Answer to problem 1

a) We can find the 3D location as the intersection of the ray from camera 1's focal point to (12, 12) and the ray from camera 2's focal point to (1, 12). We will use the formula $o = td$ for a ray, where o is the origin and d is the direction of the ray.

Since (x, y) = (12, 12) is really (-8, 12, 1) in world coordinates, and (u, v) = (1, 12) is really (21, 12, 1) in world coordinates, the direction of the camera 1 ray is (-8, 12, 1) - (-20, 0, 0) = (12, 12, 1) and the direction of the camera 2 ray is (21, 12, 1) - (20, 0, 0) = (1, 12, 1). Then the ray for camera 1 is $(-20, 0, 0) + t_1(12, 12, 1)$ and the ray for camera 2 is $(20, 0, 0) + t_2(1, 12, 1)$. The intersection of these rays can be found by setting their equations equal to each other and solving for t_1 and t_2 (note that there are actually three equations that come out of this equality, since we have 3D vectors).

$$
\begin{aligned}
-20 + 12t_1 &= 20 + t_2 \\
12t_1 &= 12t_2 \\
t_1 &= t_2
\end{aligned}
$$

Solving, we have $t_1 = t_2 = \frac{40}{11}$. And we can find the 3D intersection point by plugging this value of t_1 back into the camera 1 ray equation:

$$(-20, 0, 0) + \frac{40}{11}(12, 12, 1) = (\frac{260}{11}, \frac{480}{11}, \frac{40}{11})$$

b) Since the focal point is at $y = 0$ and all points on the line are at $y = 0$, there is no disparity in the y-direction. So we can think of everything as being on the xz-plane only.

Following the convention of the slides from lecture 11 (entitled "binocular stereo system"), the x projection of a point onto camera 1's image plane is $f(x + 20)/z$. Meanwhile, the x projection of a point onto camera 2's image plane is $f(x - 20)/z = u$. The disparity d is the difference between these two:

$$d = \frac{f(x + 20)}{z} - u$$

Since $f = 1$,

$$d = \frac{x + 20}{z} - u$$

Next, we know that points lie on the line $z = -x$. So we can substitute this into our equation and obtain an analytic expression for the disparity d of a point on the given line:

$$d = -\frac{x + 20}{x} - u$$

Currently, this x is in world coordinates. We want to make things in terms of u, which is related to x (x in world coordinates) as $u = f(x - 20)/z = (x - 20)/(-x) = -1 + 20/x$. (Thus $x = 20/(u + 1)$.) Substituting, we obtain our final analytic expression for the disparity d in terms of u:

$$d = -\frac{\frac{20}{u+1} + 20}{\frac{20}{u+1}} - u = -\left(\frac{20}{u+1} + 20\right)\left(\frac{u+1}{20}\right) - u = -1 - u - 1 - u = -2 - 2u$$

Epipolar Rectification: (**errata e3 = e1Xe2)

1.3.1 Solution 2:

a)

i. The essiential matrix E is $[t]R$,

$$E = [t]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$E^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix}$$

Because $Ee_B = 0$ and $E^T e_A = 0$, we have $e_A = [1,0,0]$, $e_B = [1,0,0]$.

ii. Let $(u, v, 1)$ in l_B in I_B, we have,

$$\begin{aligned} 0 &= (x_A, y_A, 1)E(u, v, 1)^T && (1) \\ &= t_x(v - y_A) && (2) \end{aligned}$$

Therefore, l_B is $v = y_A$.

iii. For $t = [tx, 0, 0]^T$, $R = I$, we have,

$$e_1 = \frac{t}{|t|} = [1,0,0]$$

$$e_2 = \frac{1}{\sqrt{tx^2}}[0, tx, 0] = [0,1,0]$$

$$e_3 = e_1 \times e_3 = [0,0,1]$$

Then, we have $R_{recon} = [e_1^T; e_2^T; e_3^T] = I$

Therefore, we have $HA = R_{recon} = I$, $HB = RR_{recon} = I$.

b)

i. The essiential matrix E is $[t]R$,

$$E = [t]R = \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

$$E^T = \begin{bmatrix} 0 & 0 & -t_y \\ 0 & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix}$$

Because $Ee_B = 0$ and $E^T e_A = 0$, we have $e_A = [t_x, t_y, 0]$, $e_B = [t_x, t_y, 0]$.

ii. Let $(u, v, 1)$ in l_B in I_B, we have,

$$0 = (x_A, y_A, 1)E(u, v, 1)^T \tag{3}$$
$$= t_x(y_A - v) + t_y(x_A - u) \tag{4}$$

Therefore, l_B is $t_x(y_A - v) + t_y(x_A - u) = 0$.

