

VIENNA UNIVERSITY OF TECHNOLOGY

105.625 PR ADVANCED ECONOMICS PROJECT

Double Sided Matching

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1 Introduction

Double sided matching is about finding a match between two sets of elements. The first algorithm which should solve this problem was described by David Gale and Lloyd S. Shapley in 1962 [1]. The focus will lie on the examination of this algorithm and the creation of a simulation model for it, which can be applied to different economic issues. The selected economic issues will be presented in the following chapters.

1.1 Motivation and Problem Description

In many daily live situations there are hidden double sided matching problems. For example students usually have the opportunity to apply to a number of schools. In this case students and schools have to be matched together. Both parties have preferences which need to be taken into consideration. In our everyday life we usually don't have the time to find an appropriate solution.

In some cases these problems even go by without solving them. In other cases however, it is crucial to solve them, e.g. if we consider medical school graduates and open positions in hospitals. Due to multiple reasons (e.g. different preferences, high number of elements in the two sets) this problem cannot be solved intuitively or in linear time.

1.2 Aim and Objectives

The aim of this paper is to develop a simulation model prototype which can solve similar problems, like the ones discussed above. With this model we could answer questions like:

- Are there "stable" solutions for this kind of problem?
- How can it be applied to all prospective students and universities in Austria?
- Is there also a solution for the current labour supply and demand in the European Union?

1.3 Structure

In the second chapter a theoretical overview is provided regarding the double sided matching problem and what stability means in this context. The focus lies on the Gale and Shapley Algorithm, basic information about the game theory and the relationship between them.

The third chapter describes the prototype of the simulation model, which implements the previously discussed theory into practice. As a simulation environment NetLogo¹

¹Available at <https://ccl.northwestern.edu/netlogo/>, accessed April 28, 2016

will be used. The prototype will be firstly evaluated on a simple discotheque example. In this case it is assumed, that an equal number of men and women are in a discotheque and the objective is to form couples.

The fourth chapter deals with the examination of data from Austrian universities and their integration into the prototype. As a result, a solution for this double sided matching problem will be provided.

Chapter five deals with the labour market in the European Union, trying to match the labour demand and labour supply.

Finally the most important findings and results are summarized and compared in the last chapter. Additionally there is a short outlook into further research.

2 Theory

In this chapter the necessary theoretical background for the realization of this project is discussed.

2.1 The Double Sided Matching Problem

The double sided matching problem describes a problem, which has two sets of elements. These sets are disjoint and may be equal in size, but this is not mandatory. The purpose of the double sided matching algorithm is to find a solution, so every element of the first set has a corresponding counterpart in the other set. This can be best described in an example: Assume that the elements are men and women and that there are only heterosexual preferences for the individuals. The two sets are now male and female. The double sided matching algorithm shall create couples, which consists of a man and woman.

The solutions of the matchings can have different properties, which might be very important (e.g. stability). The output of the algorithm of men and women is not allowed to be unstable, i.e. there are two men α and β who are assigned to women A and B although β prefers A to B and A prefers α to β . If this does not occur, the assignment is called *stable*. If there is more than one stable solution the *optimal* one can be of particular interest.

Optimality means that a solution has the best outcome for an individual or set. In this case optimality can be achieved from the perspective of the first set of elements or from the second set. There can also be a trade-off from the perspective of the both sets, in this case optimality is achieved by weighting the expected value of both sets. These properties have been described in the paper by Gale and Shapley [1].

Generally in economics this is also known as Pareto efficiency [2, p. 46]. Pareto efficiency is a characteristic for a solution where any change which would improve one property on one hand, but will worsen another one on the other hand.

2.2 Gale and Shapley Algorithm

In the paper of Gale and Shapley [1] an algorithm was proposed for solving the double sided matching problem. Let's assume there are two sets of elements: one contains men and the other one women. The size of the two sets is equal, the preferences are heterosexual and there are no polygamies (i.e. one element is only allowed to be matched with one element from the other set). The aim of the algorithm is to find a stable way of marrying men and women. Each man and woman has their own preference for their marriage partner.

First Iteration

In the first iteration each man proposes to the woman who he ranked first. Afterwards a woman has a list of men that proposed to her. This list might be zero, if not she rejects

every man, except her highest preferred man on this list. If a man is not rejected he and the woman create a temporary couple. All men who are not in a couple are single.

Second Iteration

In the second interaction each man who is single, proposes to his highest ranked woman, who he didn't already propose to. Each woman now chooses the man which she ranks the highest out of her received proposals. If a man is not rejected he and the woman create a temporary couple (Note: after the second iteration some men, who have been in a couple from the first iteration might be now single).

Further Iterations

The third iteration is the same as the second one. This continues until no man is single any more. If this condition is satisfied a stable solution is found for the double sided matching problem. The stability of the solution of this algorithm has been proven by contradiction [1].

2.3 Relevance to Game Theory

2.3.1 General

"Game theory is about what happens when people - or genes, or nations - interact" [3, p. 1]. An important aspect for participating parties is to anticipate how the opposite party will react on certain actions. Mathematics shall help to analyse, understand and estimate outcomes of such games. Depending on the information participants have, they decide how to act basing on rules contained in their strategy. Game theory is widely applied in economics. Companies use game theory to estimate e.g. reactions of competitors or behaviour of employees. The major advantages of game theory are its precision and its universal applicability to all kind of games. [3, pp. 1-3]

In addition there are some assumptions which are made during preparation and execution by every individual who is interacting with another one:

- Well-specified choices
- Well-defined end-state
- Specified pay-off
- Perfect knowledge
- Rationality

However, these perfect preconditions will never be met in real scenarios. In the context of double sided matching, it is important to have well-specified choices defined. Nevertheless, there might be chances to get into a nearly worst-case scenario instead. But

furthermore, as a consequence of these listed assumptions above, there are two different kind of games:

- Static ones
- Dynamic ones

The two items can be combined with the following kind of knowledge

- Incomplete
- Complete (there is no private information)

Depending on the nature of the game, whether it is static or dynamic and the knowledge is complete or incomplete four different combinations can be made. For a better understanding, following examples only consist of two players (every combination can be used for a higher amount of players too):

- Static and (in)complete information

As of complete information, the players action set can be presented in an $N \times M$ matrix. Therefore all information (also private) is known to every player and each player is able to easily eliminate the pay-offs of the other players. The pay-off is the outcome of a chosen action of a player which depends on the state or previous actions of the game. The pay-off can change dynamically while the game is in progress. As of incomplete information players are forced to be uncertain about the other player's actions and pay-offs. Besides that, there are two basic steps

1. Player 1 and 2 are choosing actions out of a set, which were predefined by them before.
2. After their actions, they receive their pay-offs.

