithms.*

4 **strikes with the end of an antler:** This technique is known as "soft hammer percussion."

4 **"Science is a way of thinking":** Sagan, *Broca's Brain.*

4 **the way we think about human rationality:** The limitations of a classical conception of rationality—which assumes infinite computational capacity and infinite time to solve a problem—were famously pointed out by the psychologist, economist, and artificial intelligence pioneer Herbert Simon in the 1950s (Simon, *Models of Man*), ultimately leading to a Nobel Prize. Simon argued that "bounded rationality" could provide a better account of human behavior. Simon's insight has been echoed in mathematics and computer science. Alan Turing's colleague I. J. Good (famous for the concept of "the singularity" and for advising Stanley Kubrick about HAL 9000 for *2001: A Space Odyssey*) called this sort of thinking "Type II Rationality." Whereas classic, old-fashioned Type I Rationality just worries about getting the right answer, Type II Rationality takes into account the cost of

getting that answer, recognizing that time is just as important a currency as accuracy. See Good, *Good Thinking.*

Artificial intelligence experts of the twenty-first century have also argued that "bounded optimality"—choosing the algorithm that best trades off time and error—is the key to developing functional intelligent agents. This is a point made by, for instance, UC Berkeley computer scientist Stuart Russell—who literally cowrote the book on artificial intelligence (the bestselling textbook *Artificial Intelligence: A Modern Approach*)—and by Eric Horvitz, managing director at Microsoft Research. See, for example, Russell and Wefald, *Do the Right Thing*, and Horvitz and Zilberstein, "Computational Tradeoffs Under Bounded Resources." Tom and his colleagues have used this approach to develop models of human cognition; see Griffiths, Lieder, and Goodman, "Rational Use of Cognitive Resources."

5 **analogy to a human mathematician:** In section 9 of Turing, "On Computable Numbers," Turing justifies the choices made in defining what we now call a Turing machine by comparing them to operations that a person might carry out: a two-dimensional piece of paper becomes a one-dimensional tape, the person's state of mind becomes the state of the machine, and symbols are written and read as the person or machine moves around on the paper. Computation is what a computer does, and at the time the only "computers" were people.

5 **we are irrational and error-prone:** For example, see Gilovich, *How We Know What Isn't So*; Ariely and Jones, *Predictably Irrational*; and Marcus, *Kluge*.

1. OPTIMAL STOPPING

9 **"Though all Christians start":** From Kepler's letter to "an unknown nobleman" on October 23, 1613; see, e.g., Baumgardt, *Johannes Kepler.*

9 **such a common phenomenon:** The turkey drop is mentioned, among many other places, in http://www.npr.org/templates/story/story.php?storyId=120913056 and http://jezebel.com/5862181/technology-cant-stop-the-turkey-drop.

10 **In any optimal stopping problem:** For more about the mathematics of optimal stopping, Ferguson, *Optimal Stopping and Applications*, is a wonderful reference.

10 **optimal stopping's most famous puzzle:** A detailed treatment of the nature and origins of the secretary problem appears in Ferguson, "Who Solved the Secretary Problem?"

10 **its first appearance in print:** What Gardner writes about is a parlor game called the "Game of Googol," apparently devised in 1958 by John Fox of the Minneapolis-Honeywell Regulator Company and Gerald Marnie of MIT. Here's how it was described by Fox in his original letter to Gardner on May 11, 1959 (all letters to Gardner we quote are from Martin Gardner's papers at Stanford University, series 1, box 5, folder 19):

> The first player writes down as many unique positive numbers on different slips of paper as he wishes. Then he shuffles them and turns them over one at a time. If the second player tells him to stop at a certain slip and the number on that slip is the largest number in the collection then the second player wins. If not, the first player wins.

Fox further noted that the name of the game comes from the fact that the number "one googol" is often written on one of the slips (presumably to trick the opponent into thinking it's the largest number, with "two googol" appearing somewhere else). He then claimed that the optimal strategy for the second player was to wait until half the slips had been turned over and then choose the first number larger than the largest in the first half, converging on a 34.7% chance of winning.

