Assignment 5

March 6, 2019

1 CSE 252B: Computer Vision II, Winter 2019 – Assignment 5

1.0.1 Instructor: Ben Ochoa

1.0.2 Due: Wednesday, March 20, 2019, 11:59 PM

1.1 Instructions

- Review the academic integrity and collaboration policies on the course website.
- This assignment must be completed individually.
- This assignment contains both math and programming problems.
- All solutions must be written in this notebook
- Math problems must be done in Markdown/LATEX. Remember to show work and describe your solution.
- Programming aspects of this assignment must be completed using Python in this notebook.
- Your code should be well written with sufficient comments to understand, but there is no need to write extra markdown to describe your solution if it is not explictly asked for.
- This notebook contains skeleton code, which should not be modified (This is important for standardization to facilate effeciant grading).
- You may use python packages for basic linear algebra, but you may not use packages that directly solve the problem. Ask the instructor if in doubt.
- You must submit this notebook exported as a pdf. You must also submit this notebook as an .ipynb file.
- Your code and results should remain inline in the pdf (Do not move your code to an appendix).
- You must submit both files (.pdf and .ipynb) on Gradescope. You must mark each problem on Gradescope in the pdf.
- It is highly recommended that you begin working on this assignment early.

1.2 Problem 1 (Math): Point on Line Closest to the Origin (5 points)

Given a line $\boldsymbol{l} = (a, b, c)^\top$, show that the point on $\boldsymbol{l}$ that is closest to the origin is the point $\boldsymbol{x} = (-ac, -bc, a^2 + b^2)^\top$ (Hint: this calculation is needed in the two-view optimal triangulation method used below).

"""your solution here"""

1.3 Problem 2 (Programming): Feature Detection (20 points)

Download input data from the course website. The file IMG_5030.JPG contains image 1 and the file IMG_5031.JPG contains image 2.

For each input image, calculate an image where each pixel value is the minor eigenvalue of the gradient matrix

$$N = \begin{bmatrix} \sum\limits_w I_x^2 & \sum\limits_w I_x I_y \\ \sum\limits_w I_x I_y & \sum\limits_w I_y^2 \end{bmatrix}$$

where w is the window about the pixel, and I_x and I_y are the gradient images in the x and y direction, respectively. Calculate the gradient images using the fivepoint central difference operator. Set resulting values that are below a specified threshold value to zero (hint: calculating the mean instead of the sum in N allows for adjusting the size of the window without changing the threshold value). Apply an operation that suppresses (sets to 0) local (i.e., about a window) non-maximum pixel values in the minor eigenvalue image. Vary these parameters such that around 1350–1400 features are detected in each image. For resulting nonzero pixel values, determine the subpixel feature coordinate using the Forstner corner point operator.

Report your final values for:

- the size of the feature detection window (i.e. the size of the window used to calculate the elements in the gradient matrix N)
- the minor eigenvalue threshold value
- the size of the local nonmaximum suppression window
- the resulting number of features detected (i.e. corners) in each image.

Display figures for:

- original images with detected features, where the detected features are indicated by a square window (the size of the detection window) about the features

In [28]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as patches
from scipy.signal import convolve2d as conv2d


def ImageGradient(I, w, t):
    # inputs:
    # I is the input image (may be mxn for Grayscale or mxnx3 for RGB)
    # w is the size of the window used to compute the gradient matrix N
    # t is the minor eigenvalue threshold
    #
    # outputs:
    # N is the 2x2xmxn gradient matrix
    # b in the 2x1xmxn vector used in the Forstner corner detector
    # J0 is the mxn minor eigenvalue image of N before thresholding
    # J1 is the mxn minor eigenvalue image of N after thresholding
```

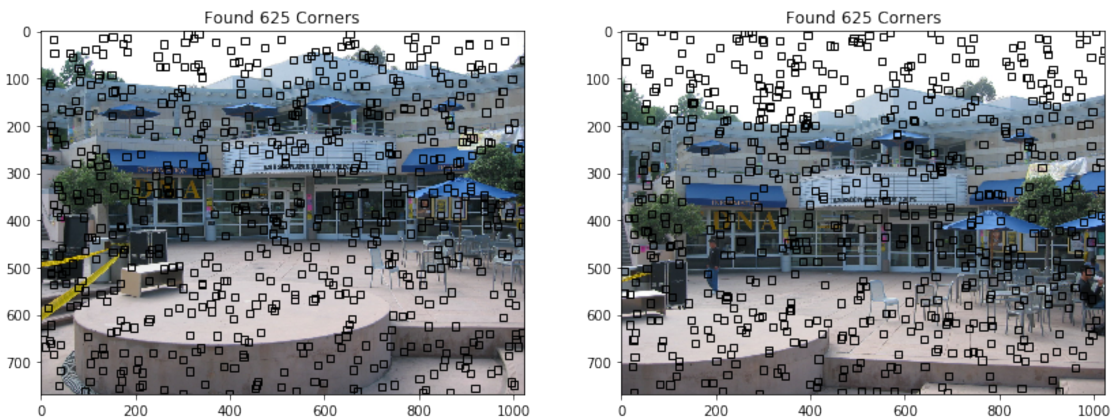
```python
    m,n = I.shape[:2]
    N = np.zeros((2,2,m,n))
    b = np.zeros((2,1,m,n))
    J0 = np.zeros((m,n))
    J1 = np.zeros((m,n))

