Homework#3 Due Nov/20/2018

problem#1

1(a) ignore $z$ in camera_P_object

$$\begin{cases} \dfrac{{}^{c_1}P_{obj}}{{}^{c_1}z} = \dfrac{(x,y)}{1} \longrightarrow \dfrac{({}^{c_1}x_{obj},\ {}^{c_1}y_{obj})}{{}^{c_1}z} = \dfrac{(12,12)}{1} \\[2ex] \dfrac{{}^{c_2}P_{obj}}{{}^{c_2}z} = \dfrac{(u,v)}{1} \longrightarrow \dfrac{({}^{c_2}u_{obj},\ {}^{c_2}v_{obj})}{{}^{c_2}z} = \dfrac{(1,12)}{1} \end{cases}$$

$\because$

$${}^{w}P_{obj} = {}^{w}P_{c_1} + {}^{c_1}P_{obj} = {}^{w}P_{c_2} + {}^{c_2}P_{obj}$$

$$\Rightarrow {}^{c_1}P_{obj} = {}^{w}P_{obj} - {}^{w}P_{c_1} = {}^{w}P_{obj} - (-20,0)$$

$$\Rightarrow {}^{c_2}P_{obj} = {}^{w}P_{obj} - {}^{w}P_{c_1} = {}^{w}P_{obj} - (20,0)$$

Also ${}^{c_1}z = {}^{c_2}z$

$\therefore$

$$\frac{({}^{w}x_{obj}+20,\ {}^{w}y_{obj})}{z} = \frac{(\cancel{1}2,\ 12)}{1}$$

$$\frac{({}^{w}x_{obj}-20,\ {}^{w}y_{obj})}{z} = \frac{(1,\ 12)}{1}$$

$$\begin{cases} {}^{w}x_{obj}+20 = 12z \\ {}^{w}x_{obj}-20 = z \end{cases} \longrightarrow \begin{matrix} {}^{w}x_{obj}+20 = 12z \\ -12\,{}^{w}x_{obj}+240 = -12z \end{matrix}$$

$$\Rightarrow \begin{cases} {}^{w}x_{obj} = 260/11 \\ z = 40/11 \\ {}^{w}y_{obj} = 12z = 480/11 \end{cases}$$

(b) $x + 8 = 0 \quad y = 0 \quad 8 > 1$

① Consider: in camera 2 setting

$$_{c_2}P_{obj} = {}_{c_2}P_w + {}_wP_{obj} = -{}_wP_{c_2} + {}_wP_{obj}$$

$$= -(20, 0, 0) + (-8, 0, 8)$$

$$= (-20-8, 0, 8) \quad 8 > 1$$

on camera 2 image

$$((-20-8)/8, 0, 1) = (u, v, 1)$$

② Consider in Camer 1 setting

$$_{c_1}P_{obj} = {}_{c_1}P_w + {}_wP_{obj} = -{}_wP_{c_1} + {}_wP_{obj}$$

$$= -(-20, 0, 0) + (-8, 0, 8)$$

$$= (-8+20, 0, 8)$$

on camera 1 image

$$((20-8)/8, 0, 1) = (x, y, 1)$$

There fore

$$\begin{cases} x = 20/8 - 1 \\ u = -20/8 - 1 \end{cases} \Rightarrow x + u = -2$$

Also $d = x - u$

$$\therefore d = -2 - u - u = -2(1 + u)$$

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problem #2

$${}^{B}_{A}T = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \qquad H_A e_A = H_B e_B = [1, 0, 0]^T$$

(a) $t = [t_x, 0, 0]^T \qquad R = I$

① $$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} = [t]R \qquad B \xrightarrow{E} A$$

$$E e_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} e_{1A} \\ e_{2A} \\ e_{2A} \end{bmatrix} = \vec{0} \qquad E^T e_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix} \begin{bmatrix} e_{1B} \\ e_{2B} \\ e_{3B} \end{bmatrix} = \vec{0}$$

$$\begin{cases} e_{1A} = e_{1A} \\ -t_x e_{3A} = 0 \\ t_x e_{2A} = 0 \end{cases} \Rightarrow \vec{e_A} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} e_{1B} = e_{1B} \\ t_x e_{3B} = 0 \\ -t_x e_{2B} = 0 \end{cases} \Rightarrow \vec{e_B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