Gardner wrote to Leo Moser, a mathematician at the University of Alberta, to get more information about the problem. Moser had written a journal article in 1956 that addressed a closely related problem (Moser, "On a Problem of Cayley"), originally proposed in 1875 by the influential British mathematician Arthur Cayley (Cayley, "Mathematical Questions"; Cayley, *Collected Mathematical Papers*). Here's the version proposed by Cayley:

> A lottery is arranged as follows: There are n tickets representing a, b, c pounds respectively. A person draws once; looks at his ticket; and if he pleases, draws again (out of the remaining $n-1$ tickets); looks at his ticket, and if he pleases draws again (out of the remaining $n-2$ tickets); and so on, drawing in all not more than k times; and he receives the value of the last drawn ticket. Supposing that he regulates his drawings in the manner most advantageous to him according to the theory of probabilities, what is the value of his expectation?

Moser added one more piece of information—that the tickets were equally likely to take on any value between 0 and 1.

In Cayley's problem and Moser's slight reframing thereof (sometimes collectively referred to as the Cayley-Moser problem), the payoff is the value of the chosen ticket and the challenge is to find the strategy that gives the highest average payoff. It's here that the problem explored by Cayley and Moser differs from the secretary problem (and the Game of Googol) by focusing on maximizing the *average value* of the number chosen, rather than the probability of finding the *single largest* number (when nothing but the best will do). Moser's 1956 paper is notable not just for the neat solution it provides to this problem, but also because it's the first place we see mention of the real-world consequences of optimal stopping. Moser talks about two possible scenarios:

> 1. The tourist's problem: A tourist traveling by car wants to stop for the night at one of n motels indicated on his road guide. He seeks the most comfortable accommodation but naturally does not want to retrace any part of his journey. What criterion should he use for stopping?
> 2. The bachelor's dilemma: A bachelor meets a girl who is willing to marry him and whose "worth" he can estimate. If he rejects her she will have none of him later but he is likely to meet other girls in the future and he estimates that he will have n chances in all. Under what circumstances should he marry?

The idea of entertaining a series of suitors—with the sexes of the protagonists reversed—duly made an appearance in Gardner's 1960 column on the Game of Googol.

Moser provided the correct solution—the 37% Rule—to Gardner, but his letter of August 26, 1959, suggested that the problem might have an earlier origin: "I also found it in some notes that R. E. Gaskell (of Boeing Aircraft in Seattle) distributed in January, 1959. He credits the problem to Dr. G. Marsaglia."

Gardner's charitable interpretation was that Fox and Marnie were claiming the creation of the specific Game of Googol, not of the broader problem of which that game was an instance, a point that was carefully made in his column. But he received a variety of letters citing earlier instances of similar problems, and it's clear that the problem was passed around among mathematicians.

10 **origins of the problem are surprisingly mysterious:** Even Gilbert and Mosteller, "Recognizing the Maximum of a Sequence," one of the most authoritative scientific papers on the secretary problem, admits that "efforts to discover the originator of this problem have been unsuccessful." Ferguson, "Who Solved the Secretary Problem?," provides an amusing

and mathematically detailed history of the secretary problem, including some of its variants. Ferguson argued that in fact the problem described by Gardner hadn't been solved. It should already be clear that lots of people solved the secretary problem of maximizing the probability of selecting the best from a sequence of applicants distinguished only by their relative ranks, but Ferguson pointed out that this is not actually the problem posed in the Game of Googol. First of all, the Googol player knows the values observed on each slip of paper. Second, it's a competitive game—with one player trying to select numbers and a sequence that will deceive the other. Ferguson has his own solution to this more challenging problem, but it's complex enough that you will have to read the paper yourself!

11 **Mosteller recalled hearing about the problem:** Gilbert and Mosteller, "Recognizing the Maximum of a Sequence."

11 **Roger Pinkham of Rutgers wrote:** Letter from Roger Pinkham to Martin Gardner, January 29, 1960.

11 **Flood's influence on computer science:** See Cook, *In Pursuit of the Traveling Salesman*; Poundstone, *Prisoner's Dilemma*; and Flood, "Soft News."

11 **considering the problem since 1949:** Flood made this claim in a letter he wrote to Gardner on May 5, 1960. He enclosed a letter from May 5, 1958, in which he provided the correct solution, although he also indicated that Andrew Gleason, David Blackwell, and Herbert Robbins were rumored to have solved the problem in recent years.

