Project Collaboration Report

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Exercise 0: Collaboration Summary

Our team, comprising two members, demonstrated equal contribution across the project's facets. We embraced a flexible methodology for dividing the workload, prioritizing adaptability and collective learning.

Contributions:

- **Code Development:** Both members actively engaged in the coding process. Complex sections were approached with pair programming, while distinct features were individually developed followed by peer review.
- **Report Writing:** The report was collaboratively outlined and drafted. Sections were divided for initial drafting and then swapped for review, ensuring a cohesive and clear final submission.

This strategy not only streamlined our tasks but also fostered skill enhancement through shared experiences.

Exercise 1: Describe the design of your project

1 Project Design

Our project leverages the concept of Kernelization from our course, implemented through an interactive tool developed with SageMath. This tool is designed as a teaching material to enhance understanding of kernelization and its applications in graph theory.

1.1 Interactive Tool Design

The core functionality allows users to generate random graphs with adjustable parameters for the number of vertices and sparsity. This dynamic generation helps in illustrating how different graph configurations affect the applicability and outcomes of kernelization processes.

1.2 Vertex Cover and Hamiltonian Cycle

For the Vertex Cover problem, given a value k, the tool applies the following simplification rules:

- If there exists a vertex $v \in V$ such that $d_G(v) = 0$, then set $G \leftarrow G - v$.
- If there exists a vertex $v \in V$ such that $d_G(v) = 1$, then set $G \leftarrow G - N_G[v]$ and $k \leftarrow k - 1$.
- If there exists a vertex $v \in V$ such that $d_G(v) > k$, then set $G \leftarrow G - v$ and $k \leftarrow k - 1$.
- If $d_G(v) \leq k$ for each $v \in V$ and $|E| > k \times k$, then return No.

Additionally, it evaluates the presence of a Hamiltonian Cycle, concluding that if $|V| > 4k$, the graph definitively does not possess a Hamiltonian Cycle.

1.3 Edge Clique Cover

For the Edge Clique Cover problem, given k, the tool employs simplification rules such as removing vertices with no edges or merging vertices with identical neighborhoods, aiming to reduce the graph further.

2 Benchmark Instances and Testing

2.1 Design of Tests

The testing environment was constructed to assess the tool's performance across various graph instances, particularly focusing on edge densities and vertex counts. Benchmark instances included sparse and dense graphs, with vertex counts ranging from small to significantly large sizes.

2.2 Scalability

The benchmarks were designed to understand how the kernelization algorithms scale with larger graph instances. Through systematic testing, we observed patterns in performance degradation as graph size increased, particularly for complex problems like Hamiltonian Cycle detection.

3 Conclusion

Our interactive tool provides a valuable resource for teaching and understanding kernelization within graph theory. Through practical experimentation with the tool, students can grasp the impact of kernelization on problem-solving in computational graph theory.

Exercise 2: Implementation Description

Our implementation focuses on developing an interactive tool that leverages SageMath for teaching kernelization concepts in graph theory. This section provides a detailed description of the implementation process, including challenges faced and insights gained.

3.1 Interactive Component Design

Designing interactive components for educational purposes posed initial challenges. Our breakthrough came upon exploring SageMath Interactions and discovering graph theory examples, which inspired us to create related interactive teaching materials.

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4 Challenges

One of the initial challenges was the lack of a clear direction for designing interactive components suitable for classroom use. The exploration of SageMath Interactions, especially the examples on graph theory, provided the necessary inspiration to tackle this challenge effectively.

4.1 Screenshots

Below are placeholders for screenshots showcasing various stages of the tool's usage:

