EE 282

Assignment.

E/19/166

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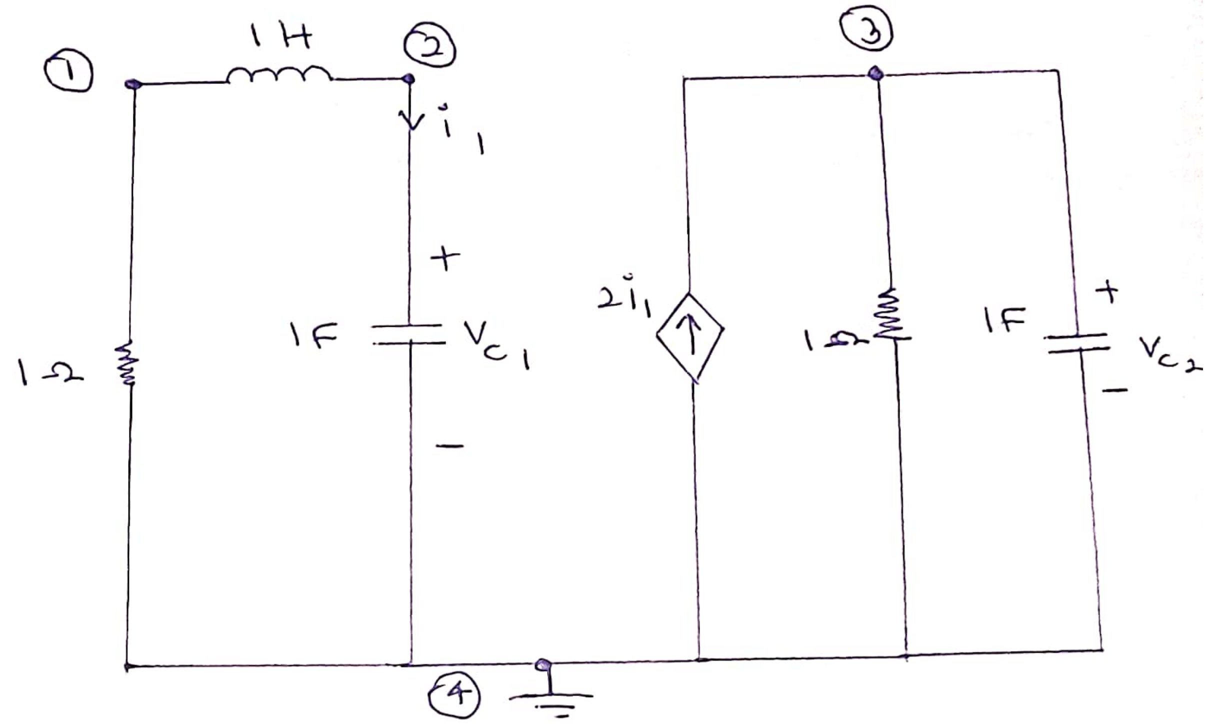
1 a.



Apply KCl for the circuit (Nodal Analysis)

Time Domain :-

KCl for node 1

$$\frac{e_1(t)}{1} = -i_1(t) \quad -①$$

KCL for node 2

$$i_1(t) = \frac{1}{1}\frac{d}{dt}(e_2(t)) \quad -②$$

KCl for node 3

$$2i_1(t) = \frac{e_3(t)}{1} + \frac{1}{1}\frac{d}{dt}e_3(t) \quad -③$$

equation for the inductor

$$e_1(t) - e_2(t) = \frac{d}{dt}i_1(t) \quad -④$$

1

Laplace domain:-

① $\Rightarrow$ $E_1(s) = -I_1(s)$

② $\Rightarrow$ $I_1(s) = sE_2(s) - e_2(0^-)$

③ $\Rightarrow$ $2I_1(s) = E_3(s) + sE_3(s) - e_3(0^-)$

④ $\Rightarrow$ $E_1(s) - E_2(s) = sI_1(s) - i_1(0^-)$

b) $$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -s & 0 & 1 \\ 0 & 0 & -(1+s) & 2 \\ 1 & -1 & 0 & -s \end{bmatrix}}_{P(s)} \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_3(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -e_2(0^-) \\ -e_3(0^-) \\ -i_1(0^-) \end{bmatrix}$$

find natural frequencies

$$\det\{[P(s)]\} = 0$$

$$0 = 1\,(-(1+s)(s^2+1)) - 1\,(1(s(1+s))$$

$$0 = -1\,(1+s)(s^2+s+1)$$

$$s_1 = -1 \;//, \quad s_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \;//, \quad s_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \;//$$