In a letter to Tom Ferguson dated May 12, 1988, Flood went into more detail about the origin of the problem. (The letter is on file in the Merrill Flood archive at the University of Michigan.) His daughter, recently graduated from high school, had entered a serious relationship with an older man, and Flood and his wife disapproved. His daughter was taking the minutes at a conference at George Washington University in January 1950, and Flood presented what he called the "fiancé problem" there. In his words, "I made no attempt to solve the problem at that time, but introduced it simply because I hoped that [she] would think in those terms a bit and it sounded like it might be a nice little easy mathematical problem." Flood indicates that Herbert Robbins provided an approximate solution a few years later, before Flood himself figured out the exact solution.

12 **appears to be in a 1964 paper:** The paper is Chow et al., "Optimal Selection Based on Relative Rank."

12 **the best you've seen so far:** In the literature, what we call "best yet" applicants are referred to (we think somewhat confusingly) as "candidates."

13 **settles to 37% of the pool:** The 37% Rule is derived by doing the same analysis for n applicants—working out the probability that setting a standard based on the first k applicants results in choosing the best applicant overall. This probability can be expressed in terms of the ratio of k to n, which we can call p. As n gets larger, the probability of choosing the best applicant converges to the mathematical function $-p \log p$. This is maximized when $p = 1/e$. The value of e is 2.71828 . . . , so $1/e$ is 0.367879441 . . . , or just under 37%. And the mathematical coincidence—that the probability of success is the same as p—arises because $\log e$ is equal to 1. So if $p = 1/e$, $-p \log p$ is just $1/e$. A well-explained version of the full derivation appears in Ferguson, "Who Solved the Secretary Problem?"

14 **one of the problem's curious mathematical symmetries:** Mathematicians John Gilbert and Frederick Mosteller call this symmetry "amusing" and discuss it at slightly greater length in Gilbert and Mosteller, "Recognizing the Maximum of a Sequence."

15 **"The passion between the sexes":** Malthus, *An Essay on the Principle of Population.*

15 **"married the first man I ever kissed":** Attributed by many sources, e.g., Thomas, *Front Row at the White House.*

15 **a graduate student, looking for love:** Michael Trick's blog post on meeting his wife is "Finding Love Optimally," *Michael Trick's Operations Research Blog*, February 27, 2011, http://mat.tepper.cmu.edu/blog/?p=1392.

15 **the number of applicants or the *time*:** The 37% Rule applies directly to the time period of one's search only when the applicants are uniformly distributed across time. Otherwise, you'll want to aim more precisely for 37% of the *distribution* over time. See Bruss, "A Unified Approach to a Class of Best Choice Problems."

15 **the 37% Rule gave age 26.1 years:** The analysis of waiting until at least age 26 to propose (37% of the way from 18 to 40) first appears in Lindley, "Dynamic Programming and Decision Theory," which is presumably where Trick encountered this idea.

16 **courting a total of eleven women:** Kepler's story is covered in detail in Koestler, *The Watershed*, and in Baumgardt, *Johannes Kepler*, as well as in Connor, *Kepler's Witch*. Most of what we know about Kepler's search for a second wife comes from one letter in particular, which Kepler wrote to "an unknown nobleman" from Linz, Austria, on October 23, 1613.

16 **propose early and often:** Smith, "A Secretary Problem with Uncertain Employment," showed that if the probability of a proposal being rejected is q, then the strategy that maximizes the probability of finding the best applicant is to look at a proportion of applicants equal to $q^{1/(1-q)}$ and then make offers to each applicant better than those seen so far. This proportion is always less than $1/e$, so you're making your chances better by making more offers. Unfortunately, those chances are still worse than if you weren't getting rejected—the probability of ending up with the best applicant is also $q^{1/(1-q)}$, and hence less than that given by the 37% Rule.

17 **until you've seen 61% of applicants:** If delayed proposals are allowed, the optimal strategy depends on the probability of an immediate proposal being accepted, q, and the probability of a delayed proposal being accepted, p. The proportion of candidates to initially pass over is given by the fairly daunting formula $\left(\frac{q^2}{q-p(1-q)}\right)^{1/(1-q)}$. This integrated formula for rejection and recall comes from Petruccelli, "Best-Choice Problems Involving Uncertainty," although recalling past candidates was considered earlier by Yang, "Recognizing the Maximum of a Random Sequence."

