Assignment 2

January 23, 2019

1 CSE 252B: Computer Vision II, Winter 2019 – Assignment 2

1.0.1 Instructor: Ben Ochoa

1.0.2 Due: Wednesday, February 6, 2019, 11:59 PM

1.1 Instructions

- Review the academic integrity and collaboration policies on the course website.
- This assignment must be completed individually.
- This assignment contains both math and programming problems.
- All solutions must be written in this notebook
- Math problems must be done in Markdown/LATEX. Remember to show work and describe your solution.
- Programming aspects of this assignment must be completed using Python in this notebook.
- Your code should be well written with sufficient comments to understand, but there is no need to write extra markdown to describe your solution if it is not explictly asked for.
- This notebook contains skeleton code, which should not be modified (This is important for standardization to facilate effecant grading).
- You may use python packages for basic linear algebra, but you may not use packages that directly solve the problem. Ask the instructor if in doubt.
- You must submit this notebook exported as a pdf. You must also submit this notebook as an .ipynb file.
- Your code and results should remain inline in the pdf (Do not move your code to an appendix).
- You must submit both files (.pdf and .ipynb) on Gradescope. You must mark each problem on Gradescope in the pdf.
- It is highly recommended that you begin working on this assignment early.

1.2 Problem 1 (Math): Line-plane intersection (5 points)

The line in 3D defined by the join of the points $X_1 = (X_1, Y_1, Z_1, T_1)^\top$ and $X_2 = (X_2, Y_2, Z_2, T_2)^\top$ can be represented as a Plucker matrix $L = X_1 X_2^\top - X_2 X_1^\top$ or pencil of points $X(\lambda) = \lambda X_1 + (1 - \lambda)X_2$ (i.e., X is a function of λ). The line intersects the plane $\pi = (a, b, c, d)^\top$ at the point $X_L = L\pi$ or $X(\lambda_\pi)$, where λ_π is determined such that $X(\lambda_\pi)^\top \pi = 0$ (i.e., $X(\lambda_\pi)$ is the point on π). Show that X_L is equal to $X(\lambda_\pi)$ up to scale.

Your solution here

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1.3 Problem 2 (Math): Line-quadric intersection (5 points)

In general, a line in 3D intersects a quadric Q at zero, one (if the line is tangent to the quadric), or two points. If the pencil of points $X(\lambda) = \lambda X_1 + (1 - \lambda)X_2$ represents a line in 3D, the (up to two) real roots of the quadratic polynomial $c_2\lambda_Q^2 + c_1\lambda_Q + c_0 = 0$ are used to solve for the intersection point(s) $X(\lambda_Q)$. Show that $c_2 = X_1^\top Q X_1 - 2X_1^\top Q X_2 + X_2^\top Q X_2$, $c_1 = 2(X_1^\top Q X_2 - X_2^\top Q X_2)$, and $c_0 = X_2^\top Q X_2$.

Your solution here

1.4 Problem 3 (Programming): Linear Estimation of the Camera Projection Matrix (15 points)

Download input data from the course website. The file hw2_points3D.txt contains the coordinates of 50 scene points in 3D (each line of the file gives the $\tilde{X}_i$, $\tilde{Y}_i$, and $\tilde{Z}_i$ inhomogeneous coordinates of a point). The file hw2_points2D.txt contains the coordinates of the 50 corresponding image points in 2D (each line of the file gives the $\tilde{x}_i$ and $\tilde{y}_i$ inhomogeneous coordinates of a point). The scene points have been randomly generated and projected to image points under a camera projection matrix (i.e., $x_i = PX_i$), then noise has been added to the image point coordinates.

Estimate the camera projection matrix P_{DLT} using the direct linear transformation (DLT) algorithm (with data normalization). You must express $x_i = PX_i$ as $[x_i]^\perp PX_i = 0$ (not $x_i \times PX_i = 0$), where $[x_i]^\perp x_i = 0$, when forming the solution. Return P_{DLT}, scaled such that $||P_{DLT}||_{Fro} = 1$