- Dynamic and complete information

When basic assumptions are made on complete information, players will create a strategy to ensure to get the highest return of each action. Although the strategy might be changed during the game after each turn. Incomplete information leads to communication of uninformed parties (i.e. closed private information) which might end up to under- or overestimate the others pay-offs. In general there will be a Nash Equilibrium between those parties. This equilibrium is created by players because they choose a strategy which do not depend on the other players. There are three basic steps:

1. Player 1 chooses an action out of his/her predefined set.
2. Player 2 observes player 1's action and chooses an action based on his/her observation.
3. After their actions, they receive their pay-offs.

The previously described theory is based on information of the paper "An introduction to applicable game theory" [4].

2.3.2 Example for the Application of Game Theory

Game theory is also very popular in the field of economics. Many companies primarily apply game theory to estimate the reactions of their competitors. In the last auction for broadband internet frequencies in Germany, all major providers hired game theorists out of two reasons. Firstly, they wanted to win the auction for the desired frequency and secondly, they also did not want to overpay for the frequency. The bidding behaviour of a provider indicated the interest rate for a frequency. As a consequence, other providers retained their bids for parts of this frequency so the interested provider did not need to overpay for it. The positive result for the providers was, that the 700-MHz frequency parts were sold for the minimum bidding price. [5]

2.3.3 Double Sided Matching Problem and Game Theory

Like described in chapter 2, the goal of the three main problems, which will be simulated in the course of this project, is to bring different parties together. In the original marriage problem discussed by Shapley, the goal was to match men and women so the overall situation is stable. This idea can also be applied to other areas like the labour market or university applications. In each of these three problems the parties need to have a strategy how to react to moves of the counterpart. If, for example, a university offers a place to a student and this is not the student's favourite one, he/she has to decide whether to accept the place (to be on the safe side) or still hope to get accepted from the favourite university (and as a consequence accept to be on the waiting list there). Not only the student needs to have a strategy how to deal with these situations but also universities have to incorporate different reactions of students into their application process (e.g. how many students to invite) [1].

The double sided matching problem is a dynamic game. The players of the two sets have incomplete information. The algorithm uses the information of both sets, therefore it has complete information.

An important aspect of the game or matching is that the rules have to be clear, which was mentioned by Roth and Sotomayor. The way agents are matched to each other influences the analysis of the problem. A possible rule might be that individuals like a student and university are only brought together if both parties agree to the matching. Other norms of a game might contain the way an individual proposes to another one or whether there exists a moderating individual [6, p. 492].

2.3.4 Double Sided Matching Problem and Optimal Stopping

The double sided matching problem also relates to the optimal stopping problem. The optimal stopping problem concerns itself with choosing the optimal time to stop an undertaken action and at the same time to minimize the expected costs and maximize the profit. A concrete example in this case is if a men is trying to find a women to spend the rest of his life with, when should he stop looking?

A few algorithms as a possible solution for the optimal stopping problem have been presented by Christian & Griffiths 2016 [7]. For the previously mentioned example of partner looking, one algorithm proposes to invest 37% of the time in looking and immediately after to stop looking and take the next best offer. 37% is the provably optimal solution [7, p. 2]. This algorithm is recommended in the case of *no-information games* (e.g. in the case of partner searching, it is hard to quantify the information, because it is hard to compare people to each other. There is no concrete criteria, one can only assume that one person is better or more suitable than another person, but not by how much) [7, p. 18].

Another possible solution for the previously mentioned problem is to set a threshold from the beginning and to take the first option that exceeds it. Christian & Griffiths 2016 [7] recommended this approach in the case of *full-information games*. A suitable example for this case is that of a house selling: the value of the house is known and there is also information about the state of the market, which allows to predict a range of offers. In this case a threshold is set and the first offer that exceeds this threshold can be accepted. This example does not include waiting costs. In the case of waiting costs, they also have to be taken into account and it is recommended to decline an offer if the chance of a better offer multiplied by how much better it is compensate the waiting costs.

Finally Christian & Griffiths 2016 [7, pp. 28-30] recommend to always stop looking at a certain point.

3 Prototype

A prototype for solving the double sided matching problem was developed in NetLogo². In this chapter the prototype is evaluated on a discotheque example. In this case, it is assumed that an equal number of men and women are in a discotheque and the objective is to form couples based on the individual preferences. Afterwards the prototype will be evaluated on more complex problems (e.g., matching students and universities and matching labour supply and labour demand). In the following, the data model, the data generation script, the graphical user interface (GUI) and code from NetLogo will be described.

3.1 Data Model

According to the problem description we defined a generic data model for every individual. This data model is valid for the discotheque example, the university and labour market example, and contains the following attributes:

- id: unique object identifier
- name: object name (e.g. male1 or female1)
- maxMatchesInt: integer value representing the maximum number of matchings for this individual (e.g. in the disco example each person is maximally matched with one other person)
- sideInt: integer helper variable assigning an individual to one of the two participant groups
- partnerList: list of identifiers of the partners, ordered according to the preference of an individual
- rankList: list of ranks (values between 1 to 0) for the partners from the partnerList; the list is ordered from highest to lowest preferences. If the value is equal to 1 this partner is a perfect match.
- hasProposedToList: list of identifiers representing the individuals to which an individual has proposed
- gotProposedByList: list of identifiers representing the individuals from which an individual received proposals. The gotProposedByList is only valid for one iteration.
- tmpMatchList: list containing the identifiers of individuals to whom a temporary match was established
- activeFlag: boolean value stating if an individual is matched or not respectively if the individual gave up (they already proposed to all individuals from the partnerList but only received rejections)

²Available at <https://ccl.northwestern.edu/netlogo/>, accessed April 28, 2016

3.2 Data Generation

In order to generate random data, which represents the individuals and their preferences the statistical computing language R³ has been used. The generated data is exported as a CSV list, which is then read from NetLogo. The R script accepts the following parameters:

- seed: integer value, which is used as a seed by the number generator. The default value is 123.
- numberOfMen: integer value representing the number of men which should be created. The default value is 10.
- numberOfWomen: integer value representing the number of women which should be generated. The default value is 10.
- pickyLower: float value between 0 and 1 representing the lower bound which will be used in order to generate a number that represents how picky a person is. The value 0 means accepts every possible match, the value 1 excepts none.
- pickyUpper: float value between 0 and 1 representing the upper bound which will be used in order to generate a number that represents how picky a person is. The value 0 means excepts every possible match, the value 1 excepts none.