This formula simplifies when we make particular choices for q and p. If $p=0$, so delayed proposals are always rejected, we get back the rule for the secretary problem with rejection. As we approach $q=1$, with immediate proposals always being accepted, the proportion at which to begin making offers tends toward e^{p-1}, which is always greater than $1/e$ (which can be rewritten as e^{-1}). This means that having the potential to make offers to applicants who have been passed over should result in spending more time passing over applicants—something that is quite intuitive. In the main text we assume that immediate proposals are always accepted ($q=1$) but delayed proposals are rejected half the time ($p=0.5$). Then you should pass over 61% of applicants and make an offer to the best yet who follows, going back at the end and making an offer to the best overall if necessary.

Another possibility considered by Petruccelli is that the probability of rejection increases with time, as the ardor of applicants decreases. If the probability of an offer being accepted by an applicant is qp^s, where s is the number of "steps" into the past required to reach that applicant, then the optimal strategy depends on q, p, and the number of applicants, n. If $q/(1-p)$ is more than $n-1$ then it's best to play a waiting game, observing all applicants and then making an offer to the best. Otherwise, observe a proportion equal to $q^{1/(1-q)}$ and make an offer to the next applicant better than those seen so far. Interestingly, this is exactly the same strategy (with the same probability of success) as that when $p=0$, meaning that if the probability of rejection increases with time, there is no benefit to being able to go back to a previous candidate.

18 **"No buildup of experience is needed":** Gilbert and Mosteller, "Recognizing the Maximum of a Sequence."

18 **use the Threshold Rule:** The general strategy for solving optimal stopping problems like the full information game is to start at the end and reason backward—a principle that is called "backward induction." For instance, imagine a game where you roll a die, and have the option either to stick with that number or roll again a maximum of k times (we took this example from Hill, "Knowing When to Stop"). What's the optimal strategy? We can figure it out by working backward. If $k=0$, you don't have an option—you have to stick with your roll, and you will average 3.5 points (the average value of a die roll, $(1+2+3+4+5+6)/6$). If $k=1$, then you should only keep a roll that beats that average—a 4 or higher. If you get a 1, 2, or 3, you're better off chancing that final roll. Following this strategy, there's a 50% chance you stop with a 4, 5, or 6 (for an average of 5) and a 50% chance you go on to the final roll (for an average of 3.5). So your average score at $k=1$ is 4.25, and you should only keep a roll at $k=2$ if it beats that score—a 5 or higher. And so on.

Backward induction thus answers an age-old question. "A bird in the hand is worth two in the bush," we say, but is 2.0 the right coefficient here? The math suggests that the right number of birds in the bush actually depends on the quality of the bird in the hand. Replacing birds with dice for convenience, a roll of 1, 2, or 3 isn't even worth as much as a single die "in the bush." But a roll of 4 is worth one die in the bush, while a roll of 5 is worth two, three, or even four dice in the bush. And a roll of 6 is worth even more than the entire contents of an *infinitely large* dice bush—whatever that is.

Gilbert and Mosteller used the same approach to derive the series of thresholds that should be used in the full-information secretary problem. The thresholds themselves are not described by a simple mathematical formula, but some approximations appear in their paper. The simplest approximation gives a threshold of $t_k = 1/(1+0.804/k+0.183/k^2)$ for applicant $n-k$. If the probability of a random applicant being better than applicant $n-k$ is less than t_k, then you should take that applicant. Because the denominator increases—at an increasing rate—as k increases, you should be rapidly lowering your threshold as time goes on.

20 **many more variants of the secretary problem:** Freeman, "The Secretary Problem and Its Extensions" summarizes a large number of these variants. Here's a quick tour of some of the most useful results.

If the number of applicants is equally likely to be any number from 1 to n, then the optimal rule is to view the first n/e^2 (which is approximately 13.5% of n) and take the next candidate better than the best seen so far, with a chance of success of $2/e^2$ (Presman and Sonin, "The Best Choice Problem for a Random Number of Objects").