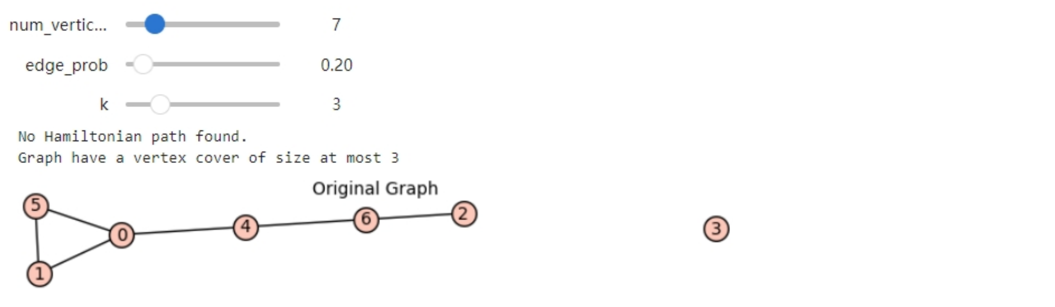

Figure 1: Placeholder 1: VC-HC

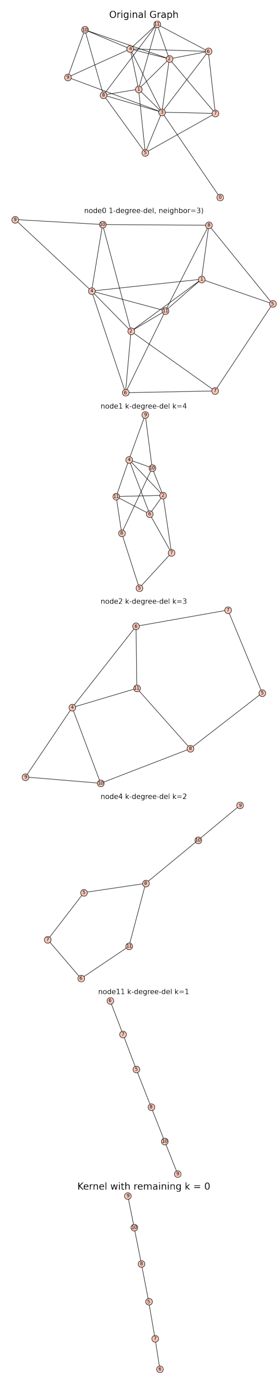

Figure 2: Placeholder 2: VC2

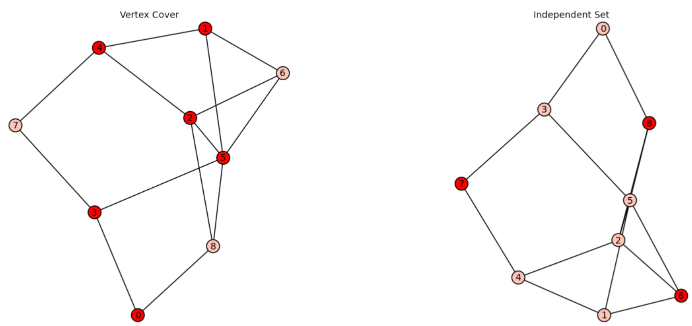

Figure 3: Placeholder 3: HC

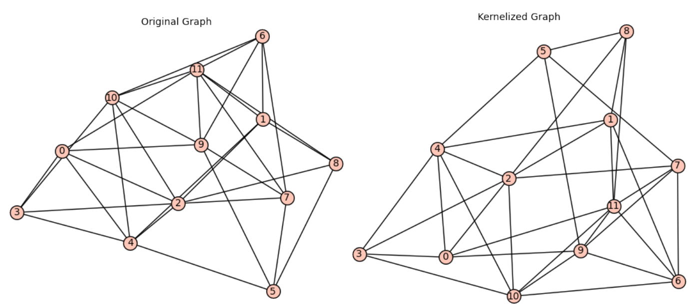

Figure 4: Placeholder 4: ECC

Exercise 3 Describe the outcomes of your project

5 Outcomes

The project successfully developed an interactive tool utilizing the concept of Kernelization within SageMath, aimed at enhancing the teaching and understanding of graph theory. The tool allows users to dynamically generate graphs with adjustable vertex counts and sparsity, applying simplification rules for the Vertex Cover problem and assessing the presence of Hamiltonian Cycles.

6 Merits

The primary merits of the project include:

- Enhanced Educational Experience: By providing an interactive way to explore graph theory concepts, the tool facilitates a deeper understanding among learners.
- Accessibility: Users can easily adjust graph parameters, making complex graph theory concepts more accessible and comprehensible.

- Encouragement of Critical Thinking: The tool prompts users to think critically about how different parameters and simplification rules affect graph properties and problem solutions.

7 Future Work

Improvements identified for future development include:

- **Visualization of vc-Hamiltonian Cycles:** Currently, the tool can determine the existence of Hamiltonian Cycles but lacks the capability to visualize these cycles when they exist. Developing a feature for visual representation of vc-Hamiltonian Cycles would significantly enhance the tool's educational value.

8 Conclusions

The project represents a valuable step forward in the educational technology for graph theory, providing a practical and user-friendly platform for exploring and understanding complex concepts. With future enhancements, particularly in the visualization of Hamiltonian Cycles, it has the potential to become an even more essential resource for students and educators alike.