The script can be called as follows from the command line:

```
Rscript initDisco.R 123 10 10 0 0
```

The script generates a CSV which looks as follows:

```
1  "";"id";"name";"maxMatchesInt";"sideInt";"partnerList";"rankList"
2  "1";1;"male1";1;1;"13#18#14#17#16#11#20#19#12#15";
   "0.96#0.95#0.9#0.68#0.57#0.45#0.33#0.25#0.1#0.04"
3  ...
4  "11";11;"female1";1;2;"3#9#5#4#10#8#2#1#6#7";
   "0.92#0.82#0.7#0.67#0.48#0.41#0.35#0.25#0.22#0.05"
```

³Available at <https://www.r-project.org/>, accessed April 28, 2016

3.3 GUI

One of the main goals in the design phase was to keep the GUI simple and clear. The GUI was structured in two main parts - the control and the information section.



Figure 3.1: The control section for the disco example

Figure 3.1 shows the control section in the GUI. With the buttons **Setup** and **Reset** the simulation model can be initialized or reset after a simulation run. The button **Next** advises the program to perform one simulation step, if **Go** is activated the simulation continues until a termination criterion is met. In the disco example the main termination criterion is if every proposing person is already matched to another person. Another criterion for termination is if the proposing person already proposed to every person in his partnerList but got rejected every time. The **debug** switch enables or disables debug messages which are printed during the setup phase and simulation run. The speed of a simulation run can be increased if the **debug** switch is set to Off. The second switch, **switching**, controls, if the two parties propose to each other alternatively (On) or if only one party proposes to the other one (Off). With the drop-down list **starter** the user can determine which party starts proposing. The reporting box **Current Ticks** shows the current number of simulation steps. The button change input file name raises a dialogue box where the user can enter the filename of an input file which shall be used in the simulation. Together with the number of ticks the file name will also be used as the name for the CSV export file.



Figure 3.2: The information section for the disco example

Figure 3.2 shows the information section of the disco example GUI. All created humans are shown in the world area. Men are coloured blue and women are red. On setup of the simulation the proposing site is shown in the top row. As soon as two people are matched their colour changes to green and a link between the two is established. If a couple is divorced the link is removed and the colour changes back to the original one. The plot named **Number of Pairs (%)** illustrates at each point in time how many of the possible couples exist. If the simulation contains 10 men and 10 women and currently 4 couples exist, the plot shows that 40% are matched. The reporting box **Number of Pairs** shows the absolute number of couples. The other reporting box, **Number of Rejections**, indicates how many couples were divorced over time. With the button **Export CSV** the current properties of all humans are exported to a CSV file. The file name is a combination of the input file name, the starting side, if the switching is activated and the current number of ticks.

3.4 NetLogo Code

At the beginning of the NetLogo code the extensions, the data model and the needed global variables are defined. The rest of the code is based on procedures (i.e. a sequence of NetLogo commands which begins with *to* and ends with *end*). The most important procedures are the following:

- *setup* and *reset*: both clear all results and reset the whole model to an initial empty state.
- *open-file*: procedure used to open a CSV list which represents the individuals and their preferences.
- *setup-globals*: procedure used to set up all global variables (e.g., current number of pairs, shape of the humans, etc.)
- *step*: procedure which represents going one step in the double sided matching algorithm. The proposing site proposes and the proposals are also processed.
- *go*: calls *step* until a termination criterion is met.
- *calc-stats*: calculate some statistical information (e.g., number of pairs, number of rejections).
- *clear-before-match*: empties the list of the proposals as a preparation for the next matching round.
- *propose-to[sender receiver]*: sends the proposals.
- *process-proposals [tmpId]*: processes the proposals.
- *reject-proposals [tmpId rejectList]*: rejects the proposals.
- *create-tmpCouples [tmpId acceptList]*: creates temporary couples.

3.5 Simulation Runs

The discotheque problem was simulated three times: once with a simple and small dataset of 20 persons (10 men and 10 women) and twice with a dataset of 200 persons (100 men and 100 women).

3.5.1 Simulation Run 1: Small Dataset

In the first simulation a dataset which represents 20 individuals (10 males and 10 females) was used. This dataset was generated using the previously described R script. After the first simulation step there were already 6 couples (see figure 3.3).

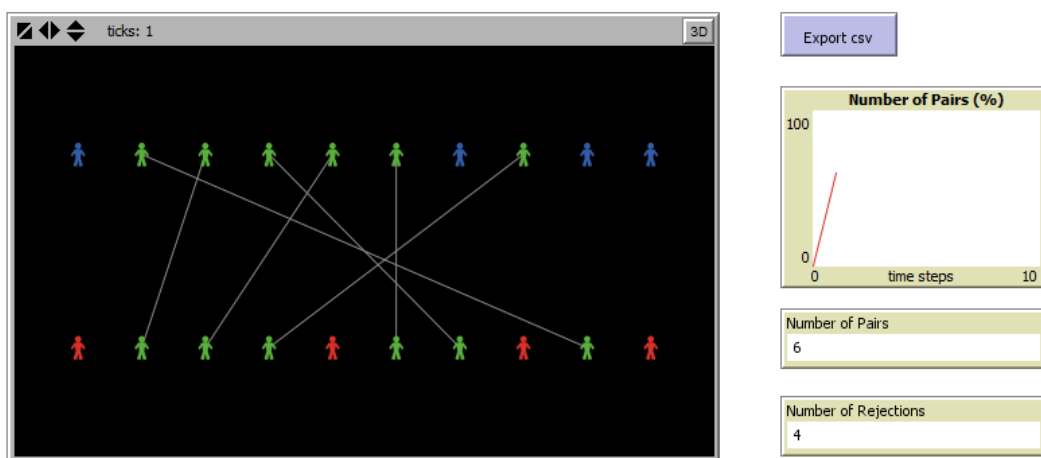


Figure 3.3: Simulation 1, step 1

In the next few steps the number of couples continued to increase by 1 at each step. After the third step 8 couples were formed (see figure 3.4).

