

VIENNA UNIVERSITY OF TECHNOLOGY

105.625 PR ADVANCED ECONOMICS PROJECT

# Double Sided Matching

9702170, Karin Leithner  
0725439, Florin Bogdan Balint  
1025735, Clemens Proyer  
1027143, Mattias Haberbusch  
1027433, Thomas Solich

March 23, 2016

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
1.1	The problem . . . . .	2
1.2	The aim . . . . .	2
1.3	Structure . . . . .	2
<b>2</b>	<b>Theory</b>	<b>4</b>
2.1	The Double Sided Matching Problem . . . . .	4
2.2	Gale and Shapley Algorithm . . . . .	4
2.3	Relevance to Game Theory . . . . .	4
2.3.1	General . . . . .	4
2.3.2	Economic Applications . . . . .	5
2.3.3	Double Sided Matching Problem and Game Theory . . . . .	5
<b>3</b>	<b>Prototype</b>	<b>7</b>
3.1	Subsection . . . . .	7
<b>4</b>	<b>Prototype Evaluation: College Applications in Austria</b>	<b>8</b>
4.1	Subsection . . . . .	8
<b>5</b>	<b>Prototype Evaluation: Labor Market in the European Union</b>	<b>9</b>
5.1	Subsection . . . . .	9
<b>6</b>	<b>Summary</b>	<b>10</b>
6.1	Subsection . . . . .	10
<b>7</b>	<b>Example</b>	<b>11</b>
7.1	Heading on level 2 (subsection) . . . . .	12
7.1.1	Heading on level 3 (subsubsection) . . . . .	12
7.2	Example for list (3*itemize) . . . . .	12
7.3	Example for list (enumerate) . . . . .	13
	<b>List of Figures</b>	<b>14</b>
	<b>List of Tables</b>	<b>15</b>
	<b>References</b>	<b>16</b>

## Introduction

This paper deals with the investigation of the double sided matching along with its application in economics.

Add more  
introduc-  
tion text

## The problem

In many daily live expierences there is an hidden double sided matching. For example But we don't have the time or energy to find the perfect or optimal solution. Whereas in some field this could be done more easily or because this is mandatory, like in the universities in germany. They have an admission restriction for the students. Only the students with the best grandes in high school can go on the best universities in germany. But these universitites can only take a limited amount of students.

Find a  
good ex-  
ample

Is this  
true?

Define the  
problem a  
bit better

## The aim

The aim of the paper is to develop a prototype, which could solve the previous discussed problem and similar ones. With this software we could solve questions like: What would be an optimal solution for the german students and universities?

Add more  
questions

## Structure

The seconds chapter deals with the theory overview and background of the double sided matching problem. We will take a closer look at the Gale and Shapley Algorithm and what this has to do with game theory.

The third chapter describes our prototype, which implements the previous discussed theory into practice.

In the fourth chapter we use data from Austria with our prototype to make an statment over the college applications.

rework  
shortly  
before  
we are  
finished,  
because  
structure  
changes  
most time

add dy-  
namic  
linking to  
chapters

Chaper five deals deals with the labor market in the european union. Our prototype will get this data and we will discuss the results of it.

In the summary we will sum up the important findings and compare the outputs of our prototype.

## Theory

### The Double Sided Matching Problem

Clemens & Flo will be here!

The double sided matching problem...

The stable matching problem refers to the problem of finding a matching between two sets of elements which may be equally sized. In [1, p. 9] this problem is firstly described based on an example of college admission: a college is considering a set of  $n$  applicants of which it can admit only a quota of  $q$ .

The assignment of students and colleges is not allowed to be unstable, i.e. there are two applicants  $\alpha$  and  $\beta$  who are assigned to colleges  $A$  and  $B$  although  $\beta$  prefers  $A$  to  $B$  and  $A$  prefers  $\alpha$  to  $\beta$ . If this does not occur, the assignment is called *stable*. In case there is more than one stable solution the *optimal* one is of particular interest. In the previously mentioned college example a stable assignment is called *optimal* if every applicant is at least well off as it would be under any other stable assignment [1, p. 10]. In Economics this is also known as pareto efficiency [2, p. 46].

### Gale and Shapley Algorithm

Clemens & Flo will be here!

### Relevance to Game Theory

...work in progress...

### General

"Game theory is about what happens when people - or genes, or nations - interact". An important aspect for participating parties is to anticipate how the opposite party will react on certain actions. Mathematics shall help to analyze,

understand and estimate outcomes of such games. Depending on the information participants have, they choose how to act basing on rules contained in their strategy. Game theory is widely applied in economics. Companies use game theory to estimate e.g. reactions of competitors or behavior of employees. The major advantages of game theory are its precision and that it can be applied to analyze all kind of games. [3, p. 1ff]

In addition there are some assumptions which are made during preparation and execution by every individual who are interacting together [4]

- Well-specified choices
- Well-defined end-state
- Specified payoff
- Perfect knowledge
- Rationality

These perfect preconditions will never be met in real scenarios. But furthermore, as a consequence of these listed assumptions above, there are two different kind of games:

- Static ones and
- Dynamic ones

The two items can be combined with the following kind of knowledge

- Incomplete
- Complete

## **Economic Applications**

### **Double Sided Matching Problem and Game Theory**

Like described in chapter XXX, the goal of the three main problems, which will be simulated in the course of this project, is to bring two different parties together.

