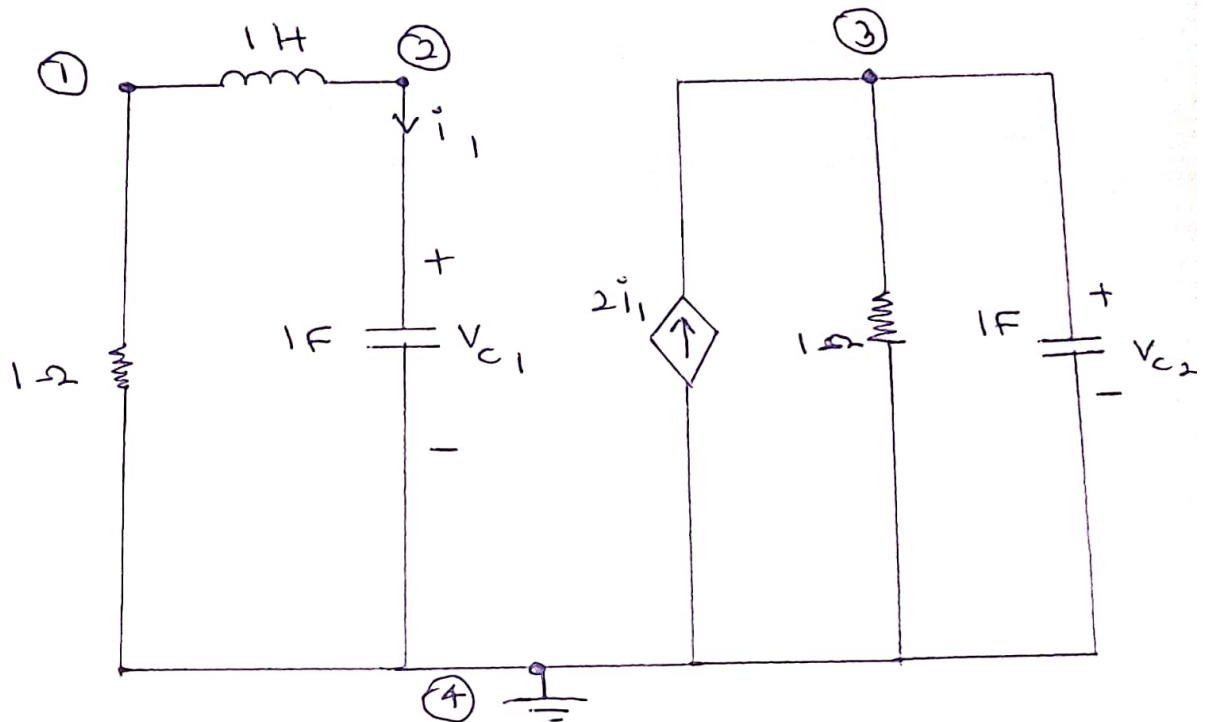


EE 282
Assignment.

E/19/166

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1 a.



Apply KCL for the circuit (Nodal Analysis)

Time Domain :-

KCL for node 1

$$\frac{e_1(t)}{1} = -i_1(t) \quad \text{--- (1)}$$

KCL for node 2

$$i_1(t) = \frac{1}{1} \frac{d}{dt} (e_2(t)) \quad \text{--- (2)}$$

KCL for node 3

$$2i_1(t) = \frac{e_3(t)}{1} + \frac{1}{1} \frac{d}{dt} e_3(t) \quad \text{--- (3)}$$

equation for the inductor

$$e_1(t) - e_2(t) = \frac{d}{dt} i_1(t) \quad \text{--- (4)}$$

1

Laplace domain:-

$$\textcircled{1} \Rightarrow E_1(s) = -I_1(s)$$

$$\textcircled{2} \Rightarrow I_1(s) = sE_2(s) - e_2(0^-)$$

$$\textcircled{3} \Rightarrow 2I_1(s) = E_3(s) + sE_3(s) - e_3(0^-)$$

$$\textcircled{4} \Rightarrow E_1(s) - E_2(s) = sI_1(s) - i_1(0^-)$$

$$b) \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -s & 0 & 1 \\ 0 & 0 & -(1+s) & 2 \\ 1 & -1 & 0 & -s \end{bmatrix}}_{P(s)} \begin{bmatrix} E_1(s) \\ E_2(s) \\ E_3(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -e_2(0^-) \\ -e_3(0^-) \\ -i_1(0^-) \end{bmatrix}$$