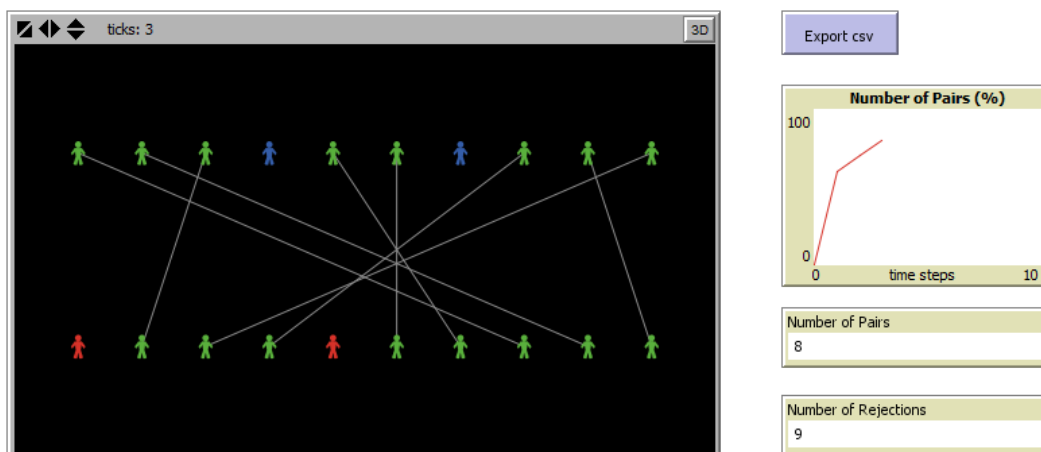


Figure 3.4: Simulation 1, step 3

Afterwards the number of couples continued to increase slowly and some of them changed according to the individuals preferences. As we can see in the figure 3.5 the male number 10 (upper right corner) was in a couple with female number 3, however male number 10 is now single, because female number 3 is now in a couple with male number 6 which is better according to their preferences.

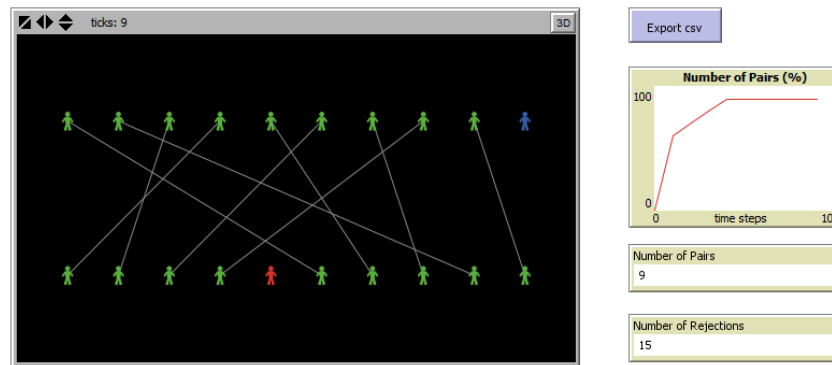


Figure 3.5: Simulation 1, step 9

In the further steps, the couples continued to change according to the individuals preferences and finally after 27 steps the double sided matching problem was solved and 10 couples have been created (see figure 3.6).

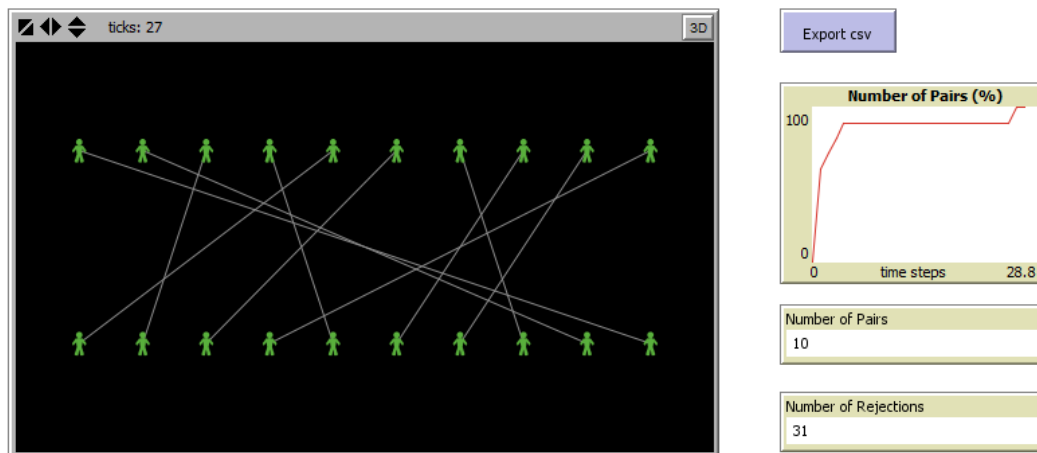


Figure 3.6: Simulation 1, step 27

3.5.2 Simulation Run 2: Large Dataset

The second simulation consisted of a dataset representing 100 men and 100 women. In the first step of the simulation 49 couples were formed (see figure 3.7).

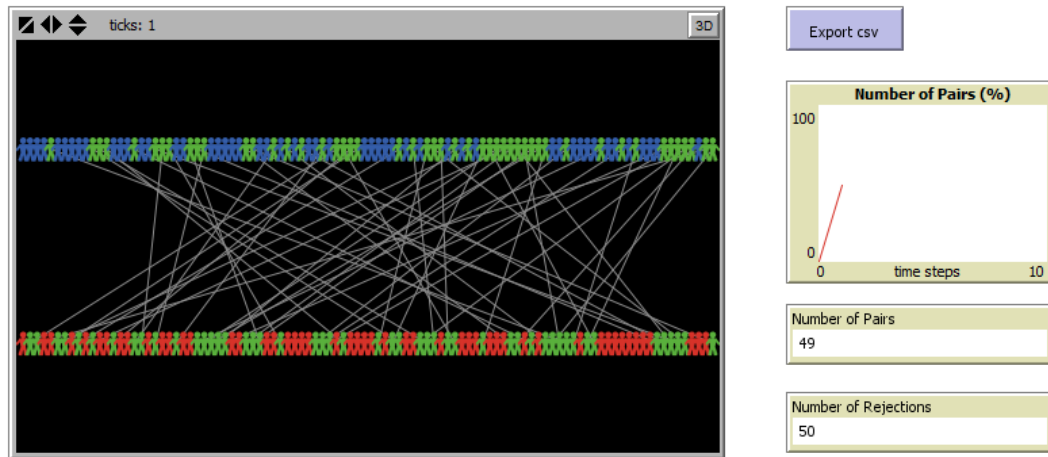


Figure 3.7: Simulation 2, step 1

As observed in the previous simulation, in the next few steps the number of formed couples continued to increase (see figure 3.8).