However, a requirement is that the matching of these parties is stable. In the original marriage problem discussed by Shapley the goal was to match men and women so the overall situation is stable. Stability in this case means that these men and women are in a better situation after the matching than they were before alone. This idea can also be expanded to other areas like the labour market or university applications. In each of these three problems the parties need to have a strategy how to react to moves of the counterparty. If, for example, a university offers a place to a student and this is not the student's favorite university, he/she has to decide whether to accept the place (to be on the safe side) or still hope to get accepted from the favorite university (and maybe accept to be on the waiting list there). Not only the student needs to have a strategy how to deal with these situations but also universities have to incorporate different reactions of students into their application process (e.g. how many students will be invited). [1]

An important aspect Roth and Sotomayor mention is, that the rules of the game/matching have to be clear. The way agents are matched to each other influences the analysis of the problem. A possible rule might be that individuals like a student and university are only brought together if both parties agree to the matching. Other rules of a game might be the way one individual proposes to another, whether there exists a moderating individual and many more. [5, p. 492]

## **Prototype**

### **Subsection**

Text...



## **Prototype Evaluation: College Applications in Austria**

### **Subsection**

Text...

## **Prototype Evaluation: Labor Market in the European Union**

### **Subsection**

Text...

## **Summary**

### **Subsection**

Text...

## Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Aenean commodo ligula eget dolor. Aenean massa. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec quam felis, ultricies nec, pellentesque eu, pretium quis, sem. In enim justo, rhoncus ut, imperdiet a, venenatis vitae, justo. Nullam dictum felis eu pede mollis pretium. Integer tincidunt. Cras dapibus. Vivamus elementum semper nisi. Aliquam lorem ante, dapibus in, viverra quis, feugiat a, tellus:

$$\begin{aligned}(x+y)^3 &= (x+y)^2(x+y) \\ &= (x^2 + 2xy + y^2)(x+y) \\ &= (x^3 + 2x^2y + xy^2) + (x^2y + 2xy^2 + y^3) \\ &= x^3 + 3x^2y + 3xy^2 + y^3\end{aligned}\tag{7.1}$$

Phasellus viverra nulla ut metus varius laoreet. Quisque rutrum. Aenean imperdiet. Etiam ultricies nisi vel augue. Curabitur ullamcorper ultricies

Figure 7.1: One angry bird.



## Heading on level 2 (subsection)

Lorem ipsum dolor sit amet, consectetur adipiscing elit.

$$A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix} \quad (7.2)$$

Aenean commodo ligula eget dolor. Aenean massa. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec quam felis, ultricies nec, pellentesque eu, pretium quis, sem.

## Heading on level 3 (subsubsection)

Nulla consequat massa quis enim. Donec pede justo, fringilla vel, aliquet nec, vulputate eget, arcu. In enim justo, rhoncus ut, imperdiet a, venenatis vitae, justo. Nullam dictum felis eu pede mollis pretium. Integer tincidunt. Cras dapibus. Vivamus elementum semper nisi. Aenean vulputate eleifend tellus. Aenean leo ligula, porttitor eu, consequat vitae, eleifend ac, enim.

**Heading on level 4 (paragraph)** Lorem ipsum dolor sit amet, consectetur adipiscing elit. Aenean commodo ligula eget dolor. Aenean massa. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec quam felis, ultricies nec, pellentesque eu, pretium quis, sem. Nulla consequat massa quis enim.

## Example for list (3\*itemize)

- First item in a list
  - First item in a list
    - \* First item in a list
    - \* Second item in a list
  - Second item in a list
- Second item in a list

**Example for list (enumerate)**

1. First item in a list
2. Second item in a list
3. Third item in a list

**List of Figures**

7.1 One angry bird. . . . . 11

## **List of Tables**



## References

- [1] D. Gale and L. S. Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.
- [2] Nicholas Barr. *Economics of the Welfare State*. Oxford University Press, 2012.
- [3] Colin Camerer. *Behavioral game theory: Experiments in strategic interaction*. Princeton University Press, 2003.
- [4] Robert Gibbons. An introduction to applicable game theory. *The Journal of Economic Perspectives*, 11:127–149, 1997.
- [5] Alvin E Roth and Marilda A Oliveira Sotomayor. *Two-sided matching: A study in game-theoretic modeling and analysis*. Number 18. Cambridge University Press, 1992.