find natural frequencies

$$\det\{[P(s)]\} = 0$$

$$0 = 1(- (1+s)(s^2+1)) - 1(1(s(1+s)))$$

$$0 = -1(1+s)(s^2+s+1)$$

$$s_1 = -1 //, s_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i //, s_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i //$$

\therefore Natural frequencies are $-1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ //

since $\text{Re}\{s_i\} < 0$ for $i = 1, 2, 3$
The circuit is exponentially stable.

c) The mode $s = -1$ is set up

$$P(-1) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix}$$

\underline{w}_1 is obtained by $P(-1)\underline{w}_1 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$w_{11} + w_{14} = 0 \quad \text{--- (1)}$$

$$w_{12} + w_{14} = 0 \quad \text{--- (2)}$$

$$w_{14} = 0 \quad \text{--- (3)}$$

$$w_{11} + w_{12} + w_{14} = 0 \quad \text{--- (4)}$$

$$\therefore \text{--- (1)} \Rightarrow w_{11} = 0$$

$$\text{--- (2)} \Rightarrow w_{12} = 0$$

$$\text{Let } w_{13} = k$$

$$\therefore \underline{w}_1 = \begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix}$$

\therefore for initial conditions

$$\begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ i_1(t) \end{bmatrix}_{t=0}$$

Initial conditions to set up modes $s = -1$

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ i_1(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix} e^{-t} \Rightarrow \begin{bmatrix} e_1(0^-) \\ e_2(0^-) \\ e_3(0^-) \\ i_1(0^-) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k \\ 0 \end{bmatrix}$$

Similarly

$$P(s_2) \underline{w}_2 = 0 \quad \text{for } s_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}j$$

$$P\left(-\frac{1}{2} + \frac{\sqrt{3}j}{2}\right) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right) & 0 & 1 \\ 0 & 0 & -\left(\frac{1}{2} + \frac{\sqrt{3}j}{2}\right) & 2 \\ 1 & -1 & 0 & \left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right)j \end{pmatrix}$$

\underline{w}_2 is obtained by $P\left(-\frac{1}{2} + \frac{\sqrt{3}j}{2}\right) \underline{w}_2 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right) & 0 & 1 \\ 0 & 0 & -\left(\frac{1}{2} + \frac{\sqrt{3}j}{2}\right) & 2 \\ 1 & -1 & 0 & \left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right)j \end{bmatrix} \begin{bmatrix} w_{21} \\ w_{22} \\ w_{23} \\ w_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$w_{21} + w_{24} = 0 \quad \text{--- (1)}$$

$$\left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right) w_{22} + w_{24} = 0 \quad \text{--- (2)}$$

$$-\left(\frac{1}{2} + \frac{\sqrt{3}j}{2}\right) w_{23} + 2w_{24} = 0 \quad \text{--- (3)}$$

$$w_{21} - w_{22} + \left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right) w_{24} = 0 \quad \text{--- (4)}$$

$$w_{21} = K^*$$

$$\text{(1)} \Rightarrow w_{24} = -w_{21}$$

$$\begin{aligned}
 \textcircled{2} \Rightarrow w_{22} &= \frac{-w_{24}}{\left(\frac{1}{2} - \frac{\sqrt{3}j}{2}\right)} \\
 &= \frac{-2w_{24}(1 + \sqrt{3}j)}{2} \\
 &= \frac{(1 + \sqrt{3}j)}{2} w_{21}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \Rightarrow w_{23} &= \frac{2w_{24}}{\left(\frac{1}{2} + \frac{\sqrt{3}j}{2}\right)} \\
 &= \frac{2w_{24}(1 - \sqrt{3}j)}{2} \\
 &= (-1 + \sqrt{3}j) w_{21}
 \end{aligned}$$

$$\underline{w}_2 = \begin{bmatrix} 1 \\ \frac{(1 + \sqrt{3}j)}{2} \\ (-1 + \sqrt{3}j) \\ -1 \end{bmatrix}$$

$$\begin{aligned}
 s_2 &= \overline{s_3} \\
 w_3 &= \overline{w_2}
 \end{aligned}$$

s_2 and $s_3 = \overline{s_2}$ modes are excited by initial conditions \underline{w}_1