② given point $[x_A, y_A, 1]$ in A

$$P_B^T E P_A = 0 \longrightarrow (E P_A)^T \cdot P_B = 0$$

where $$(E P_A)^T = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \cdot \begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} \right\}^T = \begin{bmatrix} 0 \\ -t_x \\ t_x y_A \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 0 & -t_x & t_x y_A \end{bmatrix} \begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = 0 \qquad \therefore t_x y_B = t_x y_A$$

$$y_B = y_A$$

③ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = H_A e_A = \begin{bmatrix} a^T \\ b^T \\ c^T \end{bmatrix} e_A \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

i> $\underbrace{\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}}_{a^T} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1 \Rightarrow a^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

ii> $\underbrace{\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}}_{b^T} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0$

$\Rightarrow b^T = \frac{1}{\sqrt{e_{1A}^2 + e_{2A}^2}} \begin{bmatrix} -e_{2A}, & e_{1A}, & 0 \end{bmatrix} = \begin{bmatrix} 0, & 1, & 0 \end{bmatrix}$

iii> $\underbrace{\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}}_{c^T} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0$

$\Rightarrow c^T = \vec{a} \times \vec{b} = \begin{bmatrix} 0, & 0, & 1 \end{bmatrix}$

$\Rightarrow H_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad H_B = RH_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

rectification is possible.

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1b) $t = [t_x, t_y, 0]^T$ ; $R = I$

(1) $E = [t]_\times R = \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$

$E e_A = \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} e_{1A} \\ e_{2A} \\ e_{3A} \end{bmatrix} = 0 \qquad E^T e_B = \begin{bmatrix} 0 & 0 & -t_y \\ 0 & 0 & +t_x \\ +t_y & -t_x & 0 \end{bmatrix} \begin{bmatrix} e_{1B} \\ e_{2B} \\ e_{3B} \end{bmatrix} = 0$

$$\begin{cases} t_y e_{3A} = 0 \\ -t_x e_{3A} = 0 \\ -t_y e_{1A} + t_x e_{2A} = 0 \end{cases} \Rightarrow \begin{cases} e_{1A} = \frac{t_x}{t_y} e_{2A} \\ e_{2A} = e_{2A} \\ e_{3A} = 0 \end{cases} \Rightarrow \vec{e_A} = \frac{1}{\sqrt{t_x^2 + t_y^2}} \begin{bmatrix} t_x \\ t_y \\ 0 \end{bmatrix}$$

$$\begin{cases} -t_y e_{3B} = 0 \\ t_x e_{3B} = 0 \\ t_y e_{1B} - t_x e_{2B} = 0 \end{cases} \Rightarrow \begin{cases} e_{1B} = t_x/t_y \, e_{2B} \\ e_{2B} = e_{2B} \\ e_{3B} = 0 \end{cases} \Rightarrow \vec{e_B} = \frac{1}{\sqrt{t_x^2 + t_y^2}} \begin{bmatrix} t_x \\ t_y \\ 0 \end{bmatrix}$$

(2) $p_B^T E p_A \rightarrow (E p_A)^T p_B = 0$

$$(E p_A)^T = \left\{ \begin{bmatrix} 0 & 0 & t_y \\ 0 & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} \right\}^T = \begin{bmatrix} t_y \\ -t_x \\ -t_y x_A + t_x y_A \end{bmatrix}^T$$

$(E p_A)^T p_B = 0$

$\Rightarrow t_y x_B - t_x y_B - t_y x_A + t_x y_A = 0$

(3) $H_A e_A = \begin{bmatrix} a^T \\ b^T \\ c^T \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$a^T = \left[ \frac{t_x}{\sqrt{t_x^2+t_y^2}}, \frac{t_y}{\sqrt{t_x^2+t_y^2}}, 0 \right]$$

$$b^T = \left[ -e_{2A} \quad e_{1A} \quad 0 \right] = \left[ \frac{-t_y}{\sqrt{t_x^2+t_y^2}}, \frac{t_x}{\sqrt{t_x^2+t_y^2}}, 0 \right]$$