    """your code here"""



    return N, b, J0, J1


def NMS(J, w_nms):
    # Apply nonmaximum supression to J using window w
    # For any window in J, the result should only contain 1 nonzero value
    # In the case of multiple identical maxima in the same window,
    # the tie may be broken arbitrarily
    #
    # inputs:
    # J is the minor eigenvalue image input image after thresholding
    # w_nms is the size of the local nonmaximum suppression window
    #
    # outputs:
    # J2 is the mxn resulting image after applying nonmaximum suppression
    #

    J2 = J.copy()
    """your code here"""



    return J2


def ForstnerCornerDetector(J, N, b):
    # Gather the coordinates of the nonzero pixels in J
    # Then compute the sub pixel location of each point using the Forstner operator
    #
    # inputs:
    # J is the NMS image
    # N is the 2x2xmxn gradient matrix
    # b is the 2x1xmxn vector computed in the image_gradient function
    #
    # outputs:
    # C is the number of corners detected in each image
    # pts is the 2xC list of coordinates of subpixel accurate corners
```

```
    #     found using the Forstner corner detector

    """your code here"""


    pts = np.vstack((np.random.randint(0,1024,(1,625)), np.random.randint(0,768,(1,625
    C = len(pts[0])
    return C, pts


# feature detection
def RunFeatureDetection(I, w, t, w_nms):
    N, b, J0, J1 = ImageGradient(I, w, t)
    J2 = NMS(J1, w_nms)
    C, pts = ForstnerCornerDetector(J2, N, b)
    return C, pts, J0, J1, J2
```

In [69]:
```
from PIL import Image
import time

# input images
I1 = np.array(Image.open('IMG_5030.JPG'), dtype='float')/255.
I2 = np.array(Image.open('IMG_5031.JPG'), dtype='float')/255.

# parameters to tune
w = 15
t = 1
w_nms = 1

tic = time.time()

# run feature detection algorithm on input images
C1, pts1, J1_0, J1_1, J1_2 = RunFeatureDetection(I1, w, t, w_nms)
C2, pts2, J2_0, J2_1, J2_2 = RunFeatureDetection(I2, w, t, w_nms)
toc = time.time() - tic

print('took %f secs'%toc)

# display results
plt.figure(figsize=(14,24))

# show corners on original images
ax = plt.subplot(1,2,1)
plt.imshow(I1)
for i in range(C1): # draw rectangles of size w around corners
    x,y = pts1[:,i]
    ax.add_patch(patches.Rectangle((x-w/2,y-w/2),w,w, fill=False))
# plt.plot(pts1[0,:], pts1[1,:], '.b') # display subpixel corners
```

```
    plt.title('Found %d Corners'%C1)

    ax = plt.subplot(1,2,2)
    plt.imshow(I2)
    for i in range(C2):
        x,y = pts2[:,i]
        ax.add_patch(patches.Rectangle((x-w/2,y-w/2),w,w, fill=False))
    # plt.plot(pts2[0,:], pts2[1,:], '.b')
    plt.title('Found %d Corners'%C2)