Since $w_2 = \overline{w_1}$

our initial conditions are real

$$\underline{w}_2 + \underline{w}_3 = \underline{w}_2 + \overline{\underline{w}}_2$$

$$P(s_2) \underline{w}_2 = 0 \Rightarrow \overline{P(s_2)} \overline{\underline{w}}_2 = 0$$

$$P(\overline{s_2}) \overline{\underline{w}}_2 = 0 \Rightarrow P(s_3) \overline{\underline{w}}_2 = 0$$

$$P(s_2) \underline{w}_2 = 0$$

$$\underline{w}_2 = \overline{\underline{w}}_3$$

$$\therefore \underline{e}(t) = k \underline{w}_1 e^{s_1 t} + \overline{k} \underline{w}_3 e^{s_3 t}$$

$$= 2|k| e^{s_{2r} t} \left[\cos(s_{2i} t + \angle k) \underline{w}_r - \sin(s_{2i} t + \angle k) \underline{w}_i \right] \quad \text{--- (1)}$$

where $k = |k| e^{j\angle k}$

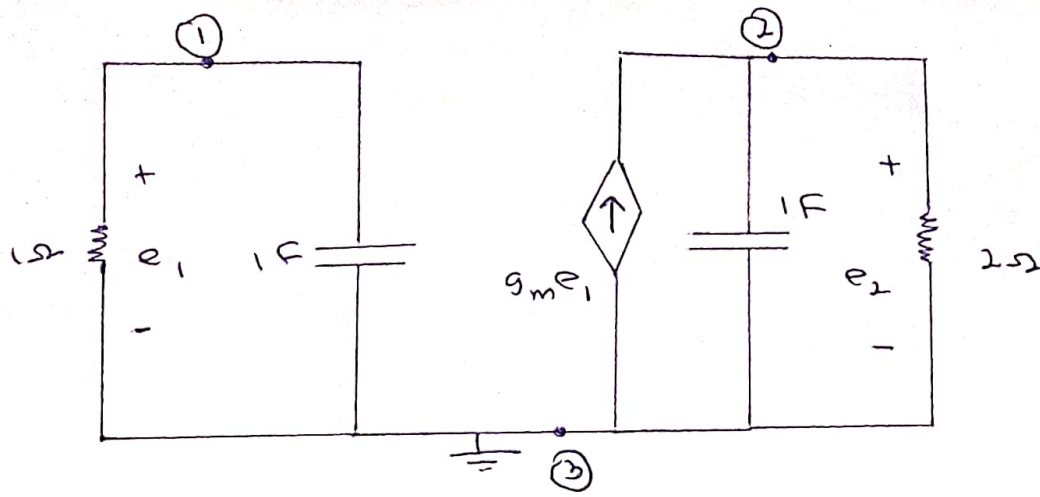
$$s_i = s_{2r} + j s_{2i}$$

$$\underline{w}_2 = \underline{w}_r + j \underline{w}_i$$

\therefore Initial conditions to set up s_2 and s_3

$$= k \begin{bmatrix} 1 \\ \frac{1 + \sqrt{3}j}{2} \\ -1 + \sqrt{3}j \\ -1 \end{bmatrix} + \overline{k} \begin{bmatrix} 1 \\ \frac{1 - \sqrt{3}j}{2} \\ -1 - \sqrt{3}j \\ -1 \end{bmatrix}$$

2



a) Apply nodal analysis.