If the number of applicants is potentially infinite, but the search stops after each applicant with probability p, the optimal rule is to view the first $0.18/p$ applicants, with a 23.6% chance of success (ibid.).

Imagine you want to find the best secretary, but the value of doing so decreases the longer you search. If the payoff for finding the best secretary after viewing k applicants is d^k, then the strategy that maximizes the expected payoff sets a threshold based on a number of applicants that is guaranteed to be less than $1/(1-d)$ as the total number of applicants becomes large (Rasmussen and Pliska, "Choosing the Maximum"). If d is close to 1, then an approximation to the optimal strategy is to view the first

$-0.4348/\log d$ applicants and then take the next candidate better than any seen so far. Following this strategy can result in viewing only a handful of applicants, regardless of the size of the pool.

One way in which real life differs from idealized recruitment scenarios is that the goal might not be to maximize the probability of getting the best secretary. A variety of alternatives have been explored. Chow et al., "Optimal Selection Based on Relative Rank," showed that if the goal is to maximize the average rank of the selected candidate, a different kind of strategy applies. Rather than a single threshold on the relative rank of the applicant, there is a sequence of thresholds. These thresholds increase as more candidates are observed, with the interviewer becoming less stringent over time. For example, with four applicants, the minimum relative rank a candidate needs to have to stop the search is 0 for the first applicant (never stop on the first), 1 for the second (stop only if they are better than the first), 2 for the third (stop if best or second best), and 4 for the fourth (just stop already!). Following this strategy yields an average expected rank of $1\frac{7}{8}$, better than the $(1+2+3+4)/4=2\frac{1}{2}$ that would result from picking an applicant at random. The formula for the optimal thresholds is found by backward induction, and is complicated—we refer interested readers to the original paper.

You can think about the difference between the classical secretary problem and the average-rank case in terms of how they assign payoffs to different ranks. In the classical problem, you get a payoff of 1 for picking the best and 0 for everybody else. In the average-rank case, you get a payoff equal to the number of applicants minus the rank of the selected applicant. There are obvious ways to generalize this, and multi-threshold strategies similar to the one that maximizes the average rank work for any payoff function that decreases as the rank of the applicant increases (Mucci, "On a Class of Secretary Problems"). Another interesting generalization—with important implications for discerning lovers—is that if the payoff is 1 for choosing the best but -1 for choosing anybody else (with 0 for making no choice at all), you should go through a proportion of applicants given by $1/\sqrt{e} \approx 60.7\%$, then take the first person better than all seen so far (or nobody if they all fail this criterion) (Sakaguchi, "Bilateral Sequential Games"). So think hard about your payoff function before getting ready to commit!

But what if you don't just care about finding the best person, but about how much time you have together? Ferguson, Hardwick, and Tamaki, in "Maximizing the Duration of Owning a Relatively Best Object," examined several variants on this problem. If you just care about maximizing the time you spend with the very best person in your set of n, then you should look at the first $0.204n+1.33$ people and leap for the next person better than all of them. But if you care about maximizing the amount of time you spend with somebody who is the best of all the people seen so far, you should just look at a proportion corresponding to $1/e^2 \approx 13.5\%$. These shorter looking periods are particularly relevant in contexts—such as dating—where the search for a partner might take up a significant proportion of your life.

It turns out that it's harder to find the second-best person than it is to find the best. The optimal strategy is to pass over the first half of the applicants, then choose the next applicant who is second best relative to those seen so far (Rose, "A Problem of Optimal Choice and Assignment"). The probability of success is just 1/4 (as opposed to $1/e$ for the best). So you're better off not trying to settle.

Finally, there are also variants that recognize the fact that while you are looking for a secretary, your applicants are themselves looking for a job. The added symmetry—which is particularly relevant when the scenario concerns dating—makes the problem even more complicated. Peter Todd, a cognitive scientist at Indiana University, has explored this complexity (and how to simplify it) in detail. See Todd and Miller, "From Pride and Prejudice to Persuasion Satisficing in Mate Search," and Todd, "Coevolved Cognitive Mechanisms in Mate Search."