    plt.show()
```

```
took 0.076840 secs
```



Final values for parameters

- w =
- t =
- w_nms =
- C1 =
- C2 =

1.4 Problem 3 (Programming): Feature matching (15 points)

Determine the set of one-to-one putative feature correspondences by performing a brute-force search for the greatest correlation coefficient value (in the range [-1, 1]) between the detected features in image 1 and the detected features in image 2. Only allow matches that are above a specified correlation coefficient threshold value (note that calculating the correlation coefficient allows for adjusting the size of the matching window without changing the threshold value). Further, only allow matches that are above a specified distance ratio threshold value, where distance is measured to the next best match for a given feature. Vary these parameters such that around 300 putative feature correspondences are established. Optional: constrain the search to coordinates in image 2 that are within a proximity of the detected feature coordinates in image 1.

Report your final values for:

- the size of the matching window
- the correlation coefficient threshold
- the distance ratio threshold
- the size of the proximity window (if used)
- the resulting number of putative feature correspondences (i.e. matched features)

Display figures for:

- pair of images, where the matched features are indicated by a square window (the size of the matching window) about the feature and a line segment is drawn from the feature to the coordinates of the corresponding feature in the other image

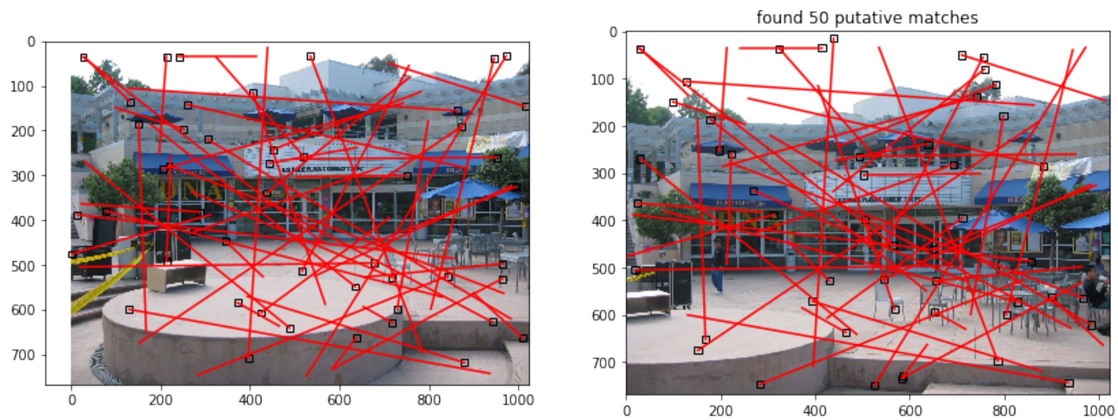
In [42]:
```
def NCC(I1, I2, pts1, pts2, w, p):
    # compute the normalized cross correlation between image patches I1, I2
    # result should be in the range [-1,1]
    #
    # inputs:
    # I1, I2 are the input images
    # pts1, pts2 are the point to be matched
    # w is the size of the matching window to compute correlation coefficients
    # p is the size of the proximity window
    #
    # output:
    # normalized cross correlation matrix of scores between all windows in
    #    image 1 and all windows in image 2

    """your code here"""


    scores = 0
    return scores


def Match(scores, t, d):
    # perform the one-to-one correspondence matching on the correlation coefficient m
    #
    # inputs:
    # scores is the NCC matrix
    # t is the correlation coefficient threshold
    # d distance ration threshold
    #
    # output:
    # list of the feature coordinates in image 1 and image 2

    """your code here"""
```

```
    inds = np.vstack((np.random.choice(625,50,replace=False),
                      np.random.choice(625,50,replace=False)))
    return inds



def RunFeatureMatching(I1, I2, pts1, pts2, w, t, d, p=0):
    # inputs:
    # I1, I2 are the input images
    # pts1, pts2 are the point to be matched
    # w is the size of the matching window to compute correlation coefficients
    # t is the correlation coefficient threshold
    # d distance ration threshold
    # p is the size of the proximity window
    #
    # outputs:
    # inds is a 2xk matrix of matches where inds[0,i] indexs a point pts1
    #     and inds[1,i] indexs a point in pts2, where k is the number of matches

    scores = NCC(I1, I2, pts1, pts2, w, p)
    inds = Match(scores, t, d)
    return inds
```

In [44]:
```
# parameters to tune
w = 15
t = 1
d = 1
p = np.inf

tic = time.time()
# run the feature matching algorithm on the input images and detected features
inds = RunFeatureMatching(I1, I2, pts1, pts2, w, t, d, p)
toc = time.time() - tic

print('took %f secs'%toc)

# create new matrices of points which contain only the matched features
match1 = pts1[:,inds[0,:]]
match2 = pts2[:,inds[1,:]]

# # display the results
plt.figure(figsize=(14,24))
ax1 = plt.subplot(1,2,1)
ax2 = plt.subplot(1,2,2)
ax1.imshow(I1)
ax2.imshow(I2)
plt.title('Found %d Putative Matches'%match1.shape[1])
for i in range(match1.shape[1]):
```