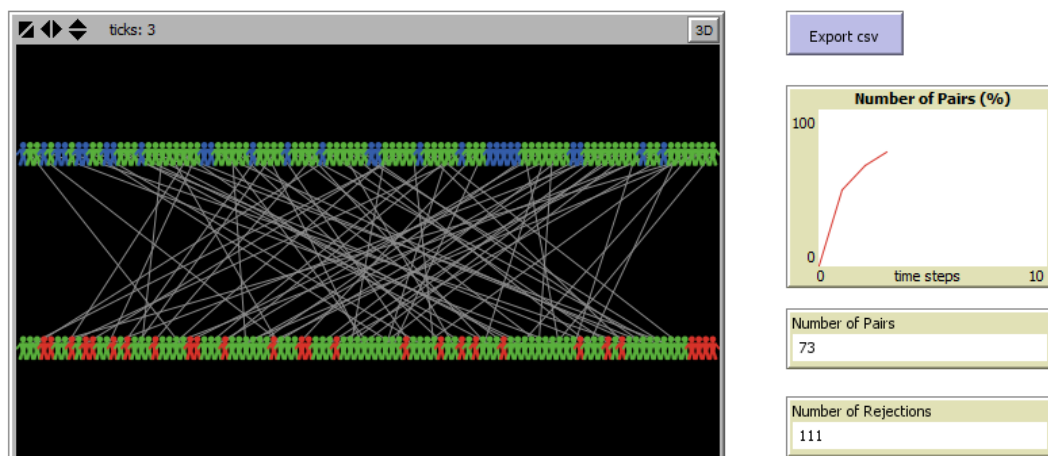


Figure 3.8: Simulation 2, step 3

As the number of steps continued to increase the number of couples increased slowly and a change in the partners was observed (see figure 3.9).

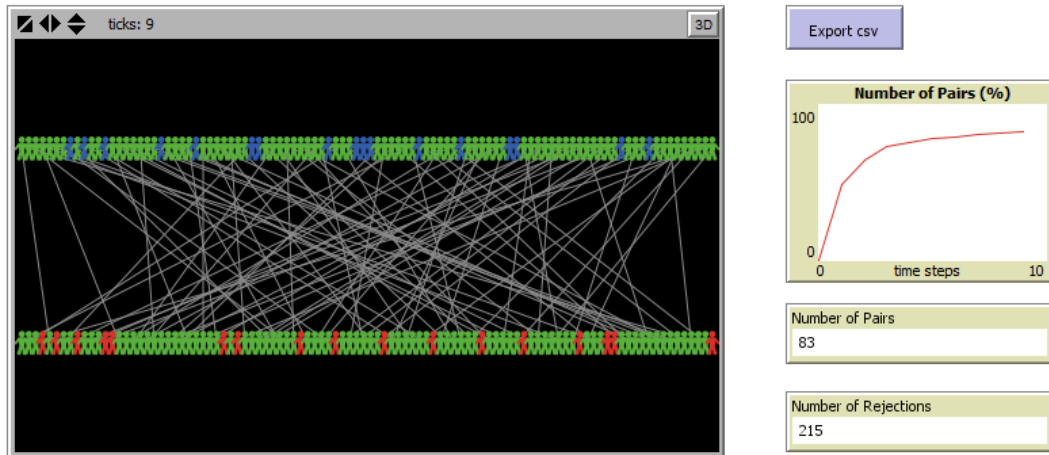


Figure 3.9: Simulation 2, step 9

As the simulation went on, the model started to reach a stable point, where the couples increased only slowly and the number of rejections increased much faster. Finally after 127 steps the model has reached a final state where 94 couples were formed. The final state of the model is illustrated in figure 3.10.

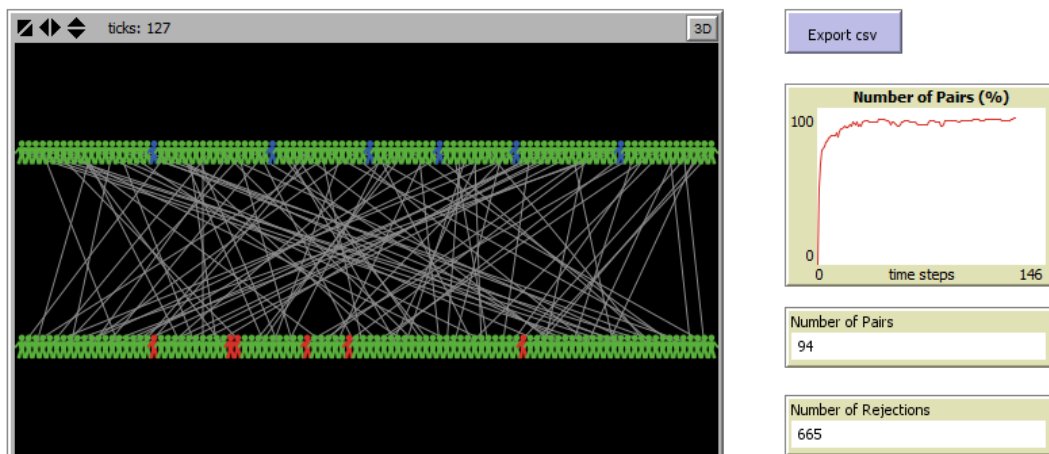


Figure 3.10: Simulation 2, step 127

3.5.3 Simulation Run 3: Large Dataset and Lower Preferences

The second simulation was repeated with a similar dataset, but where the individuals did not have too many preferences (i.e., they were not so picky) regarding their partner. During this simulation the couples were formed faster. After the first step (see figure 3.11) there were 59 couples formed (compared to 49 from the previous simulation - see figure 3.7).

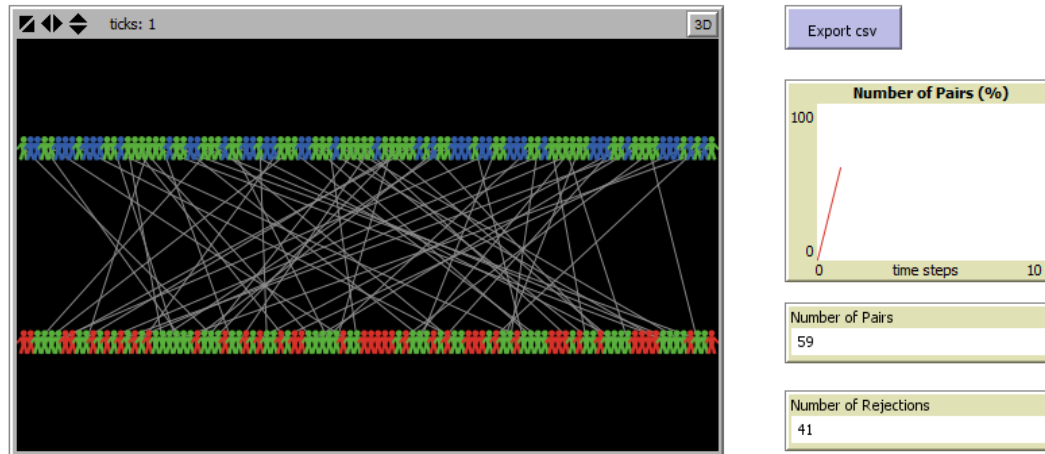


Figure 3.11: Simulation 3, step 1

After the third step there were 79 (compared to 73 from the previous simulation) and after the ninth step there were 90 (compared to 83) couples. Finally the model reached a stable state after 118 steps, where 100 couples were formed. Another observation that was made with the new dataset is that the number of rejections was much lower than in the previous simulation. In the end there were 465 rejections (see figure 3.12) compared to 665 from the previous run (see figure 3.10).

