VIENNA UNIVERSITY OF TECHNOLOGY

105.625 PR ADVANCED ECONOMICS PROJECT

Double Sided Matching

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1 Introduction

In this paper the double sided matching problem is investigated along with a few applications in economics. This problem was firstly described by David Gale and Lloyd S. Shapley in 1962 [1] and in its simplest form is about finding a match between two sets of elements.

1.1 The problem

In many daily live experiences there is a hidden double sided matching problem. For example students usually have the opportunity to apply to a number of schools. In this case students and schools have to be matched together. Both have preferences which need to be taken into consideration. In our everyday life we usually don't have the time to find an optimal solution (a student might not have enough time to apply to all preferred schools in his country).

In some cases these problems go by without solving them. In other cases however, it is crucial to solve them, e.g. if we consider medical school graduates and open positions in hospitals. Due to multiple reasons (e.g. different preferences, high number of elements in the two sets) this problem cannot be solved in linear time or intuitively.

1.2 The aim

The aim of this paper is to develop a prototype which can solve similar problems, like the one discussed previously. With this software we could solve questions like:

- What would be an optimal solution for the previously mentioned problem of medical graduate students and current open positions in hospitals.
- How can this be applied to all prospective students and universities in Austria?
- Is there also a solution for the current labor supply and demand in the European Union?

1.3 Structure

In the second chapter a theoretical overview is provided regarding the double sided matching problem. We will take a closer look at the Gale and Shapley Algorithm and it's relation to game theory.

rework shortly before we are finished, because structure changes most time

The third chapter describes our prototype, which implements the previously discussed theory into practice.

In the fourth chapter we use data from Austria with our prototype to make an statement over the college applications.

Chapter five deals deals with the labor market in the European Union, trying to match the labor demand and the labor supply.

add dynamic linking to chapters

Finally the most important findings and results are summarized and compared.

2 Theory

2.1 The Double Sided Matching Problem

The double sided matching problem describes a problem, which has two sets of elements. These sets are disjoint and may be equal in size, but this is not mandatory. The job of the double sided matching algorithm is to find a solution, that every element of the first set has a corresponding counterpart in the other set. This can be best described in an example: Assume that the elements are men and women and that there are only heterosexuals preferences for the elements. The two sets are now male and female. The double sided matching has now to create couples, which consists of a man and woman.

This solution of the matching can have some properties, which might be very important (e.g. stability). The assignment of men and women is not allowed to be unstable, i.e. there are two men α and β who are assigned to women A and B although β prefers A to B and A prefers α to β. If this does not occur, the assignment is called *stable*. If there is more than one stable solution the *optimal* one is of particular interest. In this case optimality can be achieved from the perspective of the first set of elements or from the second set. There can also be a trade-off from the perspective of the both sets, it this case optimality is achieved by weighting the expected value of both sets. These properties have been described in the paper by Gale and Shapley [1]. In Economics this is also known as pareto efficiency [2, p. 46].

2.2 Gale and Shapley Algorithm

In the paper from Gale and Shapley [1] an algorithm was proposed for solving the double sided matching problem. Lets assume there are two sets of elements: one with men and the other one with women. The size of the two sets is equal and the preferences are heterosexual and that there is no polygamies (i.e. one element is only allowed to be matched with one element from the other set). The aim of the algorithm is to find stable way of marrying the men and women. Each men and women has their own preference for their marriage partner.

First Iteration

In the first iteration each man proposes to the woman which he ranked first. Afterwards each woman rejects all men which proposed to her, excepting her highest preferred man, which proposed to her. If a man is not rejected he and his woman create a temporary couple. All man which are not in a couple are single.

Second Iteration

In the second interaction each man which is not in a couple, proposes to his highest ranked woman, which he didn't already proposed to. Each woman now chooses the man which she ranks the highest out of her received proposals. If a man is not rejected he and his woman create a temporary couple (Note: after the second iteration some men, which have been in a couple from the first iteration might be now single).

Further Iterations

The third iteration is the same as the second one. This continues until all men are in a couple. If this condition is satisfied we have achieved a stable solution of the double sided matching problem. The stability of the solution of this algorithm has been proven by contradiction [1, p. 12].

2.3 Relevance to Game Theory

...work in progress...

