Let S = s0 + s1 *x* + s2 *x*2 + s3 *x*3 +… + sn *xn + …*

Let 2*xS = 2s0* *x* + 2s1 *x*2 + 2s2 *x*3 + … + 2sn-1 *x*n + …

Let 35*x2S =* 35*s0 x2 +* 35s1 x3 + … + 35sn-2 *x*n + …

As Sn - 2Sn-1 – 35Sn-2 = 0

By addition and subtraction

S (1 - 2*x* – 35*x*2) = s0 + (s1 - 2s0) *x* + (s2 - 2s1 – 35s0) *x*2 + … + (sn - 2sn-1 – 35sn-2) *x*n + …

S(-7*x*+1)(5*x*+1) = (1 + 3*x*)

S = (1 + 3*x*) / \*

Given

(1+3*x*) / \* ≡ +

(1+3*x)* ≡ A(5*x+1) + B(-7x+1)*

*(1+3x)* ≡ 5A*x* + A - 7B*x* + B

1 = A + B

*3* = 5A - 7B

1 – A = B

3 = 5A - 7(1 – A)

3 = 5A - 7 + 7A

3 = 12A - 7

3 - 12A = -7

-12A = -10

-10 = -12A

A =

B =

Therefore,

(1 + 3*x*) / \* = () (1 + 3*x*) + () (1 + 3*x*)

= 1 + (7*x*) + (7x)2+ … +(7x)n + …

= 1 + (-5*x*) + (-5x)2 *+ …+*(-5x)n+…

S = ()[ 1 + 7x + 49x2+ … +7nxn + …] (1 + 3*x*) +  
()[ 1 - 5*x* + 25x2 *+ …+*(-5)nxn+… ] (1 + 3*x*)

S = ()(1 + 10x + 70x2+ … +[7n + 3(7n-1)]xn + …)+

()(1 - 2*x* + 10x2 *+ …*+[(-5n)+ 3(-5)n-1)]xn+… )

S = ()(1 + 10x + 70x2+ … + 10(7n-1)xn + …)+

()(1 - 2*x* + 10x2 *+ …*+(-2)(-5n-1)xn+… )

= 1 + 8x + 60x2+ … +[(5/3)(7n)+ (1/3)(-5)n]xn + …

Hence sn = (5/3)(7n)+ (1/3)(-5)n

Therefore, the closed form solution of the recurrence relation is

[(5/3)(7n)+ (1/3)(-5)n]