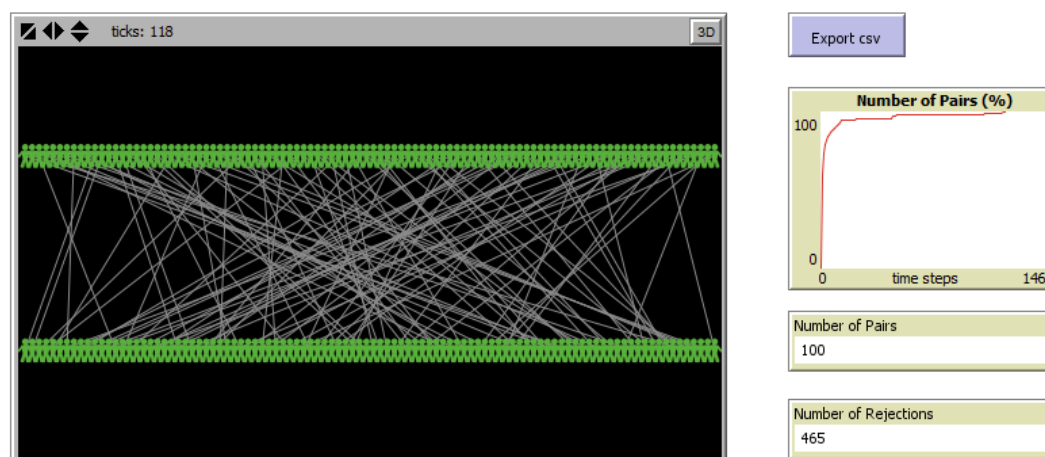


Figure 3.12: Simulation 3, step 118

4 Prototype Evaluation: College Applications in Austria

4.1 Subsection

Text...

5 Prototype Evaluation: Labour Market in the European Union

5.1 Subsection

Text...

6 Summary

6.1 Subsection

Text...

7 Appendix

7.1 Discoteque code

```
1 extensions [array]
2 extensions [string]
3 breed [humans human]
4
5 ; used for drawing a line between the couples
6 undirected-link-breed [ pairs pair ]
7
8 ; humans are turtles (common breed)
9
10 ; NetLogo is case-insensitive
11
12 humans-own [
13   id
14   name
15   maxMatchesInt
16   sideInt
17   partnerList
18   rankList
19   hasProposedToList
20   gotProposedByList
21   tmpMatchList
22   activeFlag
23 ]
24
25 ;; global variables
26 globals [
27   csv fileList ; fileList named csv
28   startSideInt ; set in setup-globals from GUI-slider
29   switchingFlag ; set in setup-globals from GUI-switch
30   debugFlag ; set in setup-globals from GUI-switch
31   user-input-filename ;
32   input-filename; set default in setup-globals or from GUI-button
33   current_nr_of_pairs
34   current_nr_of_pairs_percent
35   current_nr_of_rejects
36 ]
37
38 ;; method which is called from the setup button
39 to setup
40   clear-all
41   if debugFlag = true [
42     show "after clear-all"
43     show "clear-all"
```

```

44     show count humans]
45   setup-globals
46   if debugFlag = true [show "after open-file "
47     show "count humans after open-file "
48     show count humans]
49   reset-ticks
50 end
51
52 to reset
53   clear-ticks
54   clear-turtles
55   clear-patches
56   clear-drawing
57   clear-all-plots
58   clear-output
59   if debugFlag = true [show "after clear-all "
60     show "clear-all "
61     show count humans]
62   setup-globals
63   if debugFlag = true [show "after open-file "
64     show "count humans after open-file "
65     show count humans]
66   reset-ticks
67 end
68
69
70 ;; stackoverflow.com/questions/27096948/how-to-read-a-csv-file-with
   -netlogo
71 to open-file
72   file-open (word input-filename ".csv")
73   set fileList []
74
75   while [not file-at-end?] [
76     set csv file-read-line
77     set csv word csv ";" ; add comma for loop termination
78
79     let myList [] ; list of values
80     let i 0
81     while [not empty? csv]
82     [
83       let $x position ";" csv
84       if i > 0 [
85         let $item substring csv 0 $x ; extract item
86         carefully [set $item read-from-string $item][] ; convert if
           number
87         set myList lput $item myList ; append to list
88       ]

```



```

89     set csv substring csv ($x + 1) length csv ; remove item and
      comma
90     set i i + 1
91 ]
92 if debugFlag = true [show mylist]
93 if item 0 mylist != "id"[
94     create-humans 1 [
95         set id item 0 mylist
96         set name item 1 mylist
97         set maxMatchesInt item 2 mylist
98         set sideInt item 3 mylist
99         let tmpPartnerListString string:split-string item 4 mylist
            "#"
100        set partnerList read-from-string (word tmpPartnerListString)
101        ; set partnerList string:split-int item 4 mylist "#"
102        let tmpRankListString string:split-string item 5 mylist "#"
103        set rankList read-from-string (word tmpRankListString)
104        ; set rankList string:split-int item 5 mylist "#"
105        set hasProposedToList []
106        set gotProposedByList []
107        set tmpMatchList []
108        set activeFlag true
109    ]
110 ]
111 set fileList lput mylist fileList
112 if debugFlag = true [show "count humans at end of open-file"
113     show count humans
114     show "fileList at end of open-file"
115     show fileList]
116 ]
117 file-close
118 end
119
120 to setup-globals
121     ifelse starter = "Men" [set startSideInt 1] [set startSideInt 2]
122     set debugFlag debug
123     set switchingFlag switching
124     ifelse user-input-filename = 0 [set input-filename "
        disco100NotPicky"] [set input-filename user-input-filename]
125     set current_nr_of_rejects 0
126     set current_nr_of_pairs 0
127     set current_nr_of_pairs_percent 0
128     let current_world_width world-width
129
130     open-file ; and read initialisation data from csv file
131     ; define starting position and start color
132     let number_women count humans with [sideInt = 2]