2.3.1 General

"Game theory is about what happens when people - or genes, or nations - interact". An important aspect for participating parties is to anticipate how the opposite party will react on certain actions. Mathematics shall help to analyze, understand and estimate outcomes of such games. Depending on the information participants have, they choose how to act basing on rules contained in their strategy. Game theory is widely applied in economics. Companies use game theory to estimate e.g. reactions of competitors or behavior of employees. The major advantages of game theory are its precision and that it can be applied to analyze all kind of games. [3, p. 1ff]

In addition there are some assumptions which are made during preparation and execution by every individual who are interacting together [4]

- Well-specified choices
- Well-defined end-state
- Specified payoff
- Perfect knowledge
- Rationality

These perfect preconditions will never be met in real scenarios, as a relation to the Double-Sided-Matching, well-specified-choices are good to have defined, but in this context, there might be chances to get into a nearly worst-case scenario instead. But furthermore, as a consequence of these listed assumptions above, there are two kind of basic games:

- Static ones and
- Dynamic ones

The two items can be combined with the following kind of knowledge

- Incomplete
- Complete (there is no private information)

As a result of these different properties, four different game combinations can be made, whether the game is static or dynamic and the knowledge is incomplete or complete. In addition to better understanding we take only two players (of course every combination can be used for a higher amount of players):

- Static and (in)complete information
  As of complete information, players actions can be presented in N x M matrix. Therefore all (also private) information is known to every other player, each player is able to eliminate easily the payoff of the other players.

On the other hand incomplete information forces player to be uncertain about the other players actions and payoff. Furthermore there are two basic steps

1. Player 1 and 2 are choosing actions out of a set, they were predefined of them before.
2. After their actions, they receive their payoffs.

- Dynamic and complete information
  When basic assumptions are made on complete information, than players will create a Nash Equlibrium, otherwhise it will be a one-shot game. As of incomplete information leads to communicate (also misleading) to uninformed parties (closed private informaiton) to under- or overestimate the others payoffs, but indeed there are three basic steps:

  1. Player 1 chooses an action of his predefined set.
  2. Player 2 observes the player 1's action and chooses based on his observation an action.
  3. after their actions, they receive their payoffs.

2.3.2 Economic Applications

2.3.3 Double Sided Matching Problem and Game Theory

Like described in chapter XXX, the goal of the three main problems, which will be simulated in the course of this project, is to bring two different parties together. However, a requirement is that the matching of these parties is stable. In the original marriage problem discussed by Shapley the goal was to match men and women so the overall situation is stable. Stability in this case means that these men and women are in a better situation after the matching than they were before alone. This idea can also be expanded to other areas like the labour market or university applications. In each of these three problems the parties need to have a strategy how to react to moves of the counterparty. If, for example, a university offers a place to a student and this is not the student's favorite university, he/she has to decide whether to accept the place (to be on the safe side) or still hope to get accepted from the favorite university (and maybe accept to be on the waiting list there). Not only the student needs to have a strategy how to deal with these situations but also universities have to incorporate different reactions of students into their application process (e.g. how many students will be invited) [1].

An important aspect Roth and Sotomayor mention is, that the rules of the game/matching have to be clear. The way agents are matched to each other influences the analysis of the problem. A possible rule might be that individuals like a student and university are only brought together if both parties agree to the matching. Other rules of a game might be the way one individual proposes to another, whether there exists a moderating individual and many more [5, p. 492].

3 Prototype

3.1 Subsection

Text...

4 Prototype Evaluation: College Applications in Austria

4.1 Subsection

Text...

5 Prototype Evaluation: Labor Market in the European Union

5.1 Subsection

Text...

6 Summary

6.1 Subsection

Text...

7 Example

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$$\begin{aligned}(x+y)^3 &= (x+y)^2(x+y)\\ &= (x^2+2xy+y^2)(x+y)\\ &= (x^3+2x^2y+xy^2)+(x^2y+2xy^2+y^3)\\ &= x^3+3x^2y+3xy^2+y^3\end{aligned} \tag{7.1}$$

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Figure 7.1: One angry bird.

7.1 Heading on level 2 (subsection)

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$$A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix} \tag{7.2}$$

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7.1.1 Heading on level 3 (subsubsection)

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7.2 Example for list (3*itemize)

- First item in a list
  - First item in a list
    - First item in a list
    - Second item in a list
  - Second item in a list
- Second item in a list

7.3 Example for list (enumerate)

1. First item in a list
2. Second item in a list
3. Third item in a list

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References

[1] David Gale and Lloyd S. Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.

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