$\therefore$ Natural frequencies are $-1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ //

since $Re\{s_i\} < 0$ for $i = 1, 2, 3$

The circuit is exponentially stable.

c) The mode $s_1 = -1$ is set up

$$P(-1) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix}$$

$\underline{w}_1$ is obtained by $P(-1)\underline{w}_1 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$w_{11} + w_{14} = 0$ — (1)

$w_{12} + w_{14} = 0$ — (2)

$w_{14} = 0$ — (3)

$w_{11} - w_{12} + w_{14} = 0$ — (4)

$\therefore$ (1) $\Rightarrow w_{11} = 0$

(2) $\Rightarrow w_{12} = 0$

Let $w_{13} = k$

$$\therefore \underline{w}_1 = \begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix}$$

$\therefore$ for initial conditions

$$\begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ i_1(t) \end{bmatrix}_{t=0}$$

Initial conditions to set up modes $= -1$

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ i_1(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix} e^{-t} \quad // \qquad \Rightarrow \qquad \begin{bmatrix} e_1(0^-) \\ e_2(0^-) \\ e_3(0^-) \\ i_1(0^-) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix} \quad //$$

Similarly

$$P(s_2)\,\underline{w}_2 = 0 \quad \text{for } s_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}j$$

$$P\left(-\frac{1}{2} + \frac{\sqrt{3}j}{2}\right) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right) & 0 & 1 \\ 0 & 0 & -\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) & 2 \\ 1 & -1 & 0 & \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)j \end{pmatrix}$$

$\underline{w}_2$ is obtained by $P\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)\underline{w}_2 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right) & 0 & 1 \\ 0 & 0 & -\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right) & 2 \\ 1 & -1 & 0 & \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)j \end{bmatrix} \begin{bmatrix} w_{21} \\ w_{22} \\ w_{23} \\ w_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$w_{21} + w_{24} = 0 \quad -(1)$$

$$\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right) w_{22} + w_{24} = 0 \quad -(2)$$

$$-\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right) w_{23} + 2w_{24} = 0 \quad -(3)$$

$$w_{21} - w_{22} + \left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right) w_{24} = 0 \quad (4)$$

$$w_{21} = k^*$$

$$(1) \Rightarrow \quad w_{24} = -w_{21}$$

(2) $\Rightarrow$ $$w_{22} = \frac{-w_{24}}{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)}$$

$$= \frac{-\cancel{2}^{1}\, w_{24}\,(1+\sqrt{3}j)}{\cancel{4}\,2}$$

$$= \frac{(1+\sqrt{3}j)}{2}\, w_{21}$$

(3) $\Rightarrow$ $$w_{23} = \frac{2\, w_{24}}{\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)}$$

$$= \frac{\cancel{4}\,\cancel{w}(1-\sqrt{3}j)\, w_{24}}{\cancel{4}}$$

$$= \cancel{(-1)}\,(-1+\sqrt{3}j)\, w_{21}$$

$$\underline{w}_2 = \begin{bmatrix} 1 \\ \frac{(1+\sqrt{3}j)}{2} \\ (-1+\sqrt{3}j) \\ -1 \end{bmatrix}$$

$S_2 = \overline{S_3}$

$w_3 = \overline{w_2}$

S_2 and $S_3 = \overline{S_2}$ modes are excited by initial conditions $\underline{w}_1$

since $w_2 = \overline{w_1}$

our initial conditions are real

$$\underline{w}_2 + \underline{w}_3 = \underline{w}_2 + \overline{\underline{w}}_2$$