```
        x1,y1 = match1[:,i]
        x2,y2 = match2[:,i]
        ax1.plot([x1, x2],[y1, y2],'-r')
        ax1.add_patch(patches.Rectangle((x1-w/2,y1-w/2),w,w, fill=False))
        ax2.plot([x2, x1],[y2, y1],'-r')
        ax2.add_patch(patches.Rectangle((x2-w/2,y2-w/2),w,w, fill=False))

    plt.show()

    print('unique points in image 1: %d'%np.unique(inds[0,:]).shape[0])
    print('unique points in image 2: %d'%np.unique(inds[1,:]).shape[0])
```

```
took 0.000456 secs
```




```
unique points in image 1: 50
unique points in image 2: 50
```

Final values for parameters

- w =
- t =
- d =
- p =
- num_matches =

1.5 Problem 4 (Programming): Outlier Rejection (20 points)

The resulting set of putative point correspondences should contain both inlier and outlier correspondences (i.e., false matches). Determine the set of inlier point correspondences using the M-estimator Sample Consensus (MSAC) algorithm, where the maximum number of attempts to find a consensus set is determined adaptively. For each trial, you must use the 7-point algorithm (as described in lecture) to estimate the fundamental matrix, resulting in 1 or 3 solutions. Calculate the (squared) Sampson error as a first order approximation to the geometric error.

Hint: this problem has codimension 1

Also: fix a random seed in your MSAC. If I cannot reproduce your results, you will lose points.

Report your values for:

- the probability p that as least one of the random samples does not contain any outliers
- the probability α that a given point is an inlier
- the resulting number of inliers
- the number of attempts to find the consensus set
- the tolerance for inliers
- the cost threshold
- random seed

Display figures for:

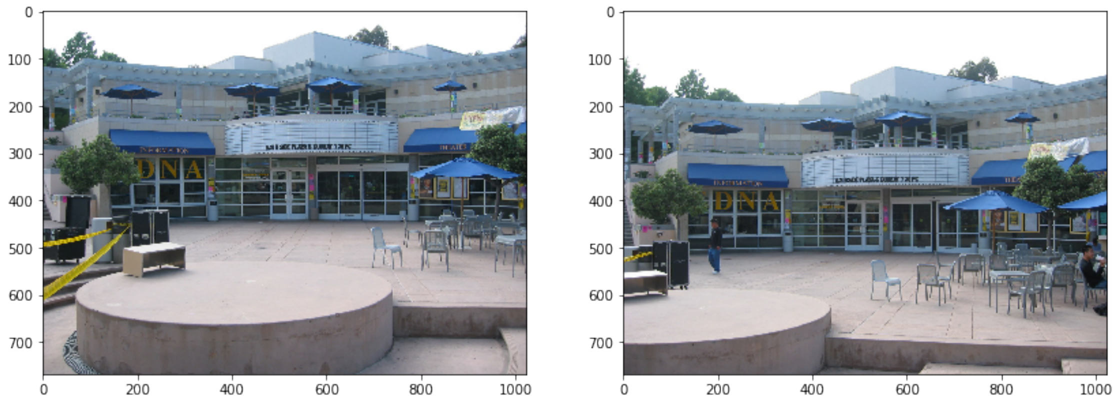
- pair of images, where the inlier features in each of the images are indicated by a square window about the feature and a line segment is drawn from the feature to the coordinates of the corresponding feature in the other image

In [50]:
```
from scipy.stats import chi2

def MSAC(pts1, pts2, thresh, tol, p):
    # Inputs:
    #    pts1 - matched feature correspondences in image 1
    #    pts2 - matched feature correspondences in image 2
    #    thresh - cost threshold
    #    tol - reprojection error tolerance
    #    p - probability that as least one of the random samples does not contain any
    #
    # Output:
    #    consensus_min_cost - final cost from MSAC
    #    consensus_min_cost_model - fundamental matrix F
    #    inliers - list of indices of the inliers corresponding to input data
    #    trials - number of attempts taken to find consensus set

    """your code here"""



    trials = 0
    max_trials = np.inf
    consensus_min_cost = np.inf
    consensus_min_cost_model = np.zeros((3,4))
    inliers = np.random.randint(0, 200, size=100)
    return consensus_min_cost, consensus_min_cost_model, inliers, trials
```

```python
    # MSAC parameters
    thresh = 0
    tol = 0
    p = 0
    alpha = 0

    tic=time.time()

    cost_MSAC, F_MSAC, inliers, trials = MSAC(pts1, pts2, thresh, tol, p)