```

```

133   let xposHumansStart current_world_width / 2 ; starting position
      for humans
134   let i 1
135   foreach sort humans with [sideInt = startSideInt] [
136     ask ? [
137       let number_starting count humans with [sideInt = startSideInt]
138       set shape "person"
139       set size 1
140       set heading 0
141       ; positioning and color
142       set ycor 4
143       ifelse sideInt = 1 [set color blue] [set color red]
144       let xposHumans i / (number_starting + 1) * current_world_width
          - xposHumansStart
145       set xcor xposHumans
146       set i i + 1
147     ]
148   ]
149   set i 1
150   foreach sort humans with [sideInt != startSideInt] [
151     ask ? [
152       let number_notstarting count humans with [sideInt !=
          startSideInt]
153       set shape "person"
154       set size 1
155       set heading 0
156       ; positioning and color
157       set ycor -4
158       ifelse sideInt = 1 [set color blue] [set color red]
159       let xposHumans i / (number_notstarting + 1) *
          current_world_width - xposHumansStart
160       set xcor xposHumans
161       set i i + 1
162     ]
163   ]
164 end
165
166 to go
167   if count humans with [activeFlag = true and (sideInt =
      startSideInt or switchingFlag = true)] = 0 [stop]
168   step
169 end
170
171
172
173 to step
174   if debugFlag = true [show "----- begin of step
      -----"]

```

```

175     show "count humans at begin of step"
176     show count humans]
177 clear-before-match
178 foreach sort humans with [activeFlag = true and sideInt =
    startSideInt] [ ; start of proposing
179 ask ? [
180     if debugFlag = true [show "count humans at begin of ask humans
        with activeFlag=true and sideInt=startSideInt"
181     show count humans with [activeFlag = true and sideInt =
        startSideInt]
182     show "myId"
183     show id
184     show "myName"
185     show name
186     show "partnerList"
187     show partnerList
188     show "hasProposedToList"
189     show hasProposedToList]
190     let tmpPotentialPartnersList list-difference partnerList
        hasProposedToList ; set-difference of partnerList \
        hasProposedToList
191     if length tmpPotentialPartnersList = 0 [
192         set activeFlag false ; this human has no potential partners
            to propose to
193         stop ; break
194     ]
195     if length tmpMatchList >= maxMatchesInt [
196         set activeFlag false ; this human has enough current matches
197         stop
198     ]
199     let myPreferredPartner item 0 tmpPotentialPartnersList ; most
        preferred partner from tmp... List
200     propose-to id myPreferredPartner
201 ]
202 ] ; end of proposing
203 if debugFlag = true [show "end of proposing"]
204 foreach sort humans with [length gotProposedByList > 0 and sideInt
    != startSideInt] [
205     ask ? [
206         process-proposals id
207     ]
208 ]
209 if switchingFlag = true [set startSideInt ((startSideInt + 1) mod
    2)]
210 if startSideInt = 0 [set startSideInt 2]
211 if debugFlag = true [show "switchingFlag"
212     show switchingFlag
213     show "startSideInt"

```

```

214     show startSideInt]
215   calc-stats
216   tick
217   if debugFlag = true [show "##### end of step
    #####"]
218 end
219
220 to calc-stats
221   set current_nr_of_pairs 0
222   ask humans with [sideInt = startSideInt] [
223     if length tmpMatchList = maxMatchesInt [
224       set current_nr_of_pairs current_nr_of_pairs + 1
225     ]
226   ]
227   let countedHumans count humans with [sideInt = startSideInt]
228   ifelse countedHumans = 0 [
229     set current_nr_of_pairs_percent 0
230   ] [
231     set current_nr_of_pairs_percent current_nr_of_pairs /
      countedHumans
232     set current_nr_of_pairs_percent current_nr_of_pairs_percent *
      100
233     if current_nr_of_pairs = 0 [set current_nr_of_pairs_percent 0]
234   ]
235   if debugFlag = true [ show "Current Nr of Pairs"
236     show current_nr_of_pairs
237     show "Current Nr of Pairs Percent"
238     show current_nr_of_pairs_percent
239   ]
240 end
241
242 to clear-before-match
243   ask humans [
244     set gotProposedbyList [] ; delete gotProposedbyList (in
      preparation for next matching-round)
245   ]
246 end
247
248 to propose-to[sender receiver]
249   if debugFlag = true [show "----- begin of propose-to
    -----"]
250   ask humans with [id = sender] [
251     set hasProposedToList lput receiver hasProposedToList
252     if debugFlag = true [show "hasProposedToList"
253       show hasProposedToList
254       show "receiver"
255       show receiver]
256     ask humans with [id = receiver] [

```

```

257     set gotProposedByList lput sender gotProposedByList
258     if debugFlag = true [show "female side"
259         show "gotProposedByList"
260         show gotProposedByList]
261     ]
262 ]
263 if debugFlag = true [show "##### end of propose-to
    #####"]
264 end
265
266 to process-proposals [tmpId]
267     if debugFlag = true [show "----- begin of process-
        proposals -----"
268         show "count humans at begin of process-proposals"
269         show count humans]
270     ask humans with [id = tmpId] [
271         if debugFlag = true [show "gotProposedByList"
272             show gotProposedByList
273             show "sideInt"
274             show sideInt
275             show "maxMatchesInt"
276             show maxMatchesInt]
277         let tmpPotentialCoupleList []
278         ifelse length tmpMatchList = 0 [
279             set tmpPotentialCoupleList gotProposedByList
280         ] [
281             set tmpPotentialCoupleList list-union-set tmpMatchList
                gotProposedByList
282         ]
283         if debugFlag = true [show "tmpPotentialCoupleList"
284             show tmpPotentialCoupleList]
285         let tmpRejectList []
286         let tmpCoupleList []
287         ifelse length tmpPotentialCoupleList > maxMatchesInt [
288             set tmpCoupleList list-order tmpPotentialCoupleList rankList
                partnerList maxMatchesInt
289             if debugFlag = true [show "has more proposals than willing to
                accept"
290                 show "tmpCoupleList"
291                 show tmpCoupleList]
292             set tmpRejectList list-difference tmpPotentialCoupleList
                tmpCoupleList
293         ] [
294             set tmpCoupleList tmpPotentialCoupleList
295         ]
296         if length tmpRejectList > 0 [
297             reject-proposals id tmpRejectList
298         ]