$P(s_2)\underline{w}_2 = 0 \quad \Rightarrow \quad \overline{P(s_2)}\,\overline{\underline{w}_2} = 0$

$P(\overline{s}_2)\overline{\underline{w}}_2 = 0 \quad \Rightarrow \quad P(s_3)\overline{\underline{w}}_2 = 0$

$$P(s_2)\underline{w}_2 = 0$$

$$w_2 = \overline{w}_3$$

$$\therefore \underline{e}(t) = k\,\underline{w}_1 e^{s_2 t} + \overline{k}\,\underline{w}_3 e^{s_3 t}$$

$$= 2|k|\, e^{s_{2r}t}\left[\cos(s_{2i}t + \measuredangle k)\underline{w}_r - \sin(s_{2i}t + \measuredangle k)\underline{w}_i\right] \quad -(1)$$

where $k = |k| e^{j\measuredangle k}$

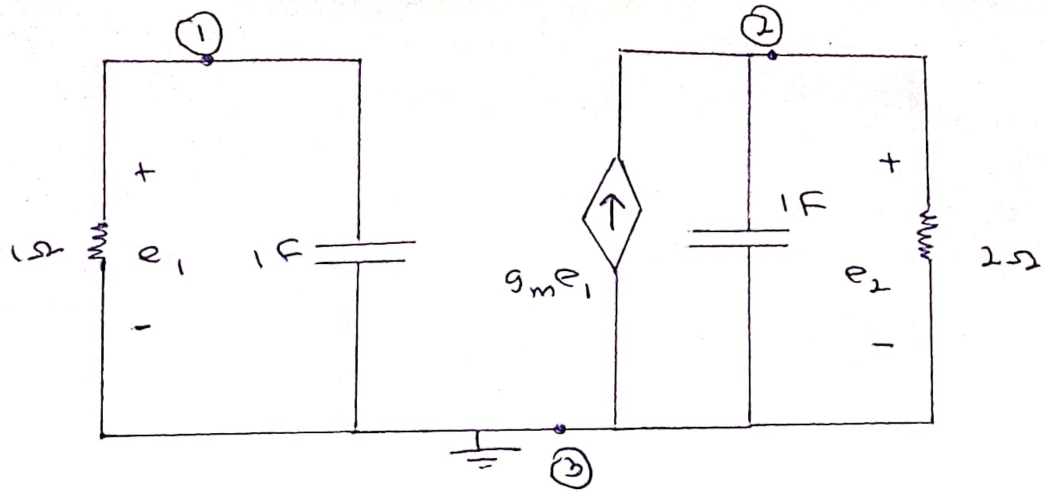
$s_i = s_{2r} + j s_{2i}$

$\underline{w}_1 = \underline{w}_r + j\,\underline{w}_i$

$\therefore$ Initial conditions to set up s_2 and s_3

$$= k\begin{bmatrix} 1 \\ \frac{1+\sqrt{3}j}{2} \\ -1+\sqrt{3}j \\ -1 \end{bmatrix} + \overline{k}\begin{bmatrix} 1 \\ \frac{1-\sqrt{3}j}{2} \\ -1-\sqrt{3}j \\ -1 \end{bmatrix}$$

2



a) Apply nodal analysis.

KCl for node 1

$$\frac{e_1(t)}{1} = \frac{d}{dt} e_1(t) \quad \text{—} ①$$

KCl for node 2

$$e_1(t) = \frac{d}{dt} e_2(t) + \frac{e_2(t)}{2} \quad \text{—} ②$$

Convert ① & ② to laplace domain.

