SPARSE LEVENBERG-MARQUARDT

PARAMETER VECTOR $P = (a^{(1)T}, \ldots, a^{(m)T}, b_1^T, \ldots, b_n^T)^T$

MEASUREMENT VECTOR $X = (X_1^{(1)T}, \ldots, X_n^{(1)T}, \ldots, X_1^{(m)T}, \ldots, X_n^{(m)T})^T$

SUCH THAT $X_i^{(j)}$ ARE UNCORRELATED

JACOBIAN $\boxed{\dfrac{\partial \hat{X}}{\partial P}}$

$$
\begin{array}{c|cccccc|}
 & a^{(1)} & \cdots & a^{(m)} & b_1 & \cdots & b_n \\
\hline
\hat{X}_1^{(1)} & A_1^{(1)} & & & B_1^{(1)} & & \\
\vdots & \vdots & & & & \ddots & \\
\hat{X}_n^{(1)} & A_n^{(1)} & & & & & B_n^{(1)} \\
\vdots & & \ddots & & & \vdots & \\
\hat{X}_1^{(m)} & & & A_1^{(m)} & B_1^{(m)} & & \\
\vdots & & & \vdots & & \ddots & \\
\hat{X}_n^{(m)} & & & A_n^{(m)} & & & B_n^{(m)} \\
\hline
\end{array}
$$

$$A_i^{(j)} = \frac{\partial \hat{X}_i^{(j)}}{\partial a^{(j)}}$$

$$B_i^{(j)} = \frac{\partial \hat{X}_i^{(j)}}{\partial b_i}$$

NORMAL EQUATIONS MATRIX $\boxed{J^T \Sigma_X^{-1} J}$

$$
\begin{bmatrix}
U^{(1)} & & & W_1^{(1)} & \cdots & W_n^{(1)} \\
 & \ddots & & & \vdots & \\
 & & U^{(m)} & W_1^{(m)} & \cdots & W_n^{(m)} \\
W_1^{(1)T} & & W_1^{(m)T} & V_1 & & \\
\vdots & \cdots & \vdots & & \ddots & \\
W_n^{(1)T} & & W_n^{(m)T} & & & V_n
\end{bmatrix}
$$

MATRIX IS SYMMETRIC

$$U^{(j)} = \sum_i A_i^{(j)T} \Sigma_{X_i^{(j)}}^{-1} A_i^{(j)}$$

$$V_i = \sum_j B_i^{(j)T} \Sigma_{X_i^{(j)}}^{-1} B_i^{(j)}$$

$$W_i^{(j)} = A_i^{(j)T} \Sigma_{X_i^{(j)}}^{-1} B_i^{(j)}$$

$U^{(j)}$ AND V_i ARE SYMMETRIC

NORMAL EQUATIONS VECTOR $\boxed{\mathbf{J}^T \boldsymbol{\Sigma}_{\mathbf{x}}^{-1} \boldsymbol{\epsilon}}$

$$\begin{bmatrix} \boldsymbol{\epsilon}_{\mathbf{a}^{(1)}} \\ \vdots \\ \boldsymbol{\epsilon}_{\mathbf{a}^{(m)}} \\ \boldsymbol{\epsilon}_{\mathbf{b}_1} \\ \vdots \\ \boldsymbol{\epsilon}_{\mathbf{b}_n} \end{bmatrix}$$

$$\boldsymbol{\epsilon}_{\mathbf{a}^{(j)}} = \sum_i \mathbf{A}_i^{(j)T} \boldsymbol{\Sigma}_{\mathbf{x}_i^{(j)}}^{-1} \boldsymbol{\epsilon}_i^{(j)}$$

$$\boldsymbol{\epsilon}_{\mathbf{b}_i} = \sum_j \mathbf{B}_i^{(j)T} \boldsymbol{\Sigma}_{\mathbf{x}_i^{(j)}}^{-1} \boldsymbol{\epsilon}_i^{(j)}$$

WHERE $\boldsymbol{\epsilon}_i^{(j)} = \mathbf{x}_i^{(j)} - \hat{\mathbf{x}}_i^{(j)}$

AUGMENTED NORMAL EQUATIONS MATRIX $\boxed{\mathbf{S} = \mathbf{U}^* - \mathbf{W}\mathbf{V}^{*-1}\mathbf{W}^T}$

MATRIX IS SYMMETRIC

$$\begin{bmatrix} \mathbf{S}^{(1,1)} & \cdots & \mathbf{S}^{(1,m)} \\ \vdots & \ddots & \vdots \\ \mathbf{S}^{(m,1)} & \cdots & \mathbf{S}^{(m,m)} \end{bmatrix}$$

$$\mathbf{S}^{(j,j)} = \mathbf{U}^{(j)*} - \sum_i \mathbf{W}_i^{(j)} \mathbf{V}_i^{*-1} \mathbf{W}_i^{(j)T}$$

$$= \mathbf{U}^{(j)*} - \sum_i \mathbf{Y}_i^{(j)} \mathbf{W}_i^{(j)T}$$

$$\mathbf{S}^{(j,k)} = -\sum_i \mathbf{W}_i^{(j)} \mathbf{V}_i^{*-1} \mathbf{W}_i^{(k)T}$$