KCL for node 1

$$\frac{e_1(t)}{1} = \frac{d}{dt} e_1(t) \quad \text{--- (1)}$$

KCL for node 2

$$e_1(t) = \frac{d}{dt} e_2(t) + \frac{e_2(t)}{2} \quad \text{--- (2)}$$

Convert ① & ② to laplace domain.

$$\text{①} \Rightarrow E_1(s) = sE_1(s) - e_1(0^-)$$

$$\text{②} \Rightarrow E_1(s) = sE_2(s) - e_2(0^-) + \frac{E_2(s)}{2}$$

$$\begin{bmatrix} 1-s & 0 \\ 1 & -(s+\frac{1}{2}) \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} = \begin{bmatrix} -e_1(0^-) \\ -e_2(0^-) \end{bmatrix}$$

find natural frequencies

$$\det\{CP(s)\} = -(1-s)(s+\frac{1}{2}) = 0$$

$$s_1 = 1, s_2 = -\frac{1}{2}$$

\therefore Natural frequencies are $1, -1/2$ //

$$b) P(s)^{-1} = \frac{1}{\frac{1}{2}(1-s)(s+\frac{1}{2})} \begin{bmatrix} -(s+\frac{1}{2}) & -0 \\ -1 & (1-s) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1-s} & 0 \\ \frac{+1}{(1-s)(s+\frac{1}{2})} & \frac{1}{(s+\frac{1}{2})} \end{bmatrix}$$

$$\begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{1-s} & 0 \\ \frac{+1}{(1-s)(s+\frac{1}{2})} & \frac{1}{(s+\frac{1}{2})} \end{bmatrix} \begin{bmatrix} -e_1(0^-) \\ -e_2(0^-) \end{bmatrix}$$

$$\therefore E_1(s) = \frac{1}{(1-s)} (-e_1(0^-)) \quad \text{--- (A)}$$

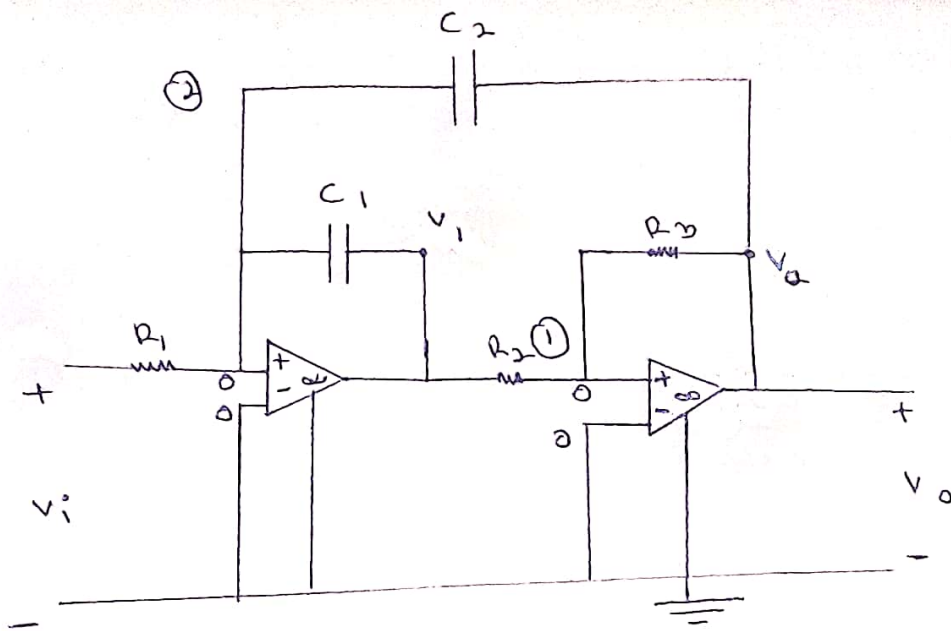
$$E_2(s) = \frac{1}{(1-s)(s+\frac{1}{2})} (-e_1(0^-)) + -\frac{1}{(s+\frac{1}{2})} e_2(0^-) \quad \text{--- (B)}$$

from (A) and (B),

The stable mode $e^{-1/2 t}$ is not observable at e_1 .

