

CENG 223

Discrete Computational Structures

Fall 2024-2025

Take Home Exam 5

Due date: January 12, 23:59

Question 1

(20 pts)

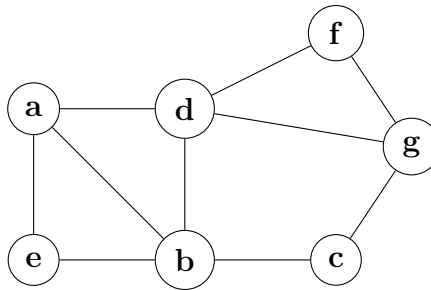


Figure 1: Graph G in Q1

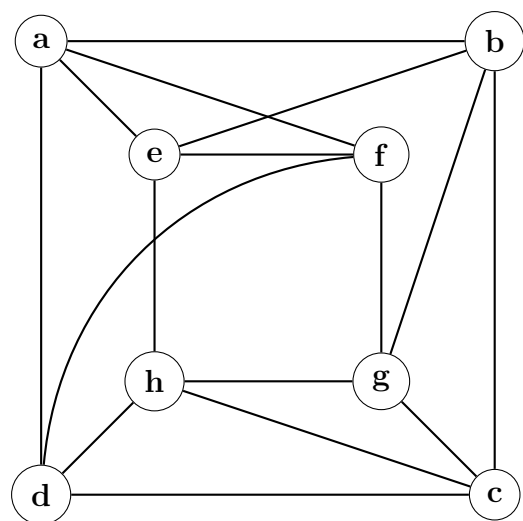
For the graph G in Figure 1, answer the following questions;

- Is there an Euler path in G ? If yes, give such a path; if no, state the reason.
- Is there an Euler circuit in G ? If yes, give such a circuit. If no, can you create an Euler circuit by adding another edge to the G ?
- Does G have a Hamilton path? If yes, find such a path; if no, justify your answer.
- Does G have a Hamilton circuit? If yes, find such a circuit; if no, justify your answer.

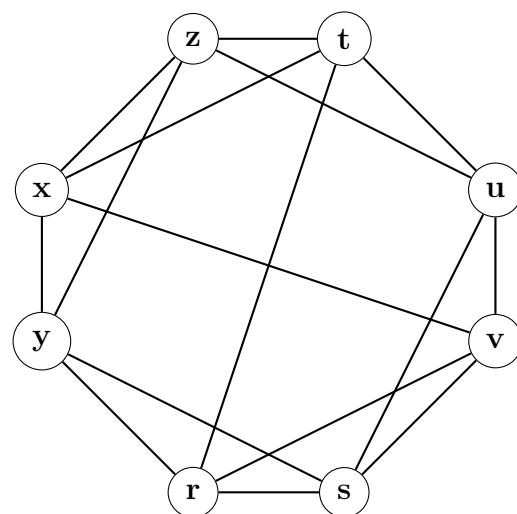
Question 2

(15 pts)

Given the graphs G and H in Figure 2.



(a) G



(b) H

Figure 2: Graph G and H in Q2.

Determine whether G and H are isomorphic, or not. Explain your answer.

Question 3

(35 pts)

- Draw a bipartite graph representing a real-world example you observed in your life. (Please avoid a graph that has more than 15-20 vertices).
- Give the definition of a bipartite graph in predicate logic. You can use $G = (V, E)$ to denote the graph and V_1, V_2 for the two vertex sets.
- The regular graph is a graph where each vertex has the same degree. Give the definition of a regular graph in predicate logic.
- If a regular graph has more than two vertices, then it can't be a tree. Why? Provide a formal proof.
- If a bipartite graph is regular, then the two subsets of vertices has the same size. Why? Provide a formal proof.

Question 4

(30 pts)

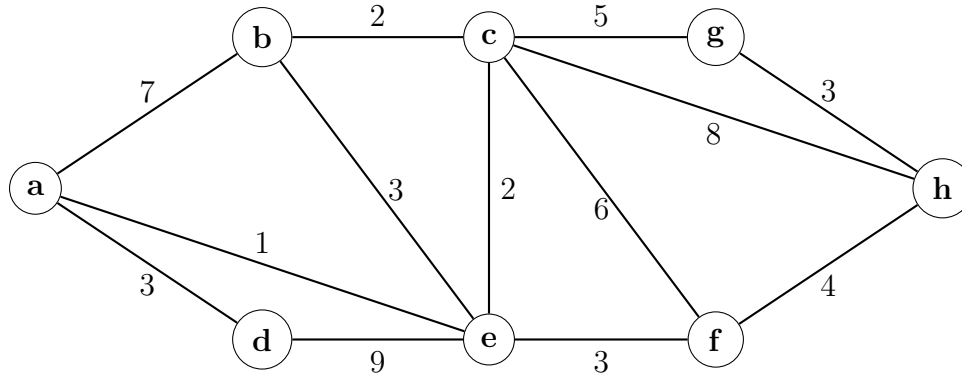


Figure 3: Graph G in Q4.

For the graph G in Figure 3, answer the following questions;

- Find a minimum spanning tree (MST) using Prim's algorithm starting at vertex a . Write the order in which the edges are added to the tree and draw the resulting MST.
- Find a minimum spanning tree (MST) using Kruskal's algorithm. Write the order in which the edges are added to the tree and draw the resulting MST.
- Using Dijkstra's algorithm, find the shortest path from vertex a to vertex h . Show all steps of the Dijkstra's algorithm. Is this shortest path unique? Briefly comment.

Regulations

- You have to write your answers to the provided sections of the template answer file given. **Hand-written solutions will not be accepted.**
- Late Submission: Not allowed!**
- Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
- Submit a single PDF file named eXXXXXXX.pdf (7-digit student number).
- You may ask your questions in the course forum or by sending a mail to "mdyumus@ceng.metu.edu.tr".