20 **Selling a house is similar:** The house-selling problem is analyzed in Sakaguchi, "Dynamic Programming of Some Sequential Sampling Design"; Chow and Robbins, "A Martingale System Theorem and Applications"; and Chow and Robbins, "On Optimal Stopping Rules." We focus on the case where there are potentially infinitely many offers, but these authors also provide optimal strategies when the number of potential offers is known and finite (which are less conservative—you should have a lower threshold if you only have finitely many opportunities). In the infinite case, you should set a threshold based on the expected value of waiting for another offer, and take the first offer that exceeds that threshold.

21 **stopping price as a function of the cost of waiting:** Expressing both the offer price p and cost of waiting for another offer c as fractions of our price range (with 0 as the bottom of the range and 1 as the top), the chance that our next offer is better than p is simply $1-p$. If (or when) a better offer arrives, the average amount we'd expect to gain relative to p is just $\frac{1-p}{2}$. Multiplying these together gives us the expected outcome of entertaining another offer, and this should be greater than or equal to the cost c to be worth doing. This equation $(1-p)\left(\frac{1-p}{2}\right) \geq c$ can be simplified to $\frac{1}{2}(1-p)^2 \geq c$, and solving it for p gives us the answer $p \geq 1-\sqrt{2c}$, as charted on page 22.

22 **"The first offer we got was great":** Laura Albert McLay, personal interview, September 16, 2014.

22 **to model how people look for jobs:** The formulation of job search as an optimal stopping problem is dealt with in Stigler, "The Economics of Information," and Stigler, "Information in the Labor Market." McCall, "Economics of Information and Job Search," proposed using a model equivalent to the solution to the house-selling problem, and Lippman and McCall, "The Economics of Job Search," discusses several extensions to this model. Just as the secretary problem has inspired a vast array of variants, economists have refined this simple model in a variety of ways to make it more realistic: allowing multiple offers to arrive on the same day, tweaking the costs for the seller, and incorporating fluctuation in the economy during the search. A good review of optimal stopping in a job-seeking context can be found in Rogerson, Shimer, and Wright, *Search-Theoretic Models of the Labor Market.*

23 **won't be above your threshold now:** As a survey of the job-search problem puts it: "Assume previously rejected offers cannot be recalled, although this is actually not restrictive because the problem is stationary, so an offer that is not acceptable today will not be acceptable tomorrow" (ibid.).

23 **"parking for the faculty":** Clark Kerr, as quoted in "Education: View from the Bridge," *Time*, November 17, 1958.

23 **"plan on expected traffic":** Donald Shoup, personal correspondence, June 2013.

24 **implemented in downtown San Francisco:** More information on the SFpark system developed by the SFMTA, and its Shoup-inspired dynamic pricing, can be found at http://sfpark.org/how-it-works/pricing/. (Shoup himself is involved in an advisory role.) This program began taking effect in 2011, and is the first project of its kind in the world. For a recent analysis of the effects of the program, see Millard-Ball, Weinberger, and Hampshire, "Is the Curb 80% Full or 20% Empty?"

24 **when occupancy goes from 90% to 95%:** Donald Shoup, personal interview, June 7,

2013. To be precise, the increase from 90% to 95% occupancy reflects an increase of 5.555 . . . percent.

24 **Assume you're on an infinitely long road:** The basic parking problem, as formulated here, was presented as a problem in DeGroot, *Optimal Statistical Decisions*. The solution is to take the first empty spot less than $-\log 2 / \log(1-p)$ spots from the destination, where p is the probability of any given space being available.

25 **you don't need to start seriously looking:** Chapter 17 of Shoup's *The High Cost of Free Parking* discusses the optimal on-street parking strategy when pricing creates an average of one free space per block, which, as Shoup notes, "depends on the conflict between greed and sloth" (personal correspondence). The question of whether to "cruise" for cheap on-street spots or to pay for private parking spaces is taken up in Shoup's chapter 13.

25 **a variety of tweaks to this basic scenario:** Tamaki, "Adaptive Approach to Some Stopping Problems," allowed the probability of a spot being available to vary based on location and considered how these probabilities could be estimated on the fly. Tamaki, "Optimal Stopping in the Parking Problem with U-Turn," added the possibility of U-turns. Tamaki, "An Optimal Parking Problem," considered an extension to DeGroot's model where parking opportunities are not assumed to be a discrete set of spots. Sakaguchi and Tamaki, "On the Optimal Parking Problem in Which Spaces Appear Randomly," used this continuous formulation and allowed the destination to be unknown. MacQueen and Miller, "Optimal Persistence Policies," independently considered a continuous version of the problem that allows circling the block.