$$= -\sum_i \mathbf{Y}_i^{(j)} \mathbf{W}_i^{(k)T}$$

WHERE $\mathbf{Y}_i^{(j)} = \mathbf{W}_i^{(j)} \mathbf{V}_i^{*-1}$

AUGMENTED NORMAL EQUATIONS VECTOR $\boxed{\mathbf{e} = \boldsymbol{\epsilon}_{\mathbf{a}} - \mathbf{W}\mathbf{V}^{*-1}\boldsymbol{\epsilon}_{\mathbf{b}}}$

$$\mathbf{e}^{(j)} = \boldsymbol{\epsilon}_{\mathbf{a}^{(j)}} - \sum_i \mathbf{W}_i^{(j)} \mathbf{V}_i^{*-1} \boldsymbol{\epsilon}_{\mathbf{b}_i}$$

$$= \boldsymbol{\epsilon}_{\mathbf{a}^{(j)}} - \sum_i \mathbf{Y}_i^{(j)} \boldsymbol{\epsilon}_{\mathbf{b}_i}$$ WHERE $\mathbf{Y}_i^{(j)} = \mathbf{W}_i^{(j)} \mathbf{V}_i^{*-1}$

$$\mathbf{e} = (\mathbf{e}^{(1)T}, \mathbf{e}^{(2)T}, \ldots, \mathbf{e}^{(m)T})^T$$

$$\boxed{\begin{array}{l} \underset{\sim}{S}\,\underset{\sim}{\delta_a} = \underset{\sim}{e}, \text{ SOLVE FOR } \underset{\sim}{\delta_a} \\ \underset{\sim}{\delta_b} = \underset{\sim}{V}^{*-1}(\underset{\sim}{\epsilon_b} - \underset{\sim}{W}^T \underset{\sim}{\delta_a}) \quad \text{BACK-SUBSTITUTION} \end{array}}$$

$$\underset{\sim}{e} = (\underset{\sim}{e}^{(1)T}, \dots, \underset{\sim}{e}^{(m)T})^T$$

$$\underset{\sim}{S}\,\underset{\sim}{\delta_a} = \underset{\sim}{e}, \text{ SOLVE FOR } \underset{\sim}{\delta_a}$$

$$\underset{\sim}{\delta_a} = (\underset{\sim}{\delta}_{a(1)}^T, \dots, \underset{\sim}{\delta}_{a(m)}^T)^T$$

$$\underset{\sim}{\delta}_{b_i} = \underset{\sim}{V}_i^{*-1}\left(\underset{\sim}{\epsilon}_{b_i} - \sum_j \underset{\sim}{W}_i^{(j)T} \underset{\sim}{\delta}_{a(j)}\right) \quad \text{BACK-SUBSTITUTION}$$

PROJECTION, PLANE TO PLANE

PARAMETERIZED POINTS ON A PLANE

THE POINTS $\underset{\sim}{X}_{\pi}$ ON THE PLANE π MAY BE WRITTEN AS

$\underset{\sim}{X}_{\pi} = \underset{\sim}{M} \underset{\sim}{x}_{\pi}$, WHERE $\underset{\sim}{M}$ IS THE NULL SPACE OF π^{T} (I.E., $\pi^{T} \underset{\sim}{M} = \underset{\sim}{0}^{T}$)

$(\underset{\sim}{x}_{\pi} = \underset{\sim}{M}^{+} \underset{\sim}{X}_{\pi})$

PROJECTION

$\underset{\sim}{x} = \underset{\sim}{P} \underset{\sim}{X}_{\pi}$

$\underset{\sim}{x} = \underset{\sim}{P} \underset{\sim}{M} \underset{\sim}{x}_{\pi}$

$\underset{\sim}{x} = \underset{\sim}{H}_{\pi} \underset{\sim}{x}_{\pi}$, WHERE $\underset{\sim}{H}_{\pi} = \underset{\sim}{P} \underset{\sim}{M}$

PROJECTION, QUADRIC TO CONIC

THE CONTOUR GENERATOR Γ IS THE SET OF POINTS X ON THE SMOOTH SURFACE S AT WHICH RAYS ARE TANGENT TO THE SURFACE. THE CORRESPONDING IMAGE APPARENT CONTOUR γ IS THE SET OF POINTS x THAT ARE THE IMAGE OF X, I.E., γ IS THE IMAGE OF Γ.

THE CONTOUR GENERATOR Γ OF A QUADRIC Q IS A PLANE CONIC C_Π. ITS IMAGE γ IS ALSO A CONIC C. THE PLANE OF Γ FOR A QUADRIC Q AND CAMERA P WITH CENTER C IS GIVEN BY

CAMERA CENTER, NOT CONIC

$$\Pi_\Gamma = Q C$$

PARAMETERIZED POINTS ON A PLANE

THE POINTS X_Π ON THE PLANE Π MAY BE WRITTEN AS

$X_\Pi = M x_\Pi$, WHERE M IS THE NULL SPACE OF Π^T (I.E., $\Pi^T M = 0^T$)

PROJECTION

$$X_\Pi^T Q X_\Pi = 0$$

$$(M x_\Pi)^T Q M x_\Pi = 0$$

$$x_\Pi^T M^T Q M x_\Pi = 0$$

$$x_\Pi^T C_\Pi x_\Pi = 0$$

WHERE $C_\Pi = M^T Q M$

$$x = P X_\Pi$$

$$x = P M x_\Pi$$

$$x = H_\Pi x_\Pi$$

WHERE $H_\Pi = P M$

$$C = H_\Pi^{-T} C_\Pi H_\Pi^{-1}$$

$$C = (PM)^{-T} M^T Q M (PM)^{-1}$$