The following helper functions may be useful in your DLT function implementation. You are welcome to add any additional helper functions.

```
In [6]: import numpy as np
        import time


        def Homogenize(x):
            # converts points from inhomogeneous to homogeneous coordinates
            return np.vstack((x,np.ones((1,x.shape[1]))))


        def Dehomogenize(x):
            # converts points from homogeneous to inhomogeneous coordinates
            return x[:-1]/x[-1]


        def Normalize(pts):
            # data normalization of n dimensional pts
            #
            # Input:
            #    pts - is in inhomogeneous coordinates
            # Outputs:
            #    pts - data normalized points
            #    T - corresponding transformation matrix
            """your code here"""
```

```python
        T = np.eye(pts.shape[0]+1)
        return pts, T

    def ComputeCost(P, x, X):
        # Inputs:
        #    x - 2D inhomogeneous image points
        #    X - 3D inhomogeneous scene points
        #
        # Output:
        #    cost - Total reprojection error
        n = x.shape[1]
        covarx = np.eye(2*n)

        """your code here"""



        cost = np.inf
        return cost

In [ ]: def DLT(x, X, normalize=True):
        # Inputs:
        #    x - 2D inhomogeneous image points
        #    X - 3D inhomogeneous scene points
        #    normalize - if True, apply data normalization to x and X
        #
        # Output:
        #    P - the (3x4) DLT estimate of the camera projection matrix
        P = np.eye(3,4)+np.random.randn(3,4)/10

        # data normalization
        if normalize:
            x, T = Normalize(x)
            X, U = Normalize(X)
        else:
            x = Homogenize(x)
            X = Homogenize(X)

        """your code here"""



        # data denormalize
        if normalize:
            P = np.linalg.inv(T) @ P @ U

        return P
```

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```python
def displayResults(P, x, X, title):
    print(title+' =')
    print (P/np.linalg.norm(P)*np.sign(P[-1,-1]))

# load the data
x=np.loadtxt('hw2_points2D.txt').T
X=np.loadtxt('hw2_points3D.txt').T


# compute the linear estimate without data normalization
print ('Running DLT without data normalization')
time_start=time.time()
P_DLT = DLT(x, X, normalize=False)
cost = ComputeCost(P_DLT, x, X)
time_total=time.time()-time_start
# display the results
print('took %f secs'%time_total)
print('Cost=%.9f'%cost)


# compute the linear estimate with data normalization
print ('Running DLT with data normalization')
time_start=time.time()
P_DLT = DLT(x, X, normalize=True)
cost = ComputeCost(P_DLT, x, X)
time_total=time.time()-time_start
# display the results
print('took %f secs'%time_total)
print('Cost=%.9f'%cost)
```

```
In [2]: # Report your P_DLT value here!
        displayResults(P_DLT, x, X, 'P_DLT')

P_DLT =
[[ 0.65286942 -0.14254182  0.00360256 -0.0186023 ]
 [-0.05558557  0.51246608  0.03691019 -0.05223975]
 [-0.04035564 -0.0640988   0.52603414  0.02885442]]
```

1.5 Problem 4 (Programming): Nonlinear Estimation of the Camera Projection Matrix (30 points)

Use P_{DLT} as an initial estimate to an iterative estimation method, specifically the Levenberg-Marquardt algorithm, to determine the Maximum Likelihood estimate of the camera projection matrix that minimizes the projection error. You must parameterize the camera projection matrix as a parameterization of the homogeneous vector $p = vec(P^{\top})$. It is highly recommended to implement a parameterization of homogeneous vector method where the homogeneous vector is of arbitrary length, as this will be used in following assignments.

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Report the initial cost (i.e. cost at iteration 0) and the cost at the end of each successive iteration. Show the numerical values for the final estimate of the camera projection matrix P_{LM}, scaled such that $||P_{LM}||_{Fro} = 1$.

The following helper functions may be useful in your LM function implementation. You are welcome to add any additional helper functions.

Hint: LM has its biggest cost reduction after the 1st iteration. You'll know if you are implementing LM correctly if you experience this.