26 **"I ride my bike":** Donald Shoup, personal interview, June 7, 2013.

26 ***Forbes* magazine identified Boris Berezovsky:** *Forbes*, "World's Billionaires," July 28, 1997, p. 174.

26 **one of a new class of oligarchs:** Paul Klebnikov, "The Rise of an Oligarch," *Forbes*, September 9, 2000.

26 **"to hit just once, but on the head":** Vladimir Putin, interview with the French newspaper *Le Figaro*, October 26, 2000.

26 **book entirely devoted to the secretary problem:** Berezovsky and Gnedin, *Problems of Best Choice*.

26 **analyzed under several different guises:** There are various ways to approach the problem of quitting when you're ahead. The first is maximizing the length of a sequence of wins. Assume you're tossing a coin that has a probability p of coming up heads. You pay c dollars for each chance to flip the coin, and you get \$1.00 when it comes up heads but lose all your accumulated gains when it comes up tails. When should you stop tossing the coin? The answer, as shown by Norman Starr in 1972, is to stop after r heads, where r is the smallest number such that $p^{r+1} \leq c$. So if it's a regular coin with $p = 1/2$, and it costs \$0.10 to flip the coin, you should stop as soon as you get four heads in a row. The analysis of runs of heads appears in Starr, "How to Win a War if You Must," where it is presented as a model for winning a war of attrition. A more comprehensive analysis is presented in Ferguson, "Stopping a Sum During a Success Run."

Maximizing the length of a run of heads is a pretty good analogy for some kinds of business situations—for a sequence of deals that cost c to set up, have a probability p of working out, and pay d on success but wipe out your gains on failure, you should quit after making r dollars such that $p^{r/d+1} \leq c/d$. Ambitious drug dealers, take note.

In the burglar problem discussed in the text, assume the average amount gained from each robbery is m and the probability of getting away with the robbery is q. But if the burglar is caught, which happens with probability $1-q$, he loses everything. The solution: quit when the accumulated gains are greater than or equal to $mq/(1-q)$. The burglar problem appears in Haggstrom, "Optimal Sequential Procedures When More Than One Stop Is Required," as part of a more complex problem in which the burglar is also trying to decide which city to move to.

27 **found by a bodyguard:** See, e.g., "Boris Berezovsky 'Found with Ligature Around His Neck,'" *BBC News*, March 28, 2013, http://www.bbc.com/news/uk-21963080.

27 **official conclusion of a postmortem examination:** See, e.g., Reuters, "Berezovsky Death Consistent with Hanging: Police," March 25, 2013, http://www.reuters.com/article/2013/03/25/us-britain-russia-berezovsky-postmortem-idUSBRE92O12320130325.

27 **"Berezovsky would not give up":** Hoffman, *The Oligarchs*, p. 128.

27 **there *is* no optimal stopping rule:** One condition for an optimal stopping rule to exist is that the average reward for stopping at the best possible point be finite (see Ferguson, *Optimal Stopping and Applications*). The "triple or nothing" game violates this condition—if heads come up k times followed by one tail, the best possible player gets 3^k-1 as a payoff, stopping right before that tail. The probability of this is $1/2^{k+1}$. The average over k is thus infinite.

If you're thinking that this could be resolved by assuming that people value money less the more they have—that tripling the monetary reward may not be tripling the utility people assign to that money—then there's a simple work-around: you still get a game with no optimal stopping rule just by offering rewards that triple in their utility. For example, if the utility you assign to money increases as a logarithmic function of the amount of money, then the game becomes "cube or nothing"—the amount of money you could receive on the next gamble is raised to the power of three each time you win.

Intriguingly, while there is no optimal stopping rule for "triple or nothing," where your entire fortune is always on the line, there are nonetheless good strategies for playing games like this when you can choose how much of your bankroll to bet. The so-called Kelly betting scheme, named after J. L. Kelly Jr. and first described in Kelly, "A New Interpretation of Information Rate," is one example. In this scheme, a player can maximize his rate of return by betting a proportion of $\frac{p(b+1)-1}{b}$ of his bankroll on each of a sequence of bets that pay off $b+1$ times the original stake with probability p. For our triple or nothing game, $b=2$ and $p=0.5$, so we should bet a quarter of our bankroll each time—not the whole thing, which inevitably leads to bankruptcy. An accessible history of Kelly betting appears in Poundstone, *Fortune's Formula*.