    # choose just the inliers
    x1 = pts1[:,inliers]
    x2 = pts2[:,inliers]
    outliers = np.setdiff1d(np.arange(pts1.shape[1]),inliers)

    toc=time.time()
    time_total=toc-tic

    # display the results
    print('took %f secs'%time_total)
    print('%d iterations'%trials)
    print('inlier count: ',len(inliers))
    print('inliers: ',inliers)
    print('MSAC Cost = %.9f'%cost_MSAC)
    print('F_MSAC = ')
    print(F_MSAC)

    # display the figures
    plt.figure(figsize=(14,8))
    ax1 = plt.subplot(1,2,1)
    ax2 = plt.subplot(1,2,2)
    ax1.imshow(I1)
    ax2.imshow(I2)
    plt.show()
    """your code here"""
```

```
took 0.001343 secs
0 iterations
inlier count:  100
inliers:  [ 55  50  99  49 116  67   0 139  51  70 189  28  83 100  11 124 125 134
  79 129 109  48  21  11  31 124  60 134 111  68  80 156 187  24 156 124
  20   3 178 126 121 108  79  10 178 196 150  68 139 193 194 184 125  16
  37  97 174  83 186 177  99  39 159  20   1  83 114  21  40 135  29 198
 134   5  88 173  38 120  85   4  97 158 188 170  66  84  87 141  93  20
  14 123 114  31  88  47  61 164 103 197]
MSAC Cost = inf
F_MSAC =
```

```
[[0. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]]
```



```
Out[50]: 'your code here'
```

Final values for parameters

- random seed =
- p =
- α =
- tolerance =
- threshold =
- num_inliers =
- num_attempts =
- consensus_min_cost =

1.6 Problem 5 (Programming): Linear Estimation of the Fundamental Matrix (15 points)

Estimate the fundamental matrix $\boldsymbol{F}_{\text{DLT}}$ from the resulting set of inlier correspondences using the direct linear transformation (DLT) algorithm (with data normalization). Include the numerical values of the resulting $\boldsymbol{F}_{\text{DLT}}$, scaled such that $||\boldsymbol{F}_{\text{DLT}}||_{\text{Fro}} = 1$

```
In [62]: def Homogenize(x):
             # converts points from inhomogeneous to homogeneous coordinates
             return np.vstack((x,np.ones((1,x.shape[1]))))

         def Dehomogenize(x):
             # converts points from homogeneous to inhomogeneous coordinates
             return x[:-1]/x[-1]
```

```python
def DLT(x1, x2, normalize=True):
    # Inputs:
    #    x1 - inhomogeneous inlier correspondences in image 1
    #    x2 - inhomogeneous inlier correspondences in image 2
    #    normalize - if True, apply data normalization to x1 and x2
    #
    # Outputs:
    #    F - the DLT estimate of the fundamental matrix

    """your code here"""

    # data normalization
    if normalize:
        x1 = x1
        x2 = x2

    # data denormalization
    if normalize:
        x1 = x1
        x2 = x2


    F = np.eye(3)
    return F


# compute the linear estimate with data normalization
print ('DLT with Data Normalization')
time_start=time.time()
F_DLT = DLT(x1, x2, normalize=True)
time_total=time.time()-time_start

# display the resulting F_DLT, scaled with its frobenius norm
print('F_DLT =')
print(F_DLT)
```

```
DLT with Data Normalization
F_DLT =
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]
```

1.7 Problem 6 (Programming): Nonlinear Estimation of the Fundamental Matrix (70 points)

Retrieve the camera projection matrices $\boldsymbol{P} = [\boldsymbol{I} \,|\, \boldsymbol{0}]$ and $\boldsymbol{P}' = [\boldsymbol{M} \,|\, \boldsymbol{v}]$, where $\boldsymbol{M}$ is full rank, from $\boldsymbol{F}_{\text{DLT}}$. Use the resulting camera projection matrix $\boldsymbol{P}'$ associated with the second image and the triangulated 3D points as an initial estimate to an iterative estimation method, specifically the sparse Levenberg-Marquardt algorithm, to determine the Maximum Likelihood estimate of the fundamental matrix $\boldsymbol{F} = [v]_\times \boldsymbol{M}$ that minimizes the reprojection error. The initial estimate of the 3D points must be determined using the two-view optimal triangulation method described in lecture (algorithm 12.1 in the Hartley & Zisserman book, but use the ray-plane intersection method for the final step instead of the homogeneous method). Additionally, you must parameterize the camera projection matrix $\boldsymbol{P}'$ associated with the second image and the homogeneous 3D scene points that are being adjusted using the parameterization of homogeneous vectors (see section A6.9.2 (page 624) of the textbook, and the corrections and errata).

Report the initial cost (i.e. cost at iteration 0) and the cost at the end of each successive iteration. Show the numerical values for the final estimate of the fundamental matrix $\boldsymbol{F}_{\text{LM}}$, scaled such that $||\boldsymbol{F}_{\text{LM}}||_{\text{Fro}} = 1$.