$$① \Rightarrow E_1(s) = sE_1(s) - e_1(0^-)$$

$$② \Rightarrow E_1(s) = sE_2(s) - e_2(0^-) + \frac{E_2(s)}{2}$$

$$\begin{bmatrix} 1-s & 0 \\ 1 & -(s+\frac{1}{2}) \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} = \begin{bmatrix} -e_1(0^-) \\ -e_2(0^-) \end{bmatrix}$$

find natural frequencies

$$\det\{[P(s)]\} = -(1-s)\left(s+\frac{1}{2}\right) = 0$$

$$s_1 = 1 \,, \; s_2 = -\frac{1}{2}$$

∴ Natural frequencies are $1, -1/2$ //

b) $$P(s)^{-1} = \frac{1}{(1-s)\left(s+\frac{1}{2}\right)} \begin{bmatrix} -\left(s+\frac{1}{2}\right) & 0 \\ -1 & (1-s) \end{bmatrix}$$

$$= \begin{pmatrix} \frac{1}{1-s} & 0 \\ +\frac{1}{(1-s)\left(s+\frac{1}{2}\right)} & \frac{1}{\left(s+\frac{1}{2}\right)} \end{pmatrix}$$

$$\begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} = \begin{bmatrix} 1/(1-s) & 0 \\ +\frac{1}{(1-s)\left(s+\frac{1}{2}\right)} & \frac{1}{\left(s+\frac{1}{2}\right)} \end{bmatrix} \begin{bmatrix} -e_1(0^-) \\ -e_2(0^-) \end{bmatrix}$$

$$\therefore E_1(s) = \frac{1}{(1-s)}\left(-e_1(0^-)\right) \quad -(A)$$

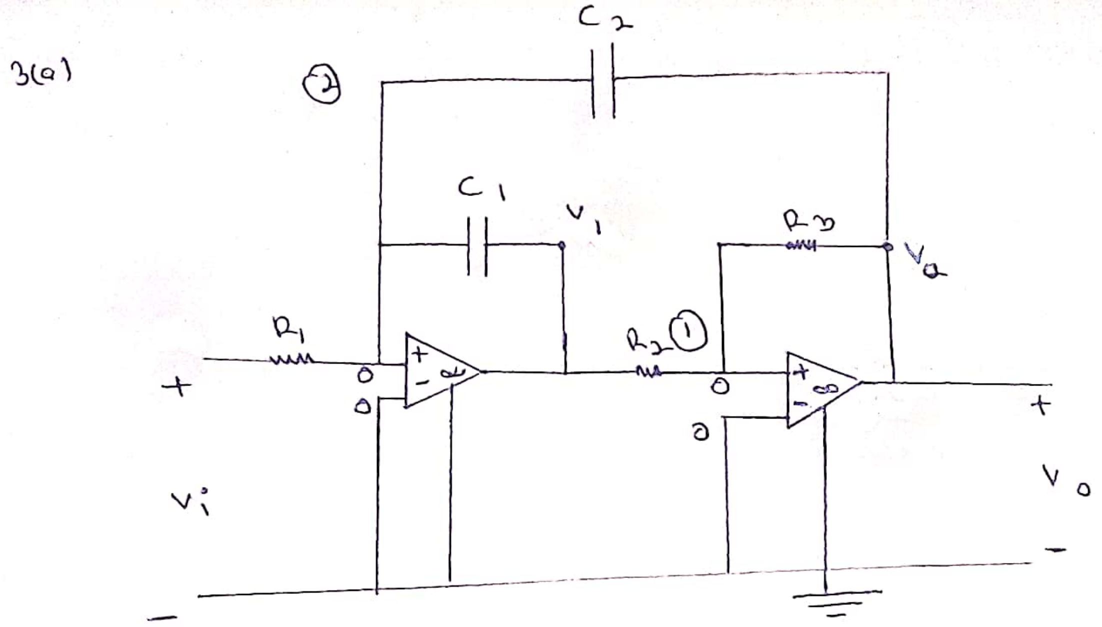
$$E_2(s) = \frac{1}{(1-s)\left(s+\frac{1}{2}\right)}\left(-e_1(0^-)\right) + -\frac{1}{\left(s+\frac{1}{2}\right)} e_2(0^-) \quad -(B)$$

from (A) and (B),

The stable mode $e^{-1/2 t}$ is not observable at e_1.

$\therefore$ Only one of the natural frequencies is observable at e_1

3(a)



Nodal analysis.