```

```

299     if length tmpCoupleList > 0 [
300         create-tmpCouples id tmpCoupleList
301     ]
302 ]
303 if debugFlag = true [show "##### end of process-
    proposals #####"]
304 end
305
306 to reject-proposals [tmpId rejectList]
307     if debugFlag = true [show "----- begin of reject-
        proposals -----"]
308     let rejecter 0
309     ask humans with [id = tmpId] [
310         set rejecter self
311         set tmpMatchList list-difference tmpMatchList rejectList
312         if debugFlag = true [show "tmpMatchList"
313             show tmpMatchList]
314         foreach rejectList [
315             ask humans with [id = ?] [
316                 set tmpMatchList list-difference tmpMatchList lput tmpId []
317                 set activeFlag true
318                 ifelse sideInt = 1 [set color blue] [set color red]
319                 set current_nr_of_rejects current_nr_of_rejects + 1
320                 remove-link self rejecter
321             ]
322             ifelse sideInt = 1 [set color blue] [set color red]
323         ]
324     ]
325     if debugFlag = true [show "##### end of reject-proposals
        #####"]
326 end
327
328 to create-tmpCouples [tmpId acceptList]
329     if debugFlag = true [show "----- begin of create-
        tmpCouples -----"]
330     ask humans with [id = tmpId] [
331         set tmpMatchList acceptList
332         if debugFlag = true [show "tmpMatchList"
333             show tmpMatchList]
334         foreach acceptList [
335             ask humans with [id = ?] [
336                 set tmpMatchList list-union-set tmpMatchList lput tmpId []
337                 set color green
338                 create-link-with myself
339             ]
340             set color green
341         ]
342     ]

```

```

343   if debugFlag = true [show "##### end of create-
      tmpCouples #####"]
344 end
345
346
347 to-report list-order [listToOrder ranking partners maxMatches]
348   if debugFlag = true [show "----- begin of list-order
      -----"]
349   set listToOrder list-overlap listToOrder partners
350   if debugFlag = true [show "listToOrder"
351     show listToOrder]
352   if length listToOrder > maxMatches [
353     let tmpRanking get-rating-of-list partnerList listToOrder
      ranking
354     let tmpList listToOrder
355     if debugFlag = true [show "tmpRanking"
356       show tmpRanking
357       show "tmpList"
358       show tmpList]
359     set listToOrder []
360     let i 0
361     loop [
362       if i >= maxMatches [report listToOrder]
363       let j position (max tmpRanking) tmpRanking
364       if debugFlag = true [show "j"
365         show j
366         show "item j tmpList"
367         show item j tmpList
368         show "tmpList"
369         show tmpList]
370       set listToOrder lput item j tmpList listToOrder
371       set tmpRanking remove-item j tmpRanking
372       set tmpList remove-item j tmpList
373       if debugFlag = true [show "listToOrder"
374         show listToOrder
375         show "tmpList"
376         show tmpList]
377       set i i + 1
378     ]
379
380   ]
381   report listToOrder
382   if debugFlag = true [show "##### end of list-order
      #####"]
383 end
384
385 ;; symmetrical difference: fullList \ toRemoveList
386 to-report list-difference [fullList toRemoveList]

```

```

387   report filter [not member? ? toRemoveList] fullList
388 end
389
390 ;; intersection of listA and listB
391 to-report list-overlap [listA listB]
392   if debugFlag = true [show "##### begin of list-overlap
      #####"
393     show listA
394     show listB
395     show filter [member? ? listB] listA]
396   if debugFlag = true [show "##### end of list-overlap
      #####"]
397   report filter [member? ? listB] listA
398 end
399
400 ;; union of listA and listB
401 to-report list-union-set [listA listB]
402   let tmpList list-difference listB listA
403   foreach tmpList [ set listA lput ? listA]
404   report listA
405 end
406
407
408 ;; gets ranking associated with fullList for partialList
409 to-report get-rating-of-list [fullList partialList ranking]
410   let tmpList []
411   foreach partialList [
412     set tmpList lput item (position ? fullList) ranking tmpList
413   ]
414   report tmpList
415 end
416
417
418
419 ;;;;;;;;;;;;;;;;;;;;;;;;;;
420 ;;; Input filename    ;;;
421 ;;;;;;;;;;;;;;;;;;;;;;;;;;
422
423 to change-input
424   set user-input-filename user-input "Type input filename (without
      extension).".
425   if debugFlag = true [show "user-input-filename"
426     show user-input-filename]
427 end
428
429
430
431

```



```

432 ;;;;;;;;;;;;;;;;;;;;;;;;;;
433 ;; Output matches ;;
434 ;;;;;;;;;;;;;;;;;;;;;;;;;;
435
436
437 ;; write-to-file taken from
438 ;; Wilensky, U. (1999). NetLogo. http://ccl.northwestern.edu/netlogo/.
439 ;; Center for Connected Learning and Computer-Based Modeling,
440   Northwestern University, Evanston, IL
441 ;; code from File/Models Library/Code Examples/File Output Example
442
443 to export-to-csv
444   let output-filename (word input-filename "_Starter_" starter "
445     _Switch_" switching "_Ticks_" ticks ".csv")
446   foreach sort humans [
447     ask ? [
448       let delimPartnerList list-concat-with-delim partnerList "#"
449       let delimRankList list-concat-with-delim rankList "#"
450       let delimHasProposedToList list-concat-with-delim
451         hasProposedToList "#"
452       let delimGotProposedByList list-concat-with-delim
453         gotProposedByList "#"
454       let delimTmpMatchList list-concat-with-delim tmpMatchList "#"
455       write-csv output-filename (list (id) (name) (maxMatchesInt) (
456         sideInt) (delimPartnerList) (delimRankList) (
457         delimHasProposedToList) (delimGotProposedByList) (
458         delimTmpMatchList) (activeFlag))
459     ]
460   ]
461 end
462
463
464
465
466 ;; http://stackoverflow.com/questions/22462168/netlogo-export-
467   tableau-issues
468 to write-csv [ #filename #items ]
469   ;; #items is a list of the data (or headers!) to write.
470   if is-list? #items and not empty? #items
471     [ file-open #filename
472       ;; quote non-numeric items
473       set #items map quote #items
474       ;; print the items
475       ;; if only one item, print it.
476       ifelse length #items = 1 [ file-print first #items ]
477       [file-print reduce [ (word ?1 ";" ?2) ] #items]
478       ;; close-up
479       file-close
480     ]
481   ]

```

```

471 end
472
473 to remove-link [human1 human2]
474   ask links with [(end1 = human1 and end2 = human2) or (end1 =
      human2 and end2 = human1)] [
475     die]
476 end
477 ;; http://stackoverflow.com/questions/22462168/netlogo-export-
      tableau-issues
478 to-report quote [ #thing ]
479   ifelse is-number? #thing
480     [ report #thing ]
481     [ report (word "\"" #thing "\"") ]
482 end
483
484 ;; https://groups.yahoo.com/neo/groups/netlogo-users/conversations/
      topics/6490
485 ;; intersperse listA with delim
486 to-report list-concat-with-delim [listA delim]
487   if length listA > 0 [report reduce [(word ?1 delim ?2)] listA]
488   report ""
489 end

```

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