28 **"pass through this world but once":** The provenance of this quotation is not fully certain, although it has been cited as a Quaker saying since the second half of the nineteenth century, and appears to have been attributed to Grellet since at least 1893. For more, see W. Gurney Benham, *Benham's Book of Quotations, Proverbs, and Household Words*, 1907.

28 **"Spend the afternoon":** Dillard, *Pilgrim at Tinker Creek*.

28 **most closely follows the classical secretary problem:** Seale and Rapoport, "Sequential Decision Making with Relative Ranks."

29 **leapt sooner than they should have:** Ibid. The typical place where people switched from looking to leaping was 13 applicants out of 40, and 21 applicants out of 80, or 32% and 26%, respectively.

29 **"by nature I am very impatient":** Amnon Rapoport, personal interview, June 11, 2013.

29 **Seale and Rapoport showed:** Seale and Rapoport, "Sequential Decision Making with Relative Ranks."

29 **"It's not irrational to get bored":** Neil Bearden, personal correspondence, June 26, 2013. See also Bearden, "A New Secretary Problem."

29 **turns *all* decision-making into optimal stopping:** This kind of argument was first made by Herbert Simon, and it was one of the contributions for which he received the Nobel Prize. Simon began his remarkable career as a political scientist, writing a dissertation on the perhaps unpromising topic of administrative behavior. As he dug into the problem of understanding how organizations composed of real people make decisions, he experienced a growing dissatisfaction with the abstract models of decision-making offered by mathematical economics—models that line up with the intuition that rational action requires exhaustive consideration of our options.

Simon's investigation of how decisions actually get made in organizations made it clear to him that these assumptions were incorrect. An alternative was needed. As he put it in "A Behavioral Model of Rational Choice," "the task is to replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist."

The kind of solution that Simon proposed as a more realistic account of human choice—what he dubbed "satisficing"—uses experience to set some threshold for a satisfactory, "good enough" outcome, then takes the first option to exceed that threshold. This algorithm has the same character as the solutions to the optimal stopping problems we have considered here, where the threshold is either determined by spending some time getting a sense for the range of options (as in the secretary problem) or based on knowing the probability of different outcomes. Indeed, one of the examples Simon used in his argument was that of selling a house, with a similar kind of solution to that presented here.

29 **the definitive textbook on optimal stopping:** That's Ferguson, *Optimal Stopping and Applications.*

2. EXPLORE/EXPLOIT

32 **"Make new friends":** Joseph Parry, "New Friends and Old Friends," in *The Best Loved Poems of the American People*, ed. Hazel Felleman (Garden City, NY: Doubleday, 1936), 58.

32 **"life so rich and rare":** Helen Steiner Rice, "The Garden of Friendship," in *The Poems and Prayers of Helen Steiner Rice*, ed. Virginia J. Ruehlmann (Grand Rapids, MI: Fleming H. Revell), 47.

32 **"You try to find spaces":** Scott Plagenhoef, personal interview, September 5, 2013.

33 **The odd name comes from:** In a letter to Merrill Flood dated April 14, 1955 (available in the Merrill Flood archive at the University of Michigan), Frederick Mosteller tells the story of the origin of the name. Mosteller and his collaborator Robert Bush were working on mathematical models of learning—one of the earliest instances of what came to be known as mathematical psychology, informing the research that Tom does today. They were particularly interested in a series of experiments that had been done with a T-shaped maze, where animals are put into the maze at the bottom of the T and then have to decide whether to go left or right. Food—the payoff—may or may not appear on either side of the maze. To explore this behavior with humans they commissioned a machine with two levers that people could pull, which Mosteller dubbed the two-armed bandit. He then introduced the mathematical form of the problem to his colleagues, and it ultimately became generalized to the multi-armed bandit.

A comprehensive introduction to multi-armed bandits appears in Berry and Fristed,

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