KCL for node 1

$$\frac{V_1}{R_2} + \frac{V_o}{R_3} = 0 \quad -(1)$$

KCL for node 2

$$C_1 \frac{dV_1}{dt} + C_2 \frac{dV_o}{dt} = -\frac{V_i}{R_1} \quad -(2)$$

For zero-state response

$$(1) \Rightarrow \frac{V_1(s)}{R_2} + \frac{V_o(s)}{R_3} = 0 \quad -(1)'$$

$$(2) \Rightarrow sC_1V_1(s) + sC_2V_o(s) = -\frac{V_i(s)}{R_1} \quad -(2)'$$

from $(1)'$ & $(2)'$

$$\therefore C_1 s\left[-\frac{V_o(s)}{R_3}\right]R_2 + sC_2V_o(s) = -\frac{V_i(s)}{R_1}$$

$$\therefore \cancel{H_s = \frac{V_o(s)}{\quad}}$$

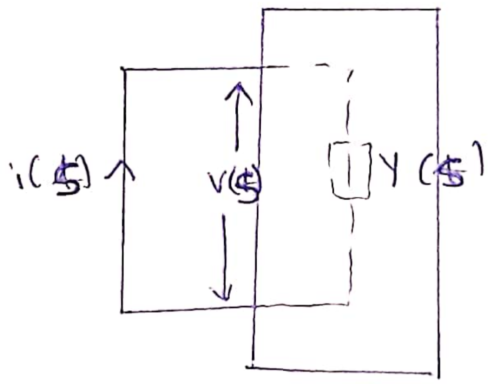
$$\frac{V_o(s)}{R_3}\left(-C_1sR_2 + sC_2R_3\right) = -\frac{V_i(s)}{R_1}$$

$$\frac{V_o(s)}{R_3}\left(sC_2R_3 - C_1 s R_2\right) = -\frac{V_i(s)}{R_1}$$

$$V_o(s)\left(C_1 s R_2 - sC_2R_3\right) = +\frac{R_3\,(V_i(s))}{R_1}$$

$$H_s = \frac{V_o(s)}{V_i(s)} = \frac{R_3}{R_1 s\left(C_1R_2 - C_2R_3\right)} \quad //$$

b. (i)



~~time domain~~

~~i(t) = v(t) y(t)~~

Laplace domain

$$I(s) = V(s).Y(s)$$

$$Y(s) = \frac{I(s)}{V(s)}$$

$\therefore$ $Y(s)$ - network function

$I(s)$ - L(zero-state response of $i(t)$)

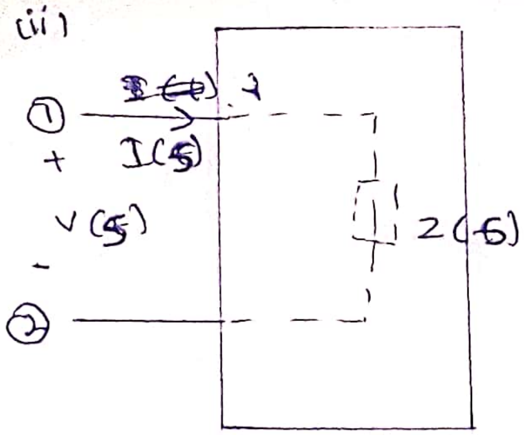
$V(s)$ - L(input)

Poles of network function ($Y(s)$) are neccessarily natural frequencies of the circuit.

$\therefore$ If s_1 is a pole of $Y(s)$, then s_1 is a natural ~~response~~ frequency of the circuit. Therefore, we may observe $i(t) = k_1 e^{s_1 t}$ as s_1 is a natural frequency of the circuit. (for some initial state)

$\therefore$ The statement ~~of~~ is true.