```
In [3]: # Note that np.sinc is different than defined in class
        def Sinc(x):
            # Returns a scalar valued sinc value
            """your code here"""




            y = x
            return y


        def Jacobian(P,p,X):
            # compute the jacobian matrix
            #
            # Input:
            #    P - 3x4 projection matrix
            #    p - 11x1 homogeneous parameterization of P
            #    X - 3n 3D scene points
            # Output:
            #    J - 2nx11 jacobian matrix
            J = np.zeros((2*X.shape[1],11))

            """your code here"""




            return J


        def Parameterize(P):
            # wrapper function to interface with LM
            # takes all optimization variables and parameterizes all of them
            # in this case it is just P, but in future assignments it will
            # be more useful
            return ParameterizeHomog(P.reshape(-1,1))


        def Deparameterize(p):
            # Deparameterize all optimization variables
            return DeparameterizeHomog(p).reshape(3,4)
```

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```python
def ParameterizeHomog(V):
    # Given a homogeneous vector V return its minimal parameterization
    """your code here"""



    return v_hat


def DeParameterizeHomog(v):
    # Given a parameterized homogeneous vector return its deparameterization
    """your code here"""



    return v_bar
```

```python
In [4]: def LM(P, x, X, max_iters, lam):
            # Input:
            #    P - initial estimate of P
            #    x - 2D inhomogeneous image points
            #    X - 3D inhomogeneous scene points
            #    max_iters - maximum number of iterations
            #    lam - lambda parameter
            # Output:
            #    P - Final P (3x4) obtained after convergence

            # data normalization
            x, T = Normalize(x)
            X, U = Normalize(X)

            """your code here"""


            # you may modify this so long as the cost is computed
            # at each iteration
            for i in range(max_iters):




                cost = ComputeCost(P, x, X)
```

```python
        print ('iter %03d Cost %.9f'%(i+1, cost))

        # data denormalization
        P = np.linalg.inv(T) @ P @ U
        return P



    # LM hyperparameters
    lam = .001
    max_iters = 100

    # Run LM initialized by DLT estimate with data normalization
    print ('Running LM with data normalization')
    print ('iter %03d Cost %.9f'%(0, cost))
    time_start=time.time()
    P_LM = LM(P_DLT, x, X, max_iters, lam)
    time_total=time.time()-time_start
    print('took %f secs'%time_total)
```

```
Running LM with data normalization
iter 000 Cost inf
iter 001 Cost inf
iter 002 Cost inf
iter 003 Cost inf
iter 004 Cost inf
iter 005 Cost inf
iter 006 Cost inf
iter 007 Cost inf
iter 008 Cost inf
iter 009 Cost inf
iter 010 Cost inf
iter 011 Cost inf
iter 012 Cost inf
iter 013 Cost inf
iter 014 Cost inf
iter 015 Cost inf
iter 016 Cost inf
iter 017 Cost inf
iter 018 Cost inf
iter 019 Cost inf
iter 020 Cost inf
iter 021 Cost inf
iter 022 Cost inf
iter 023 Cost inf
iter 024 Cost inf
iter 025 Cost inf
iter 026 Cost inf
```

```
iter 027 Cost inf
iter 028 Cost inf
iter 029 Cost inf
iter 030 Cost inf
iter 031 Cost inf
iter 032 Cost inf
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iter 034 Cost inf
iter 035 Cost inf
iter 036 Cost inf
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iter 065 Cost inf
iter 066 Cost inf
iter 067 Cost inf
iter 068 Cost inf
iter 069 Cost inf
iter 070 Cost inf
iter 071 Cost inf
iter 072 Cost inf
iter 073 Cost inf
iter 074 Cost inf
```

```
iter 075 Cost inf
iter 076 Cost inf
iter 077 Cost inf
iter 078 Cost inf
iter 079 Cost inf
iter 080 Cost inf
iter 081 Cost inf
iter 082 Cost inf
iter 083 Cost inf
iter 084 Cost inf
iter 085 Cost inf
iter 086 Cost inf
iter 087 Cost inf
iter 088 Cost inf
iter 089 Cost inf
iter 090 Cost inf
iter 091 Cost inf
iter 092 Cost inf
iter 093 Cost inf
iter 094 Cost inf
iter 095 Cost inf
iter 096 Cost inf
iter 097 Cost inf
iter 098 Cost inf
iter 099 Cost inf
iter 100 Cost inf
took 0.015998 secs
```

In [7]: # Report your P_LM final value here!
        displayResults(P_LM, x, X, 'P_LM')

```
P_LM =
[[ 0.65286942 -0.14254182  0.00360256 -0.0186023 ]
 [-0.05558557  0.51246608  0.03691019 -0.05223975]
 [-0.04035564 -0.0640988   0.52603414  0.02885442]]
```