$$c^T = a \times b = [0, 0, 1]$$

$$\Rightarrow H_A = \begin{bmatrix} \frac{t_x}{\sqrt{t_x^2+t_y^2}} & \frac{t_y}{\sqrt{t_x^2+t_y^2}} & 0 \\ \frac{-t_y}{\sqrt{t_x^2+t_y^2}} & \frac{t_x}{\sqrt{t_x^2+t_y^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$H_B = RH_A$

rectification is possible

1(c) $t = [0,\ 0,\ t_z]$ $\quad R = I$

(1) $$E = [t]_\times R = \begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E\, e_A = \begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1A} \\ e_{2A} \\ e_{3A} \end{bmatrix} \qquad E^T e_B = \begin{bmatrix} 0 & t_z & 0 \\ -t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1B} \\ e_{2B} \\ e_{3B} \end{bmatrix}$$

$$\begin{aligned} e_{1A} &= 0 \\ e_{2A} &= 0 \\ e_{3A} &= e_{3A} \end{aligned} \Rightarrow \vec{e}_A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{aligned} e_{1B} &= 0 \\ e_{2B} &= 0 \\ e_{3B} &= 0 \end{aligned} \Rightarrow \vec{e}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(2) $$(E p_A)^T p_B = \left\{ \begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} \right\}^T \begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [-t_z y_A \quad x_A t_z \quad 0] \begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = 0$$

$$-t_z y_A x_B + t_z x_A y_B = 0$$

Rectification is possible

(3) $a^T = e_A \qquad b^T = [0 \quad 1 \quad 0]$

$$c^T = a \times b = [1 \quad 0 \quad 0]$$

$$H_A = H_B = R H_A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

1(d) $t = [0, 0, 0]$ $R$ = Arbitrary

① $E = [t]_\times R = 0$

$E e_A = 0$ $E^T e_B = 0$

$\Rightarrow$ $e_A$ and $e_B$ can be any unit length vector

② $(E p_A)^T p_B = 0$ $\Rightarrow$ $0 \cdot p_B = 0$

no such line exists in image B

③ $H_A$ and $H_B = R H_A$ Do not exist

Rectification is impossible, as $H_A$ & $H_B$ do not exist problem #3

1(a)

$$I_{output} = \Phi_{derivative} * \Phi_{smooth} * I_{input}$$

$$= (\Phi_{derivative\,x} * \Phi_{smooth}) * I_{input}$$

where $\Phi_{smooth} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$\Phi_{derivative\,x} = \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

It is like Dot product

$$\begin{bmatrix} -1/27 & -1/27 & 0 & -1/27 & -1/27 \\ 2/27 & 2/27 & 0 & -2/27 & -2/27 \\ 1/9 & 1/9 & 0 & -1/9 & -1/9 \\ 2/27 & 2/27 & 0 & -2/27 & -2/27 \\ 1/27 & 1/27 & 0 & -1/27 & -1/27 \end{bmatrix} = \Phi_{smooth} * \Phi_{derivative\,x}$$

(b) Considering most efficient way to do convolution in both before separation & After seperation

before seperation

$(n-5+1) \times (n-5+1)$ is the output image size

$\Rightarrow$ (7 additions + 1 multiplication) x 2 + (3 addition + 1 multiplication)

= 17 additions + 3 multiplication

$(n-4)(n-4) \times$ (17 additions + 3 multiplication)

$(n^2-8n+16) \times 17$ additions + $(n^2-8n+16) \times 3$ multiplication

After seperation

1) output image size After smoothing $(n-2)(n-2)$

(8 additions + 1 multiplication) $\times (n^2-4n+4)$

$= (n^2-4n+4) \times 8$ additions + $(n^2-4n+4) \times 1$ multiplication

2) output image size After derivative $(n-4)(n-4)$

(5 additions + 1 multiplication) $\times (n^2-8n+16)$

$(n^2-8n+16) \times 5$ additions + $(n^2-8n+16) \times 1$ multiplication

$\Rightarrow (13n^2-72n+112)$ additions + $(2n^2-12n+20)$ multiplication

2) Before seperation

$(17n^2-136n+272)$ additions + $(3n^2-24n+48)$ multiplicatio

After seperation.

$(13n^2-72n+112)$ additions + $(2n^2-12n+20)$ multiplication

Therefore for large $n$, After seperation is preferable

and for small $n$ Before sepperation is preferable