(ii)



$V(s) = I(s) Z(s)$

$Z(s) = \dfrac{V(s)}{I(s)}$

$\therefore$ $Z(s)$ - network function

$\cancel{V}(s)$ - L (zero state response of i(t))

$I(s)$ - L (input)

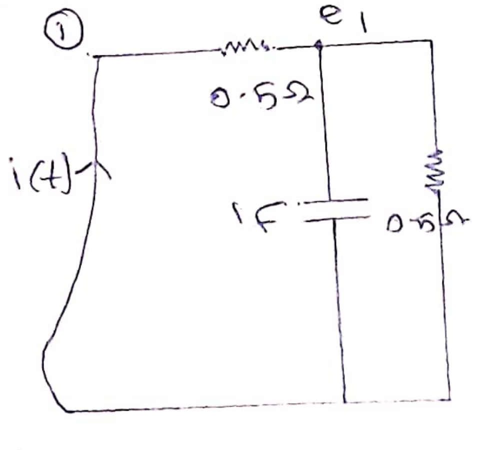
Poles of network function ($Z(s)$) are necessarily natural frequencies of the circuit.

$\therefore$ If s_2 is a pole of $z(s)$, then s_2 is a natural frequency of the circuit.

$\therefore$ We may observe $v(t) = k_1 e^{s_2 t}$ as s_2 is a natural frequency of the circuit. (for some initial state)

$\therefore$ The statement is true.

(iii) <u>when short circuited</u>



KCl for e_1,

time domain

$$\frac{e_1(t)}{0.5} + \frac{d}{dt} e_1(t) + \frac{e_1(t)}{0.5} = 0$$

$$\frac{d}{dt} e_1(t) + 4e_1(t) = 0$$

Laplace domain

$$sE_1(s) - e_1(0^-) + 4E_1(s) = 0$$

$$(4+s)E_1(s) = e_1(0^-)$$

$$E_1(s) = \frac{e_1(0^-)}{(s+4)}$$

Apply inverse laplace transform

$$E_1(t) = e_1(0^-)e^{-4t}$$

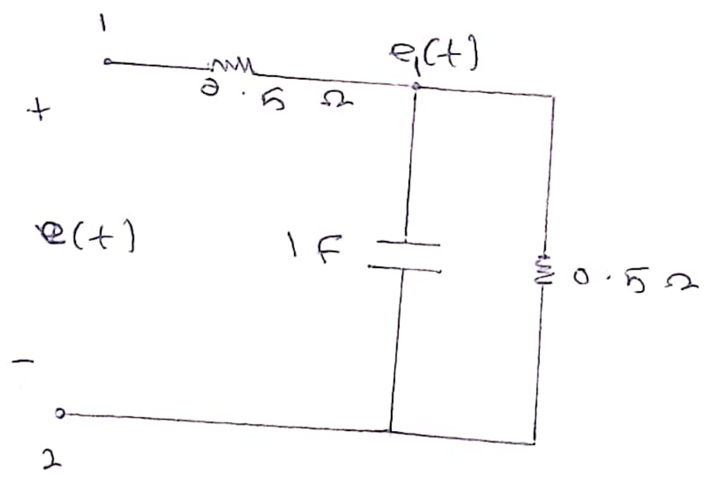
$$\therefore \quad i(t) = -\frac{e_1(0^-)e^{-4t}}{0.5}$$

$s_1 = -4$

$$i(t) = -2e(0^-)e^{-4t} \quad //$$

$\therefore$ by part (i), for some initial state, we may observe $i(t) = k_1 e^{s_1 t} = -2e(0^-)e^{-4t}$ //
and $s_1 = -4$ is a pole of $E_1(s)$.

When open circuited.



KCL for e_1,

time domain

$$\frac{d}{dt}e_1(t) + \frac{e_1(t)}{0.5} = 0$$

laplace domain

$$sE_1(s) - e_1(0^-) + 2E_1(s) = 0$$

$$E_1(s)(s+2) = e_1(0^-)$$

$$E_1(s) = \frac{e_1(0^-)}{s+2}$$

Applying inverse laplace transform

$$e_1(t) = e_1(0^-) e^{-2(t)}$$

Let $e_1(t) = v(t) \Rightarrow v(t) = v(0^-) e^{-2t}$

$$s_2 = -2$$

$\therefore$ by part (ii), for some initial state, we may observe $v(t) = v(0^-) e^{-2t}$ //

and $s_2 = -2$ is a pole of $E_1(s)$ //