iii. For $t = [tx, ty, 0]^T$, $R = I$, we have,

$$e_1 = \frac{t}{|t|} = [\frac{tx}{\sqrt{tx^2 + ty^2}}, \frac{ty}{\sqrt{tx^2 + ty^2}}, 0]$$

$$e_2 = \frac{1}{\sqrt{tx^2 + ty^2}}[-ty, tx, 0] = [\frac{-ty}{\sqrt{tx^2 + ty^2}}, \frac{tx}{\sqrt{tx^2 + ty^2}}, 0]$$

$$e_3 = e_1 \times e_3 = [0, 0, 1]$$

Then, we have,

$$R_{recon} = [e_1^T; e_2^T; e_3^T] \tag{5}$$
$$= \begin{bmatrix} \frac{tx}{\sqrt{tx^2+ty^2}} & \frac{ty}{\sqrt{tx^2+ty^2}} & 0 \\ \frac{-ty}{\sqrt{tx^2+ty^2}} & \frac{tx}{\sqrt{tx^2+ty^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Therefore, we have $HA = R_{recon}$, $HB = RR_{recon} = R_{recon}$, which is a rotation matrix.

c)

i. The essiential matrix E is $[t]R$,

$$E = [t]R = \begin{bmatrix} 0 & -tz & 0 \\ tz & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E^T = \begin{bmatrix} 0 & tz & 0 \\ -tz & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Because $Ee_B = 0$ and $E^T e_A = 0$, we have $e_A = [0, 0, 1]$, $e_B = [0, 0, 1]$.

ii. Let $(u, v, 1)$ in l_B in I_B, we have,

$$0 = (x_A, y_A, 1)E(u, v, 1)^T \tag{7}$$
$$= t_z v x_A - t_z u y_A \tag{8}$$

Therefore, l_B is $x_A v - y_A u = 0$.

iii. For $t = [0, 0, tz]^T$, $R = I$, we have,

$$e_1 = \frac{t}{|t|} = [0, 0, 1]$$

$$e_2 = \frac{1}{\sqrt{0^2 + 0^2}}[0, 0, 0]$$

e_2 do not exist, we cannot find R_{recon}, so as HA and HB.

d)

i. The essiential matrix E is $[t]R$,

$$E = [t]R = 0$$
$$E^T = 0$$

Because $E = 0$, e_A, e_B can be any vectors, or they do not exist.

ii. Let $(u, v, 1)$ in l_B in I_B, because $E = 0$, we have,

$$(x_A, y_A, 1)E(u, v, 1)^T = 0$$

Therefore, l_B does not exist.

iii. For $t = [0, 0, 0]^T$, we have,

$$e_1 = \frac{t}{|t|} = \frac{t}{0}$$

e_1 do not exist, we cannot find R_{recon}, so as HA and HB.

Answer to problem 3

a) Since convolution is associative, we can compute a single convolution kernel that will perform the desired operation as the convolution of a 3x3 box filter and a "central differences" x-derivative filter. To combine the two kernels, we will use a "full" convolution (speaking in NumPy/SciPy terms) to ensure that commutativity and associativity hold.

We use zero padding when necessary (i.e. we have designed this filter for a zero padding use case.)

$$\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} * \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} = \begin{bmatrix} 1/9 & 1/9 & 0 & -1/9 & -1/9 \\ 1/9 & 1/9 & 0 & -1/9 & -1/9 \\ 1/9 & 1/9 & 0 & -1/9 & -1/9 \end{bmatrix}$$

b) An example of a separable filter is the 3x3 box filter from the last example.

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

It can be separated into 1D row and column filters:

$$\begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix} \text{ and } \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

With separation, at each pixel in the $n \times n$ image, we will have to perform 3 multiplications and 2 additions for the first 1D filter, and 3 multiplications and 2 additions for the second 1D filter. This comes out to 10 operations for each of n^2 pixels, or $10n^2$ arithmetic operations total (and $6n^2$ multiplications total).

Without separation, at each pixel in the $n \times n$ image, we will have to perform 9 multiplications and 8 additions (since 9 entries in the window / 9 multiplications, and then 8 additions to add all the products up). This comes out to 17 operations for each of n^2 pixels, or $17n^2$ arithmetic operations total (and $9n^2$ multiplications total).

Problem 4: Sparse Stereo Matching [22 pts]

In this problem we will play around with sparse stereo matching methods. You will work on two image pairs, a warrior figure and a figure from the Matrix movies. These files both contain two images, two camera matrices, and set sets of corresponding points (extracted by manually clicking the images). For illustration, I have run my code on a third image pair (dino1.png, dino2.png). This data is also provided for you to debug your code, but you should only report results on warrior and matrix. In other words, where I include one (or a pair) of images in the assignment below, you will provide the same thing but for BOTH matrix and warrior. Note that the matrix image pair is harder, in the sense that the matching algorithms we are implementing will not work quite as well. You should expect good results, however, on warrior.