\therefore Only one of the natural frequencies is observable at e_1 .

3(a)



Nodal analysis.

KCL for node ①

$$\frac{V_1}{R_2} + \frac{V_0}{R_3} = 0 \quad \text{--- (1)}$$

KCL for node ②

$$C_1 \frac{dV_1}{dt} + C_2 \frac{dV_0}{dt} = -\frac{V_i}{R_1} \quad \text{--- (2)}$$

For zero-state response

$$\textcircled{1} \Rightarrow \frac{V_1(s)}{R_2} + \frac{V_0(s)}{R_3} = 0 \quad \text{--- (1)'}$$

$$\textcircled{2} \Rightarrow sC_1 V_1(s) + sC_2 V_0(s) = -\frac{V_i(s)}{R_1} \quad \text{--- (2)'}$$

from ①' & ②'

$$\therefore C_1 s \left(-\frac{V_0(s)}{R_3} \right) R_2 + sC_2 V_0(s) = -\frac{V_i(s)}{R_1}$$

$$\therefore H(s) = \frac{V_0(s)}{V_i(s)}$$

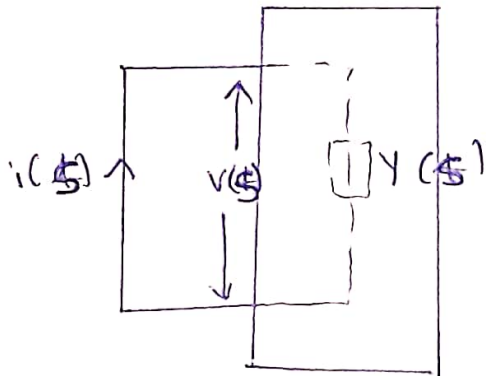
$$\frac{V_0(s)}{R_3} (-C_1 s R_2 + sC_2 R_3) = -\frac{V_i(s)}{R_1}$$

$$\frac{V_o(s)}{R_3} (sC_2 R_3 - C_1 s R_2) = -\frac{V_i(s)}{R_1}$$

$$V_o(s) (C_1 s R_2 - sC_2 R_3) = + \frac{R_3 (V_i(s))}{R_1}$$

$$H_s = \frac{V_o(s)}{V_i(s)} = \frac{R_3}{R_1 s (C_1 R_2 - C_2 R_3)} //$$

b. (i)



~~time domain~~

$$~~i(t) = v(t) \cdot y(t)~~$$

Laplace domain

$$I(s) = V(s) \cdot Y(s)$$

$$Y(s) = \frac{I(s)}{V(s)}$$

$\therefore Y(s)$ - network function

$I(s)$ - L (zero-state response of $i(t)$)

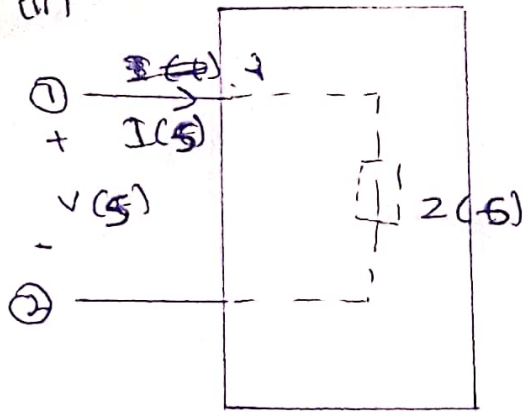
$v(s)$ - L (input)

Poles of network function ($Y(s)$) are necessarily natural frequencies of the circuit.

\therefore If s_1 is a pole of $Y(s)$, then s_1 is a natural ~~response~~ frequency of the circuit. Therefore, we may observe $i(t) = k_1 e^{s_1 t}$ as s_1 is a natural frequency of the circuit. (for some initial state)

\therefore The statement ~~of~~ is true.

