

CENG 223

Discrete Computational Structures

Fall 2024-2025

Take Home Exam 4

Due date: December 29, 23:59

Question 1

(15 pts)

For the recurrence relation

$$a_n = 3a_{n-1} + 4a_{n-2}, n \geq 2$$

where $a_0 = 2$ and $a_1 = 5$;

- a) Solve by finding the characteristic equation and the roots. (5 pts)
- b) Solve using generating functions. (10 pts)

Question 2

(15 pts)

Solve the recurrence relation

$$a_n = a_{n/2} + 6a_{n/4}$$

where $a_1 = 3$ and $a_2 = 4$, and $n = 2^m$ for $m \in \mathbb{Z}_{\geq 0}$.

(Hint: You can use a transformation $b_k = a_{2^k}$)

Question 3

(15 pts)

Find the generating function (in closed form) for the sequence $\langle 3, 9, 18, 39, 96, 261 \dots \rangle$. Show all the steps clearly.

Question 4

(40 pts)

For the following statements, indicate whether the statement is true or false. If the statement is true, provide a proof. If the statement is false, give a counterexample.

- a) Every partitioning of a set induces an equivalence relation.
- b) Transitive closure of an antisymmetric relation is always antisymmetric.
- c) Every equivalence relation is also a partial order.
- d) The relation R on a set A is antisymmetric if and only if $R \cap R^{-1}$ is a subset of the diagonal relation $\Delta = \{(a, a) | a \in A\}$.
- e) If relation R is reflexive and transitive, then $R^n = R$ for all positive integers n .

Question 5

(15 pts)

$R = \{(a, b) | a \text{ divides } b\}$ is a relation defined on $A = \{2, 3, 12, 18, 36\}$.

- a) Draw the Hasse diagram of R .
- b) Let R_s be the symmetric closure of R . List all pairs (x, y) where $(x, y) \in (R_s - R)$.
- c) You are allowed to remove a single element in A and add another element. Is it possible to create a lattice? Which element would you remove and add to create a lattice?

Each item is worth 5 pts. Note that partial points may not be given to the items.

Regulations

1. You have to write your answers to the provided sections of the template answer file given.
Handwritten solutions will not be accepted.
2. **Late Submission: Not allowed!**
3. **Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
4. Submit a single PDF file named eXXXXXXX.pdf (7-digit student number).
5. You may ask your questions in the course forum or by sending a mail to "mduymus@ceng.metu.edu.tr".