```
In [66]: from scipy.linalg import block_diag

         def LM(F, x1, x2, max_iters, lam):
             # Input:
             #    F - DLT estimate of the fundamental matrix
             #    x1 - inhomogeneous inlier points in image 1
             #    x2 - inhomogeneous inlier points in image 2
             #    max_iters - maximum number of iterations
             #    lam - lambda parameter
             # Output:
             #    F - Final fundamental matrix obtained after convergence



             """your code here"""

             cost = np.inf
             print ('iter %03d Cost %.9f'%(0, cost))

             for i in range(max_iters):
                 print ('iter %03d Cost %.9f'%(i+1, cost))


             return F

         # LM hyperparameters
         lam = .001
         max_iters = 10

         # Run LM initialized by DLT estimate
         print ('Sparse LM')
         time_start=time.time()
         F_LM = LM(F_DLT, x1, x2, max_iters, lam)
         time_total=time.time()-time_start
         print('took %f secs'%time_total)
```

```
# display the resulting F_LM, scaled with its frobenius norm
print('F_LM =')
print(F_LM)
```

```
Sparse LM
iter 000 Cost inf
iter 001 Cost inf
iter 002 Cost inf
iter 003 Cost inf
iter 004 Cost inf
iter 005 Cost inf
iter 006 Cost inf
iter 007 Cost inf
iter 008 Cost inf
iter 009 Cost inf
iter 010 Cost inf
took 0.000542 secs
F_LM =
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]
```

1.8 Problem 7 (Programming): Point to Line Mapping (10 points)

Qualitatively determine the accuracy of $\boldsymbol{F}_{\text{LM}}$ by mapping points in image 1 to epipolar lines in image 2. Choose three distinct points $\boldsymbol{x}_{\{1,2,3\}}$ distributed in image 1 that are not in the set of inlier correspondences and map them to epipolar lines $\boldsymbol{l'}_{\{1,2,3\}} = \boldsymbol{F}_{\text{LM}}\boldsymbol{x}_{\{1,2,3\}}$ in the second image under the fundamental matrix $\boldsymbol{F}_{\text{LM}}$.

Include a figure containing the pair of images, where the three points in image 1 are indicated by a square (or circle) about the feature and the corresponding epipolar lines are drawn in image 2. Comment on the qualitative accuracy of the mapping. (Hint: each line $\boldsymbol{l'}_i$ should pass through the point $\boldsymbol{x'}_i$ in image 2 that corresponds to the point $\boldsymbol{x}_i$ in image 1).

```
In [59]: """your code here"""
```

1.9 Problem 8 (Programming): Projective to Euclidean Reconstruction (15 points)

You are given a Matlab file containing points obtained from applying three-view geometry techniques (using the trifocal tensor) to obtain a projective reconstruction of points from a 3D scene. Also in the file are groundtruth control points. Compute the homography transformation using the DLT along with the projected 3D scene points and control points to upgrade the projective reconstruction to a Euclidean reconstruction. Render the scene, and comment on your results. What does the scene look like? (You may have to rotate the plot to get a better view.)

```
In [70]: from mpl_toolkits.mplot3d import Axes3D
         import scipy.io as sio
```

```python
 reconstruction = sio.loadmat('ereconstruction.mat')
 X_projective = reconstruction['X_projective']
 X_projective = X_projective.T
 X_control = reconstruction['X_c']
 X_control = X_control.T
```

In [ ]:
```python
def ComputeHomography(Xp, Xc):
    """your code here"""

    print(H)
    return H

X_euclidean = H @ X_projective
Xe, Ye, Ze = X_euclidean[0,:], X_euclidean[1,:], X_euclidean[2,:]
fig = plt.figure(figsize=(14, 10))
axis = fig.add_subplot(1, 1, 1, projection="3d")
axis.scatter(Xe, Ye, Ze, marker="+", s=5)
plt.show()
```