(ii)



$$v(s) = I(s) z(s)$$

$$z(s) = \frac{v(s)}{I(s)}$$

$\therefore z(s)$ - network function

$v(s)$ - L (zero state response of $i(t)$)

$I(s)$ - L (input)

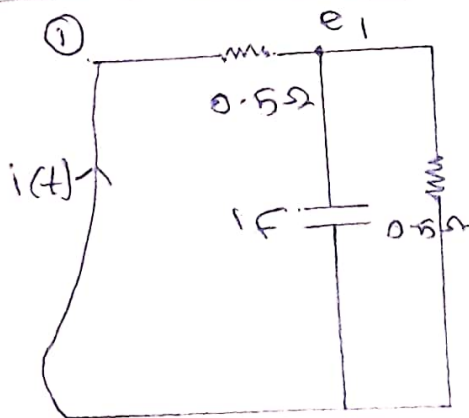
Poles of network function ($z(s)$) are necessarily natural frequencies of the circuit.

\therefore If s_1 is a pole of $z(s)$, then s_1 is a natural frequency of the circuit.

\therefore We may observe $v(t) = k_1 e^{s_1 t}$ as s_1 is a natural frequency of the circuit - (for some initial state).

\therefore The statement is true.

lii) when short circuited



KCL for e_1 ,

time domain

$$\frac{e_1(t)}{0.5} + \frac{d}{dt} e_1(t) + \frac{e_1(t)}{0.5} = 0$$

$$\frac{d}{dt} e_1(t) + 4e_1(t) = 0$$

②

Laplace domain

$$sE_1(s) - e_1(0^-) + 4E_1(s) = 0$$

||

$$(4+s)E_1(s) = e_1(0^-)$$

$$E_1(s) = \frac{e_1(0^-)}{[s+4]}$$

Apply inverse laplace transform

$$E_1(t) = e_1(0^-) e^{-4t}$$

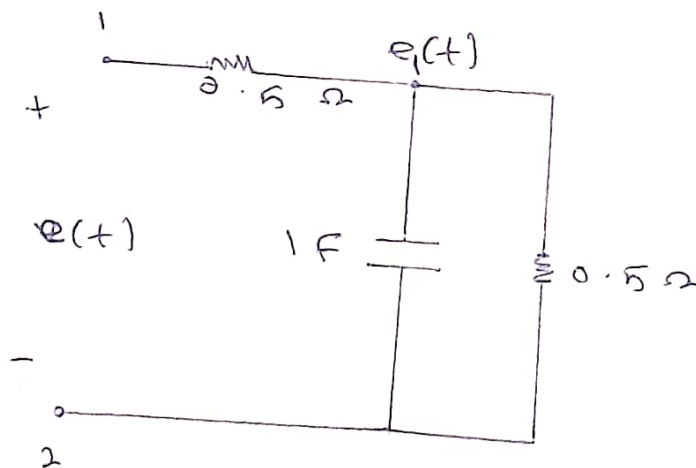
$$\therefore i(t) = \frac{-e_1(0^-) e^{-4t}}{0.5}$$

$$s_1 = -4$$

$$i(t) = -2 e(0^-) e^{-4t} //$$

Part for
by (i) Some initial state, we may observe $i(t) = k_1 e^{s_1 t} = -2 e(0^-) e^{-4t} //$
and $s_1 = -4$ is a pole of $E(s)$.

When open circuited.



KCL for e_1 ,

time domain

$$\frac{d}{dt} e_1(t) + \frac{e_1(t)}{0.5} = 0$$

laplace domain

$$sE_1(s) - e_1(0^-) + 2E_1(s) = 0$$

$$E_1(s)(s+2) = e_1(0^-)$$

$$E_1(s) = \frac{e_1(0^-)}{s+2}$$

Applying inverse Laplace transform

$$-2(t)$$

$$e_1(t) = e_1(0^-) e^{-2t}$$

$$\text{Let } e_1(t) = v(t) \Rightarrow v(t) = v(0^-) e^{-2t}$$

$$s_2 = -2$$

\therefore by part (ii), for some initial state, we may observe $v(t) = v(0^-) e^{-2t}$ //

and $s_2 = -2$ is a pole of $E_1(s)$ //